Signals And Systems
Home Assignment

1- Average power of $A\cos(\omega t+\phi)$ $\quad T \rightarrow \frac{2\pi}{\omega}$

Average power for a periodic signal is same as for one time period.

$$= \frac{1}{T}\int_0^T |x(t)|^2 dt = \frac{1}{T}\int_0^T A^2\cos^2(\omega t+\phi)\,dt$$

$$= \frac{A^2}{T}\int_0^T \frac{1+2\sin(2\omega t+2\phi)}{2}\,dt = \frac{A^2}{T}\left[\frac{t}{2}\right]_0^T + 0$$

$$= \frac{A^2}{T}\frac{T}{2} = \frac{A^2}{2} \quad \underline{\underline{\text{Ans}}}$$

Q- Given rectangular pulse $x(t) = A, \quad 0 \le t \le T$

$\qquad\qquad\qquad\qquad\qquad\quad 0, \quad$ otherwise

$y(t) = \int_0^t x(\tau)\,d\tau$. find avg power of $y(t)$

Sol- if $t<0 \rightarrow 0 = y(t)$

if $t<T$, $At = y(t)$

if AT, $T \quad t>T$

$$y(t) = \begin{cases} 0 & t<0 \\ At & t \le T \\ AT & t > T \end{cases}$$

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Average power of $y(\tau)$

$$= \lim_{\tau \to \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} |y(t)|^2 d\tau$$

$$= \lim_{\tau \to \infty} \frac{1}{2\tau} \int_0^T |At|^2 dt + \frac{1}{2\tau} \int_T^{\tau} (AT)^2 d\tau$$

$$= \lim_{\tau \to \infty} \frac{1}{2\tau} \frac{A^2 t^3}{3}\Big|_0^T + \frac{1}{2\tau} A^2 T^2 \tau \Big|_T^{\tau}$$

$$= \lim_{\tau \to \infty} \frac{A^2 T^3}{6\tau} + \frac{A^2 T^2 \tau}{2\tau} - \frac{A^2 T^2 T}{2\tau}$$

$$= \frac{A^2 T^2}{2} \quad \text{Ans}$$

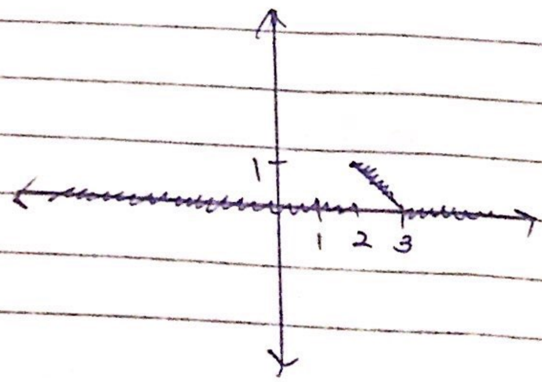
1.2

$$x(t) = \begin{cases} -1-t & -1 \le t < 0 \\ t & 0 \le t < 1 \\ 1 & 1 \le t < 2 \\ 3-t & 2 \le t < 3 \\ 0 & \text{else} \end{cases}$$

$$y(t) = \begin{cases} 1 & -2 < t < -1 \\ -1 & -1 \le t < 0 \\ t-1 & 0 \le t < 1 \\ 1 & 1 \le t < 2 \\ 0 & \text{else} \end{cases}$$

① Plot $x(t)\, y(t-4) =$

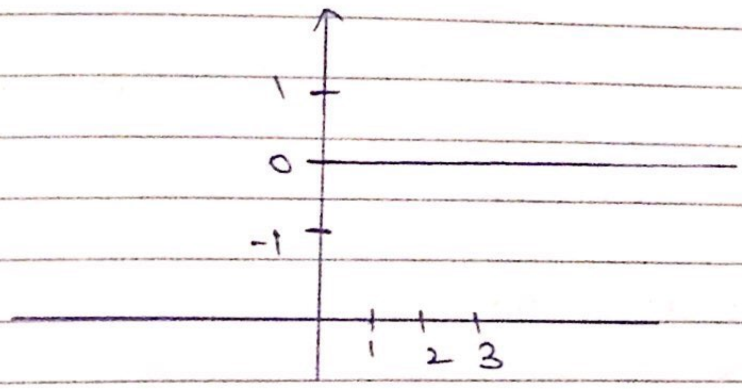
$$\begin{cases} 0 & , t < 2 \\ 3-t & , 2 \le t < 3 \\ 0 & , t \geq 3 \end{cases}$$



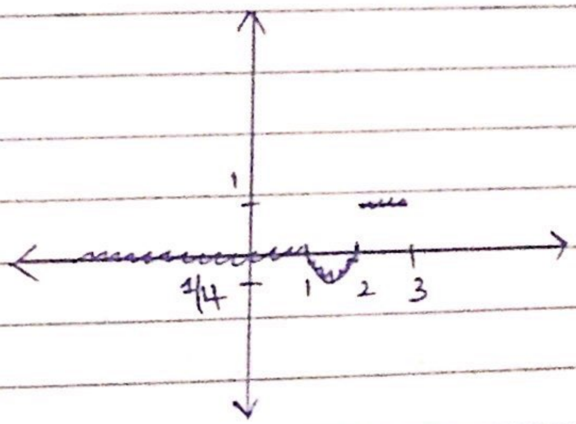
Zigzag lines are the graph

(2) $x(t+1)\, y(t-4)$ $\quad 0 \quad t \le 2$

$\quad 0 \quad t > 2$



(3) $x(4-t)\, y(t-1) = \begin{cases} t^2 - 3t + 2, & 1 \le t < 2 \\ 1, & 2 \le t < 3 \\ 0, & \text{else} \end{cases}$



Q.1 1 $y(t) = \dfrac{d}{dt} x(t)$

(a) The system is not causal as output values are determined by future values

$$\frac{d}{dt} x(t) = \lim_{l \to 0} \frac{x(t+l) - x(t)}{l}$$

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(b) The system may or may not be stable
Because if $x(t) = 2$, $x=0$
0 else
then $\frac{d}{dt} x(t) = \text{not bounded} \Big|_{t=0}$

Although input is bounded.

c) Yes, the system is linear.
$y_1(t) = \frac{d\, x_1(t)}{dt}$ $y_2(t) = \frac{d x_2(t)}{dt}$

$a y_1(t) + b y_2(t) = \frac{d}{dt}\left[a x_1(t) + b x_2(t)\right]$

LHS = RHS
Hence Proved

(d) Time Invariant
$y(t+t_0) = \frac{d}{dt}(x(t))\Big|_{t=t_0}$

$y(t+t_0) = y(t)$ But $x(t)$ is shifted
So, it is not time invariant.

II $y(n) = x(2-n)$
(a) for $n=0$, $y(0) = x(2)$
Since the signal is dependent on future values,
it is non causal.

(b) As $y(n) = x(2-n)$
if $|x(n)| \le B \Rightarrow x(n)$ is stable
then $|x(2-n)| \le B$
Hence $y(n)$ is stable.

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(c) Linearity

$y_1(n) = x_1(2-n)$, $y_2(n) = x_2(2-n)$

$a y_1(n) + b y_2(n) = a x_1(2-n) + b x_2(2-n)$

LHS = RHS

Hence, Linearity property is satisfied.

(d) Time Invariance

$y(n-n_0) = x(2-n+n_0)$

$\neq x(2-n-n_0)$

Hence, by shifting $x(n)$, $y(n)$ is not changed by same amount. Hence, it is not ~~linear~~ time invariant.

2.2 Linear Convolution

1. $x(n) = \cos\left[\frac{\pi}{2}(n)\right]$ $\qquad h(n) = \left(\frac{1}{2}\right)^n U(n-2)$

Sol: $y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

$= \sum_{k=-\infty}^{\infty} \cos\left(\frac{\pi k}{2}\right)\left(\frac{1}{2}\right)^{n-k} U(n-k+2)$

$= \sum_{k=-\infty}^{n-2} \cos\left(\frac{\pi k}{2}\right)\left(\frac{1}{2}\right)^{n-k}$

if $n = 4k$, $y(n) = \frac{-1}{4} + \frac{1}{16} - \frac{1}{64} \dots = \frac{-1}{4} \frac{1}{1+\frac{1}{4}} = \frac{-1}{5}$

$n = 4k+1$, $y(n) = \frac{-1}{8} + \frac{1}{32} - \frac{1}{128} = \frac{-1}{8} \times \frac{1}{1+\frac{1}{4}} = \frac{-1}{10}$

$n = 4k+2$, $y(n) = \frac{1}{4} - \frac{1}{16} + \frac{1}{64} + \dots = \frac{1}{4} \times \frac{1}{1+\frac{1}{4}} = \frac{1}{5}$

$n = 4k+3$, $y(n) = \frac{1}{8} - \frac{1}{32} + \frac{1}{128} + \dots = \frac{1}{8} \times \frac{1}{1+1/4} = \frac{1}{10}$

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$$y(n) = \begin{cases} -1/5 & , n = 4k \\ -1/10 & , n = 4k+1 \\ 1/5 & , n = 4k+2 \\ 1/10 & , n = 4k+3 \end{cases}$$

~~J.f~~

2.2 Q-2. Find convolution of the signals

$x(n) = 1 \quad -5 \le n \le 1$
$\quad\quad\quad 0 \quad$ otherwise

$h(n) = 1 \quad 0 \le n \le 3$
$\quad\quad\quad 2 \quad 4 \le n \le 8$
$\quad\quad\quad 0 \quad$ otherwise

Sol: $y(n) = x(n) * h(n)$

$x(n) \downarrow$ / $h(n) =$	1	1	1	1	2	2	2	2
1	1	1	1	1	2	2	2	2
1	1	1	1	1	2	2	2	2
1	1	1	1	1	2	2	2	2
1	1	1	1	1	2	2	2	2
1	1	1	1	1	2	2	2	2
1	1	1	1	1	2	2	2	2
1	1	1	1	1	2	2	2	2

$$y(n) = \left(1\ 2\ 3\ 4\ 6\ \underset{\substack{\uparrow \\ n=0}}{8}\ 10\ 11\ 12\ 11\ 10\ 8\ 6\ 4\ 2\right)$$

2. Fourier Representation of Signals -

$x(t) = 1 - \frac{|t|}{3}$, $|t| < 1$ $\qquad T = 3$, $\omega \Rightarrow \frac{2\pi}{3}$

Sol: $a_k = \frac{1}{T}\int_{\infty}^{\infty} x(t) \cdot e^{-j\omega k t}\, dt$

$= \frac{1}{3}\int_{-1}^{1}\left(1 - \frac{|t|}{3}\right) e^{-j\omega k t}\, dt$

$= \frac{1}{3}\frac{e^{-j\omega k t}}{-j\omega k}\Big|_{-1}^{1} - \frac{1}{3\times3}\int_{0}^{1} t\, e^{-j\omega k t}\, dt + \frac{1}{9}\int_{-1}^{0} t\, e^{-j\omega t k}\, dt$

$= \frac{2}{3}\frac{\sin \omega k}{\omega k} - \frac{1}{9}\, t\frac{e^{-j\omega k t}}{-j\omega k}\Big|_{0}^{1} + \frac{1}{9}\, t\frac{e^{-j\omega k t}}{-j\omega k}\Big|_{-1}^{0}$

$+ \frac{1}{9}\int_{0}^{1}\frac{e^{-j\omega k t}}{-j\omega k}\, dt - \frac{1}{9}\int_{0}^{-1}\frac{e^{-jk\omega t}}{-jk\omega}\, dt$

$a_k = \frac{2}{3}\frac{\sin \omega k}{3\omega k}\times 3 - \frac{1}{9}\frac{-2j\sin \omega k}{-j\omega k} + \frac{1}{9}\frac{e^{-j\omega k t}}{-\omega^2 k^2}\Big|_{0}^{1} - \frac{1}{9}\frac{e^{-j\omega k t}}{-\omega^2 k^2}\Big|_{0}^{-1}$

$a_k = \frac{6}{9}\frac{\sin \omega k - 2\sin \omega k}{\omega k} + \frac{2}{9}\frac{\cos \omega k}{-\omega^2 k^2} + \frac{1}{9}\frac{(2)}{-\omega^2 k^2}$

$$\boxed{a_k = \frac{4}{9}\frac{\sin(\omega k)}{(\omega k)} - \frac{2}{9}\frac{\cos(\omega k)}{(\omega k)^2} + \frac{2}{9(\omega k)^2}}$$ Ans

$$\boxed{a_0 = \frac{5}{9}}$$

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(2) $x(n) = \cos\left(\frac{\pi n}{5}\right) \quad |n| \le 2 \quad N = 5$

$$\Rightarrow a_k = \frac{1}{N} \sum_{k=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$= \frac{1}{5}\left[\cos\frac{\pi}{5} e^{-j\frac{2\pi}{5}k} + \cos\left(\frac{-\pi}{5}\right) e^{j\frac{2\pi}{5}k} + \cos\frac{2\pi}{5} e^{-j4\pi k/5} + \cos -\frac{2\pi}{5} e^{j\frac{2\pi}{5}\cdot 2k}\right]$$

$$= \frac{1}{5}\left[1 + 2\cos\frac{\pi}{5}\cos\frac{2\pi k}{5} + 2\cos\frac{2\pi}{5}\cos\frac{4\pi k}{5}\right]$$

$a_0 = 0.6471$

$a_1 = 0.200$ $\qquad a_4 = 0.2$

$a_2 = -0.0236$

$a_3 = -0.0236$

Fourier Transform

(1) $x(t) = 2 \quad |t| < 1$

$\quad 0 \quad$ otherwise,

Sol: $X(\omega) = \int_{-1}^{1} x(t)\, e^{-j\omega t} dt = \int_{-1}^{1} 2\, e^{-j\omega t} dt = 2\int_{-1}^{1} e^{-j\omega t} dt$

$$= 2\left|\frac{e^{-j\omega t}}{-j\omega}\right|_{-1}^{1} = 2\left(\frac{-2j\sin\omega}{-j\omega}\right) = \frac{4\sin(\omega)}{\omega} \text{ Ans}$$

~~(2) $x(t) = e^{-\frac{|t|}{2}} e^{-j\omega t} dt$~~

(2) $x(t) = e^{-|t|/2} \quad |t| < 1$

$\quad 0 \quad$ else

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Sol $X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt = \int_{-1}^{1} e^{-(|t|)/2}\, e^{-j\omega t} dt$

$= \int_{0}^{1} \frac{e^{-j\omega t - t/2}}{-j\omega - 1/2} (-j\omega - 1/2)\, dt + \int_{-1}^{0} e^{-j\omega t + t/2}\, dt$

$= \left. \frac{e^{-j\omega t - t/2}}{-j\omega - 1/2} \right|_{0}^{1} + \left. \left(\frac{e^{-j\omega t + t/2}}{-j\omega + 1/2} \right|_{-1}^{0}$

$= \frac{e^{-j\omega - 1/2}}{-j\omega - 1/2} + \frac{e^{-j\omega + 1/2}}{-j\omega + 1/2} + \frac{8j\omega}{1 - 4j^2\omega^2}$

$= \frac{-\left(e^{-j\omega - 1/2}\right)(-j\omega + 1/2) + (j\omega + 1/2)\, e^{-j\omega + 1/2}}{\left(\frac{1}{2}\right)^2 - (j\omega)^2} + \frac{8j\omega}{1+4\omega^2}$

$= \frac{\left(j\omega e^{-j\omega - 1/2} - \frac{1}{2} e^{-j\omega - 1/2}\right) + j\omega e^{-j\omega + 1/2} + \frac{1}{2} e^{-j\omega + 1/2}}{1/4 + \omega^2} + \frac{8j\omega}{1+4\omega^2}$

$= \frac{4\left[j\omega e^{-j\omega - 1/2} - \frac{1}{2} e^{-j\omega - 1/2} + j\omega e^{-j\omega + 1/2} + \frac{1}{2} e^{-j\omega + 1/2} \right] + 8j\omega}{\cancel{1+4}\ 1+4\omega^2}$ Ans

Inverse Fourier Transform

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① $X(\omega) = \mathrm{Sinc}^2(\omega)$

Sol: $x(t) = 1 - \frac{|t|}{T} \quad |t| < T$

$\qquad\quad 0 \quad$ otherwise

$$X(\omega) = \int_{-T}^{0}\left(1+\frac{t}{T}\right)e^{-j\omega t}\,dt + \int_{0}^{T}\left(1-\frac{t}{T}\right)e^{-j\omega t}\,dt$$

$$= \frac{e^{-j\omega t}}{-j\omega}\Big|_{-T}^{0} + \frac{1}{T}\left[t\,\frac{e^{-j\omega t}}{-j\omega}\Big|_{-T}^{0} + \frac{1}{\omega^2}e^{-j\omega t}\Big|_{-T}^{0}\right] + \frac{e^{-j\omega t}}{-j\omega}\Big|_{0}^{T}$$

$$-\frac{1}{T}\left[t\,\frac{e^{-j\omega t}}{-j\omega}\Big|_{0}^{T} + \frac{1}{\omega^2}e^{-j\omega t}\Big|_{0}^{T}\right]$$

$$= \frac{1}{-j\omega} + \frac{e^{j\omega T}}{j\omega} + \frac{-e^{-j\omega T}\,T}{-j\omega T} + \frac{e^{-j\omega T}}{j\omega^2} + \frac{1}{\omega^2} + \frac{1}{j\omega} +$$

$$\frac{e^{-j\omega T}}{-j\omega} + \frac{1}{\omega^2} - \frac{e^{-j\omega T}}{T\omega^2} - \frac{T e^{-j\omega T}}{j\omega}$$

$$= 2\sin\frac{\omega T}{\omega} - 2\sin\frac{\omega T}{\omega} + 2\left(\frac{1-\cos\omega T}{T\omega^2}\right)$$

$$= T\,\frac{4\sin^2(\omega T/2)}{(T\omega)^2} = T\,\frac{\sin^2(\omega t/2)}{(\omega T/2)^2} = 2\,\mathrm{sinc}^2(\omega)$$

By using duality

if $x(t) \to X(\omega)$

$X(\omega) \to 2\pi\, x(-\omega)$

So, Inverse fourier Transform of $\mathrm{Sinc}^2(\omega)$ is

$\pi\left(1-\frac{|t|}{2}\right)$ by using duality

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(3) $X(\omega) = \frac{1}{1+j\omega RC}$

Sol: Consider $x(t) = e^{-at} U(t)$

$$X(\omega) = F(x(t)) = \int_{-\infty}^{\infty} e^{-at} U(t) e^{-j\omega t} dt = \int_{0}^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-t(a+j\omega)} dt = \frac{e^{-t(a+j\omega)}}{-(a+j\omega)} \Big|_{0}^{\infty}$$

$$= \frac{0 - e^{0}}{-(a+j\omega)} = \frac{1}{a+j\omega}$$

Considering $a = 1/RC$, rearranging the constant, we have

$$F\left[(RC)^{-1} e^{-1/RC\, t} U(t)\right] = X(\omega) = \frac{1}{1+j\omega RC}$$

So $$F^{-1}\left[\frac{1}{1+j\omega RC}\right] = \boxed{\frac{e^{-t/RC} U(t)}{RC}}$$ Ans

Discrete Fourier Transform →

DFT

$$W(n) = 0.42 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right) + 0.08 \cos\left(\frac{4\pi n}{N-1}\right) \quad \boxed{N=5}$$

Sol $$W(k) = \frac{1\times 5}{5}\left[\sum_{n=0}^{4} (0.42) - 0.5 \cos\left(\frac{\pi}{2} n\right) + 0.08 \cos(\pi n)\right] e^{-jk\frac{2\pi}{5} n}$$

$$W(k) = 0 + 0.34\, e^{-jk2\pi/5} + e^{-jk4\pi/5} + 0.35\, e^{-jk\frac{6\pi}{5}}$$

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$\Rightarrow$ $w(0) = 1.68$

$w(1) = 0.34\left[e^{-j2\pi/5} + e^{-j6\pi/5}\right] + e^{-j4\pi/5}$

$w(2) = 0.34\left(e^{-j4\pi/5} + e^{-j12\pi/5}\right) + e^{-j8\pi/5}$

$w(3) = 0.34\left(e^{-j6\pi/5} + e^{-j18\pi/5}\right) + e^{-j12\pi/5}$

$w(4) = 0.34\left(e^{-j8\pi/5} + e^{-j24\pi/5}\right) + e^{-j16\pi/5}$

Circular Convolution

$x_1(n) = \{1, 2, 3, 1\}$ $\qquad x_2(n) = \{4, 3, 2, 2\}$

$x_3(n) = x_1(n) \, (N) \, x_2(n)$

Sol $x(k) = e^{-j\frac{2\pi}{4}k(0)} + 2e^{-j\frac{\pi}{2}k} + 3e^{-j\pi k} + e^{-j2\pi/4(3)}$

$x_1(1) = -(2+i)$

$x_1(2) = 1 + 2e^{-j\pi} + 3e^{-j2\pi} + e^{-j3\pi} = 1$

$x_1(3) = 2e^{-j3\pi/2} + 1 + 3e^{-j3\pi} + e^{-j9\pi/2} = -(2-i)$

$x_1(0) = 1+2+3+1 = 7$

$x_2(0) = 4+3+2+2 = 11$, $x_2(1) = 4 + 3e^{-j\pi/2} + 2e^{-j\pi} + 2e^{-j3\pi/2} = 2-i$

$x_2(2) = 4 + 3e^{-j\pi} + 2e^{-j2\pi} + e^{-j3\pi} = 1$

$x_2(3) = 4 + e^{-3\pi/2j} \times 3 + 2e^{-j3\pi} + 2e^{-j\pi/2} = 2+i$

$y = x_1 x_2 = 77, -5, 1, -5$ element wise multiplication

Ans $x(0) = \frac{1}{4}(77-5+1-5) = 17$ $\qquad x(2) = 22$

$x(1) = \frac{1}{4}(77+5j-5j-1) = 19$ $\qquad x(3) = 19$

$x(0) = 17$, $x(1) = 19$, $x(2) = 22$

$x(3) = 19$ Ans