

# Computer Assignment - 03 - Spring 2019

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## **Analysis and Synthesis**

For the given periodic signals with the period  $T = 3$ , compute the Fourier coefficients and then reconstruct the original signal.

For each of the signals, plot the following:

1. The original and reconstructed signal on the same plot.
2. The Fourier coefficients; both the real and imaginary components vs the theoretical values.
3. Demonstrate the convergence of the reconstructed signal with respect to the original signal.

**Signal:  $x(t) = t^2$  ,  $|t| < 1$**

**Solution:**

```
%function
function y = x(t)
    y = zeros(size(t));
    y(t>-1 & t<1) = t(t>-1 & t<1).^2;
end
```

```
T=3;
M=100;
w0 = 2*pi/T;
t = -(T/2):0.001:(T/2);
u = -M:M;
a = zeros(size((u)));

%calculating fourier coefficients
for k = 1:length(u)
    basis = exp(-1*i*t*w0*u(k));
```

```

    a(k) = trapz(t,x(t).*basis)/T;
end
plot(t,x(t),'blue')
hold on

%re-construction
y = zeros(size(t));
for i = 1:length(t)
    for j = 1:length(u)
        y(i) = y(i)+(a(j)*exp(1i*u(j)*w0*t(i)));
    end
end

plot(t,real(y),'red')
hold off
legend('Original signal','Reconstructed signal');
title('Original signal vs Reconstructed signal');

%calculating theoretical fourier coefficients
theoretical_fourier_coefficients = zeros(size(u));
for k = 1:length(u)
    theoretical_fourier_coefficients(k) =
    (2*(((u(k)*u(k)*w0*w0)-2)*sin(u(k)*w0)+2*u(k)*w0*cos(u(k)*w0))
)/(T*u(k)*u(k)*u(k)*w0*w0*w0);
end
theoretical_fourier_coefficients(101)=0.2219;
stem(u,real(a));
title('Real(calculated fourier coefficients)');
stem(u,real(theoretical_fourier_coefficients));
title('Real(theoretical fourier coefficients)');
stem(u,imag(a));
ylim([-0.2 0.2])
title('Imag(calculated fourier coefficients)');

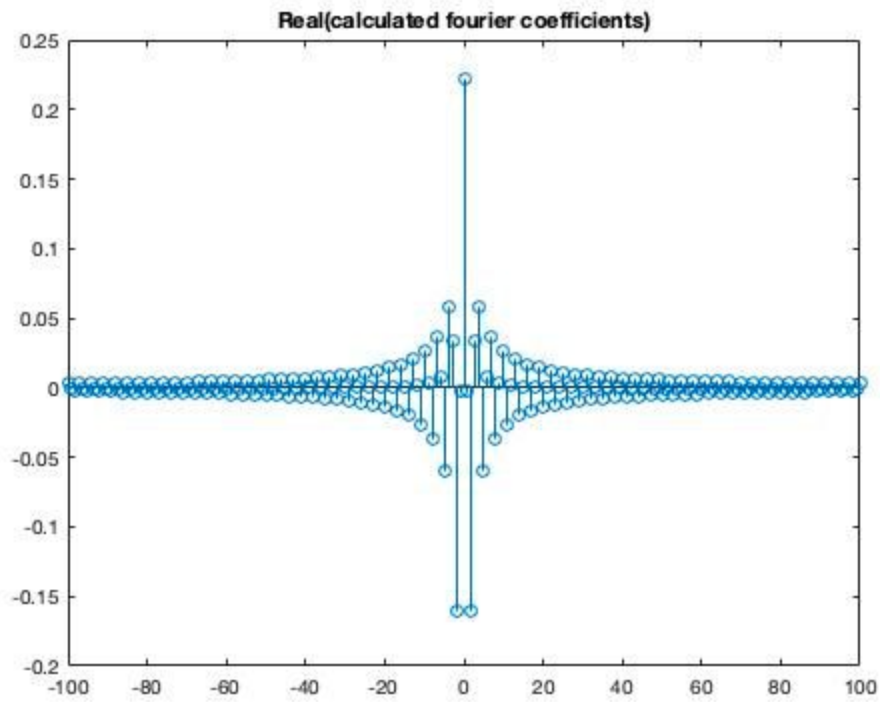
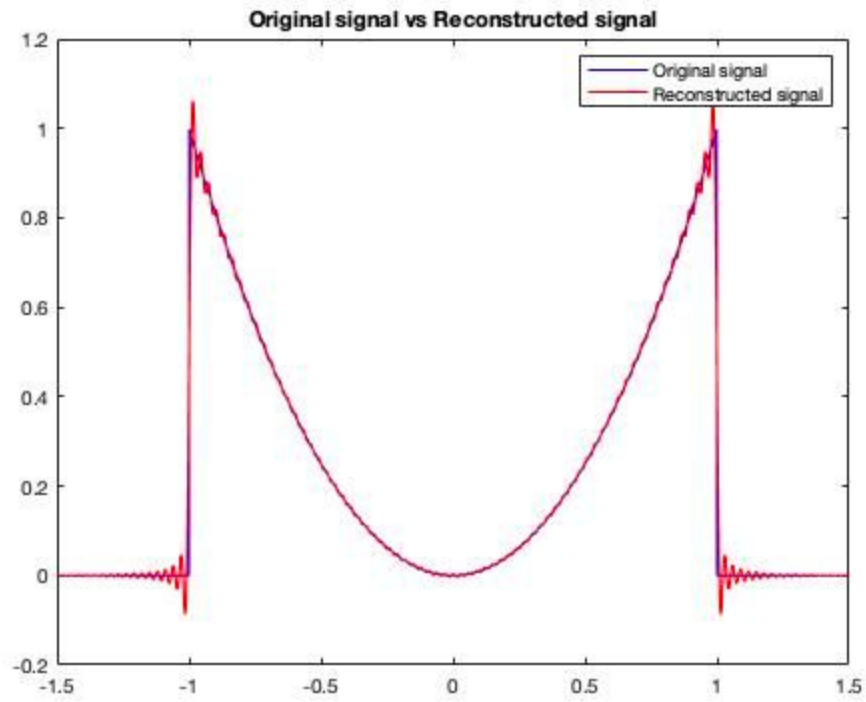
```

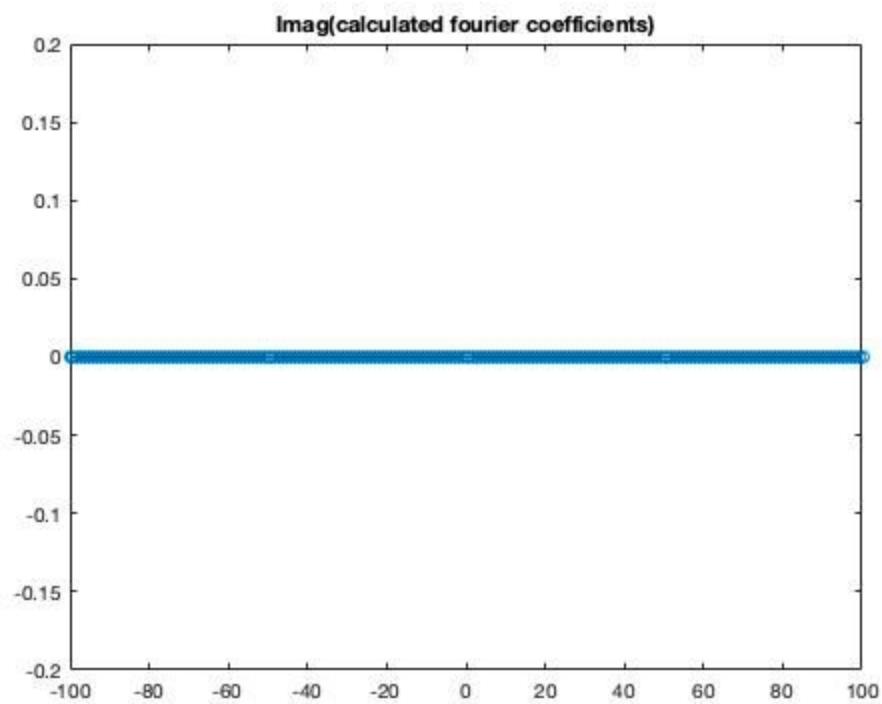
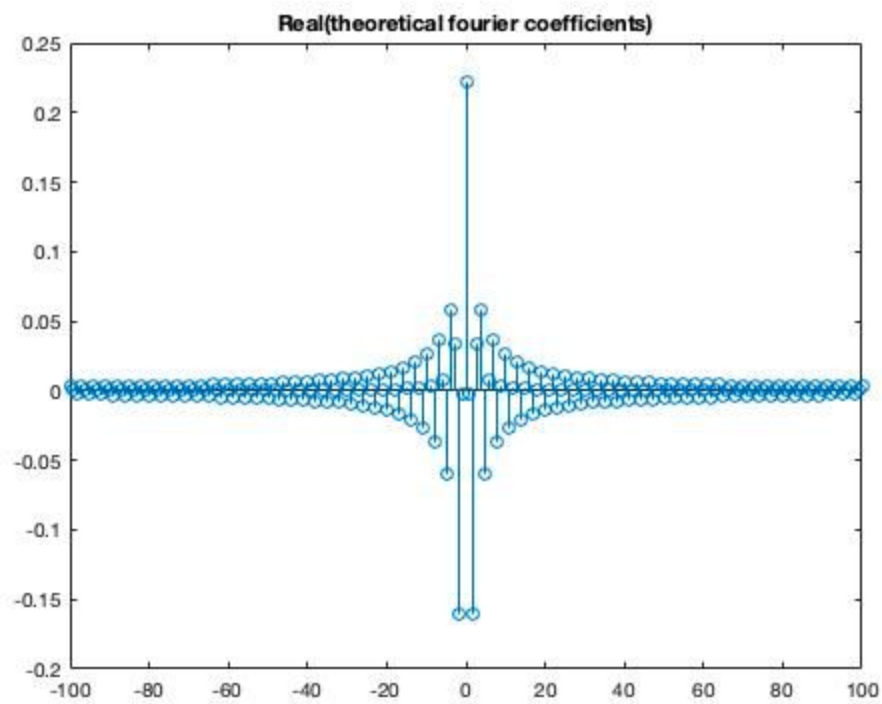
```

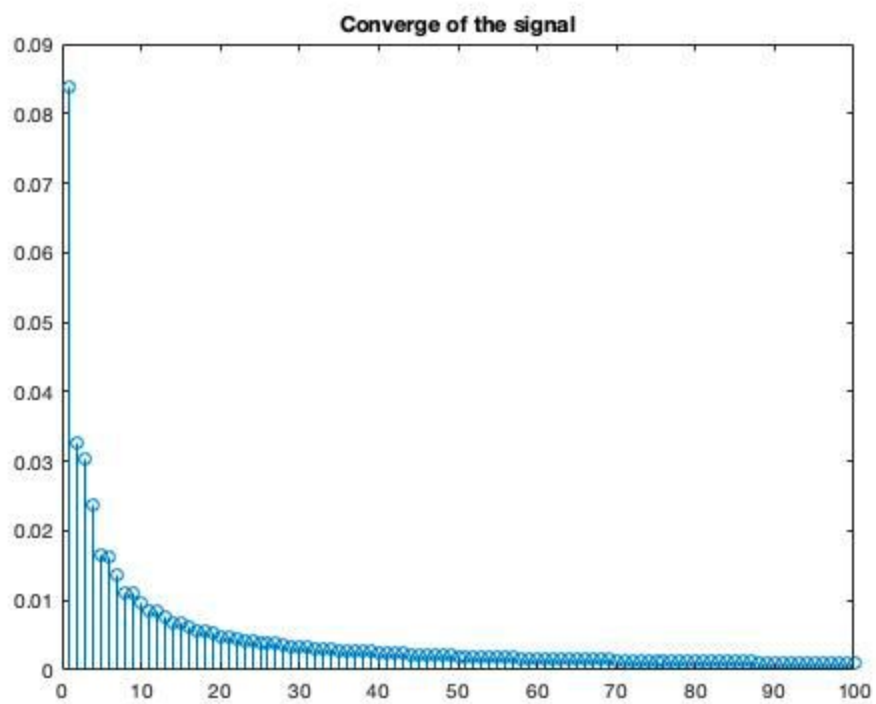
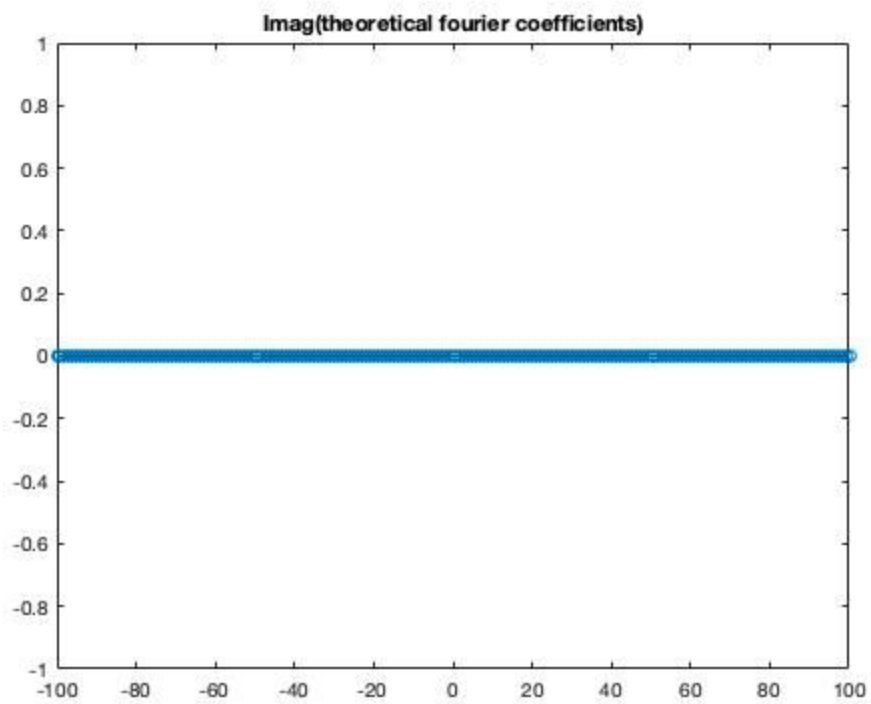
stem(u,imag(theoretical_fourier_coefficients));
title('Imag(theoretical fourier coefficients)');

%convergence
time_m = 1:100;
final_converge = zeros(size(time_m));
for i = 1:length(time_m)
    u = -i:i;
    a = zeros(size(u));
    for k = 1:length(u)
        basis = exp(-1*i*t*w0*u(k));
        a(k) = trapz(t,x(t).*basis)/T;
    end
    y = zeros(size(t));
    for m = 1:length(t)
        for n = 1:length(u)
            y(m) = y(m)+(a(n)*exp(1i*u(n)*w0*t(m)));
        end
    end
    final_converge(i) = trapz(t,(abs(x(t)-y)).^2)/T;
end
stem(time_m,final_converge);
title('Converge of the signal')

```







$$q_k = I = \frac{1}{3} \int_{-1}^1 t^2 e^{-jkw t} dt$$

$$I = \frac{1}{3} \left[ \int_{-1}^1 t^2 e^{-jkw t} dt \right] = \frac{1}{3} \left[ \left. \frac{t^2 e^{-jkw t}}{-jkw} \right|_{-1}^1 - \int_{-1}^1 \frac{2t e^{-jkw t}}{-jkw} dt \right]$$

$$\text{let } I_1 = \int_{-1}^1 2t e^{-jkw t} dt$$

$$I_1 = 2 \left[ \left. \frac{t e^{-jkw t}}{-jkw} \right|_{-1}^1 - \frac{e^{-jkw t}}{(jkw)^2} \right]_{-1}^1 = 2 \left[ \frac{\frac{-jkw}{e} + \frac{jkw}{e}}{-jkw} - \frac{e^{-jkw} - e^{jkw}}{(jkw)^2} \right]$$

$$= 2 \left[ \frac{2 \cos kw}{-jkw} + \frac{2j \sin kw}{(-1)(kw)^2} \right]$$

$$\text{So } I = \frac{1}{3} \left[ \left( \frac{e^{-jkw} - e^{jkw}}{-jkw} \right) - \frac{j4 \cos kw}{(-j)(kw)^2} + \frac{4j \sin kw}{(kw)^2 (kw)(-j)} \right]$$

$$I = \frac{1}{3} \left[ \frac{-2j \sin kw}{-jkw} + \frac{4 \cos kw}{kw^2} + \frac{(-4) \sin kw}{(kw)^3} \right]$$

$$I = \frac{2}{3} \left[ \frac{(kw^2 - 2) \sin(kw) + 2kw \cos(kw)}{kw^3} \right]$$

**Signal:  $x(t) = 1 - |t|/3$  ,  $|t| < 1$**

**Solution:**

```
function y = x(t)
    y = zeros(size(t));
    y(t>-1 & t<1) = 1-(abs(t(t>-1 & t<1))/3);
end

T=3;
M=100;
w0 = 2*pi/T;
t = -(T/2):0.001:(T/2);
u = -M:M;
a = zeros(size(u));
for k = 1:length(u)
    basis = exp(-1*i*t*w0*u(k));
    a(k) = trapz(t,x(t).*basis)/T;
end
plot(t,x(t),'blue')
hold on

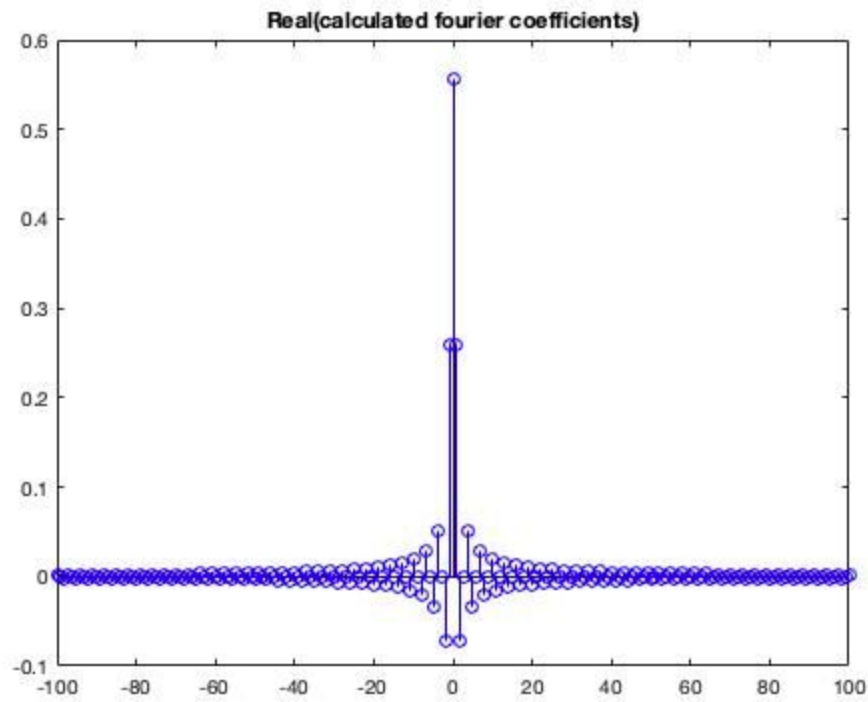
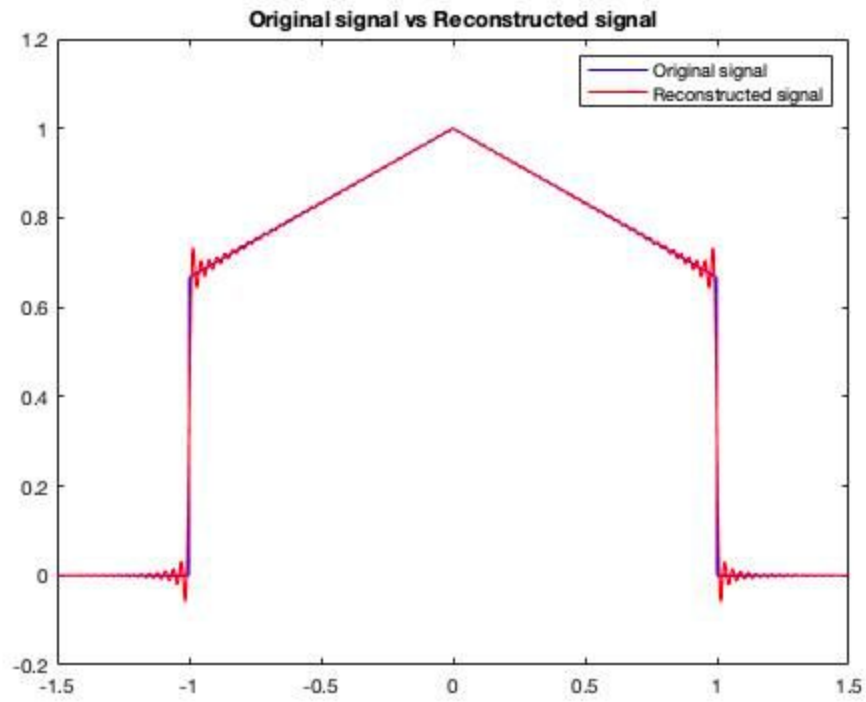
y = zeros(size(t));
for i = 1:length(t)
    for j = 1:length(u)
        y(i) = y(i)+(a(j)*exp(1i*u(j)*w0*t(i)));
    end
end
plot(t,real(y),'red')
hold off
legend('Original signal','Reconstructed signal');
title('Original signal vs Reconstructed signal');
theoretical_fourier_coefficients = zeros(size(u));
for k = 1:length(u)
```

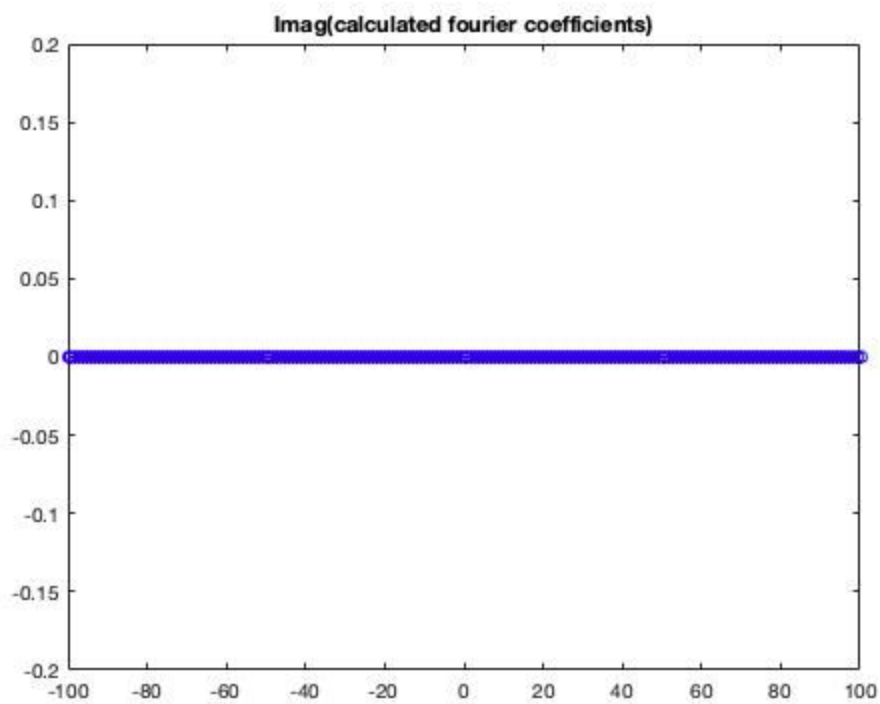
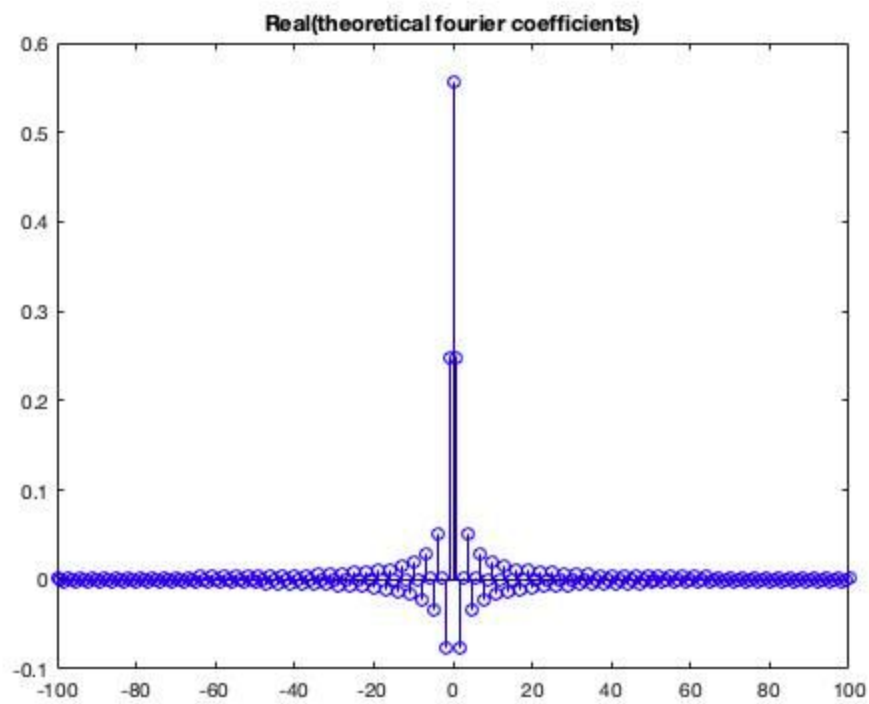


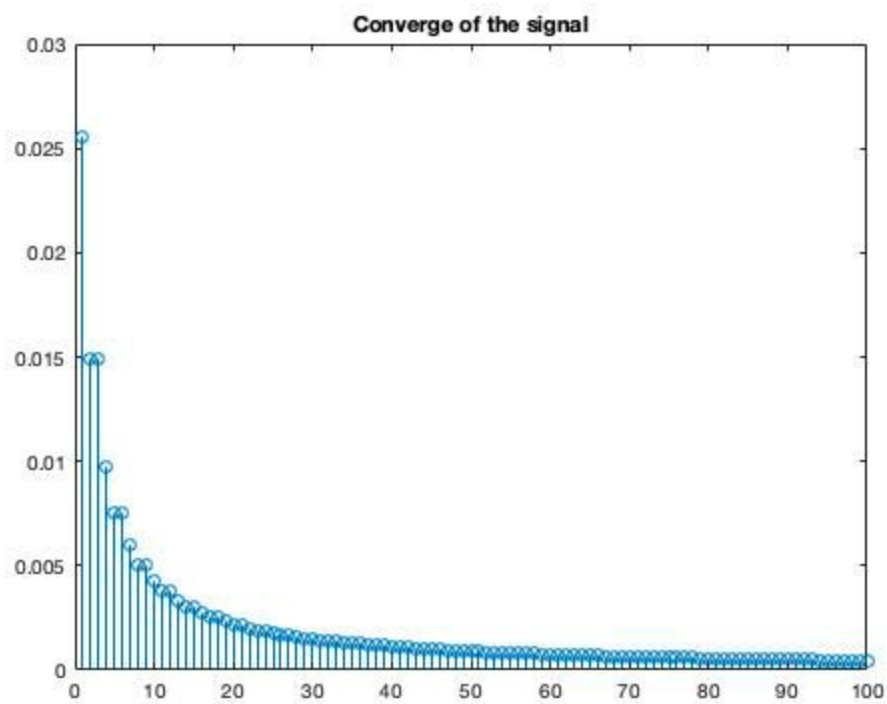
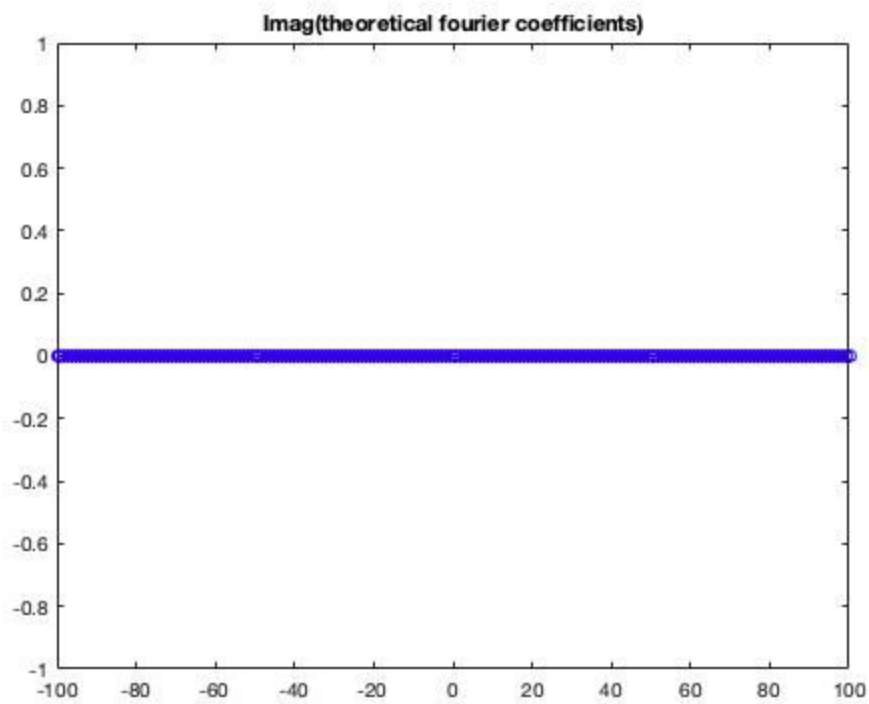
```

    theoretical_fourier_coefficients(k) =
    (((4/3)*(sin(u(k)*w0)/(u(k)*w0))) +
    ((1/3)*(2-2*cos(u(k)*w0))/(u(k)*u(k)*w0*w0)))/T;
end
theoretical_fourier_coefficients(101)=0.5553;
stem(u,real(a),'blue');
title('Real(calculated fourier coefficients)');
stem(u,real(theoretical_fourier_coefficients),'blue');
title('Real(theoretical fourier coefficients)');
stem(u,imag(a),'blue');
ylim([-0.2 0.2])
title('Imag(calculated fourier coefficients)');
stem(u,imag(theoretical_fourier_coefficients),'blue');
title('Imag(theoretical fourier coefficients)');
time_m = 1:100;
final_converge = zeros(size(time_m));
for i = 1:length(time_m)
    u = -i:i;
    a = zeros(size(u));
    for k = 1:length(u)
        basis = exp(-1*i*t*w0*u(k));
        a(k) = trapz(t,x(t).*basis)/T;
    end
    y = zeros(size(t));
    for m = 1:length(t)
        for n = 1:length(u)
            y(m) = y(m)+(a(n)*exp(1i*u(n)*w0*t(m)));
        end
    end
    final_converge(i) = trapz(t,(abs(x(t)-y)).^2)/T;
end
stem(time_m,final_converge);
title('Converge of the signal')

```







$$\begin{aligned}
 a_k = I &= \frac{1}{3} \int_{-1}^1 \left(1 - \frac{|t|}{3}\right) \times e^{-jk\omega t} dt \\
 I &= \frac{1}{3} \int_{-1}^0 \left(1 + \frac{t}{3}\right) e^{-jk\omega t} dt + \int_0^1 \left(1 - \frac{t}{3}\right) e^{-jk\omega t} dt \\
 &= \frac{1}{3} \int_{-1}^0 e^{-jk\omega t} dt + \frac{1}{3} \int_{-1}^0 t e^{-jk\omega t} dt + \frac{1}{3} \int_0^1 e^{-jk\omega t} dt - \frac{1}{3} \int_0^1 t e^{-jk\omega t} dt \\
 &= \frac{e^{-jk\omega t}}{-jk\omega} \Big|_{-1}^0 + \frac{1}{3} \left( \frac{t e^{-jk\omega t}}{-jk\omega} \Big|_{-1}^0 - \frac{e^{-jk\omega t}}{(jk\omega)^2} \right) \Big|_{-1}^0 \\
 &\quad - \frac{1}{3} \left( \frac{t e^{-jk\omega t}}{-jk\omega} \Big|_0^1 - \frac{e^{-jk\omega t}}{(jk\omega)^2} \right) \Big|_0^1 \\
 &= \frac{1}{3} \left[ \frac{4}{3} \frac{\sin k\omega}{k\omega} + \frac{1}{3} \frac{(2 - 2\cos k\omega)}{k^2 \omega^2} \right]
 \end{aligned}$$

**Signal:**  $x(t) = \cos(\pi t)$  ,  $|t| < 1$

**Solution:**

```

function y = x(t)
    y = zeros(size(t));
    y(t>-1 & t<1) = cos(pi*t(t>-1 & t<1));
end

```

```

T=3;
M=100;
w0 = 2*pi/T;
t = -(T/2):0.001:(T/2);
u = -M:M;
a = zeros(size(u));
for k = 1:length(u)
    basis = exp(-1*i*t*w0*u(k));
    a(k) = trapz(t,x(t).*basis)/T;
end

```

```

plot(t,x(t),'blue')
hold on

y = zeros(size(t));
for i = 1:length(t)
    for j = 1:length(u)
        y(i) = y(i)+(a(j)*exp(1i*u(j)*w0*t(i)));
    end
end
plot(t,real(y),'red')
hold off
legend('Original signal','Reconstructed signal');
title('Original signal vs Reconstructed signal');
theoretical_fourier_coefficients = zeros(size(u));
for k = 1:length(u)
    theoretical_fourier_coefficients(k) =
(2*u(k)*w0*sin(u(k)*w0))/((T)*((pi*pi)-(u(k)*u(k)*w0*w0)));
end
stem(u,real(a),'blue');
title('Real(calculated fourier coefficients)');
stem(u,real(theoretical_fourier_coefficients),'blue');
title('Real(theoretical fourier coefficients)');
stem(u,imag(a),'blue');
ylim([-0.2 0.2])
title('Imag(calculated fourier coefficients)');
stem(u,imag(theoretical_fourier_coefficients),'blue');
title('Imag(theoretical fourier coefficients)');

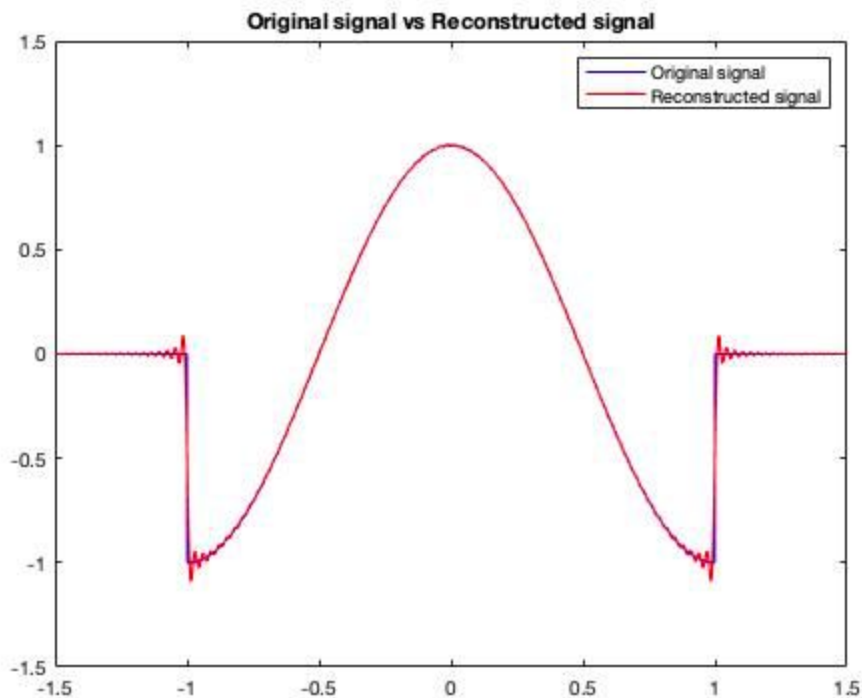
time_m = 1:100;
final_converge = zeros(size(time_m));
for i = 1:length(time_m)
    u = -i:i;
    a = zeros(size(u));

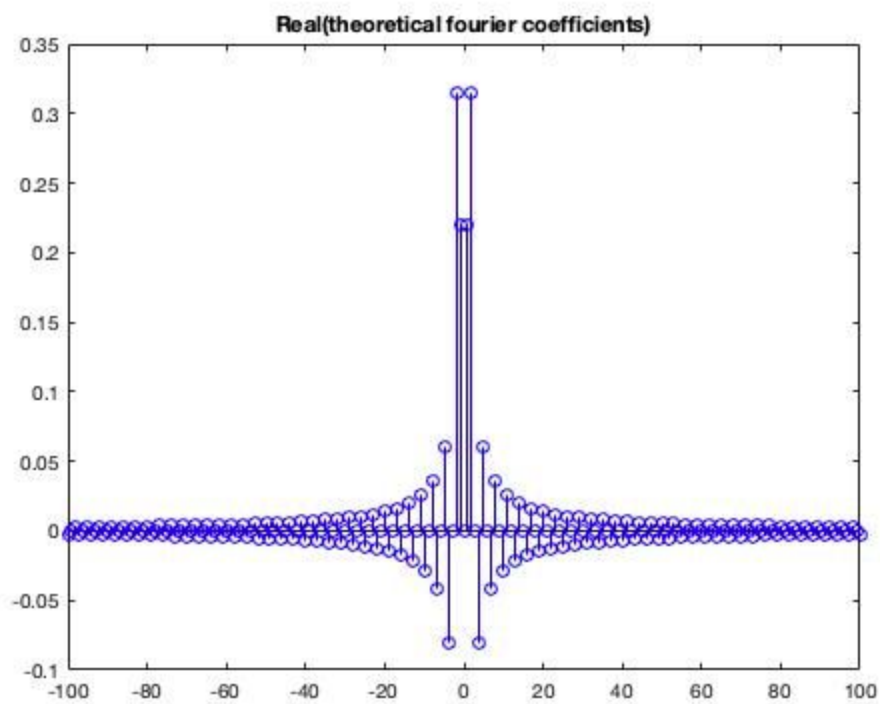
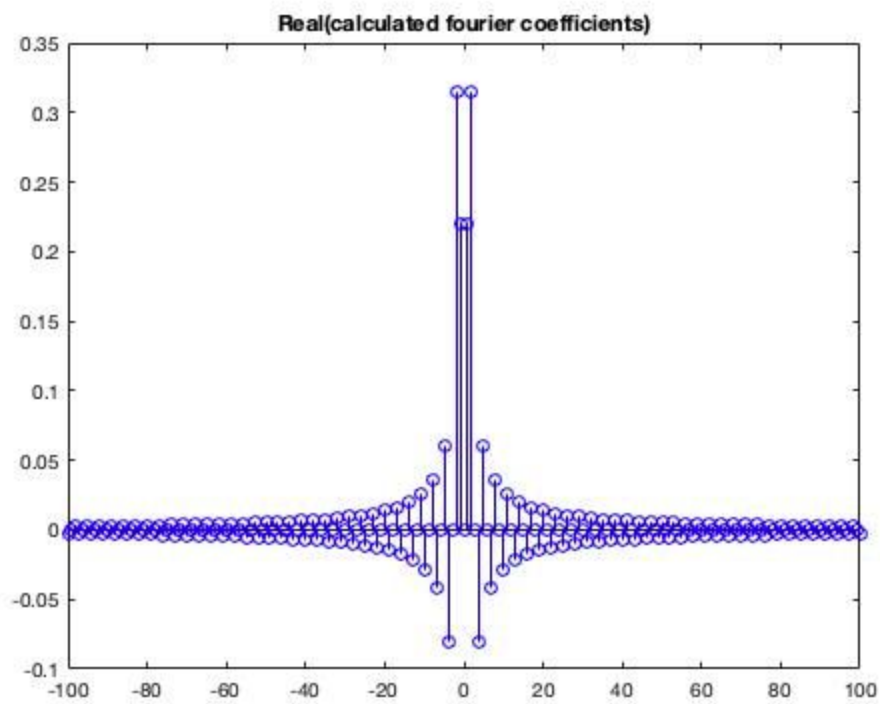
```

```

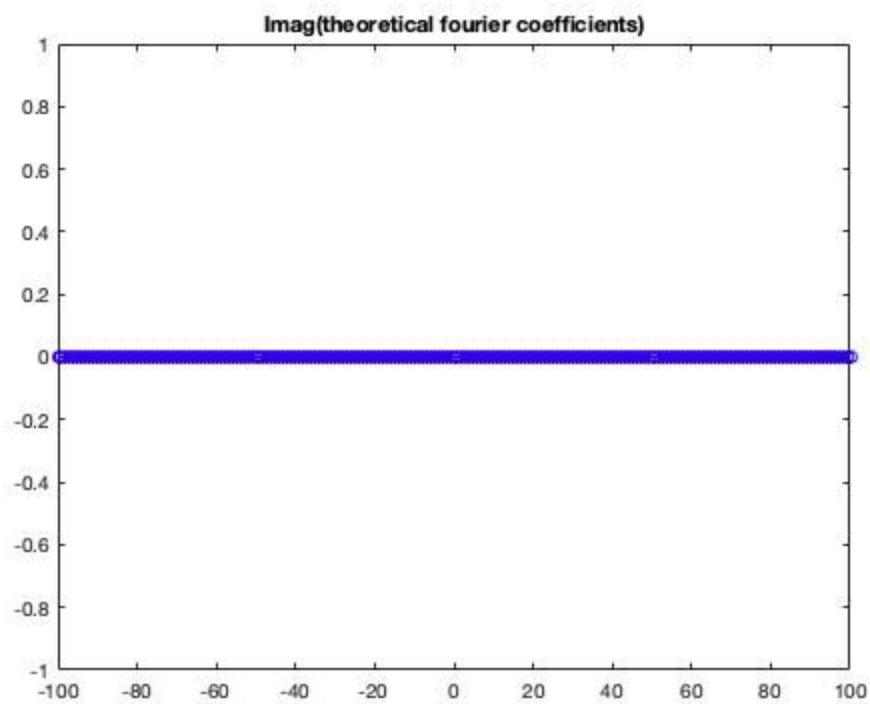
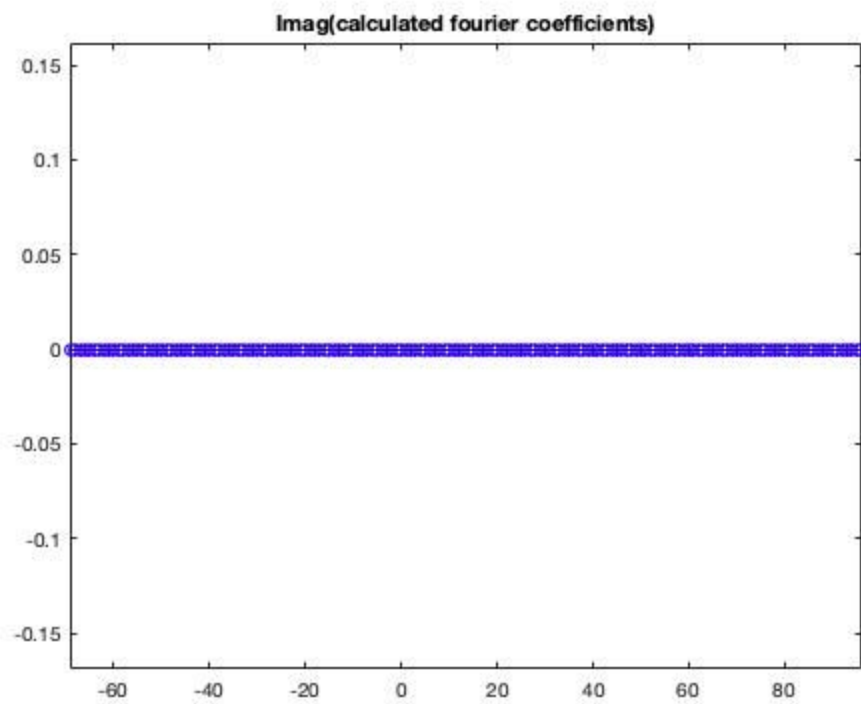
for k = 1:length(u)
    basis = exp(-1*i*t*w0*u(k));
    a(k) = trapz(t,x(t).*basis)/T;
end
y =zeros(size(t));
for m = 1:length(t)
    for n = 1:length(u)
        y(m) = y(m)+(a(n)*exp(1i*u(n)*w0*t(m)));
    end
end
final_converge(i) = trapz(t,(abs(x(t)-y)).^2)/T;
end
stem(time_m,final_converge,'blue');
title('Converge of the signal')

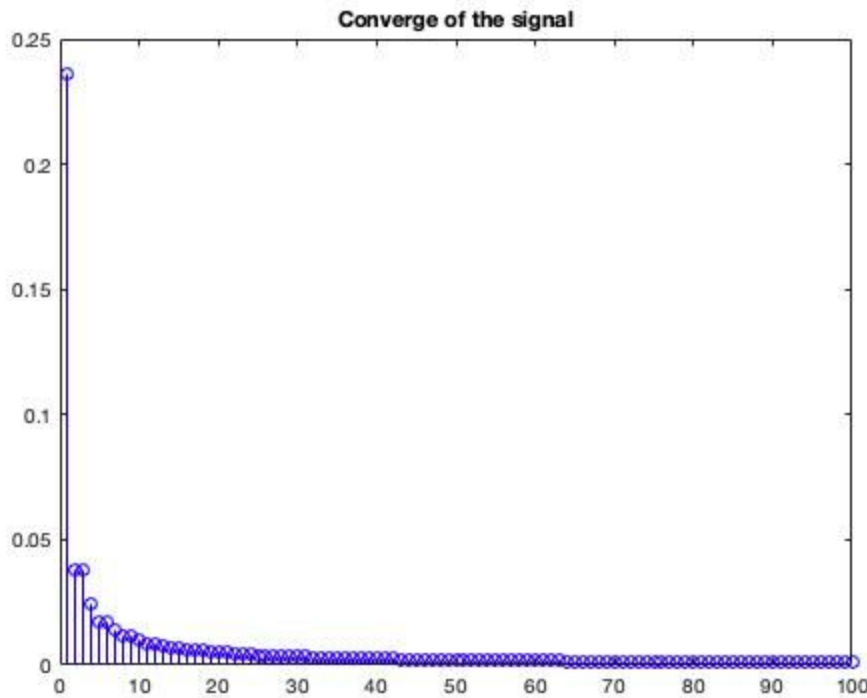
```











$$\begin{aligned}
 a_k &= \frac{1}{3} \int_{-1}^1 \cos(\pi t) e^{-jk\omega_0 t} dt = \frac{1}{3} \int_{-1}^1 \cos(\pi t) e^{-j\frac{2k\pi}{3} t} dt \\
 &= \frac{1}{6} \int_{-1}^1 \left[ e^{jt \left( \frac{3\pi + 2k\pi}{3} \right)} + e^{jt \left( \frac{3\pi - 2k\pi}{3} \right)} \right] dt \\
 &= \frac{1}{6} \left[ \frac{e^{jt \frac{\pi(2k+3)}{3}}}{-j \frac{\pi(2k+3)}{3}} + \frac{e^{jt \frac{\pi(3-2k)}{3}}}{j \frac{\pi(3-2k)}{3}} \right]_{-1}^1 \\
 &= \frac{1}{6} \left[ \frac{-e^{j \frac{\pi(2k+3)}{3}} + e^{j \frac{\pi(2k+3)}{3}}}{i \left( \frac{\pi}{3} \right) (2k+3)} \right] + \\
 &\quad \frac{1}{6} \left[ \frac{e^{j \frac{\pi(3-2k)}{3}} - e^{-j \frac{\pi(3-2k)}{3}}}{-j \left( \frac{\pi}{3} \right) (3-2k)} \right]
 \end{aligned}$$

$$= \frac{\sin \frac{\pi}{2} (2k+3)}{\pi (2k+3)} + \sin \left( \frac{\pi}{3} (3-2k) \right) \frac{1}{\pi (3-2k)}$$

$$= \frac{2k\omega \sin k\omega}{\sum (\pi^2 - k^2 \omega^2)} \quad \text{as } \omega = \frac{2\pi}{3}$$

## Fourier Transform

For the given aperiodic signals, compute the Fourier transform and then the inverse Fourier Transform.

For each of the signals, plot the following:

1. The original and reconstructed signal on the same plot.
2. The Fourier transform both the real and imaginary components vs the theoretical values.

**Signal:**  $x(t) = t^3$ ,  $|t| < 1$

**Solution:**

```
t = -1:0.01:1;
w0 = -100:100;
x = t.^3;
a = zeros(size(w0));

for i = 1:length(w0)
    basis = exp(-1i*w0(i)*t);
    a(i) = trapz(t,x.*basis);
end

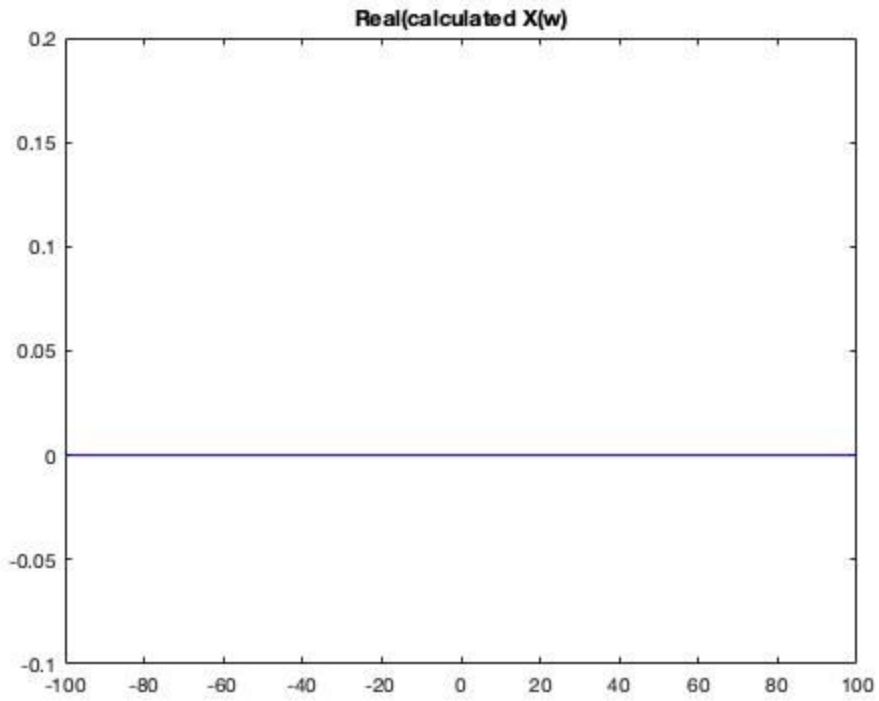
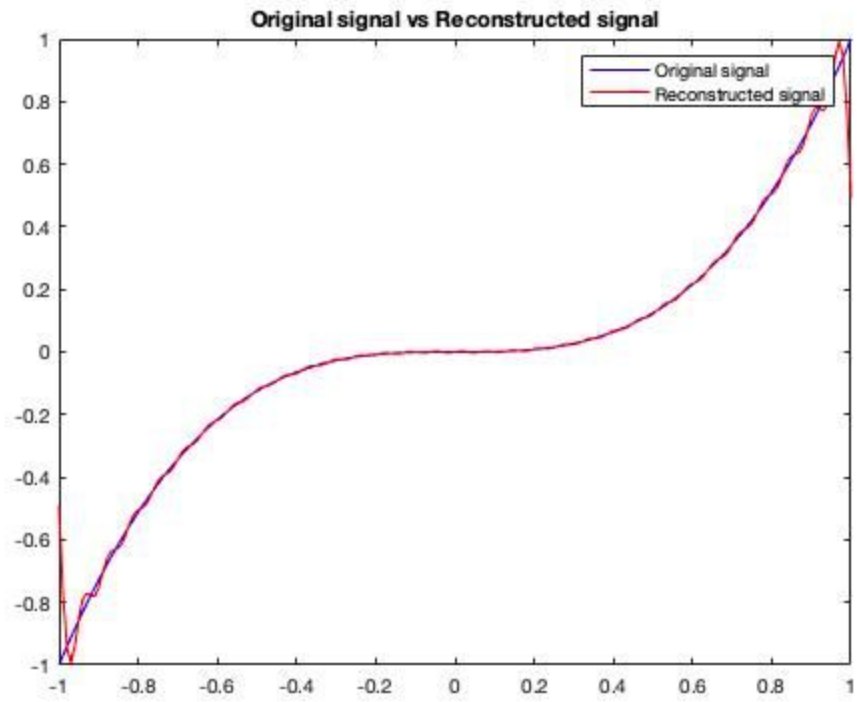
theoretical_x_w = zeros(size(w0));
for i = 1:length(w0)
    w = w0(i);
```

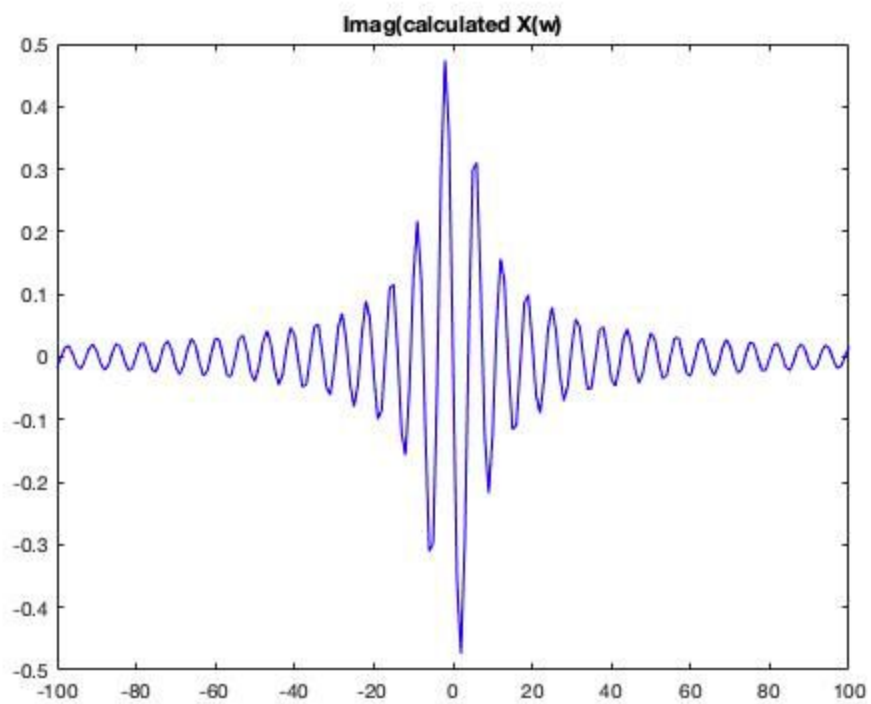
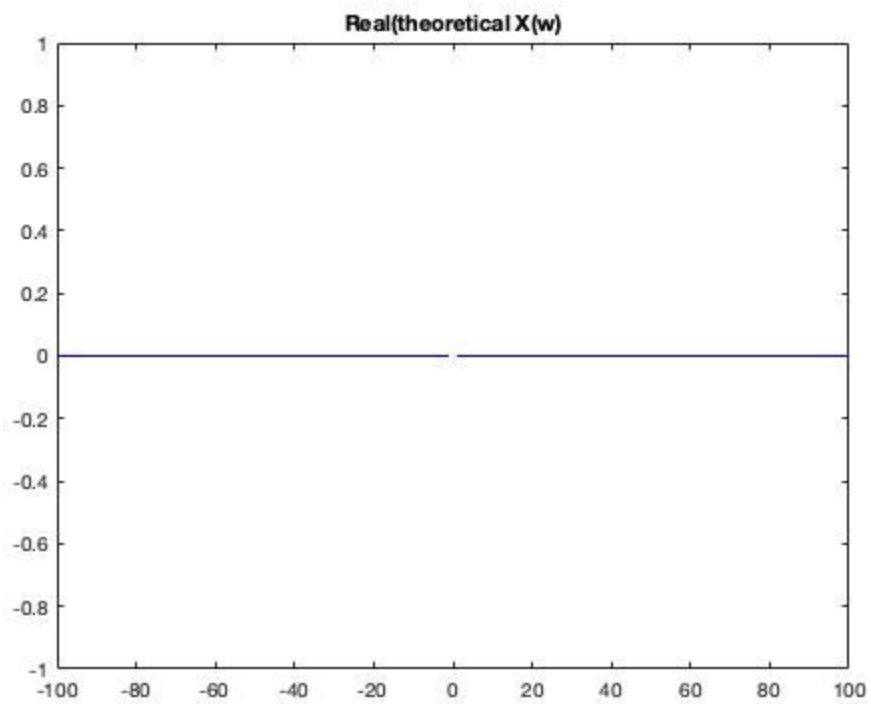
```

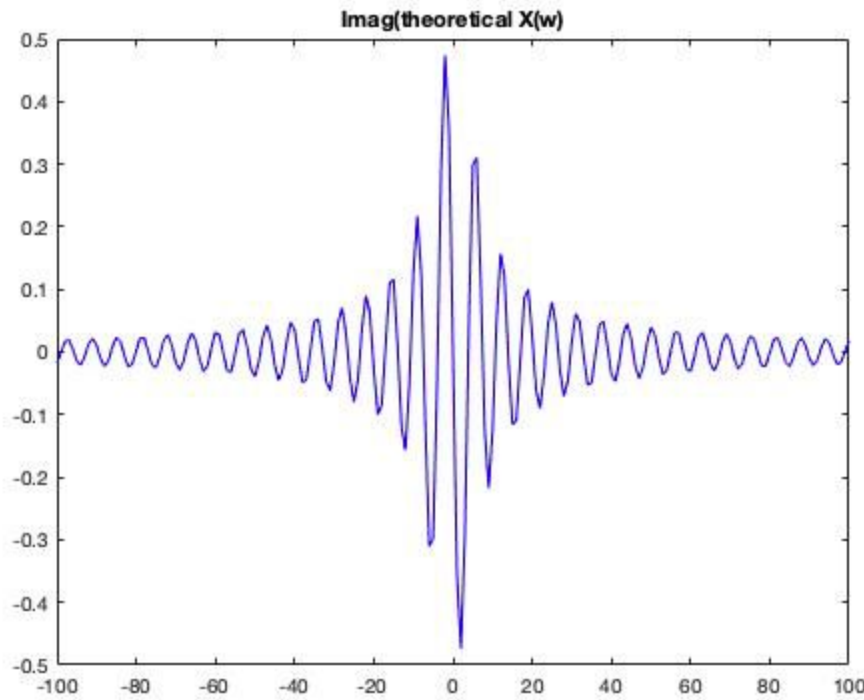
    theoretical_x_w(i) = ((2*1i*cos(w))/w) -
    ((6*1i*sin(w))/(w*w))-((12*1i*cos(w))/(w*w*w))+((12*1i*sin(w))/(
    w*w*w*w));
end
theoretical_x_w(101)=0;
plot(w0,real(a));
title('Real(calculated X(w));
ylim([-0.1 0.2])
plot(w0,real(theoretical_x_w));
title('Real(theoretical X(w));
plot(w0,imag(a));
title('Imag(calculated X(w));
plot(w0,imag(theoretical_x_w));
title('Imag(theoretical X(w));
inverse_a = zeros(size(t));
for i = 1:length(t)
    basis = exp(1i*w0*t(i));
    inverse_a(i) = (trapz(w0,basis.*a))/(2*pi);
end

plot(t,x,'blue');
hold on;
plot(t,real(inverse_a),'red');
hold off;
legend('Original signal','Reconstructed signal')
title('Original signal vs Reconstructed signal')

```







$$\begin{aligned}
 x(t) &= t^3 \quad |t| < 1 \\
 X(w) &= \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_{-1}^1 t^3 e^{-j\omega t} dt \\
 &= \left[ \frac{t^3 e^{-j\omega t}}{-j\omega} - \int_{-1}^1 3t^2 \frac{e^{-j\omega t}}{-j\omega} dt \right] \Rightarrow \\
 &= \frac{e^{-j\omega} + e^{j\omega}}{-j\omega} + \frac{3}{j\omega} \left[ \frac{t^2 e^{-j\omega t}}{-j\omega} - \int_{-1}^1 2t \frac{e^{-j\omega t}}{-j\omega} dt \right] \\
 &= \frac{2\cos\omega}{-j\omega} + \frac{3}{j\omega^2} (e^{j\omega} - e^{-j\omega}) + \frac{6}{j\omega^2} \int_{-1}^1 e^{-j\omega t} dt \\
 &= \frac{2j\cos\omega}{\omega} + \frac{3}{\omega^2} (-2j\sin\omega) - \frac{6}{\omega^2} \left[ \frac{t e^{-j\omega t}}{-j\omega} - \int_{-1}^1 \frac{e^{-j\omega t}}{-j\omega} dt \right] \\
 &= \frac{2j\cos\omega}{\omega} - \frac{6j\sin\omega}{\omega^2} - \frac{12j\cos\omega}{\omega^3} + \frac{6}{j\omega^3} \left[ \frac{e^{-j\omega t}}{-j\omega} \right]_{-1}^1 \\
 &= \frac{2j\cos\omega}{\omega} - \frac{6j\sin\omega}{\omega^2} - \frac{12j\cos\omega}{\omega^3} - \frac{6}{\omega^4} (-2j\sin\omega) \\
 &= \frac{2j\cos\omega}{\omega} - \frac{6j\sin\omega}{\omega^2} - \frac{12j\cos\omega}{\omega^3} + \frac{12j\sin\omega}{\omega^4} \quad \text{Ans}
 \end{aligned}$$

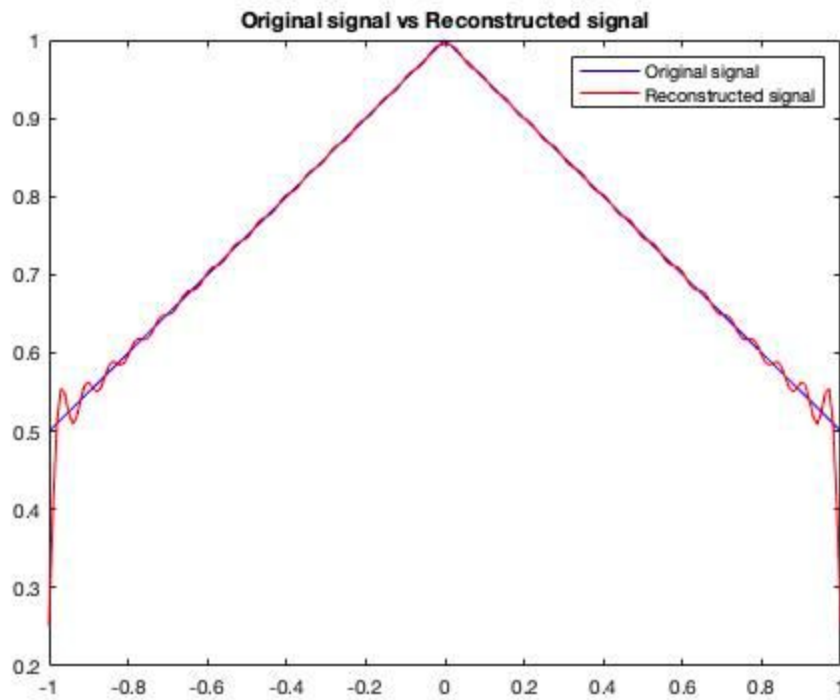
**Signal:  $x(t) = 1 - |t|/2$  ,  $|t| < 1$**

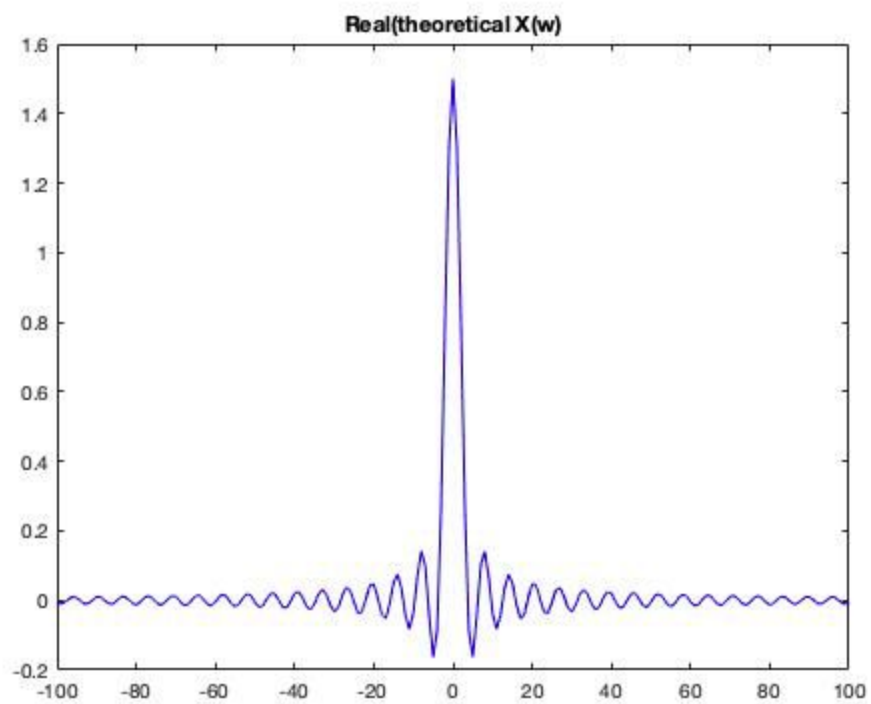
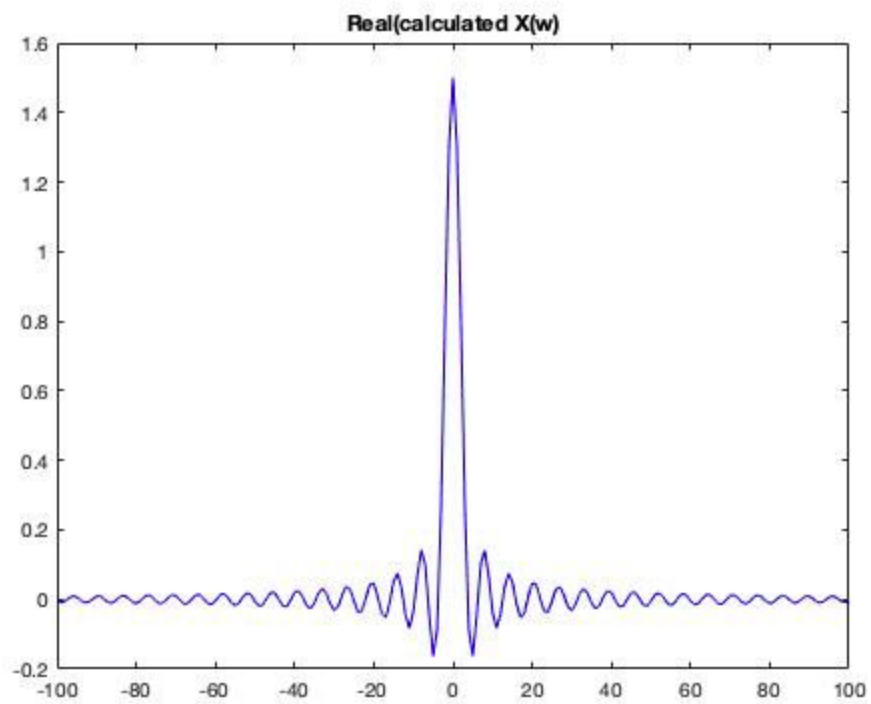
**Solution:**

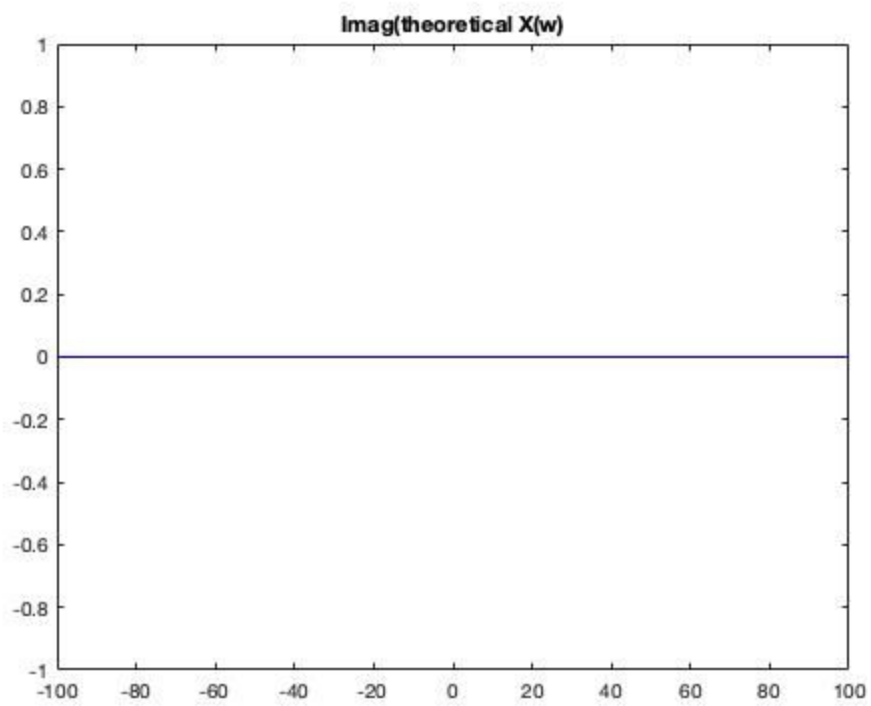
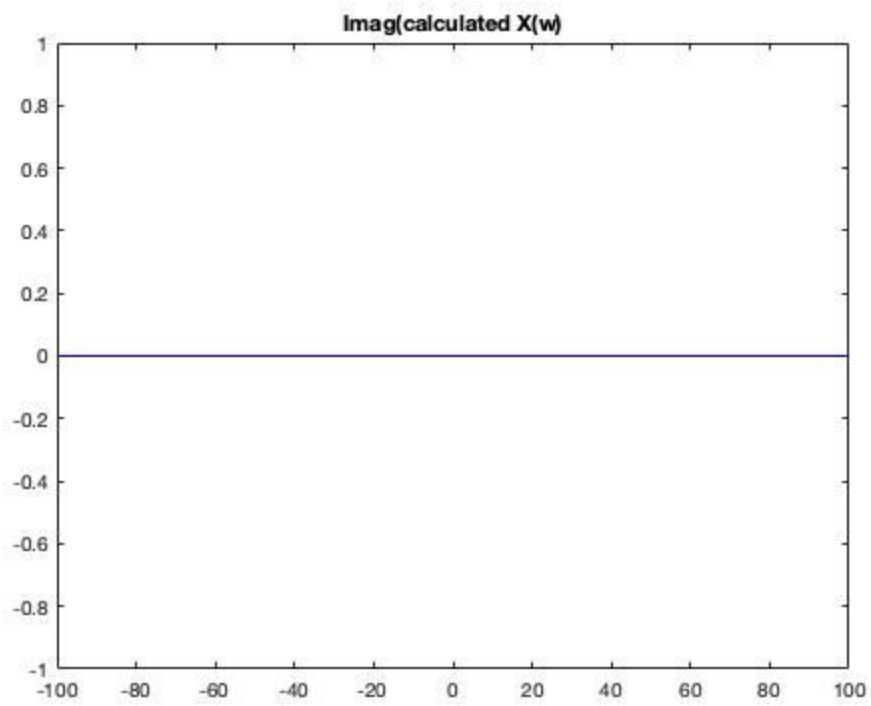
```
t = -1:0.01:1;
w0 = -100:100;
x = 1-((abs(t))/2);
a = zeros(size(w0));
for i = 1:length(w0)
    basis = exp(-1i*w0(i)*t);
    a(i) = trapz(t,x.*basis);
end
theoretical_x_w = zeros(size(w0));
for i = 1:length(w0)
    w = w0(i);
    theoretical_x_w(i) = (((sin(w)/(w))) +
((1/2)*(2-2*cos(w))/(w*w)));
end
theoretical_x_w(101)=1.5;
plot(w0,real(a),'blue');
title('Real(calculated X(w))');
plot(w0,real(theoretical_x_w),'blue');
title('Real(theoretical X(w))');
plot(w0,imag(a),'blue');
title('Imag(calculated X(w))');
ylim([-1 1])
plot(w0,imag(theoretical_x_w),'blue');
title('Imag(theoretical X(w))');
inverse_a = zeros(size(t));
for i = 1:length(t)
    basis = exp(1i*w0*t(i));
    inverse_a(i) = (trapz(w0,basis.*a))/(2*pi);
end
plot(t,x,'blue');
hold on;
```



```
plot(t,real(inverse_a),'red');  
hold off;  
legend('Original signal','Reconstructed signal')  
title('Original signal vs Reconstructed signal')
```







$$X(\omega) = \int_{-1}^1 x(t) e^{-j\omega t} dt$$

$$x(t) = 1 - \frac{|t|}{2}$$

$$= \int_{-1}^0 \left(1 + \frac{t}{2}\right) e^{-j\omega t} dt + \int_0^1 \left(1 - \frac{t}{2}\right) e^{-j\omega t} dt$$

$k=1$

$$= \int_{-1}^1 e^{-j\omega t} dt + \frac{1}{2} \int_{-1}^0 t e^{-j\omega t} dt + -\frac{1}{2} \int_0^1 t e^{-j\omega t} dt$$

$$= \frac{e^{-j\omega t}}{-j\omega} \Big|_{-1}^1 + \frac{1}{2} \left( \frac{t e^{-j\omega t}}{-j\omega} \Big|_{-1}^0 - \frac{e^{-j\omega t}}{(j\omega)^2} \Big|_{-1}^0 \right) - \frac{1}{2} \left( \frac{t e^{-j\omega t}}{-j\omega} \Big|_0^1 - \frac{e^{-j\omega t}}{(j\omega)^2} \Big|_0^1 \right)$$

$k=1$

$$= \frac{\sin \omega}{\omega} + \frac{1 - \cos \omega}{\omega^2}$$

$$= \frac{\sin \omega}{\omega} + \frac{1 - \cos \omega}{\omega^2}$$


---

**Signal:  $x(t) = \text{sinc}(t)$  ,  $|t| < 2\pi$**

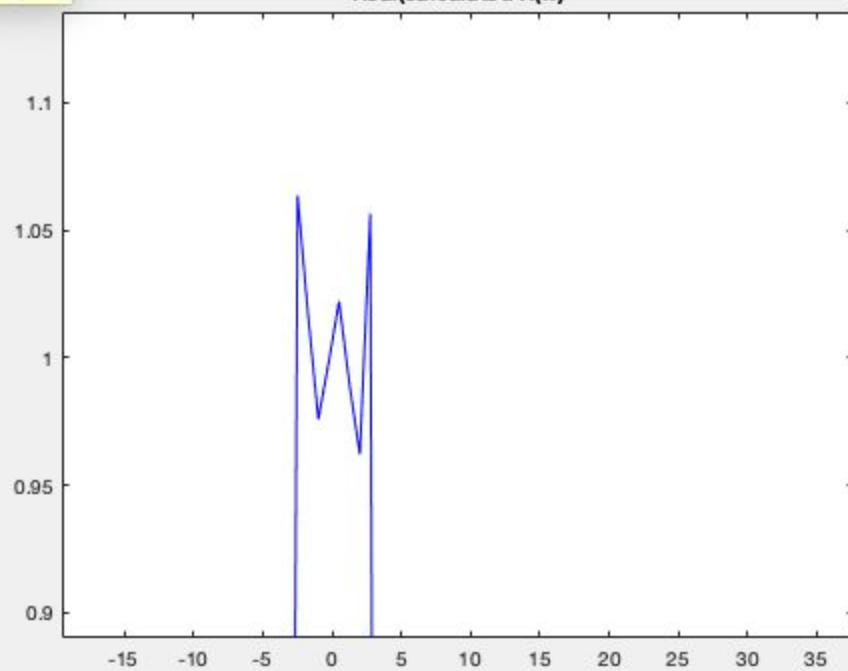
**Solution:**

```
t = -2*pi:0.001:2*pi;
w0 = -100:0.75:100;
x = sinc(t);
a = zeros(size(w0));
for i = 1:length(w0)
    basis = exp(-1i*w0(i)*t);
    a(i) = trapz(t,x.*basis);
end
plot(t,real(fftshift(x)));
title('Real(theoretical X(w))');
plot(t,imag(fftshift(x)));
title('Imag(theoretical X(w))');
plot(w0,real(a),'blue');
title('Real(calculated X(w))');

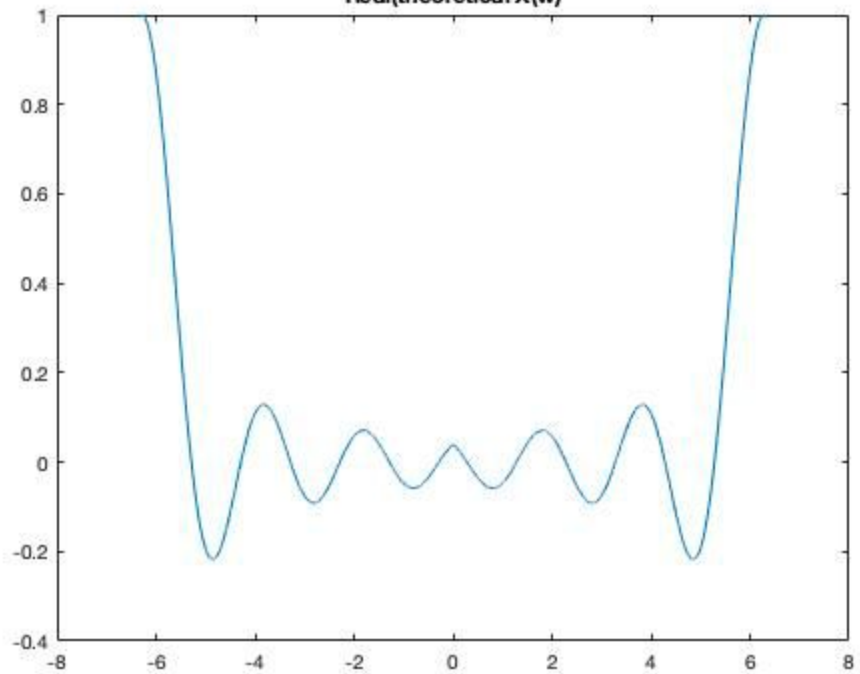
plot(w0,imag(a),'blue');
title('Imag(calculated X(w))');
ylim([-1 1])
inverse_a = zeros(size(t));
for i = 1:length(t)
    basis = exp(1i*w0*t(i));
    inverse_a(i) = (trapz(w0,basis.*a))/(2*pi);
end
plot(t,x,'blue');
hold on;
plot(t,real(inverse_a),'red');
hold off;
legend('Original signal','Reconstructed signal')
title('Original signal vs Reconstructed signal')
```

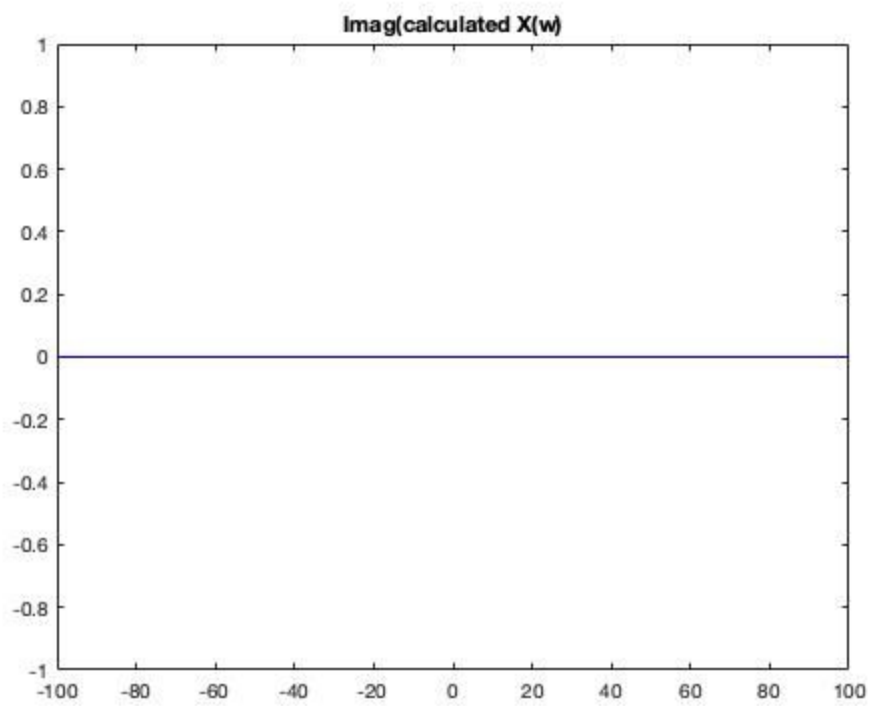
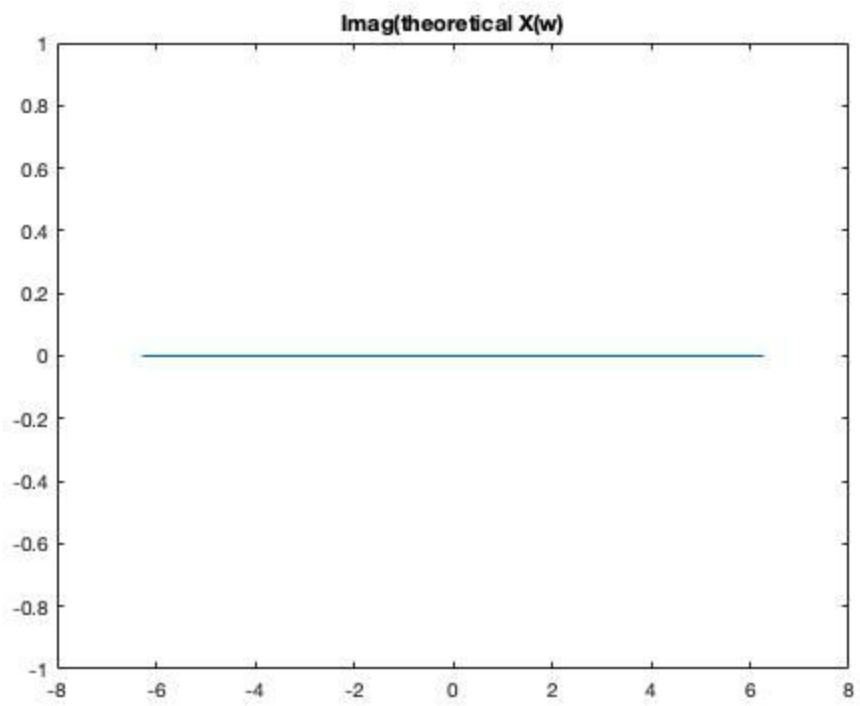
New Figure

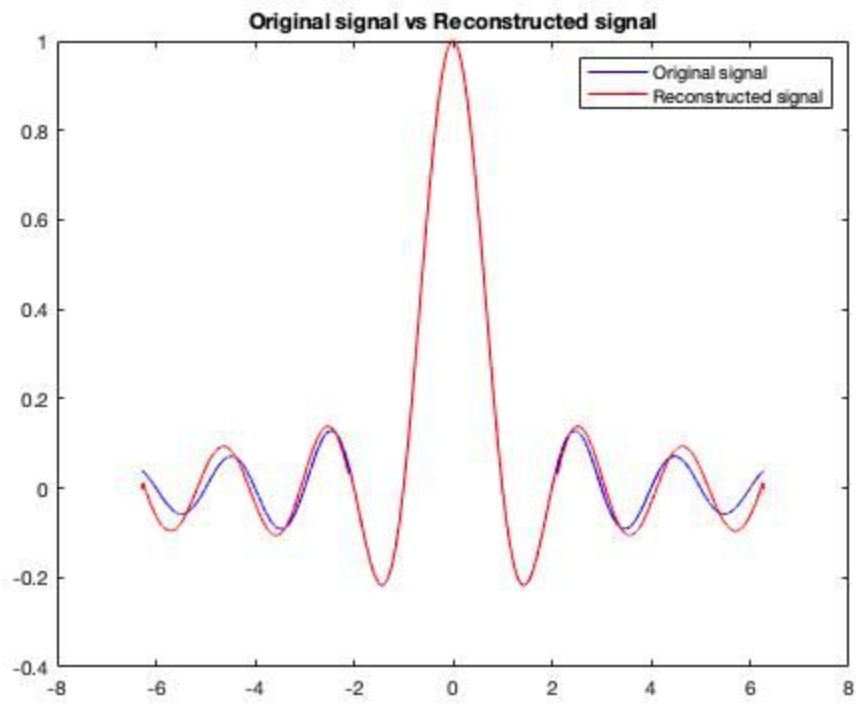
Real(calculated  $X(w)$ )



Real(theoretical  $X(w)$ )







Thank You!!