ex. 2.11

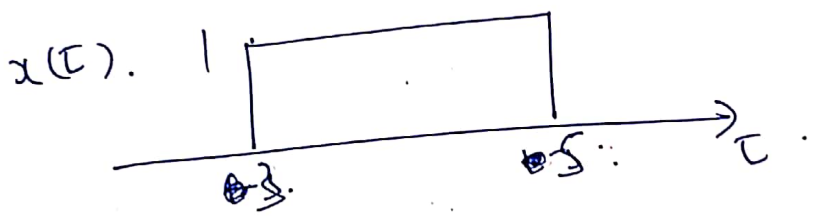
(a) $x(t) = u(t-3) - u(t-5)$.

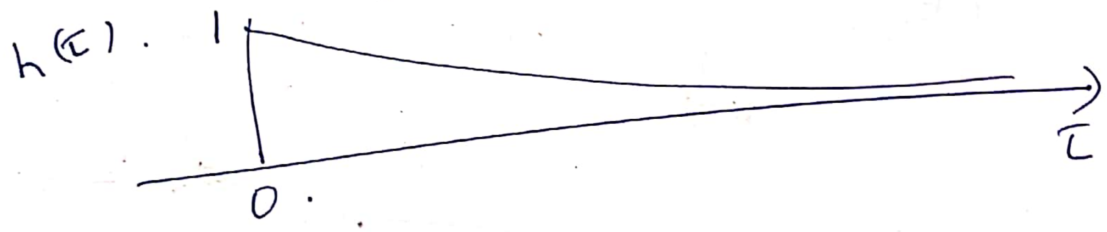
$h(t) = e^{-3t} u(t)$.

$y(t) = x(t) * h(t)$

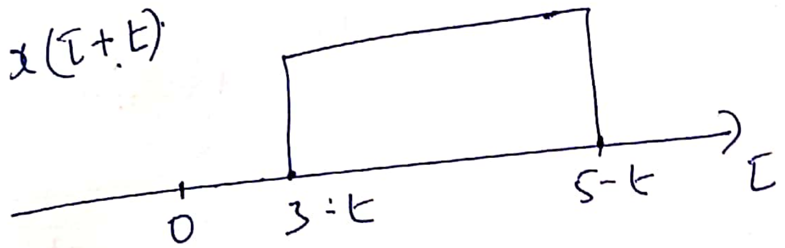
$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

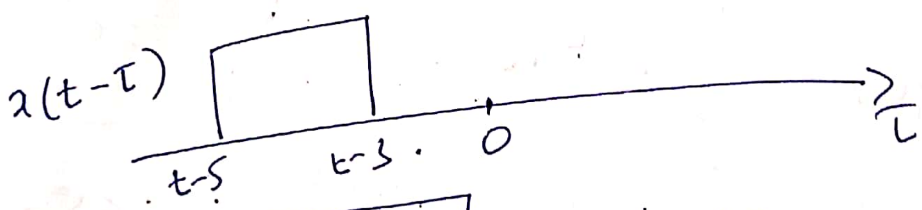
$$= \int_{-\infty}^{\infty} \big(u(t-3) - u(t-5)\big) \cdot e^{-3(t-\tau)} u(t-\tau)\, d\tau .$$





$t<0$.





$\therefore$ $\boxed{y(t) = 0.}$ $t<0$.

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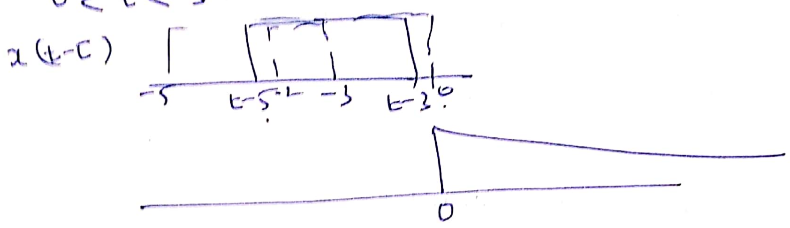
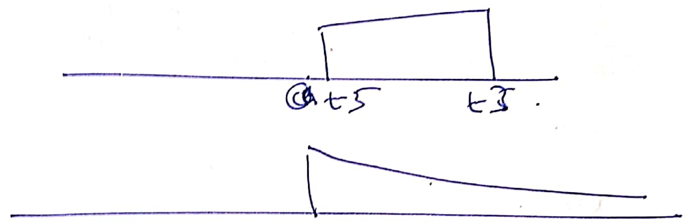
$0 < t < 3.$



$\boxed{y(t) = 0}$ for $0 < t < 3.$

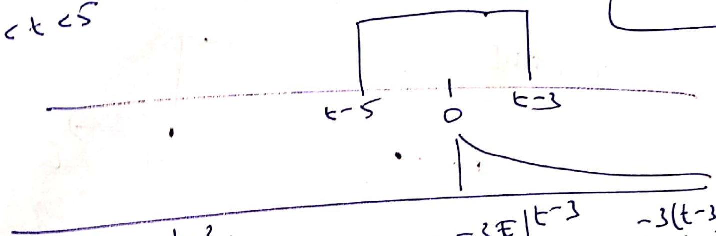
~~3 < t < 5~~. $t > 5.$



$$y(t) = \int_{t-5}^{t-3} e^{-3\tau} \cdot d\tau = \boxed{\frac{e^{9-3t}}{3}(e^{6}-1)} \quad \boxed{t > 5}$$

~~t < 5~~.

$3 < t < 5$



$$y(t) = \int_{0}^{t-3} e^{-3\tau} d\tau = \frac{e^{-3\tau}}{-3}\Bigg|_{0}^{t-3} = \frac{e^{-3(t-3)}-1}{-3}.$$

$$\boxed{y(t) = \frac{1}{3}\left[1 - e^{9-3t}\right] \quad 3 < t < 5.}$$

2.11

(b). $g(t) = \left(\frac{d}{dt} x(t)\right) * h(t)$

$\therefore \frac{d}{dt} x(t) = \delta(t-3) - \delta(t-5)$.

$\therefore g(t) = \left(\delta(t-3) - \delta(t-5)\right) * e^{-3t} u(t)$.

$g(t) = \delta(t-3) * e^{-3t} u(t) - \delta(t-5) e^{-3t} u(t)$

$g(t) = e^{-3(t-3)} u(t-3) - e^{-3(t-5)} u(t-5)$.

o

(c). Find relation between $g(t)$ & $y(t)$.

$\therefore$ recall $y(t) = \begin{cases} 0 & t \leq 3. \\ \dfrac{1-e^{-3(t-3)}}{3} & 3 < t \leq 5 \\ \dfrac{(1-e^{-6})e^{-3(t-5)}}{3} & 5 < t \leq \infty \end{cases}$

$\dfrac{dy(t)}{dt} = \begin{cases} 0 & t < 3. \\ -\dfrac{(-3)}{3} \cdot e^{-3(t-3)} & 3 < t \leq 5. \\ \dfrac{1-e^{-6}}{3} (-3) e^{-3(t-5)} & 5 < t. \end{cases}$

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$$\therefore \frac{dy(t)}{dt} = e^{-3(t-3)}\left[u(t-3) - u(t-5)\right] - (1-e^{-6})\, e^{-3(t-5)} \cdot u(t-5)$$

$$= e^{-3(t-3)} u(t-3) - \left(e^{-3(t-3)} + (1-e^{-6}) e^{-3(t-5)}\right) u(t-5)$$

$$= e^{-3(t-3)} u(t-3) - e^{-3(t-5)} \left[e^{-6} + 1 - e^{-6}\right] u(t-5)$$

$$\boxed{\therefore \frac{dy(t)}{dt} = g(t)}$$

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