BASIC ELECTRONIC CIRCUITS

Network Theorems

Contents

Linearity and Superposition

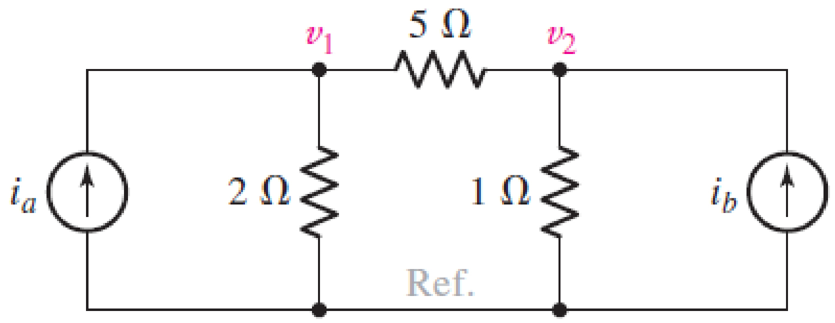
- The most important consequence of linearity is "superposition".
- Statement: the response (desired current or voltage) in a *linear circuit* having more than one independent source can be obtained by adding the responses caused by the separate independent sources *acting alone*.

Linear elements and linear circuits

- Linear element is defined as a passive element that has a "linear voltage-current relationship".
- A linear dependent source is defined as a dependent current or voltage source whose output current or voltage is proportional only to the first power of a specified current or voltage variable in the circuit (or to the sum of such quantities).
- A linear circuit is defined as a circuit composed entirely of independent sources, linear dependent sources, and linear elements, Hence "the response is proportional to the source"

The superposition principle

- Forcing functions are i_a and i_b and v1 and v2 are the response functions.



$$0.7v_1 - 0.2v_2 = i_a$$
$$-0.2v_1 + 1.2v_2 = i_b$$

- Step 1: perform experiment 'x'

$$0.7v_{1x} - 0.2v_{2x} = i_{ax}$$
$$-0.2v_{1x} + 1.2v_{2x} = i_{bx}$$

- Step 2: perform experiment 'y'

$$0.7v_{1y} - 0.2v_{2y} = i_{ay}$$
$$-0.2v_{1y} + 1.2v_{2y} = i_{by}$$

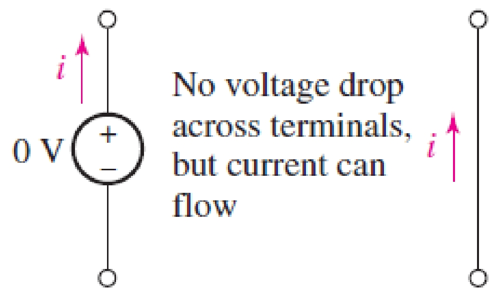
- Lets superimpose the last two set of equations:

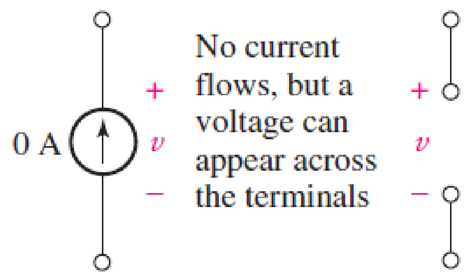
$$(0.7v_{1x} + 0.7v_{1y}) - (0.2v_{2x} + 0.2v_{2y}) = i_{ax} + i_{ay}$$
$$0.7v_1 \quad - \quad 0.2v_2 \quad = \quad i_a$$

$$-(0.2v_{1x} + 0.2v_{1y}) + (1.2v_{2x} + 1.2v_{2y}) = i_{bx} + i_{by}$$
$$-0.2v_1 \quad + \quad 1.2v_2 \quad = \quad i_b$$

- If i_{ax} and i_{ay} are selected as such that their sum is i_a and i_{bx} and i_{by} are selected as such that their sum is i_b, then the desired responses v_1 and v_2 may be found by adding v_{1x} to v_{1y} and v_{2x} to v_{2y} , respectively.

- Each independent source (and the response it generates) one at a time with the other independent sources "turned off" or "zeroed out."

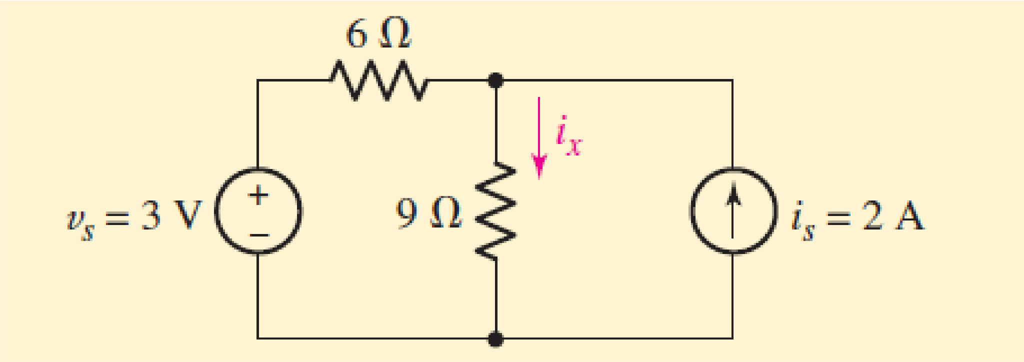




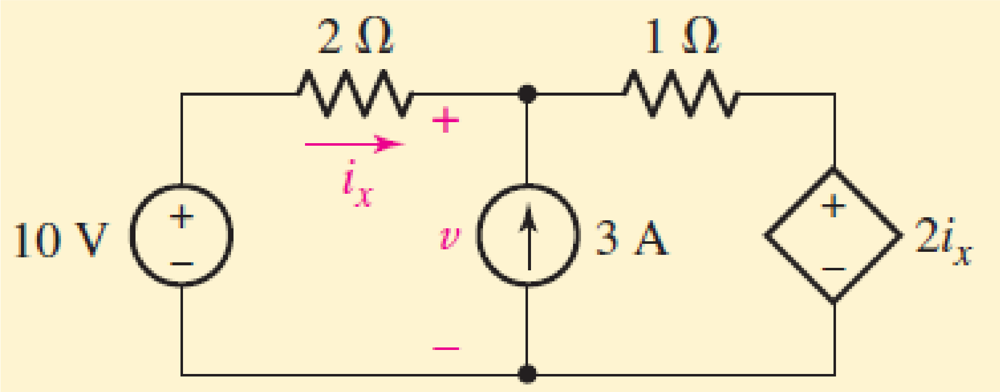
- Thus the superposition theorem states that

"In any linear resistive network, the voltage across or the current through any resistor or source may be calculated by adding algebraically all the individual voltages or currents caused by the separate independent sources acting alone, with all other independent voltage sources replaced by short circuits and all other independent current sources replaced by open circuits."

- Ex 1: Determine i_x,
- Ans: 1 A.



- Ex 2: Determine i_x,
- Ans: 1.4 A.

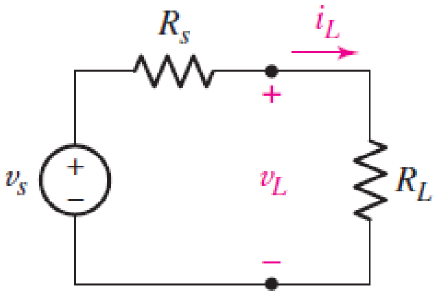


Points to be noted:

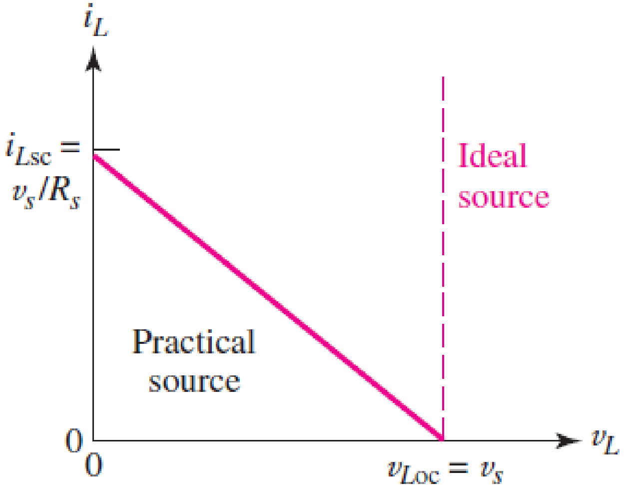
- Superposition does not generally reduce our workload, but useful in identifying the significance of various parts of a more complex circuit.
- It is applicable only to linear responses, and thus the most common nonlinear response—power—is not subject to superposition.

Source Transformation: Ideal vs Practical sources

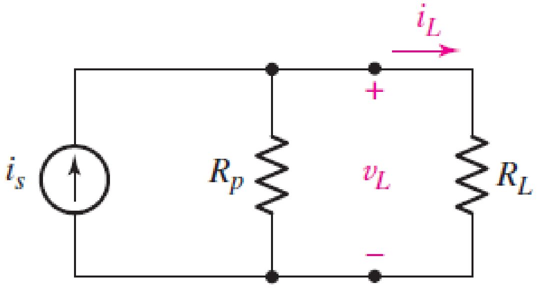
Practical voltage source



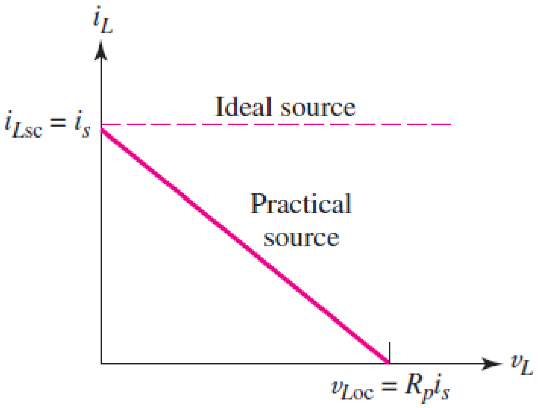
$$v_L = v_s - R_s i_L$$



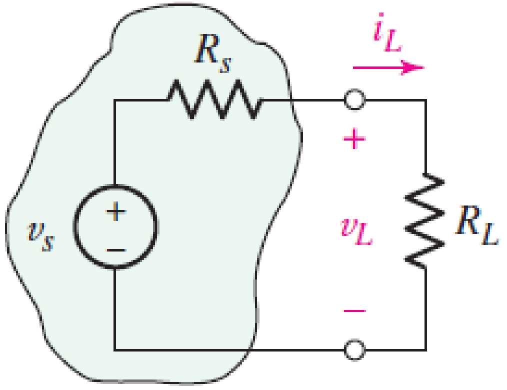
Practical current source



$$i_L = i_s - \frac{v_L}{R_p}$$



Equivalent practical sources

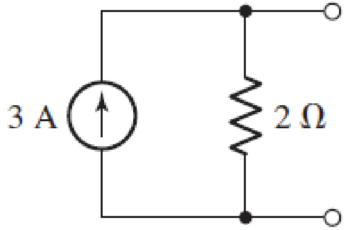


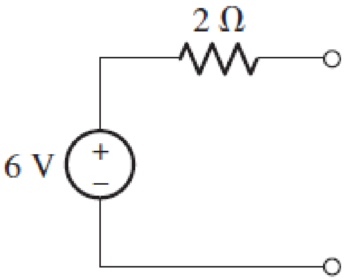
(a)

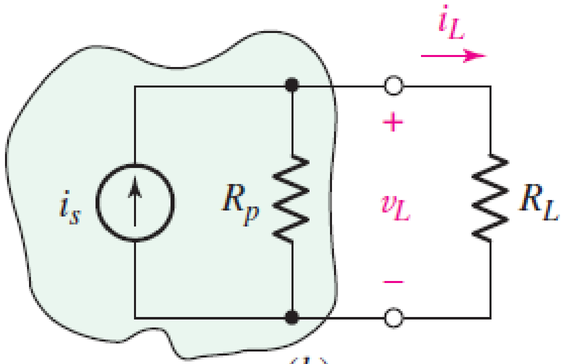
- a practical voltage source can be electrically equivalent to a practical current source

$$v_L = v_s \frac{R_L}{R_s + R_L}$$

- replace one practical source with the other
- if, $R_s = R_p$ and $v_s = R_p i_s = R_s i_s$
- Ex:





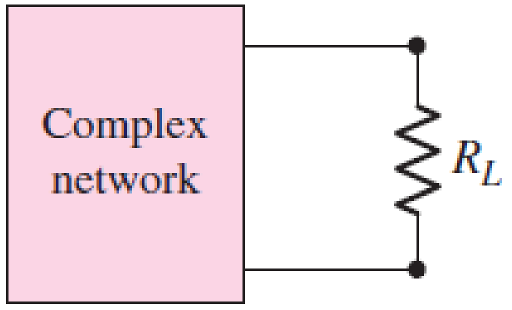


(b)

$$v_L = \left(i_s \frac{R_p}{R_p + R_L} \right) \cdot R_L$$

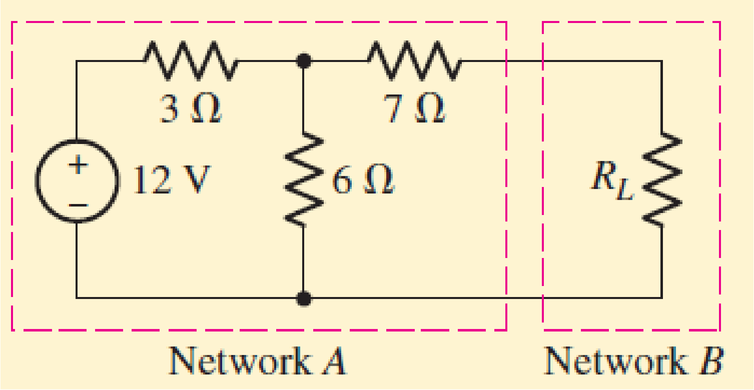
Thevenin and Norton Equivalent Circuits

- Simplification of the complex "linear circuits" are possible.
- Large part of a circuit can be replaced with a very simple equivalent.
- Helps in faster calculations of the voltage, current, and power.
- Helps in choosing the best value for the load.

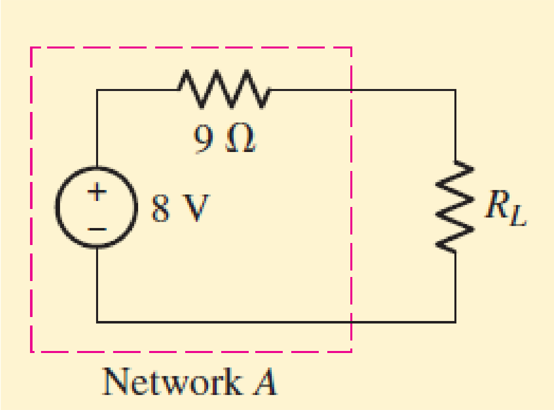


Thevenin's Theorem

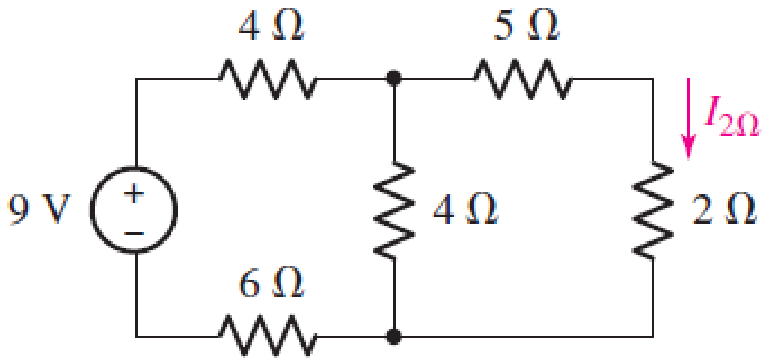
- Given any linear circuit, rearrange it in the form of two networks, A and B, connect by two wires.
- Disconnect network B,. Define a voltage v_{oc} as the voltage now appearing across the terminals of network A.
- Turn off or "zero out" every independent source in network A to form an inactive network, leave dependent sources unchanged.
- Connect an independent voltage source with value v_{oc} in series with the inactive network.
- Connect network B to the terminals of the new network A.



By using the source transformation



- Ex: Using the Thevenin's theorem calculate $I_{2\Omega}$.



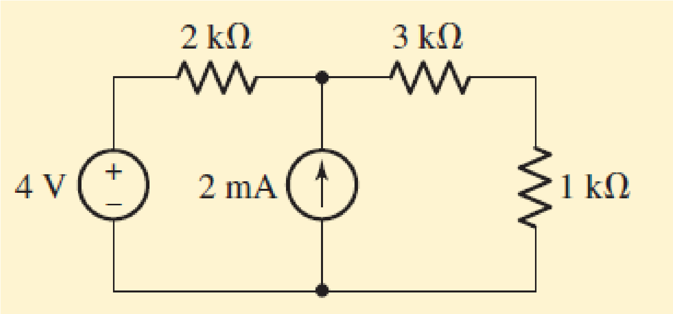
- V_{th} = 2.57 V, R_{th} = 7.857 Ω, I = 0.26 A.

Key Points

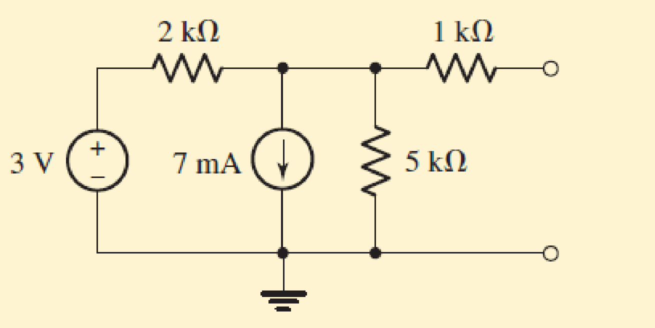
- Dependent sources (and the control element) must be local to a given network.
- No restriction on the complexity of A and B.
- The dead network (no active sources) A, is represented by a single equivalent resistance 'R_{th}'.

Norton's Equivalent Circuit

- Given any linear circuit, rearrange it in the form of two networks, A and B, connected by two wires.
- Disconnect network B and short the terminals of A.
- Turn off (zero out) every independent source in network A to form an inactive network.
- Connect an independent current source with value i_{sc} in parallel with the inactive network.
- Connect network B to the terminals of the new network A.



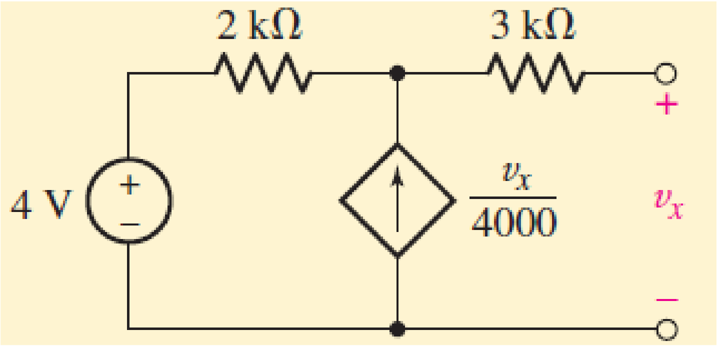
V_{oc} = 8 v, R_{th} = 5 KΩ, and I_{sc} = 1.6 mA.

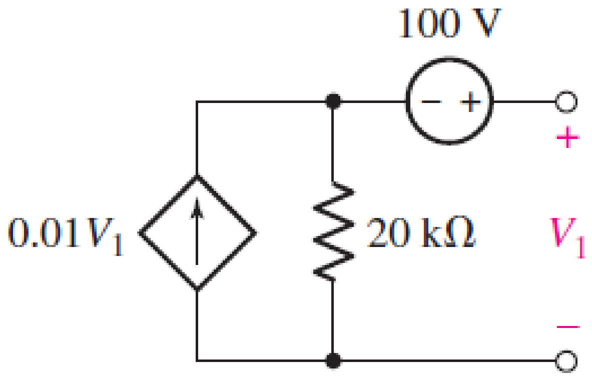


V_{oc} = -7.85 v, R_{th} = 2.43 KΩ, and I_{sc} = -3.23 mA.

When dependent sources are present

- Determine the Thevenin's equivalent of Circuit
- Ans: 8 V, 10 kΩ



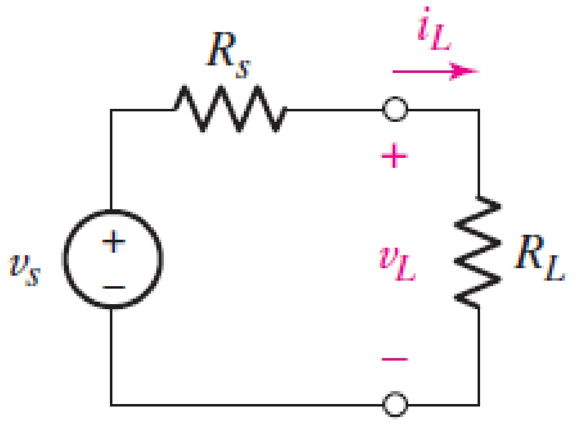


- -502 mV, -100.5 Ω

Maximum Power Transfer Theorem

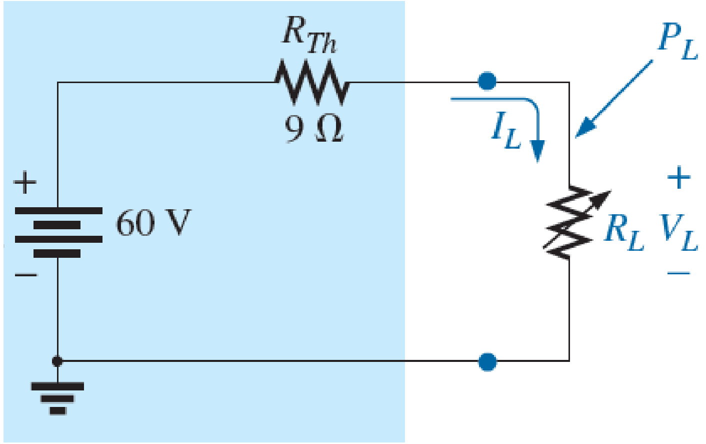
- A load will receive maximum power from a network when its resistance is exactly equal to the Thevenin resistance of the network applied to the load. That is, $R_s = R_L$
- Power delivered to load

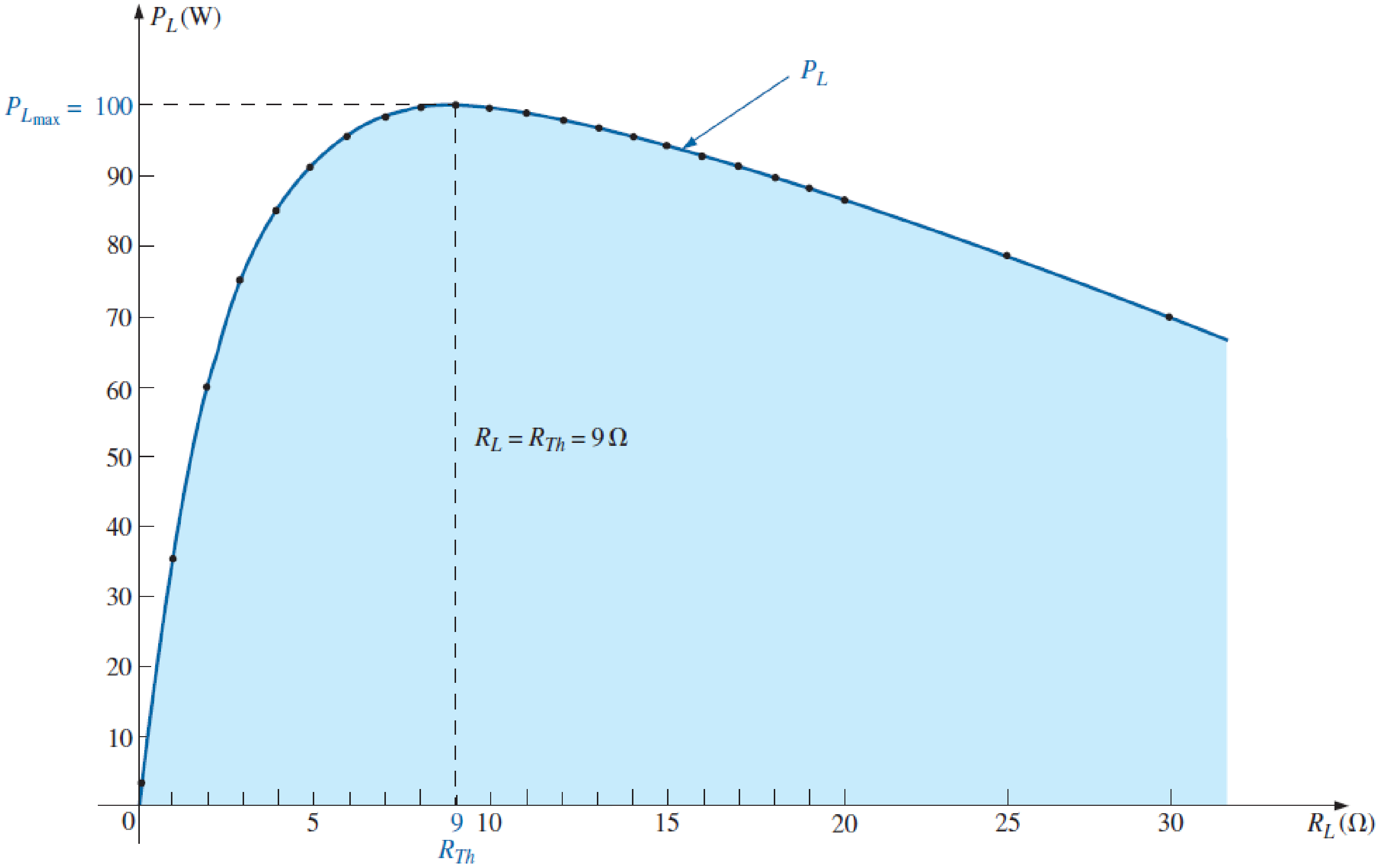
$$p_L = i_L^2 R_L = \frac{v_s^2 R_L}{(R_s + R_L)^2}$$



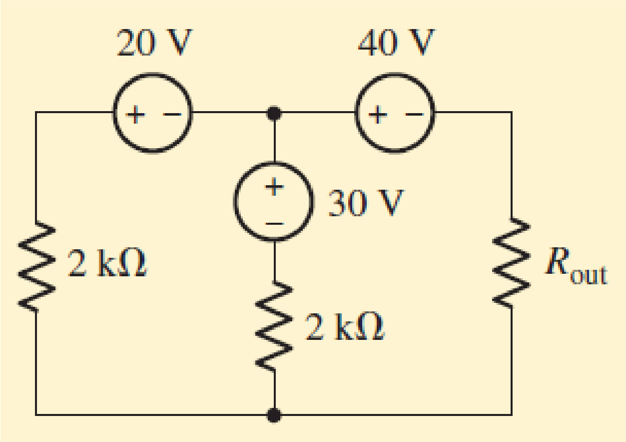
$$p_{\max}\Big|_{\text{delivered to load}} = \frac{v_s^2}{4R_s} = \frac{v_{TH}^2}{4R_{TH}}$$

R_L vs Power delivered to load



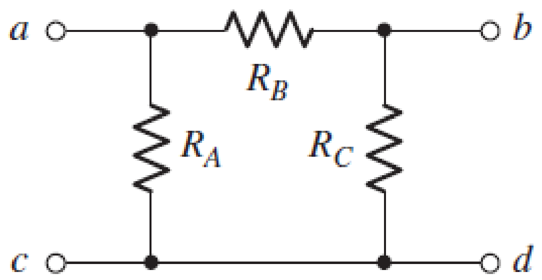


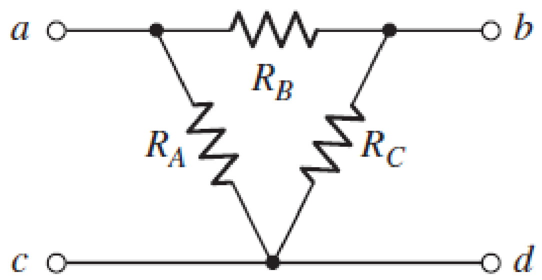
- Ex;
- If R_{out} = 3k, find the power delivered to it?
- Ans: 230 mW
- What is the maximum power can be delivered to any R_{out}?
- Ans: 306 mW
- What different values of R_{out} will have exactly 20 mW deliver to them?
- 59.2 KΩ and 16.88 Ω

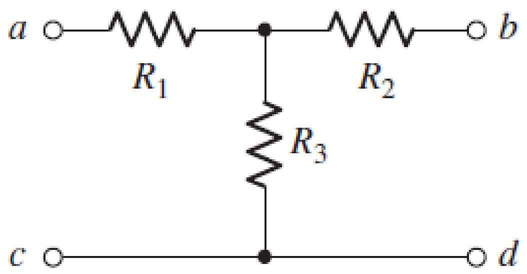


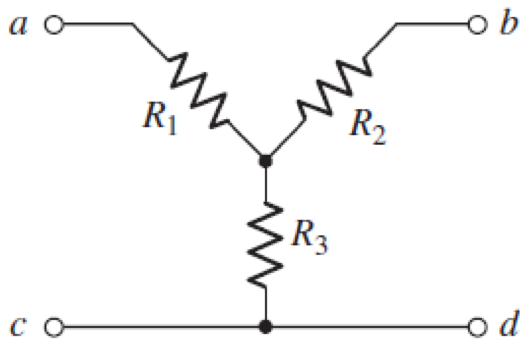
Delta-Wye Conversion

- By identifying the parallel and series connections in the given network will lead to reduction in complexity
- Often source transformation can also helps in reduction in complexity
- There is another technique Delta – Wye conversion.

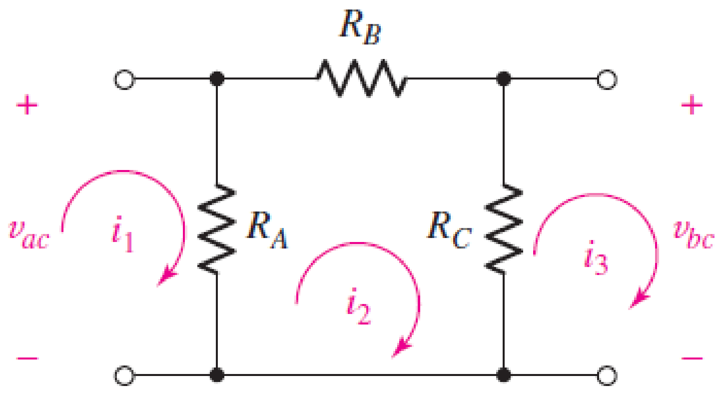






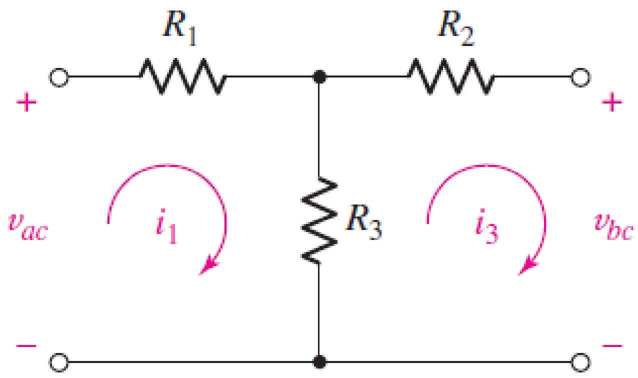


- If the two networks are equal then the terminal voltages and currents must be equal.
- By performing the mesh analysis

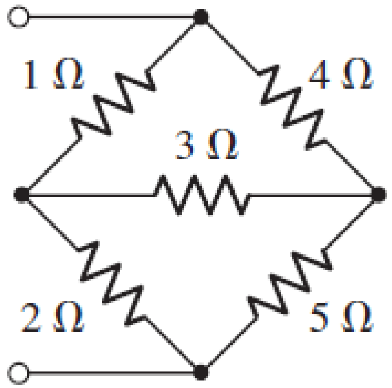


$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$
$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$
$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_1 = \frac{R_AR_B}{R_A + R_B + R_C}$$
$$R_2 = \frac{R_BR_C}{R_A + R_B + R_C}$$
$$R_3 = \frac{R_CR_A}{R_A + R_B + R_C}$$

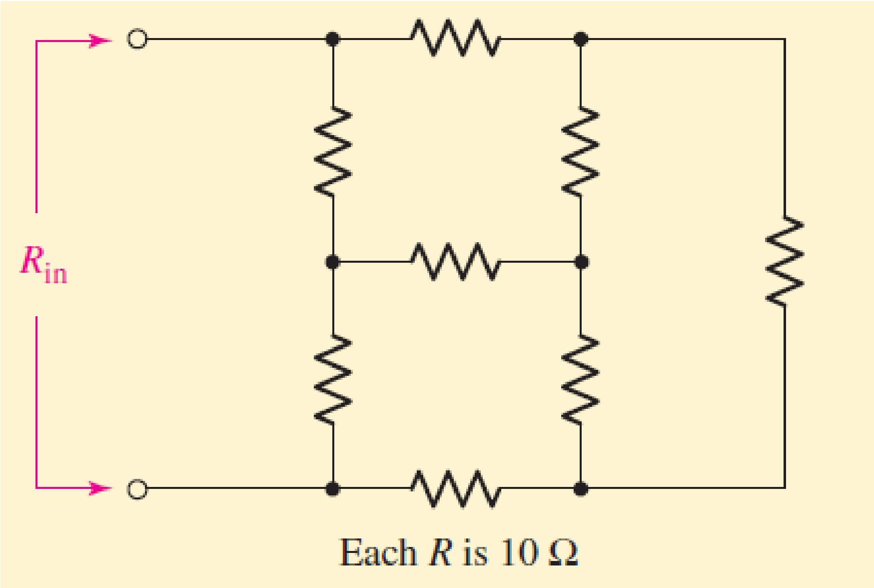


- Problems:



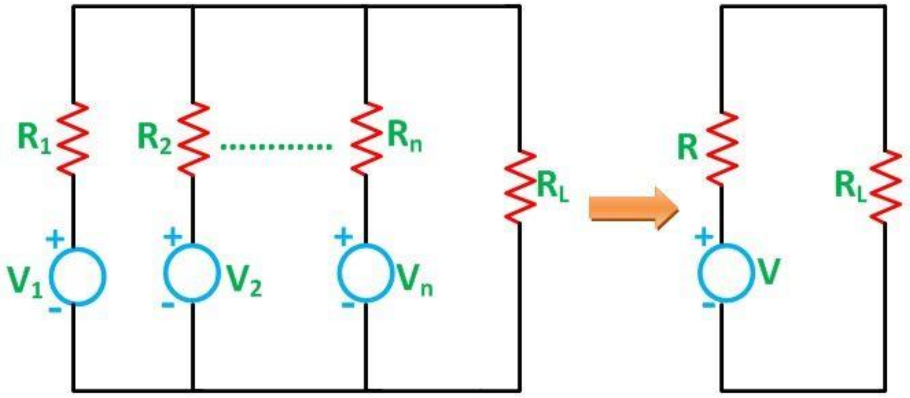
- Ans: 2.24 Ω

- Ans: 11.43 Ω



Millman's Theorem

- States that, in any network, if the voltage sources V_1, V_2, …V_n in series with internal resistances R_1, R_2, …, R_n, respectively, are in parallel, then the sources may be replaced by a single voltage source V in series with R .

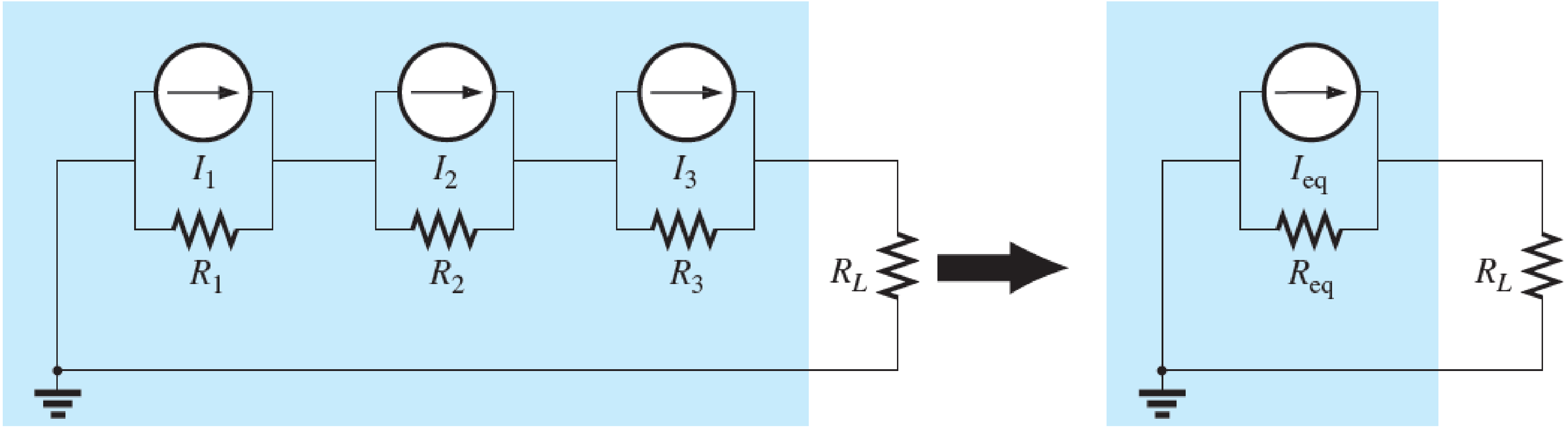


$$V = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \; \dots . + \frac{V_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \; \dots + \frac{1}{R_n}}$$

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \; \dots + \frac{1}{R_n}}$$

Millman's Theorem

- Similarly, for n current sources having internal conductances which can be replaced by a single current source I in parallel with an equivalent conductances.

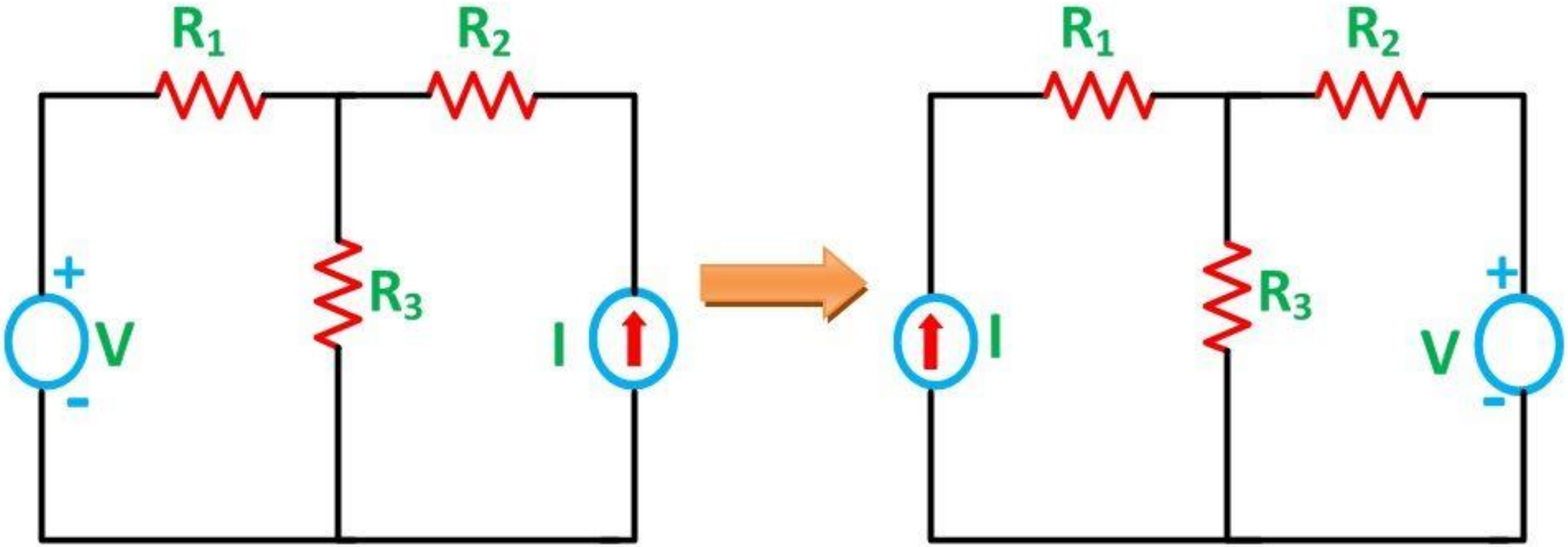


$$I = \frac{I_1R_1 + I_2R_2 + I_3R_3 + \cdots + InRn}{R_1 + R_2 + R_3 + \cdots + Rn}$$

$$G = \frac{1}{R_1 + R_2 + R_3 + \cdots + Rn}$$

Reciprocity

- In any linear bilateral network, a single voltage source in branch ‘a’ produces a current I in branch ‘b’, then if voltage source is removed and inserted in branch ‘b’ will produce a current I in branch ‘a’. The ratio of response to excitation is same for the two conditions.



- Example: