BASIC ELECTRONIC CIRCUITS

Two-port Network Parameters

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One-port Network

- A terminal is a connecting wire thorough which the electrical energy or signal that enter (input terminal) into a network or leave (output terminal) the network.
- A pair of terminals at which a signal may enter or leave network is called a "PORT".
- A network having only one such pair is called one-port network.
- More than one pair of terminals is present, "Multiport network".

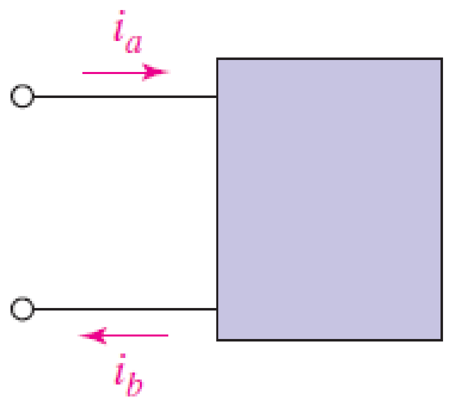


Fig. 1. One-port network.

Two-port Network

- Describes the relation between the input and output terminals.
- Linear networks
- No independent source should present in the network.
- Dependent source may present.

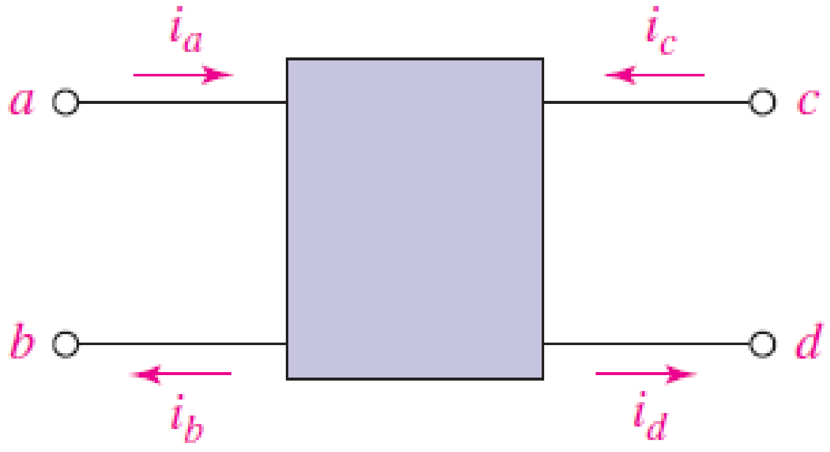
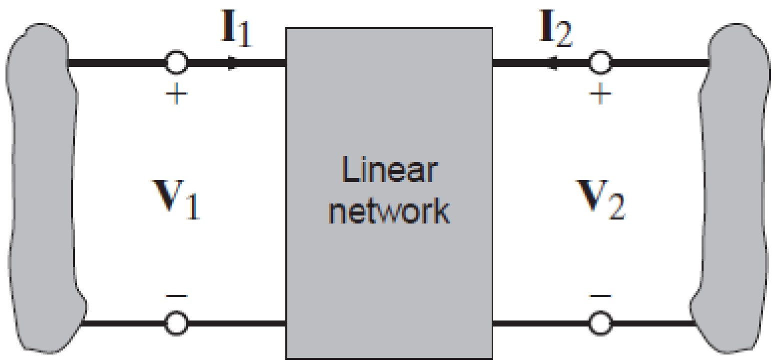


Fig. 2. Two-port network.

Admittance (Y) Parameters

- Linear network with No independent sources.
- V_1 and V_2 are applied voltage and I_1 and I_2 are the responses.



- BY using the superposition:

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

- $I_1 = Y_{11}V_1 + Y_{12}V_2$
- $I_2 = Y_{21}V_1 + Y_{22}V_2$

Y-parameters: Short circuit admittance parameters

- Short-Circuit input admittance – Y_{11}
- Short circuit transfer admittances – Y_{12}, Y_{21}
- Short-circuit output admittance – Y_{22}.
- Properties: Reciprocal ($Y_{12} = Y_{21}$); Symmetric ($Y_{11} = Y_{22}$)

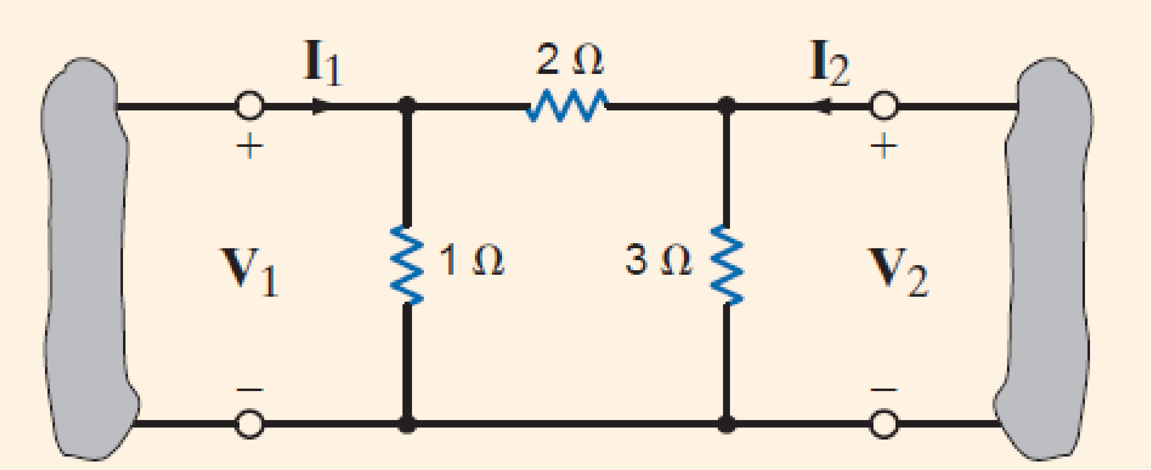
$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}$$

$$\mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}$$

$$\mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

- Ex: Determine the Y parameters of the network.



- Further, determine the current in a 4 Ω load, which is connected to the output port when a 2 A current source is applied at the input port.

$$I_1 = \frac{3}{2}V_1 - \frac{1}{2}V_2 \qquad I_2 = -\frac{1}{2}V_1 + \frac{5}{6}V_2$$

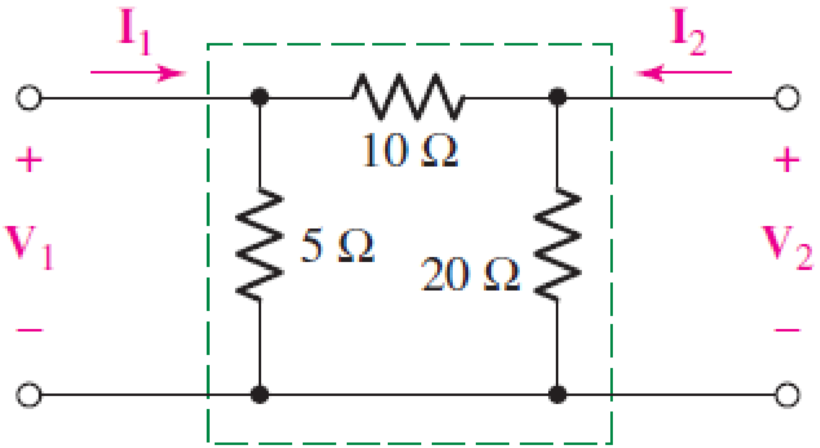
$$I_1 = 2\text{ A} \quad \text{and} \quad V_2 = -4I_2$$

$$2 = \frac{3}{2}V_1 - \frac{1}{2}V_2$$

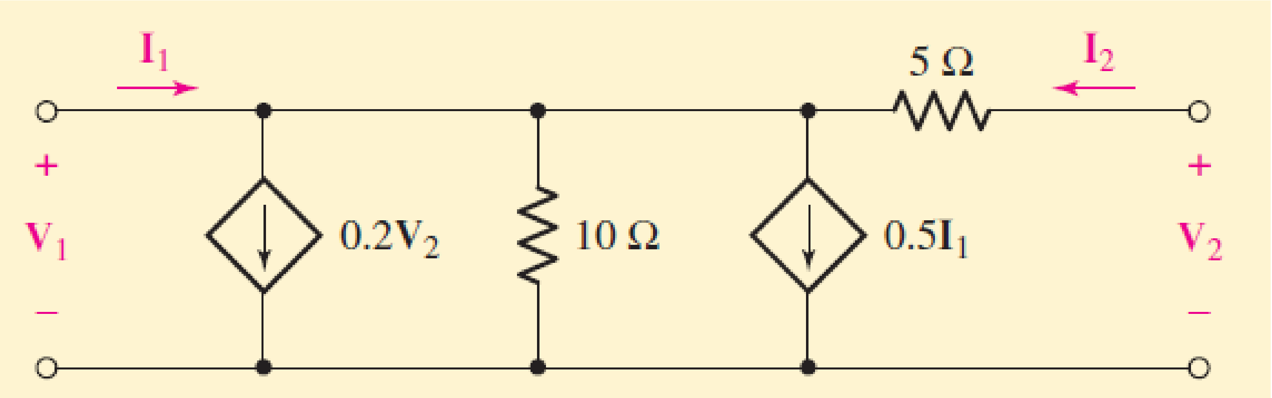
$$0 = -\frac{1}{2}V_1 + \frac{13}{12}V_2$$

$V_2 = 8/11$ V and therefore $I_2 = -2/11$ A.

- Determine the Y-parameters of the following networks.



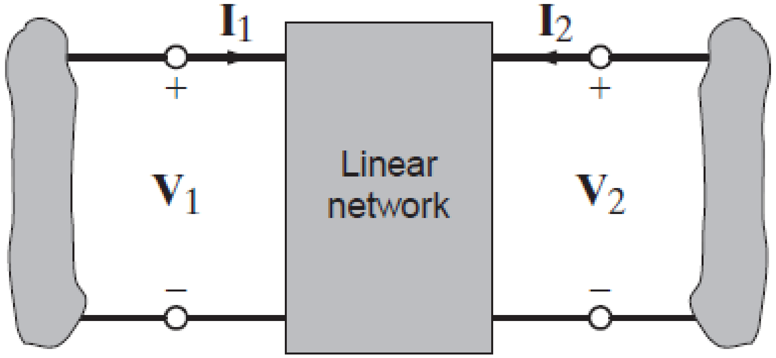
$$y = \begin{bmatrix} 0.3 & -0.1 \\ -0.1 & 0.15 \end{bmatrix}$$



$$\begin{bmatrix} 0.6 & 0 \\ -0.2 & 0.2 \end{bmatrix}$$

Impedance (Z) parameters: Open-circuit impedance parameters

- $V_1 = Z_{11}I_1 + Z_{12}I_2$
- $V_2 = Z_{21}I_1 + Z_{22}I_2$



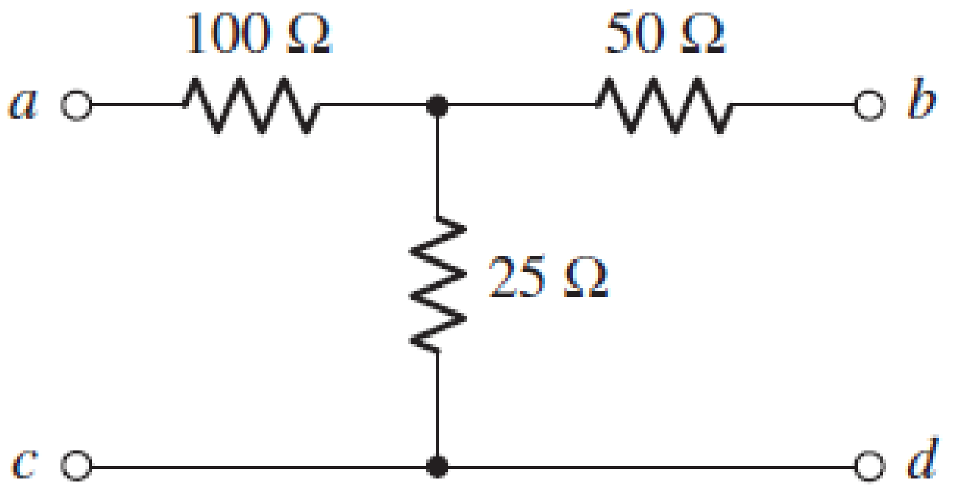
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\boxed{\begin{aligned} z_{11} &= \left.\frac{V_1}{I_1}\right|_{I_2=0} \\ z_{12} &= \left.\frac{V_1}{I_2}\right|_{I_1=0} \\ z_{21} &= \left.\frac{V_2}{I_1}\right|_{I_2=0} \\ z_{22} &= \left.\frac{V_2}{I_2}\right|_{I_1=0} \end{aligned}}$$

- Open-Circuit input impedance – Z_{11}
- Open circuit transfer impedances – Z_{12}, Z_{21}
- Open-circuit output impedance – Z_{22}.

- Ex: Determine the Z parameters of the network.

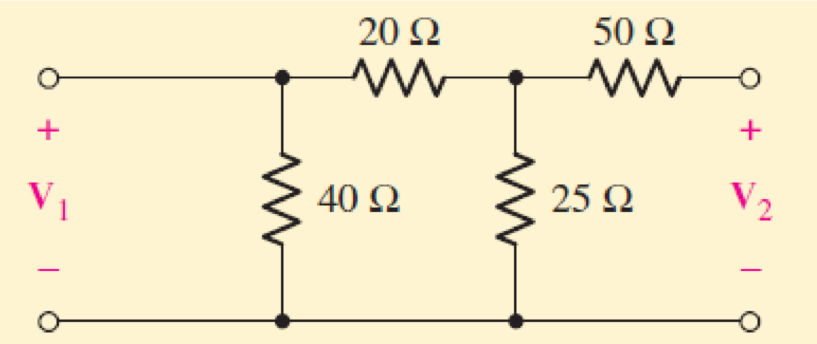
- Further, determine the currents, when a 25 Ω load is connected to the output port when a 5 V voltage source is applied at the input port.



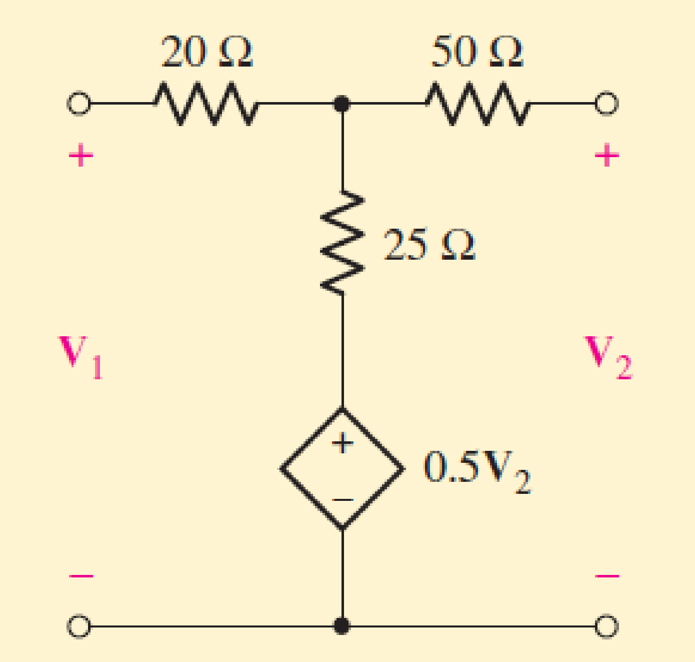
Z_{11} = 125 Ω; Z_{12} = 25 Ω; Z_{21} = 25 Ω; Z_{22} = 75 Ω.

I_2 =- 1 mA; V2 = 0.26 v; I_1 = 42 mA

- Ex: Determine the Impedance parameters:



$$\begin{bmatrix} 21.2 & 11.76 \\ 11.76 & 67.6 \end{bmatrix} (\Omega)$$



$$\begin{bmatrix} 70 & 100 \\ 50 & 150 \end{bmatrix} (\Omega)$$

Hybrid (h) Parameters

- Difficulties in measuring the transfer parameters (Z_{21}).

$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} = \text{short-circuit input impedance}$$

$$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} = \text{short-circuit forward current gain}$$

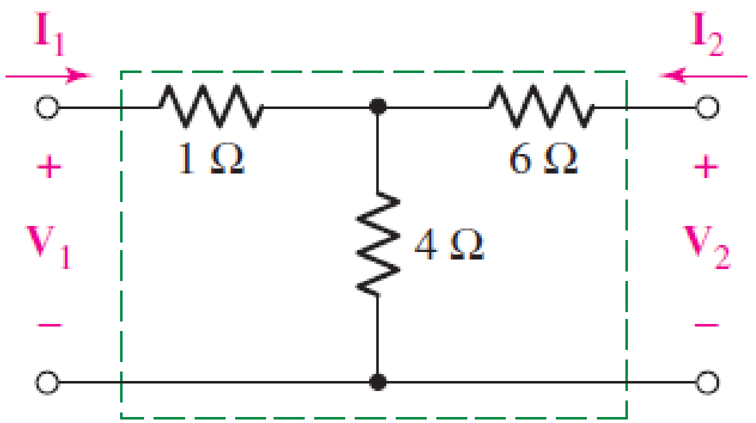
$$\mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} = \text{open-circuit reverse voltage gain}$$

$$\mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} = \text{open-circuit output admittance}$$

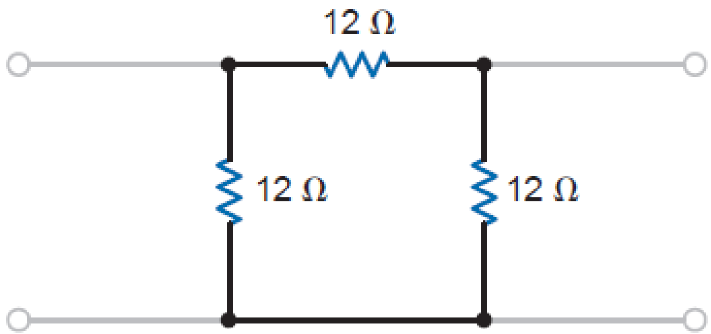
$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

- Ex: Determine the hybrid parameters of the bilateral resistive network



$$\begin{bmatrix} 3.4\ \Omega & 0.4 \\ -0.4 & 0.1\ \mathrm{S} \end{bmatrix}$$



$h_{11} = 6\ \Omega$; $h_{12} = 0.5$; $h_{21} = -0.5$; $h_{22} = 0.125$ S.

Transmission (t) or (ABCD) parameters

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

- A – Open circuit reverse voltage gain
- B – Short circuit transfer impedance
- C – Open circuit transfer admittance
- D – Short circuit reverse current gain

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

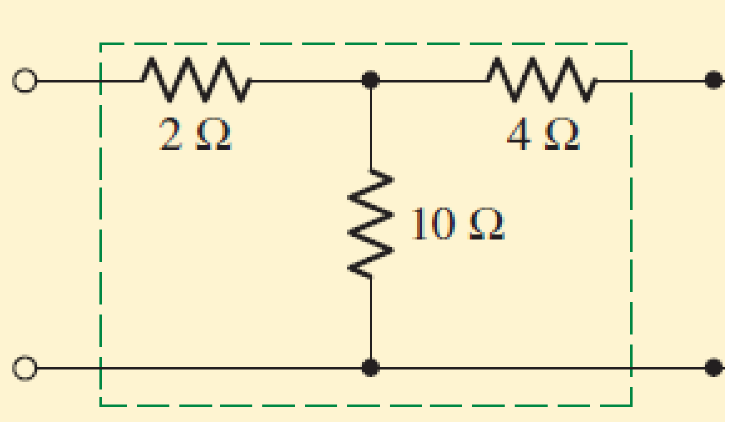
$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

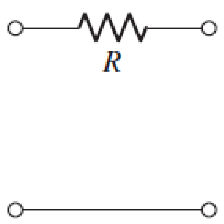
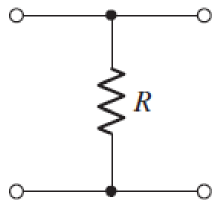
$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

- Obtain the Transmission parameters

$$\mathbf{t}_A = \begin{bmatrix} 1.2 & 6.8\ \Omega \\ 0.1\ \text{S} & 1.4 \end{bmatrix}$$

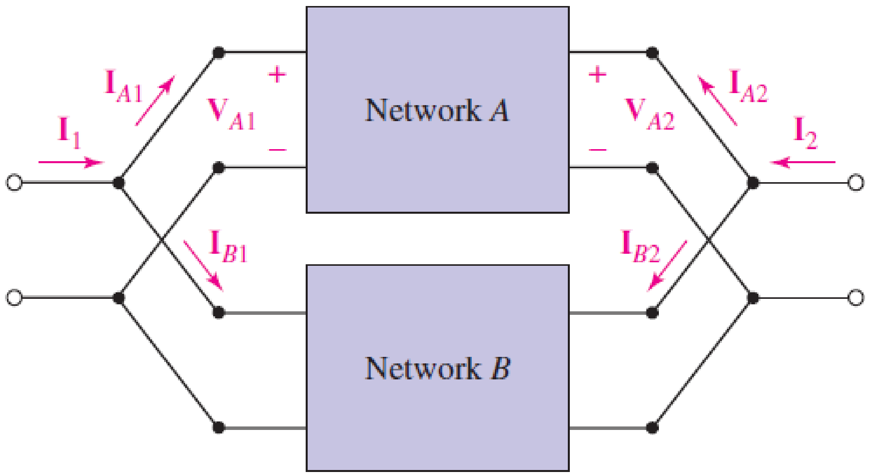


Two-port parameters of a series and shunt elements

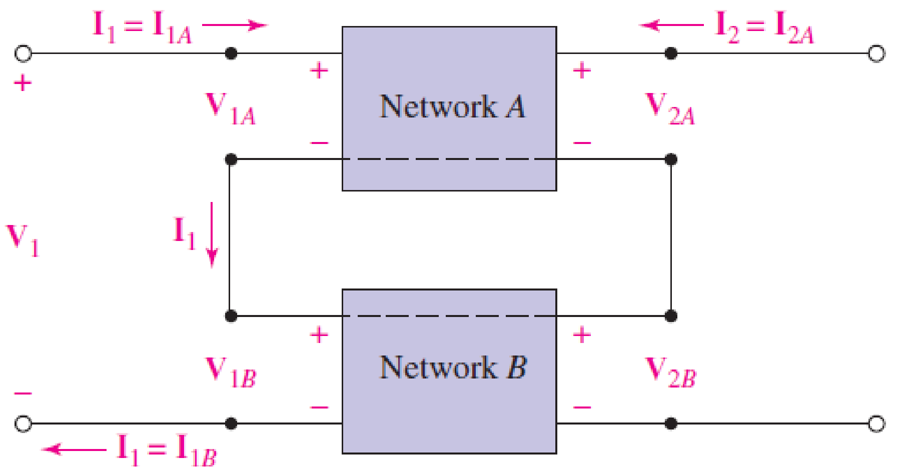
	Y-parameters	Z-parameters	h-parameters	ABCD Parameters
	$\begin{bmatrix} 1/R & -1/R \\ -1/R & 1/R \end{bmatrix}$	$\begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix}$	$\begin{bmatrix} R & 1 \\ -1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}$
	$\begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix}$	$\begin{bmatrix} R & R \\ R & R \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -1 & 1/R \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 1/R & 1 \end{bmatrix}$

Types of Interconnection of two-ports

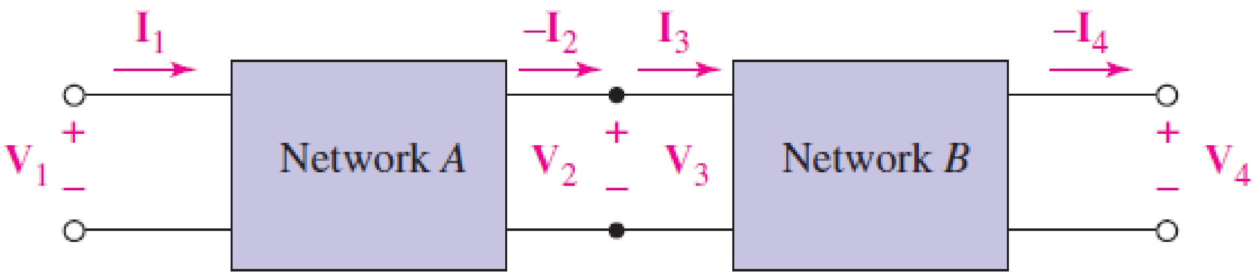
- Parallel connection:



- Series Connection:

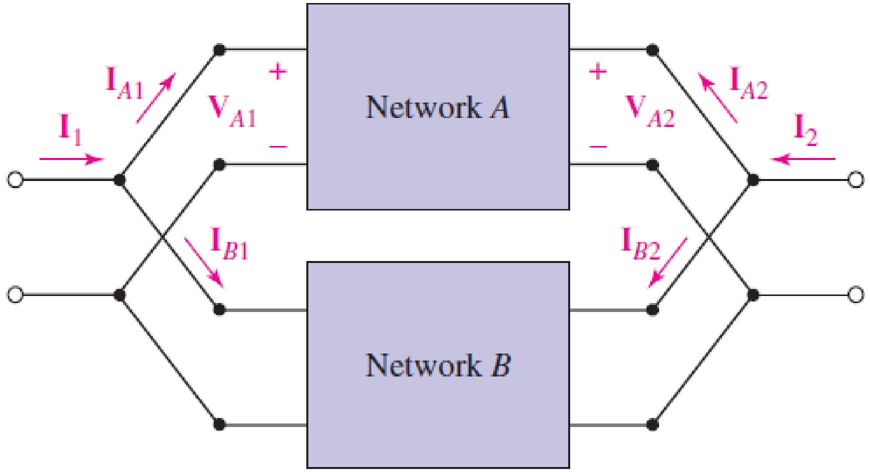


- Cascade Connection:



Analysis of Parallel Connection:

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \qquad I_2 = Y_{21}V_1 + Y_{22}V_2$$



- For parallel connection:

$$I_1 = I_{A1} + I_{B1} \qquad V_1 = V_{A1} = V_{B1}$$

$$I_2 = I_{A2} + I_{B2} \qquad V_2 = V_{A2} = V_{B2}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11A} + Y_{11B} & Y_{12A} + Y_{12B} \\ Y_{21A} + Y_{21B} & Y_{22A} + Y_{22B} \end{bmatrix}$$

- Y parameters of network A:

$$I_{A1} = Y_{11A}V_{A1} + Y_{12\mathrm{A}}V_{A2} \qquad I_{A2} = Y_{21A}V_{A1} + Y_{22A}V_{A2}$$

- Y parameters of Network B:

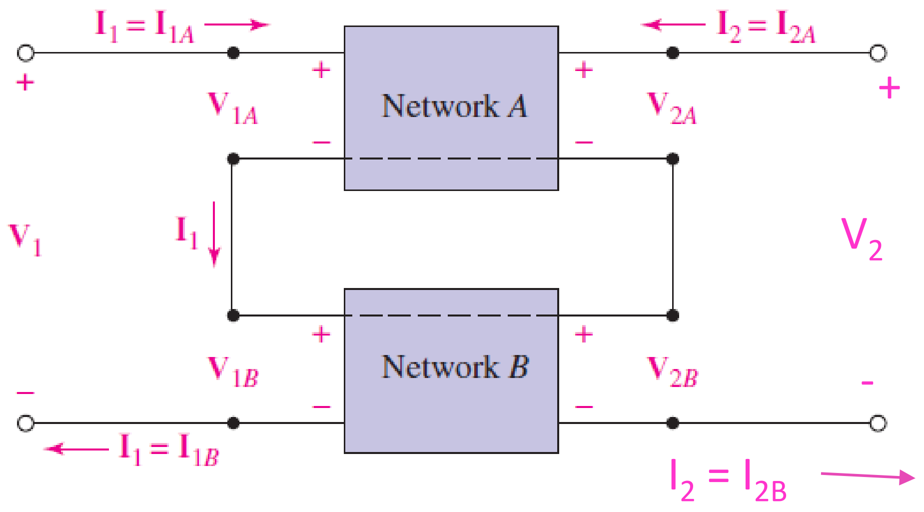
$$I_{B1} = Y_{11B}V_{\mathrm{B}1} + Y_{12\mathrm{B}}V_{\mathrm{B}2} \qquad I_{B2} = Y_{21B}V_{\mathrm{B}1} + Y_{22\mathrm{B}}V_{\mathrm{B}2}$$

- Finally,

$$I_1 = (Y_{11A} + Y_{11B})V_1 + (Y_{12A} + Y_{12B})V_2 \qquad I_2 = (Y_{21A} + Y_{21B})V_1 + (Y_{22A} + Y_{22B})V_2$$

Analysis of Series Connection:

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \qquad V_2 = Z_{21}I_1 + Z_{22}I_2$$



- For series connection:

$$V_1 = V_{1A} + V_{1B} \qquad I_1 = I_{1A} = I_{1B}$$

$$V_2 = V_{2A} + V_{2B} \qquad I_2 = I_{2A} = I_{2B}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11A} + Z_{11B} & Z_{12A} + Z_{12B} \\ Z_{21A} + Z_{21B} & Z_{22A} + Z_{22B} \end{bmatrix}$$

- Z parameters of network A:

$$V_{1A} = Z_{11A}I_{1A} + Z_{12A}I_{2A} \qquad V_{2A} = Z_{21A}I_{1A} + Z_{22A}I_{2A}$$

- Z parameters of Network B:

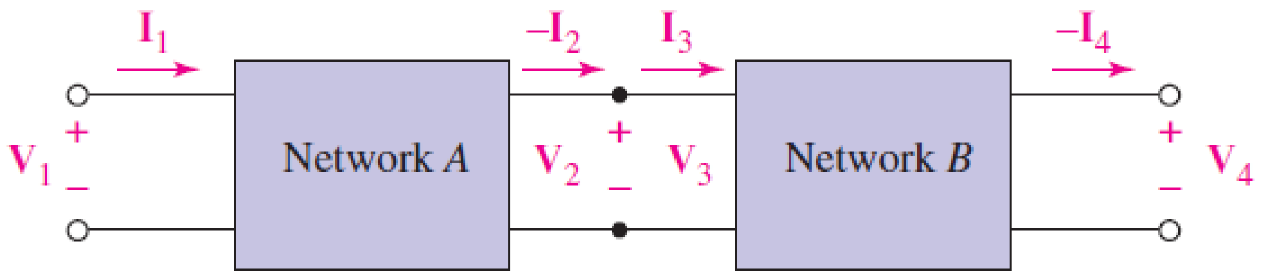
$$V_{1B} = Z_{11B}I_{1B} + Z_{12B}I_{2B} \qquad V_{2B} = Z_{21B}I_{1B} + Z_{22B}I_{2B}$$

- Finally,

$$V_1 = (Z_{11A} + Z_{11B})I_1 + (Z_{12A} + Z_{12B})I_2 \qquad V_2 = (Z_{21A} + Z_{21B})I_1 + (Z_{22A} + Z_{22B})I_2$$

Analysis of Cascade Connection:

$V_1 = AV_2 - BI_2$ $\qquad I_1 = CV_2 - DI_2$



- For a cascade connection:

$V_2 = V_3$ $\qquad -I_2 = I_3$

- ABCD parameters of network A:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_A & B_A \\ C_A & D_A \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

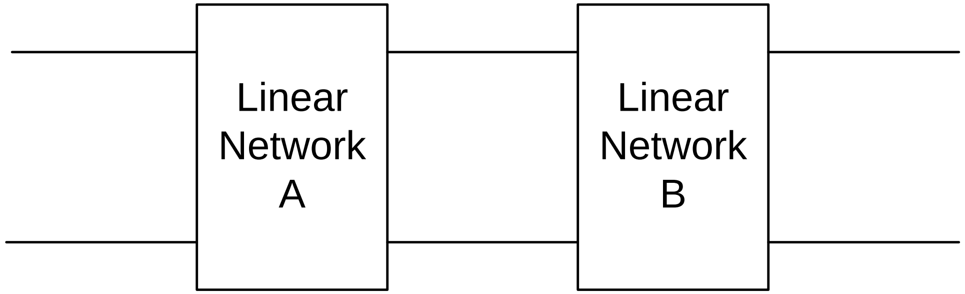
- ABCD parameters of Network B:

$$\begin{bmatrix} V_3 \\ I_3 \end{bmatrix} = \begin{bmatrix} A_B & B_B \\ C_B & D_B \end{bmatrix} \begin{bmatrix} V_4 \\ -I_4 \end{bmatrix}$$

- Finally,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_A & B_A \\ C_A & D_A \end{bmatrix} \begin{bmatrix} A_B & B_B \\ C_B & D_B \end{bmatrix} \begin{bmatrix} V_4 \\ -I_4 \end{bmatrix}$$

Parameter conversions



- Z parameters to ABCD parameters

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad (1)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad (2)$$

$$V_1 = AV_2 - BI_2 \quad (3)$$

$$I_1 = CV_2 - DI_2 \quad (4)$$

Rewriting eq. 2,

$$I_1 = \frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}I_2 \quad (5)$$

Using I_1 in eq. 1,

$$V_1 = \frac{Z_{11}}{Z_{21}}V_2 - \frac{\Delta Z}{Z_{21}}I_2 \quad (6)$$

$$\Delta Z = Z_{11}Z_{22} - Z_{21}Z_{12}$$

By comparing eq. 6 with eq. 3 and eq. 5 with eq. 4,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

• Z parameters to h parameters

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \Rightarrow (1)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \Rightarrow (2)$$

$$V_1 = h_{11}I_1 + h_{12}V_2 \Rightarrow (3)$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \Rightarrow (4)$$

Rewriting eq. 2,

$$I_2 = -\frac{Z_{21}}{Z_{22}}I_1 + \frac{1}{Z_{22}}V_2 \Rightarrow (5)$$

By comparing eq. 6 with eq. 3 and eq. 5 with eq. 4,

Using I_2 in eq. 1,

$$V_1 = \frac{\Delta Z}{Z_{22}}I_1 + \frac{Z_{12}}{Z_{22}}V_2 \Rightarrow (6)$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$$

$$\Delta Z = Z_{11}Z_{22} - Z_{21}Z_{12}$$

- Z parameters to Y parameters

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \Rightarrow 1$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \Rightarrow 2$$

In Matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Multiplication with inverse matrix of Z,

$$[Z]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z]^{-1}[Z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[Z]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Z]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Rightarrow 5$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \Rightarrow 3$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \Rightarrow 4$$

In Matrix form,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Rightarrow 6$$

From 5 and 6

$$[Y] = [Z]^{-1}$$

Finally,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{\Delta Z} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$$\Delta Z = Z_{11}Z_{22} - Z_{21}Z_{12}$$

- For a two port network, the open circuit voltage at the output terminals is 3 V when a 5 V source is connected to its input terminals (3 *A* current is supplied by 5 V source) and the open circuit voltage at the input terminals is 5 V when a 10 V source is connected to its output terminals (6 *A* current is supplied by the 10 V source), determine the transmission parameters of the two-port network.

	y		**z**		**h**		**t**	
y	y_{11}	y_{12}	$\dfrac{z_{22}}{\Delta_z}$	$\dfrac{-z_{12}}{\Delta_z}$	$\dfrac{1}{h_{11}}$	$\dfrac{-h_{12}}{h_{11}}$	$\dfrac{t_{22}}{t_{12}}$	$\dfrac{-\Delta_t}{t_{12}}$
	y_{21}	y_{22}	$\dfrac{-z_{21}}{\Delta_z}$	$\dfrac{z_{11}}{\Delta_z}$	$\dfrac{h_{21}}{h_{11}}$	$\dfrac{\Delta_h}{h_{11}}$	$\dfrac{-1}{t_{12}}$	$\dfrac{t_{11}}{t_{12}}$
z	$\dfrac{y_{22}}{\Delta_y}$	$\dfrac{-y_{12}}{\Delta_y}$	z_{11}	z_{12}	$\dfrac{\Delta_h}{h_{22}}$	$\dfrac{h_{12}}{h_{22}}$	$\dfrac{t_{11}}{t_{21}}$	$\dfrac{\Delta_t}{t_{21}}$
	$\dfrac{-y_{21}}{\Delta_y}$	$\dfrac{y_{11}}{\Delta_y}$	z_{21}	z_{22}	$\dfrac{-h_{21}}{h_{22}}$	$\dfrac{1}{h_{22}}$	$\dfrac{1}{t_{21}}$	$\dfrac{t_{22}}{t_{21}}$
h	$\dfrac{1}{y_{11}}$	$\dfrac{-y_{12}}{y_{11}}$	$\dfrac{\Delta_z}{z_{22}}$	$\dfrac{z_{12}}{z_{22}}$	h_{11}	h_{12}	$\dfrac{t_{12}}{t_{22}}$	$\dfrac{\Delta_t}{t_{22}}$
	$\dfrac{y_{21}}{y_{11}}$	$\dfrac{\Delta_y}{y_{11}}$	$\dfrac{-z_{21}}{z_{22}}$	$\dfrac{1}{z_{22}}$	h_{21}	h_{22}	$\dfrac{-1}{t_{22}}$	$\dfrac{t_{21}}{t_{22}}$
t	$\dfrac{-y_{22}}{y_{21}}$	$\dfrac{-1}{y_{21}}$	$\dfrac{z_{11}}{z_{21}}$	$\dfrac{\Delta_z}{z_{21}}$	$\dfrac{-\Delta_h}{h_{21}}$	$\dfrac{-h_{11}}{h_{21}}$	t_{11}	t_{12}
	$\dfrac{-\Delta_y}{y_{21}}$	$\dfrac{-y_{11}}{y_{21}}$	$\dfrac{1}{z_{21}}$	$\dfrac{z_{22}}{z_{21}}$	$\dfrac{-h_{22}}{h_{21}}$	$\dfrac{-1}{h_{21}}$	t_{21}	t_{22}

For all parameter sets: $\Delta_\mathbf{p} = p_{11}p_{22} - p_{12}p_{21}$.