

Tutorials

1 Determine whether or not each of following signals are periodic

$$(a) \quad x_1(t) = 1 + j e^{j5t}$$

$$(b) \quad x_2(t) = e^{j(2-3t)}$$

$$(c) \quad x_3[n] = e^{j3\pi n}$$

$$(d) \quad x_4[n] = e^{j4n}$$

2. Let $x(t)$ be a signal with $x(t) = 0$ for $t < 4$. For given signals below determine for which values of t they are guaranteed to be zero

$$(a) \quad x(t-1)$$

$$(b) \quad x(t-3) + x(2-3t)$$

$$(c) \quad x(t-1) x(2-t)$$

$$(d) \quad x(4t)$$

3. Define the signal $\delta_{\Delta}(t)$ as

$$\delta_{\Delta}(t) = \begin{cases} \frac{1}{\Delta} & 0 < t < \Delta \\ 0 & \text{else} \end{cases}$$

and
$$U_{\Delta}(t) = \begin{cases} \frac{t}{\Delta} & 0 < t < \Delta \\ 1 & t > \Delta \\ 0 & t < 0 \end{cases}$$

Given
$$U_{\Delta}(t) = \int_{-\infty}^t \delta_{\Delta}(\tau) d\tau$$

show that
$$U(t) = \int_{-\infty}^t \delta(\tau) d\tau$$

here
$$\delta(t) = \lim_{\Delta \rightarrow 0} \delta_{\Delta}(t)$$

$$U(t) = \lim_{\Delta \rightarrow 0} U_{\Delta}(t)$$

Problems

① $x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right) \right]^2$

Periodic?

$$x(t) = \frac{1 + \cos\left(4t - \frac{2\pi}{3}\right)}{2}$$

$$\therefore x(t) = \frac{1}{2} + \frac{1}{2} \cos\left(4t - \frac{2\pi}{3}\right)$$

here $\omega_0 = 4$.

A: Yes.

$$\text{Fun. Period} = T_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

② $x(t) = \sum v_i \left\{ \cos 4\pi t \cdot u(t) \right\}$
 $= \frac{1}{2} \left[\cos 4\pi t \cdot u(t) + \cos 4\pi t \cdot u(-t) \right]$

$$\therefore x(t) = \begin{cases} \frac{1}{2} \cos 4\pi t & t > 0 \\ 1 & t = 0 \\ \frac{1}{2} \cos 4\pi t & t < 0 \end{cases}$$

Not periodic.

③ $x[n] = \sin\left(\frac{6\pi}{7}n + 1\right)$

$$= \sin\left(\frac{6\pi}{7}n\right) \cdot \cos(1) + \cos\left(\frac{6\pi}{7}n\right) \cdot \sin(1)$$

$$\frac{m}{N} 2\pi = \frac{6\pi}{7} \Rightarrow N = \frac{7}{3}m$$

$$\therefore N_1 = 7 \quad \text{for } m = 3$$

4.
(1.36)

$$x[n] = x[nT] = e^{j\omega_0 nT} \rightarrow \text{sampled signal of } e^{j\omega_0 t}$$

periodicity.

$$x[n+N] = x[n]$$

$$\omega_0 = \frac{2\pi}{T_0}$$

$$e^{j\omega_0(n+N)T} = e^{j\omega_0 nT}$$

$$\therefore e^{j\omega_0 NT} = 1$$

$$\omega_0 NT = 2\pi m$$

$$\omega_0 = \frac{2\pi}{T_0} \therefore \frac{1}{T_0} \cdot T \cdot N = m$$

$$\therefore \boxed{\frac{N}{T} = \frac{1}{T_0}} \rightarrow \boxed{mT_0 = NT}$$