M4 Diff

Numerical implementation of an averager

$$x(t) \longrightarrow \boxed{\frac{1}{T}\int (\cdot)} \longrightarrow y(t)$$

$$y(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)\, d\tau$$

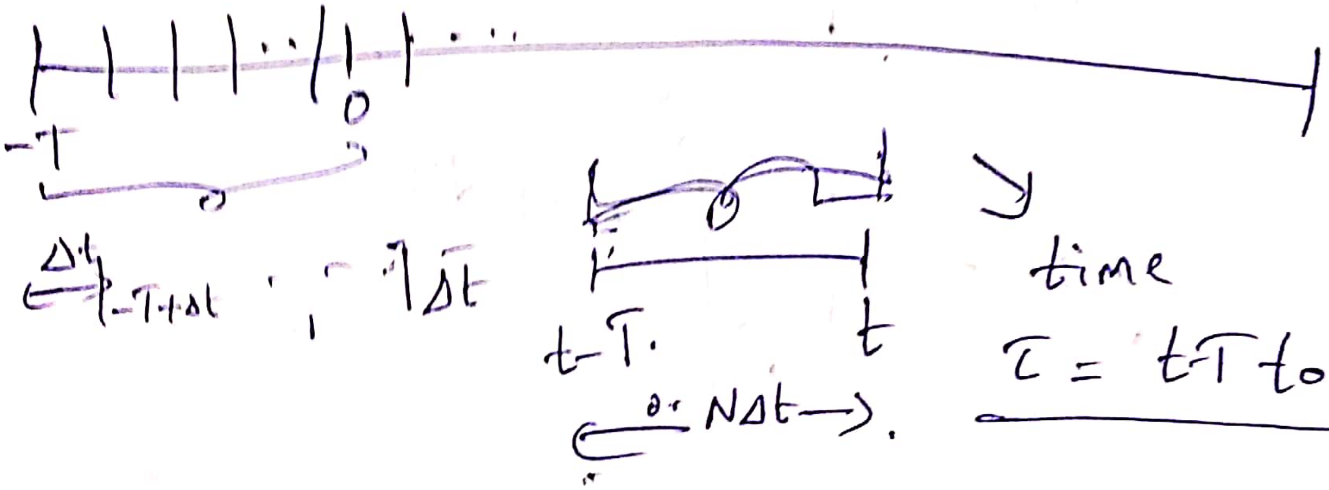
~~Sim~~ Num. Implement

→ Algorithm

- choose: Average time T.
- (find) assign sampling rate.
- ~~at~~ Create time vector 't'
- Note: locations of $t = 0$, $t = T$, $t =$ final. Value (discrete) $N = \frac{T}{\Delta t}$
- ~~assign multiply scaling parameter.~~
- def: signal. or (Note observations)
- For loop
  - starts here

MEX

- for $t = 0 : t_{end}$.
    - determine corresponding time vector



time

$\tau = t-T$ to t

- extract $x(\tau)$ for $t-T$ to t

- $y_{num}(\cdot) = \frac{1}{T}\,\mathrm{trapz}(\tau, x(\tau))$.

end.

- plot - results.

DSAA

Stability & Causality Checks.

① $h[n] = \left(-\frac{1}{2}\right)^n u[n] + (1.01)^n u[n-1]$.

(a) Causality ✓.

$h[-1] = 0 + (1.01)^{-1} u(-2) = 0$.

$\therefore h[-2] = 0 \ldots \quad h[n] = 0 \quad n < 0$

(b) Stability.

$$\sum_n |h[n]| = \sum_n \left| \left(-\frac{1}{2}\right)^n u[n] + (1.01)^n u[n-1] \right|$$

$$\leq \sum_n \left| \left(-\frac{1}{2}\right)^n u[n] \right| + \sum_n \left| (1.01)^n u(n-1) \right|$$

$$= \underbrace{\sum_{n=0}^{\infty} \left| (-1)^n \left(\frac{1}{2}\right)^n \right|}_{\text{converges}} + \underbrace{\sum_{n=1}^{\infty} (1.01)^n}_{\text{diverges}}.$$

$\therefore \sum_n |h(n)| \xrightarrow[\text{be}]{\text{may}} \infty$. $\therefore$ Not stable.

(2) $h[n] = (\frac{-1}{2})^n u[n] + (1.01)^n u[1-n]$.

Causal

$h[-1] = 0 + (1.01)^{-1} u[2] \neq 0$

$\therefore$ Not Causal

stability

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{-1} |(1.01)^n u[1-n]|$$

$$+ \sum_{n=0}^{1} |(\frac{-1}{2})^n u[n] + (1.01)^n u[1-n]|$$

$$+ \sum_{n=2}^{\infty} |(\frac{-1}{2})^n|$$

$$= \sum_{m=0}^{\infty} (1.01)^{-m} - \cancel{1} + 2 + |\frac{-1}{2} + 1.01|$$

$$+ \sum_{n=0}^{\infty} (\frac{1}{2})^n - \cancel{1} - \frac{1}{2}.$$

$$= \frac{1}{1 - \frac{1}{1.01}} + 0.01 + \frac{1}{1 - \frac{1}{2}} = \underline{\underline{103.02}} < \infty$$

$\therefore$ Stable

2.18 Consider a causal LTI system.

$$x[n] - \boxed{h(n)} - y[n].$$

With system equation

$$y[n] = \frac{1}{4} y[n-1] + x[n]. \quad — ①$$

find $y[n]$ for $x[n] = \delta[n-1]$.

Sol:

Step ① find $y(n)$ first. [explicit equation closed form.]

0 → Causal sys.

$$y[0] = 0 + x(0)$$

$$y[1] = \frac{1}{4} x[0] + x[1]$$

$$y[2] = \frac{1}{4}\left(\frac{1}{4} x[0] + x(1)\right) + x[2]$$

$$y[3] = \frac{1}{4}\left[\frac{x[0]}{16} + \frac{x[1]}{4} + x[2]\right] + x[3].$$

$\vdots$

$$y[n] = \frac{1}{4} \cdot \left[\sum_{k=0}^{n-1} x[k] \cdot \left(\frac{1}{4}\right)^{n-1-k}\right] + x[n].$$

for. $n \geq 1$ — ②

$$= \sum_{k=0}^{n} x(k) \left(\frac{1}{4}\right)^{n-k}$$

Put $x[n] = \delta[n]$

$$\therefore h[n] = \frac{1}{4}\sum_{k=0}^{n-1}\left(\frac{1}{4}\right)^{n-1-k}\delta[k] + \delta[n]. \quad -(3)$$

For $x[n] = \delta[n-1]$.

$$y_1[n] = \frac{1}{4}\sum_{k=0}^{n-1}\left(\frac{1}{4}\right)^{n-1-k}\delta[k-1] + \delta[n-1]. \quad -(4)$$

$n \geq 1$

from eq (2) $n=0$, $y_1[0] = \delta[-1] = 0$.

Extra From eq (3). & convolution.

$$y[n] = x[n] * h[n]$$

$$= \delta[n-1] * \frac{1}{4}\sum_{k=0}^{n-1}\left(\frac{1}{4}\right)^{n-1-k}\delta(k) + \delta[n-1] * \delta[n]$$

extra *

$$= \frac{1}{4}\sum_{k=0}^{n-1}\left(\frac{1}{4}\right)^{n-1-k}\delta[k-1] + \delta[n-1].$$

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ex. 2.11

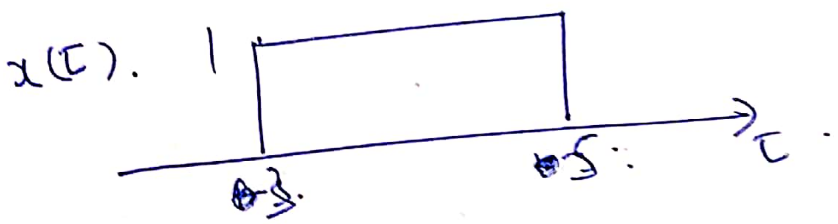
(a) $x(t) = u(t-3) - u(t-5).$

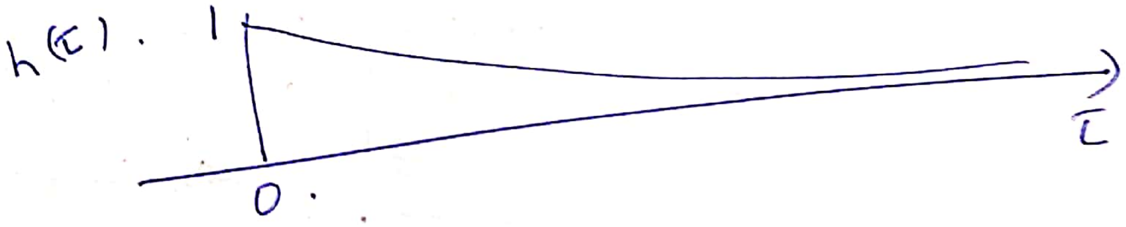
$h(t) = e^{-3t}\, u(t).$

$$y(t) = x(t) * h(t)$$

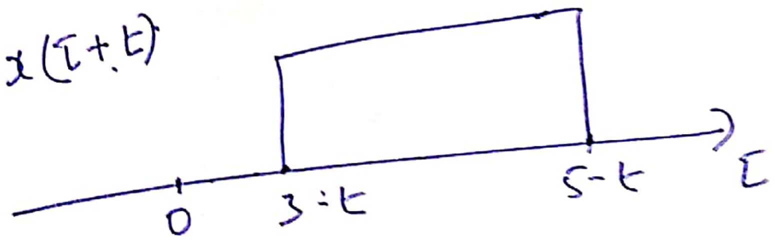
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

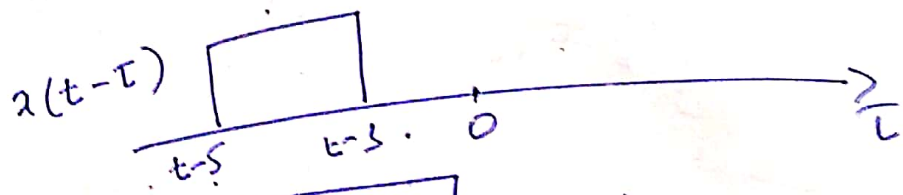
$$= \int_{-\infty}^{\infty} \left(u(t-3) - u(t-5)\right) \cdot e^{-3(t-\tau)}\, u(t-\tau)\, d\tau.$$





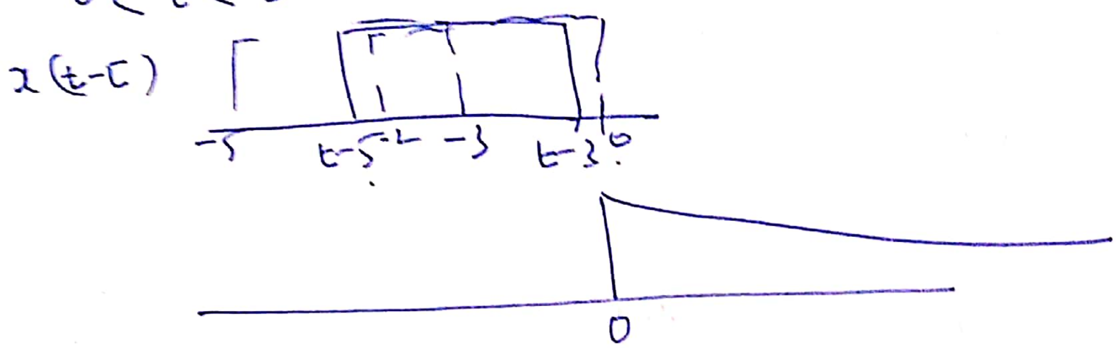
t<0.





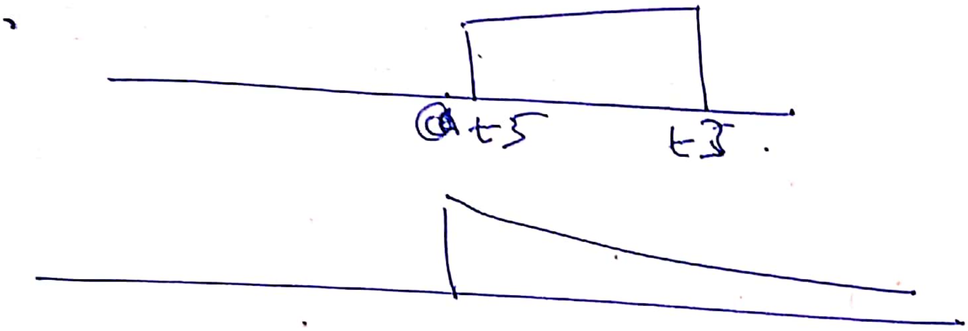
$\therefore \boxed{y(t) = 0.}$ $\quad t<0.$

$0 < t < 3.$



$$\boxed{y(t) = 0} \quad \text{for } 0 < t < 3.$$

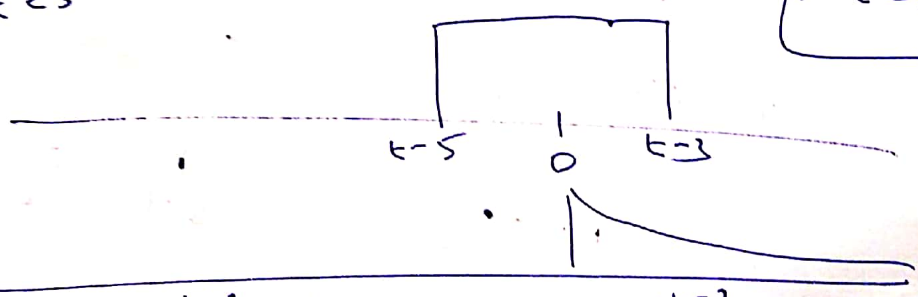
~~3 < t < 5~~. $t > 5$.



$$y(t) = \int_{t-5}^{t-3} e^{-3\tau}\,d\tau = \boxed{\frac{e^{9-3t}}{3}(e^{6}-1)} \quad \boxed{t > 5}$$

~~t < 5~~.

$3 < t < 5$



$$y(t) = \int_{0}^{t-3} e^{-3\tau}\,d\tau = \left.\frac{e^{-3t}}{-3}\right|_{0}^{t-3} = \frac{e^{-3(t-3)}-1}{-3}.$$

$$\boxed{y(t) = \frac{1}{3}\left[1 - e^{9-3t}\right] \quad 3 < t < 5.}$$

2.11

(b). $g(t) = \left(\frac{d}{dt}x(t)\right) * h(t)$

$\therefore \frac{d}{dt}x(t) = \delta(t-3) - \delta(t-5)$

$\therefore g(t) = \left(\delta(t-3) - \delta(t-5)\right) * e^{-3t}u(t).$

$g(t) = \delta(t-3) * e^{-3t}u(t) - \delta(t-5)\, e^{-3t}u(t)$

$g(t) = e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5).$

(c). Find relation between $g(t)$ & $y(t)$.

$\therefore$ recall $y(t) = \begin{cases} 0 & t \le 3. \\ \frac{1-e^{-3(t-3)}}{3} & 3 < t \le 5 \\ \frac{(1-e^{-6})e^{-3(t-5)}}{3} & 5 < t \le \infty \end{cases}$

$\frac{dy(t)}{dt} = \begin{cases} 0 & t < 3. \\ -\frac{(-3)}{3}.e^{-3(t-3)} & 3 < t \le 5. \\ \frac{1-e^{-6}}{3}(-3)e^{-3(t-5)} & 5 < t. \end{cases}$

$$\therefore \frac{d}{dt} y(t) = e^{-3(t-3)}\left[u(t-3) - u(t-5)\right]$$

$$- (1-e^{-6})\, e^{-3(t-5)} \cdot u(t-5)$$

$$= e^{-3(t-3)} u(t-3)$$

$$- \left(e^{-3(t-3)} + (1-e^{-6}) e^{-3(t-5)}\right) u(t-5)$$

$$= e^{-3(t-3)} u(t-3)$$

$$- e^{-3(t-5)} \left[e^{-6} + 1 - e^{-6}\right] u(t-5)$$

$$\boxed{\therefore \frac{dy(t)}{dt} = g(t).}$$

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① Step Response $s(n)$

a) $h(n) = \left(-\frac{1}{2}\right)^n u(n)$.

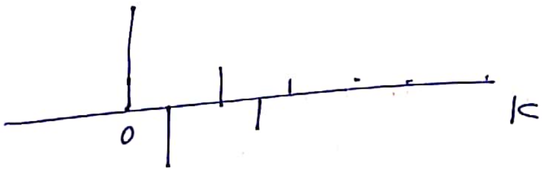
def: $s(n) = h(n) * u(n)$.

$$\therefore s(n) = \sum_{k} h(k)\, u(n-k).$$

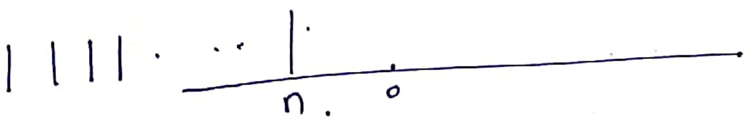
For $n \geq 0$

$$s(n) = \sum_{k=0}^{n} \left(-\frac{1}{2}\right)^k. \qquad \cancel{n \geq 0}.$$

$h(k)$



$n < 0$.



$\therefore$ No overlap.

$\therefore s(n) = 0 \quad \forall\, n<0$.

$$\therefore s(n) = \frac{1-\left(-\frac{1}{2}\right)^{n+1}}{\frac{3}{2}} = \frac{2}{3}\left(1-\left(-\frac{1}{2}\right)^{n+1}\right).$$

for $n \geq 0$.

$$\therefore \boxed{s(n) = \frac{2}{3}\left(1-\left(-\frac{1}{2}\right)^{n+1}\right) u(n)}$$

(b). $h(n) = \delta(n) - \delta[n-2]$.

$$s(n) = h(n) * u(n)$$

$$= \delta(n) * u(n) - \delta[n-2] * u[n]$$

using sifting property

$$\boxed{s[n] = u[n] - u[n-2]}.$$

(c) $h[n] = n.u[n]$

$S[n] = h[n] * u[n]$.

$n < 0$ $\quad s[n] = 0$ $\quad$ due to ~~no~~ Overlap.

for $n \geq 0$.

$S[n] = n u[n] * u[n]$.

$$= \sum_{k=-\infty}^{n} k.u[k].$$

$$= \sum_{k=1}^{n} k. = \frac{n(n+1)}{2}.$$

$$\therefore \boxed{S[n] = \frac{n(n+1)}{2}.u[n].}$$

(d) $h(t) = e^{-|t|}$.

$$S(t) = h(t) * u(t). = \int_{-\infty}^{t} h(\tau)d\tau.$$

$\therefore$ For $t < 0$ $\quad s(t) = \int_{-\infty}^{t} e^{+\tau} d\tau = e^{t}$.

$t > 0$.
$$s(t) = \int_{-\infty}^{0} e^{\tau} d\tau + \int_{0}^{t} e^{-\tau} d\tau.$$

$$= 1 + 1 - e^{-t}. = 2 - e^{-t}$$

$\therefore$ $s(t) = e^{t}$ $\quad t < 0$

$\qquad 2 - e^{-t}$ $\quad t \geq 0$.

DSPA

(i) $T(x[n]) = x[n-n_0]$

O/p is a delayed version of input for $n_0 \geq 0$

Stable - yes.

Causal - yes for $n_0 \geq 0$

no for $n_0 < 0$.

Linear - yes.

Sn- ~~$a x_1[n-n_0] + b x[n-n_0]$ a.~~

$a x_1[n] + b x_2(n) \longrightarrow a x_1[n-n_0] + b x_2(n-n_0)$

$= a y_1[n] + b y_2[n]$.

Time-invariant - yes.

$T(x(n-k)) = x[n-k-n_0]$

$y[n-k) = x[n-k-n_0]$.

Memoryless - no

except for $n_0 = 0$ ✓

(3)

Ex

(iii) $T(x[n]) = a\,x[n] + b$

Stable - yes.

Causal - no

for $x[n] = 0$ for say $n<0$

$y[n] = b$

Linear - no

$y_1(n) = a\,x_1(n) + b$

$y_2(n) = a\,x_2(n) + b$

let $x(n) = a_1 x_1(n) + a_2 x_2(n)$

$$y(n) = a(a_1 x_1(n) + a_2 x_2(n)) + b$$

$$= a_1 (a\,x_1(n)) + a_2\, a\, x_2(n) + b.$$

$$\neq a_1 y_1(n) + a_2 y_2(n).$$

Time-invariant - Yes.

$y[n-k] = a\,x[n-k] + b$

$y_1(n) = a \cdot x[n-k] + b.$

Here $y_1(n) = y[n-k]$

Memoryless - yes.

a

(iii) $T(x(n)) = x[-n]$

Stable - Yes.

Causal - NO

- $y(-2) = x[2]$ depends on future value

Linear - Yes

$y_1[n] = x_1[-n]$, $y_2[n] = x_2[-n]$.

$x[n] = a\,x_1[n] + b\,x_2[n]$

$y[n] = x[-n] = a\,x_1[-n] + b\,x_2[-n]$

$= a\,y_1[n] + b\,y_2[n]$

Time invariant - ~~Yes~~. NO

$y[n-k] = \cancel{x[-(n-k)]}\ x[-n-k]$.

$y_1[n] = x[-(n-k)]$. → Same

Memory less. - NO.

$y[2] = x[-2]$ depends on past

(3)

Please See Next Page.

iv) $T(x[n]) = x[n] + 3u[n+1]$.

Stable - yes.

Causal - yes.

Linear - NO

$x_1[n] \rightarrow y_1[n]$, $x_2[n] \rightarrow y_2[n]$.

$x[n] = a x_1[n] + b x_2[n]$

$y[n] = a x_1[n] + b x_2[n] + 3u[n+1]$

$\neq a y_1[n] + b y_2[n]$

Time-invariant - NO.

$y[n-k] = x[n-k] + 3u[n-k+1]$.

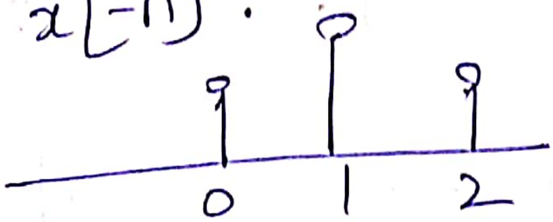
$y_1[n] = x[n-k] + 3u[n+1] \neq y[n-k]$.

Memoryless. - Yes.

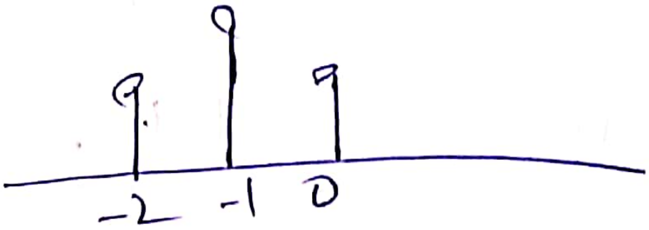
(3)

$y[n] = x[-n]$.

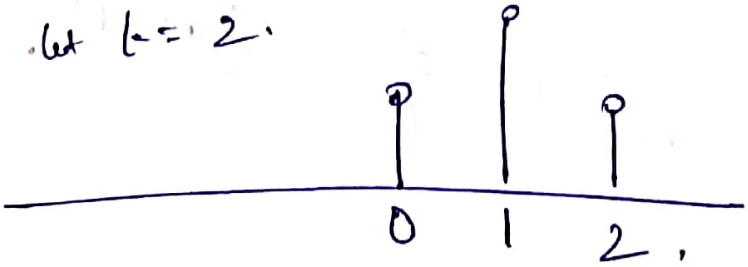
eg $x[n]$



$y[n] =$

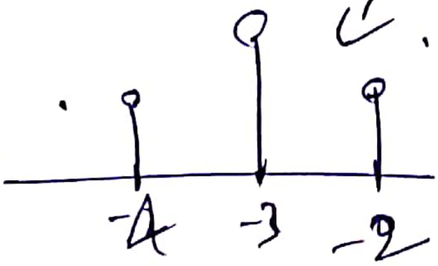


$y[n-k]$ let $k = 2$.

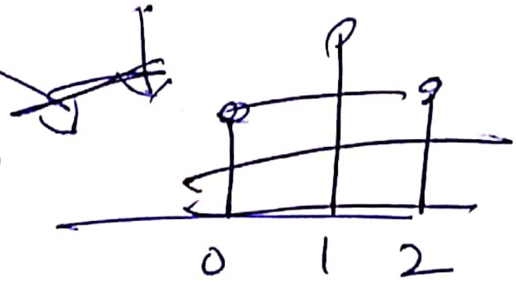


~~$x(t+k)$~~ $x[-(n+k)] = x[-2-n]$.

$y[n-2)$
$\neq x[-n-2]$

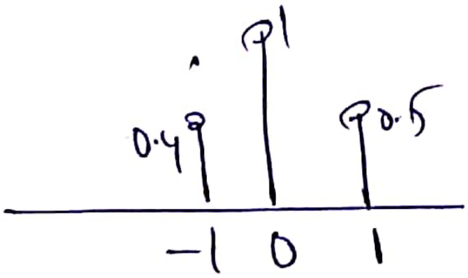


$\rightarrow$



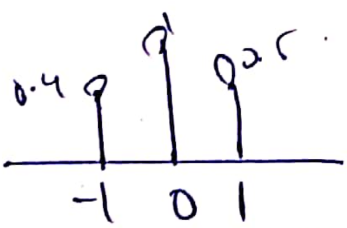
$\therefore$ for given $x[n]$; $y[n] = x[n]$

~~Moreover~~
let $k = 1$.
$y[n-1]$

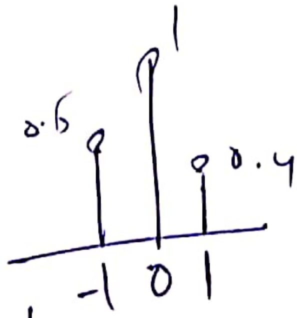


$x[-(n-1)] \rightarrow$

$x[1-n]$



$\rightarrow$

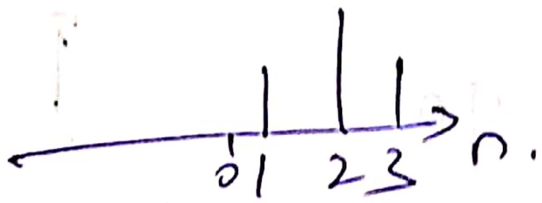


$x[1-n]$.

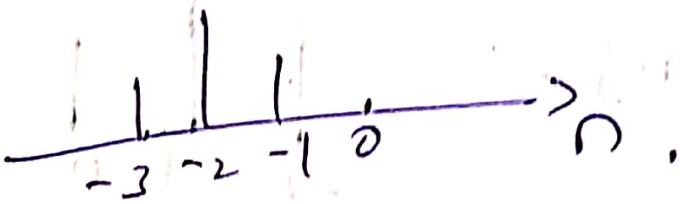
$\boxed{\therefore y[n-1] \neq x[1-n]}$

$x[-n-1]$. $\Rightarrow$ due to delay in i/p

$x[n-1] \rightarrow$



$x[-n-1]$



$\therefore\ y[n-1] \neq x[-n-1]$.

(1) Laplace transforms

Analysis.

domain
Time to
Laplace.
(s) domain

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} \, dt$$

Synthesis.

s to
time
domain.

$$x(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s) e^{st} \, ds.$$

$$x(t) \xleftrightarrow{\mathcal{L}} X(s)$$

Reasons:

(1) Eigen function. property

$$e^{st} \longrightarrow \boxed{H(s)} \longrightarrow y_e(t)$$

$$\boxed{y_e(t) = H(s) e^{st}}$$

Proof:

From definition of linear convolution for an LTI system

$$y_e(t) = H(x_e(t))$$

$$= h(t) * x(t)$$

$$= \int h(\tau)\, x(t-\tau)\, d\tau$$

For $x(t) = e^{st}$.

$$y_e(t) = \int_{-\infty}^{\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau$$

$$= e^{st} \int h(\tau)\, e^{-s\tau}\, d\tau$$

Recall $H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau$

$\therefore \underline{y_e(t) = H(s)\, e^{st}}$

reason

(2) Insights into system characteristics.

a) Transfer function.

$$x(t) \rightarrow \boxed{H(s)} \rightarrow y(t)$$

O/p. LT $Y(s) = H(s)\,X(s)$.

- result of L.transform of convolution.

$$\therefore H(s) = \frac{Y(s)}{X(s)}.$$

↳ definition of transfer function

Importance: $H(s)$ completely characterizes an LTI System.

b) Causality

Requirement $h(t) = 0$ for $t \leq 0$.

If $H(s)$ has poles on left half of s plane → decaying comp. to $h(t)$.

Th.

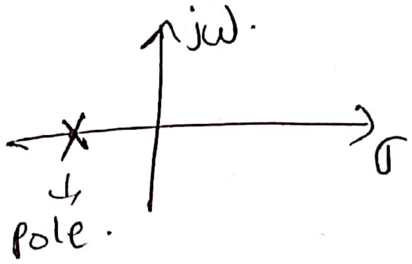
Proof:- for stability

$$H(s) = \frac{B(s)}{A(s)} = \sum_{k} \frac{C_k}{s-P_k}$$

→ partial fractions.

$$\therefore H(s) = \sum_{k} \frac{C_k}{s-P_k} = \frac{C_1}{s-P_1} + \frac{C_2}{s-P_2} + \cdots$$

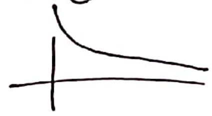
If $re(P_1) < 0$



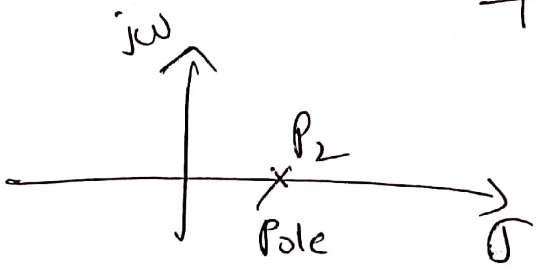
$$\mathcal{L}^{-1}\left(\frac{C_1}{s-P_1}\right) = C_1 \cdot e^{P_1 t} u(t)$$

since $re(P_1) < 0$

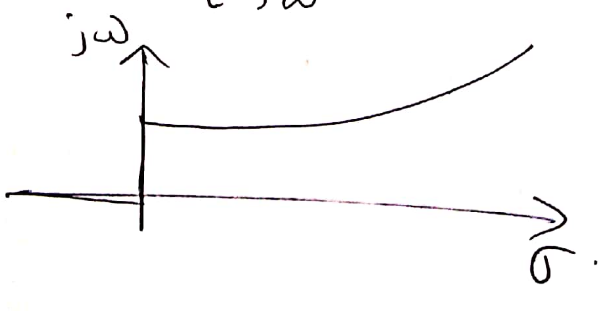
$$\lim_{t \to \infty} e^{P_1 t} \to 0$$



If $re(P_2) > 0$



$$\mathcal{L}^{-1}\left(\frac{C_2}{s-P_2}\right) = C_2 e^{P_2 t} u(t) \quad \lim_{t \to \infty} e^{P_2 t} \to \infty$$



If $H(s)$ has poles on right

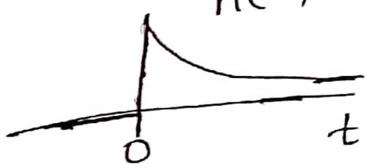
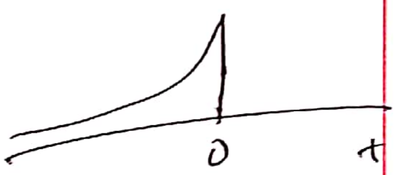
$\rightarrow$ growing component to $h(t)$.

(C) stability

def: Impulse response is absolutely integrable

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty .$$

$H(s) \rightarrow$ with poles $\rightarrow$ nature of $h(t)$.

Pole position	Nature of $h(t)$	$h(t)$
Left half of s-plane	decaying	
Right half of s-plane	growing till $t=0$.	

∴ $\boxed{\text{Causal \& Stable means. – Poles only on left half s-plane.}}$

Th.

Proof: Contd

For ~~Causal~~ & Stable

SIDE NOTE $\rightarrow \int |h(t)|\, dt < \infty$

Now $H(s) = \int h(t) e^{-st} dt$

$$\cancel{|H(s)| = \left| \int h(t) e^{-st} dt \right| \leq \int h(t) e^{-st}}$$

For $P_k < 0 \rightarrow e^{P_k t} \cdot u(t)$ in $h(t)$

$\hookrightarrow \boxed{\text{stability}}$

$P_k > 0 \rightarrow e^{P_k t}\, u(t) \rightarrow \boxed{\text{Not stable.}}$

$$\int |h(t)|\, dt = \int \left| \sum_k c_k e^{P_k t} u(t) \right| dt.$$

$$\leq \int \left[|c_1| |e^{P_1 t}| + |c_2| |e^{P_2 t}| + \ldots \right] dt$$

$$= \int_0^\infty |c_1| |e^{P_1 t}|\, dt + \int_0^\infty |c_2| |e^{P_2 t}|\, dt + \ldots$$

$$= \cancel{|c_1|\, e^{re(P_1) t}} \qquad \cancel{= \sum_k |c_k| \cdot \frac{1 - e^{re(P_k) t}}{|re(P_k)|}}$$

$$= \sum_k |c_k| \cdot \frac{1}{|re(P_k)|} < \infty. \qquad \boxed{\text{if } c_k < \infty}$$

Fourier Analysis:

Motivation.

Consider a signal $x(t)$.

$$x(t) \rightarrow \underset{(LTI)}{\boxed{h(t)}} \rightarrow y(t).$$

$$y(t) = \int x(\tau)\, h(t-\tau)\, d\tau \quad \text{—} (1)$$

Laplace transform.

$$X(s) = \mathcal{L}\{x(t)\} = \int_0^{\infty} x(t) e^{-st}\, dt. \quad \text{—} (2).$$

Now Consider input $x(t) = e^{st}$.

corresponding output

$$y(t) = \int_0^{\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau$$

$$= e^{st} . \int_0^{\infty} h(\tau) e^{-s\tau}\, d\tau$$

$$\therefore y(t) = H(s)\, e^{st}. \quad \text{—} (3)$$

$$\therefore e^{st} \rightarrow \boxed{h(t)} \rightarrow H(s)e^{st}.$$

" eigen function of LTI system "

Now consider a linear combination

$$x(t) = a_1 e^{s_1 t} + a_2 e^{s_2 t} + a_3 e^{s_3 t}$$

Clearly, using the linearity of system.

$$y(t) = a_1 H(s_1) e^{s_1 t} + a_2 H(s_2) e^{s_2 t} + a_3 H(s_3) e^{s_3 t}.$$

Now for a general signal.

$$x(t) = \sum_k a_k e^{s_k t} \quad -(4)$$

Corresponding output is

$$y(t) = \sum_k a_k H(s_k) e^{s_k t}. \quad -(5)$$

<u>Conclusion</u>:- If a_k the coefficients for decomposition of $x(t)$ are known the output $y(t)$ can be evaluated using eq. (5) :

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Definition of Fourier series.

For a periodic signal $x(t)$.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk2\pi t/T_0} = \sum_k a_k e^{jk\omega_0 t} \quad -(1)$$

For purely real signals.

$$x^*(t) = x(t)$$

$$\therefore x(t) = \sum_{k=-\infty}^{\infty} a_k^* e^{-jk\omega_0 t}$$

re-write $k = -l$.

$$= \sum_{l=-\infty}^{\infty} a_{-l}^* e^{jl\omega_0 t}. \quad -(2)$$

$\therefore$ from (1) & (2)

$$a_k = a_{-k}^* \quad \text{or} \quad a_k^* = a_{-k} \quad -(3)$$

From (1)

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k e^{jk\omega_0 t} + \sum_{k=-\infty}^{-1} a_k e^{jk\omega_0 t}$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k e^{jk\omega_0 t} + \sum_{l=1}^{\infty} a_{-l} e^{-jl\omega_0 t}$$

— using ③

$$= a_0 + \sum_{k=1}^{\infty} \left[a_k e^{jk\omega_0 t} + a_k^* e^{-jk\omega_0 t} \right]$$

$$z + z^* = Re[2z].$$

$$\therefore x(t) = a_0 + 2 \cdot \sum_{k=1}^{\infty} Re\left\{ a_k e^{jk\omega_0 t} \right\}$$

now let $a_k = A_k e^{j\theta_k}$.

$$x(t) = a_0 + 2 \sum_{k=1}^{\infty} Re\left\{ A_k e^{j(k\omega_0 t + \theta_k)} \right\}$$

$$= a_0 + 2 \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \theta_k)$$

$$= a_0 + 2 \sum_{k=1}^{\infty} A_k \cos\theta_k \cos k\omega_0 t - A_k \sin\theta_k \sin k\omega_0 t$$

Trignometric F. S.

$$\therefore x(t) = a_0 + 2 \sum_{k=1}^{\infty} \left[B_k \cos k\omega_0 t - C_k \sin k\omega_0 t \right]$$

here $a_k = B_k + jC_k = A_k(\cos\theta_k + j\sin\theta_k)$.

eg. on Fourier series.

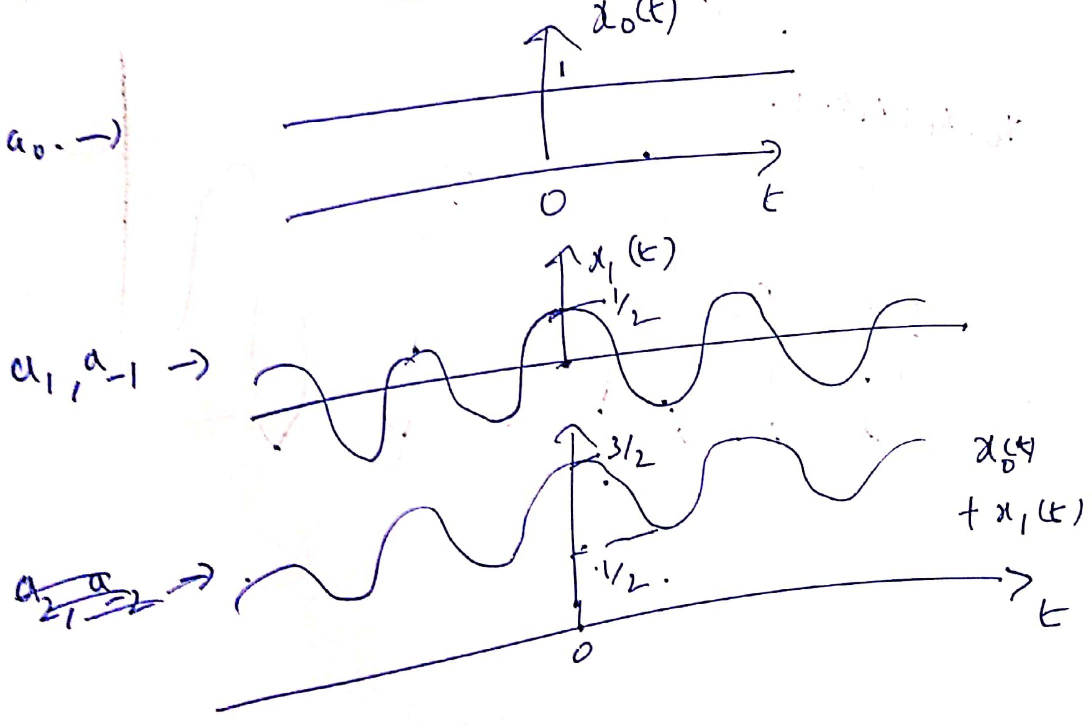
$$x(t) = \sum_{k=-3}^{3} a_k e^{jk2\pi t}$$

given $a_0 = 1$; $a_1 = a_{-1} = \frac{1}{4}$

$a_2 = a_{-2} = \frac{1}{2}$; $a_3 = a_{-3} = \frac{1}{3}$

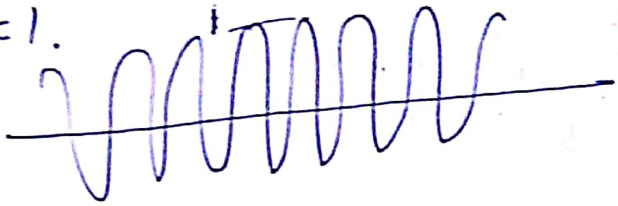
$$\therefore x(t) = 1 + \frac{1}{4}\left(e^{j2\pi t} + e^{-j2\pi t}\right) + \frac{1}{2}\left(e^{j4\pi t} + e^{-j4\pi t}\right) + \frac{1}{3}\left(e^{j6\pi t} + e^{-j6\pi t}\right)$$

$$\therefore x(t) = 1 + \frac{1}{2}\cos 2\pi t + \cos 4\pi t + \frac{2}{3}\cos 6\pi t$$

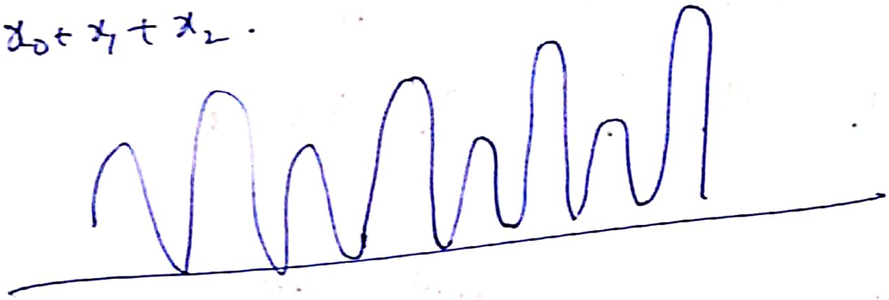


$\cos 4\pi t$.

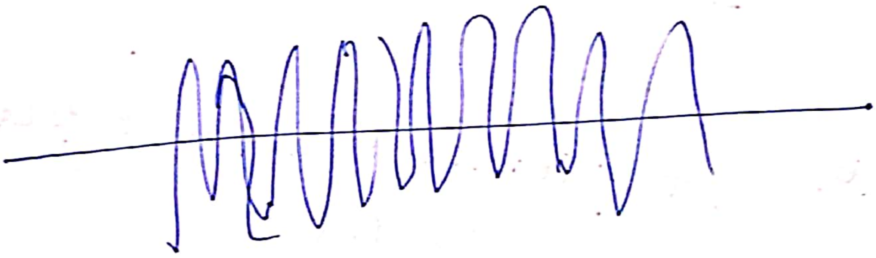
$x_2(t)$.



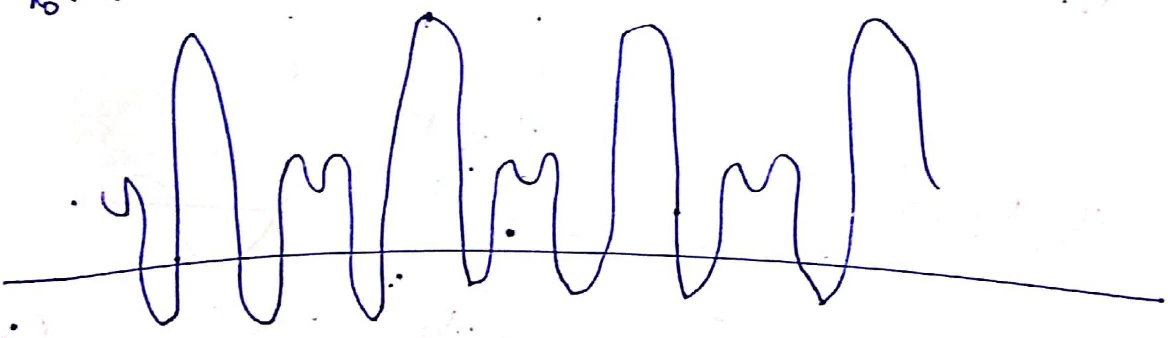
$x_0 + x_1 + x_2$.



$x_3(t) \to \frac{2}{3}\cos 6\pi t$



$x_0 + x_1 + x_2 + x_3$.

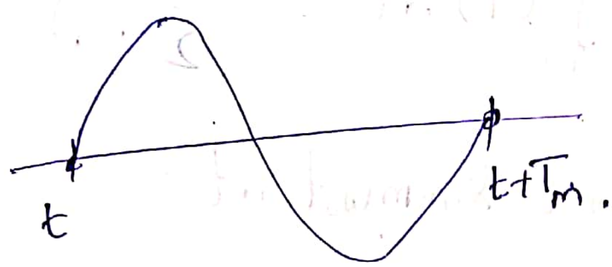


Important trigonometric Integrals.

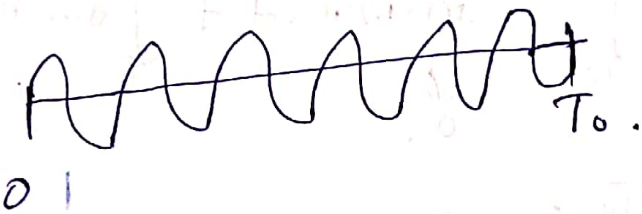
① $\int_{T_0} \sin m\omega_0 t \, dt$

$T_m = \frac{2\pi}{m\omega_0}$

$T_0 = \frac{2\pi}{\omega_0}; \; T = \frac{T_0}{m}$



$T_0 = mT$



area = 0.

② $\int_{T_0} \cos n\omega_0 t \, dt = 0$.

③ $\int_{T_0} \cos n\omega_0 t \cdot \cos m\omega_0 t \, dt$ — $\boxed{1}$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$.

① becomes $\frac{1}{2}\int_{T_0} \cos(n+m)\omega_0 t \, dt + \frac{1}{2}\int_{T_0} \cos(n-m)\omega_0 t \, dt$

$\downarrow$
0 from def ①, ②.

Fourier Series — background.

$$\frac{1}{2} \int_{T_0} \cos (n-m) \omega_0 t \cdot dt \quad - (2)$$

eq (2) $\to 0$ for $\forall$ $n \neq m$.

For $n = m$.

$$\frac{1}{2} \int_{T_0} (1) \, dt = \frac{T_0}{2}.$$

$$= \frac{T_0}{2} \delta(n-m).$$

(4) $$\int_{T_0} \cos n\omega_0 t \sin m\omega_0 t \, dt.$$

$$= \frac{1}{2} \Big[\underbrace{\int_{T_0} \sin (m+n) \omega_0 t \, dt}_{0} + \underbrace{\int_{T_0} \sin (m-n) \omega_0 t \, dt}_{} \Big]$$

1 for $m=n$.

0 for $m \neq n$.

ie $\delta[m-n]$.

$$= \frac{T_0}{2} \cancel{\delta[m-n]}. \, 0$$

(5) IIIly $$\int_{T_0} \sin n\omega_0 t \sin m\omega_0 t \, dt$$

$$= \frac{T_0}{2} \delta(n-m).$$

Motivation for F.S.

$$e^{jk\omega_0 t} \longrightarrow \boxed{h(t)} \longrightarrow y_k(t).$$

$$y_k(t) = h(t) * e^{jk\omega_0 t}.$$

$$= \int h(\tau)\, e^{jk\omega_0 (t-\tau)}\, d\tau.$$

$$= e^{jk\omega_0 t} \cdot \int h(\tau)\, e^{-jk\omega_0 \tau}\, d\tau.$$

$$\therefore y_k(t) = H(k\omega_0)\, e^{jk\omega_0 t}$$

- eigen function

recall $x(t) = \sum a_k e^{jk\omega_0 t}$ → F.S.

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t).$$

$$a_k e^{jk\omega_0 t} \longrightarrow a_k \cdot H(k\omega_0) \cdot e^{jk\omega_0 t}.$$

$$\therefore y(t) = \sum_k H(k\omega_0)\, a_k\, e^{jk\omega_0 t}$$

→ Ease of analysis.

Complex F.S. derivation

def: $x(t) = \sum_k c_k e^{jk\omega_0 t}$ — $\boxed{1}$

- defined only for periodic signals.

Fund. period $T_0 = \frac{2\pi}{\omega_0}$; ω_0 = fund. frequency

Harmonics: $\omega_k = k\omega_0$.

$c_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt$. → def.

Sol. Apply $\times e^{-jl\omega_0 t}$ and integrate over T_0.

to $\boxed{1}$

$$\int_{T_0} x(t)\, e^{-jl\omega_0 t}\, dt = \int \sum_k c_k e^{jk\omega_0 t} e^{-jl\omega_0 t}\, dt$$

$$= \sum_k c_k \int_{T_0} e^{-j(l-k)\omega_0 t}\, dt.$$

$$\int_{T_0} e^{-j(l-k)\omega_0 t}\, dt = \left. \frac{e^{-j(l-k)\omega_0 t}}{-j(l-k)\omega_0} \right|_t^{t+T_0}.$$

— $\boxed{2}$

For $l \neq k$.

$$\frac{e^{-j(l-k)\omega_0 \cdot t} - e^{-j(l-k)\omega_0(t+T_0)}}{-(l-k)j\omega_0}.$$

since $\omega_0 = \frac{2\pi}{T_0}$. $\Big|$ $e^{-j(l-k)2\pi} = 1$.

$$= \frac{e^{-j(l-k)\omega_0 t}(1-1)}{j(k-l)\omega_0} = 0.$$

For $l = k$.

LHS in $\boxed{2}$ become

$$\int_{T_0} (1)\,dt = T_0.$$

$$\therefore \int_{T_0} x(t)\, e^{-jl\omega_0 t}\, dt = \sum_k C_k\, T_0 \cdot \delta(l-k).$$

$$\therefore \boxed{C_l = \frac{1}{T_0}\int_{T_0} x(t) \cdot e^{-jl\omega_0 t}\, dt}$$

02/06/2017
(DSAA)

Fourier series rep. of a sinusoid.

$$x(t) = A\cos(\omega_0 t + \phi)$$

$$\Rightarrow x(t) = A\cos\omega_0 t \cdot \cos\phi - \sin\omega_0 t \sin\phi.$$

F.S $x(t) = \sum_k a_k e^{jk\omega_0 t}.$

$$x(t) = A\cos\phi \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} - A\sin\phi \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}.$$

group ~~all~~ terms.
$\omega_0, -\omega_0$

$$= \frac{A}{2}(\cos\phi + j\sin\phi) e^{j\omega t}$$
$$+ \frac{A}{2}(\cos\phi - j\sin\phi) e^{-j\omega t}.$$

$$\therefore a_1 = \frac{A}{2} e^{j\phi}. \qquad a_{-1} = \frac{A}{2} e^{-j\phi}.$$

$$a_0 = a_2 = \cdots = 0.$$

Special case

(i) $\phi = 0.$ (ii) $\phi = 90^\circ.$

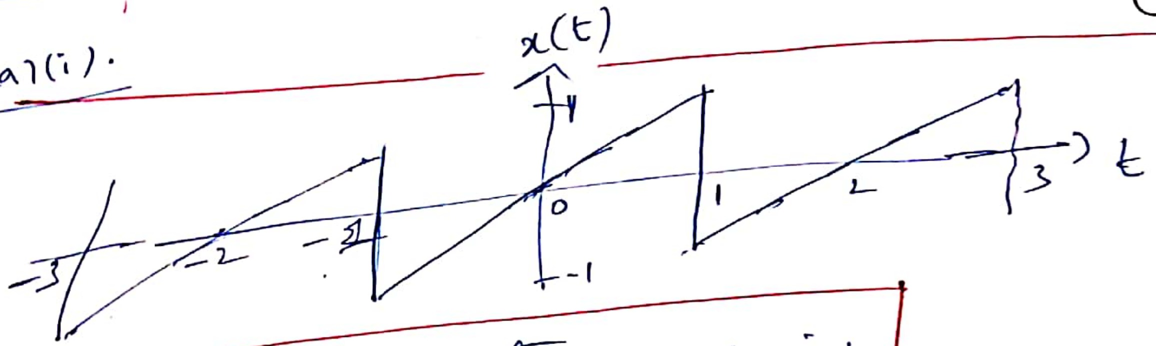
$a_1 = a_{-1} = \frac{A}{2}.$ $a_1 = j\frac{A}{2}; \; a_{-1} = \frac{-jA}{2}.$

Ex) $x(t) = A\cos\omega_0 t$ $x(t) = -A\sin\omega_0 t.$

Ex: S.K.S | A.O. | P.168 | S.4.2 |

PP.T3033 S & S Anna Univ 03/05/17 (DSAA)

12 (a)(i).



Find $\boxed{\text{exponential fourier series.}}$

Sol:

def. $x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\omega_0 n t}$

$$c_n = \frac{1}{T}\int_T x(t)\, e^{-j\omega_0 n t}\, dt.$$

From the graph. $x(t) = t \cdot \quad -1 < t < 1.$

in the fundamental interval.

$T = 2.$

$\omega_0 = \frac{2\pi}{T} = \pi.$

$$\therefore c_n = \frac{1}{2}\int_{-1}^{1} t \cdot e^{-j\omega_0 n t}\, dt.$$

$$\text{i)}\quad c_0 = \frac{1}{2}\int_{-1}^{1} t\, dt = \frac{1}{2}\left.\frac{t^2}{2}\right|_{-1}^{1} = 0.$$

$$c_n = \frac{1}{2}\left[\underbrace{\left. t\,\frac{e^{-j\omega_0 n t}}{-j\omega_0 n}\right|_{-1}^{1}}_{\text{term①}} - \underbrace{\int \frac{e^{-j\omega_0 n t}}{-j\omega_0 n}\, dt}_{\text{term②}}\right]$$

$\omega_0 = \pi$

term ① $\rightarrow$ $\frac{1}{2} \frac{e^{-j\pi n} + e^{j\pi n}}{-j\cancel{\pi} n} = j \frac{\cos \pi n}{\pi n}$

term ② $\rightarrow$ $\frac{1}{-(j\omega_0 n)^2} \cdot \frac{1}{2} e^{-j\omega_0 n t} \Big|_{-1}^{1}$

$$= \frac{1}{\pi^2 n^2} \left(\frac{e^{-j\pi n} - e^{j\pi n}}{2} \right)$$

$$= \frac{-j}{\pi^2 n^2} \sin \pi n = 0.$$

$\therefore C_0 = 0; \quad C_n = \frac{j(-1)^n}{\pi n} \quad n \neq 0.$

$$\therefore x(t) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{j(-1)^n}{\pi n} \exp(j\pi n t) \quad -①$$

From relation with trigonometric series

$a_n = \text{real}(C_n) = 0$

$b_n = \text{imag}(C_n) = \frac{(-1)^n}{\pi n}$

$$\therefore x(t) = -2 \sum_{k=1}^{\infty} \frac{(-1)^k}{\pi k} \sin(k\pi t). \quad -②$$

Matlab Program for Computing Fourier Series.

step ① e.g. $x(t) = A\cos(\omega_0 t + \phi)$.

- ~~Gene~~ define the time vector.

choose a ω_0.

Then $T_0 = \frac{2\pi}{\omega_0}$.

$\therefore t_{vec} = 0 : \frac{1}{F_s} : T_0$.

$F_s = 10\,\cancel{F_0}$.

large val.

- Define the signal

A = 1;

$\omega_0 = 2\pi$;

$\phi = \pi/8$;

$xt = A.\cos(\omega_0 t_{vec} + \phi)$

- Evaluate the F.C.

Determine the valid range for k.

(1. Let) $k_{vec} = -10 : 10$;

for each value of k_{vec}. ($i = 1$ to len(k_{vec})).

do. $k = k_{vec}(i)$.

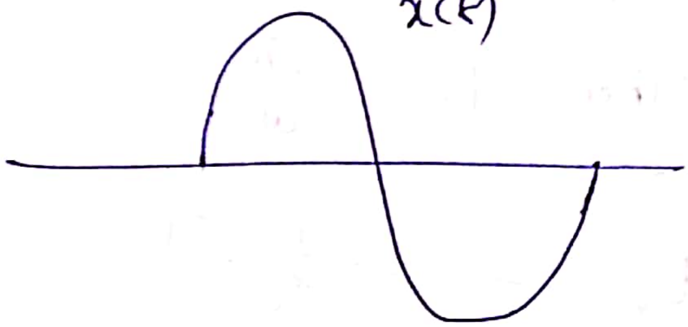
basis = exp(1i. ω_0.k.t_{vec}).

~~C kx~~

$$c(i) = \frac{1}{T}\,\mathrm{trapz}(\mathrm{tvec},\ xt \,.\!*\, \mathrm{basis});$$

end.

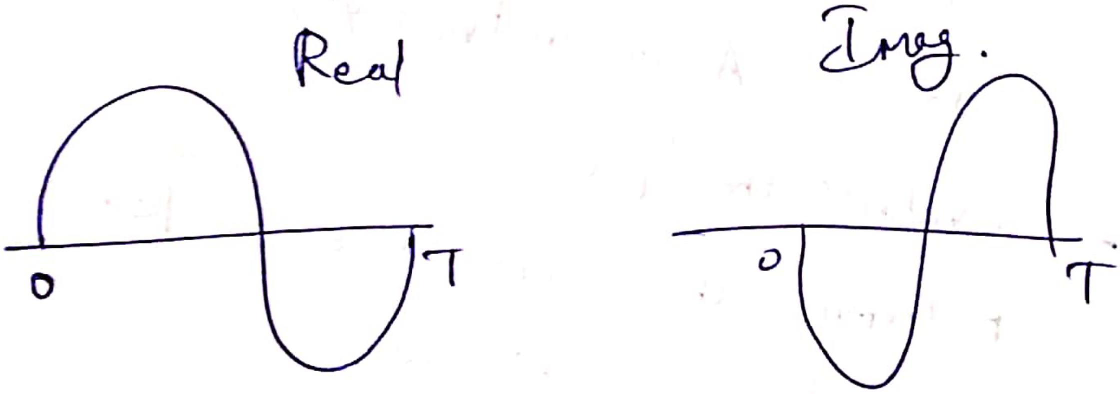
√,



$\times$

$e^{-jk\omega_0 t}$

$$= x(t)\, e^{-jk\omega_0 t}$$



T

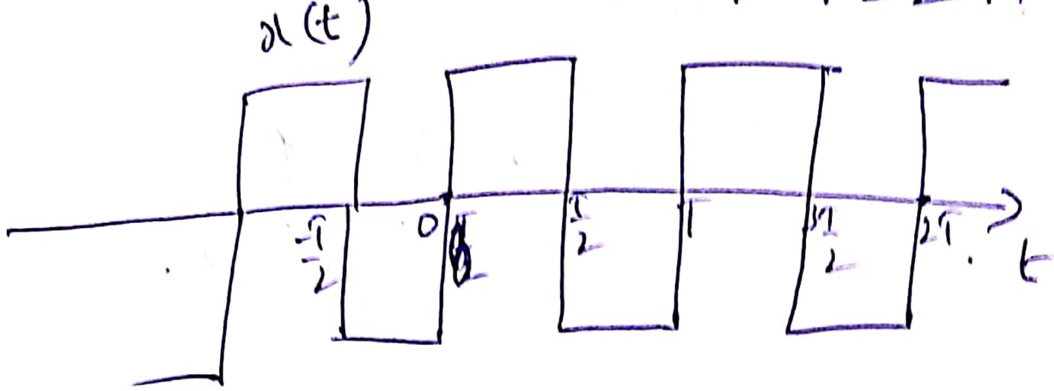
- plotting

~~plot~~ stem(kvec, c);

$$\underset{T}{x(t)} = \begin{cases} -1 & -\frac{T}{2} < t < 0 \\ 1 & 0 < t < \frac{T}{2} \end{cases}$$

Periodic signal $\}\rightarrow$

For $T = 2$, $\omega_0 = \pi$



$$C_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T}\left[\int_{-T/2}^{0} (-1)\, e^{-jk\omega_0 t}\, dt + \int_{0}^{T/2} (1)\, e^{-jk\omega_0 t}\, dt\right]$$

$$= \frac{1}{T}\left[\left.\frac{-e^{-jk\omega_0 t}}{-jk\omega_0}\right|_{-T/2}^{0} + \left.\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right|_{0}^{T/2}\right]$$

$$= \frac{1}{T}\left[\frac{-1 + e^{+jk\omega_0 T/2}}{-jk\omega_0} + \frac{e^{-jk\omega_0 \frac{T_0}{2}} - 1}{-jk\omega_0}\right]$$

$= \cancel{\cos k\omega_0}$

$$= \frac{\cos k\pi - 1}{-jk\pi}$$

$$= j\left[\frac{\cos k\pi}{k\pi} - \frac{1}{k\pi}\right].$$

$k=1;\quad \frac{\cos\pi}{\pi} - \frac{1}{\pi} = -\frac{2}{\pi}$

$$\therefore\ a_k = j\left[\frac{(-1)^k - 1}{k\pi}\right]$$

$$\therefore\ a_k = \begin{cases} -\dfrac{j2}{k\pi} & \forall\ k \to \text{odd}. \\ 0 & k \to \text{even} \end{cases}$$

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D SAA

Dirichlet conditions for F.S.

(1). Over any period $x(t)$ must be abs. integrable:

$$\int_{T_0} |x(t)|\,dt < \infty. \quad \text{— } \boxed{I}$$

since $a_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt$

if $\boxed{I}$ is true $|a_k| < \infty$ (finite)

Proof: $a_k = \frac{1}{T_0}\int x(t)\, e^{-jk\omega_0 t}\, dt.$

$$|a_k| = \frac{1}{T_0}\left|\int x(t)\, e^{-jk\omega_0 t}\, dt\right|$$

$$\leq \frac{1}{T_0}\int |x(t)|\,dt < \infty. \quad \text{from } \boxed{I}.$$

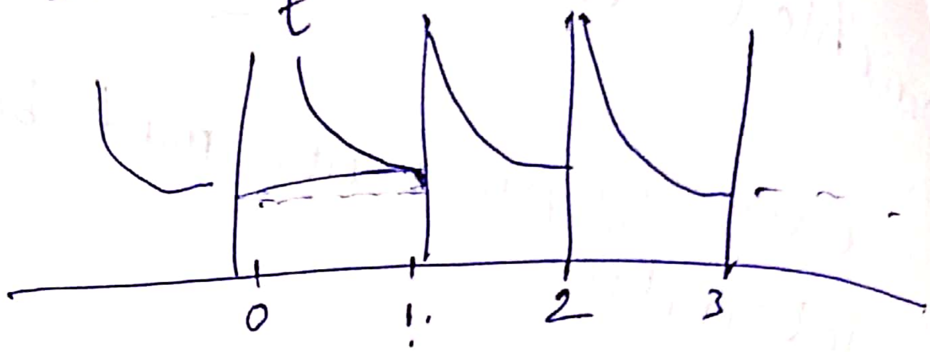
$\therefore |a_k| < \infty$

e.g. $x(t) = 1 \quad -\frac{T_0}{2} < t < \frac{T_0}{2}$

satisfies $\boxed{I}$

TH

$$x(t) = \frac{1}{t} \qquad 0 \le t \le 1.$$



$$x(t) = \frac{1}{t} \qquad 0 \le t \le 1.$$

$$\frac{1}{t-1} \qquad 1 \le t \le 2.$$

$$\frac{1}{t-2} \qquad 2 \le t \le 3.$$

$$\vdots$$

$$x_n(t) = \frac{1}{t-n}. \qquad n \le t \le n+1.$$

$$\int_0^1 |x(t)|\,dt = \int_0^1 \left|\frac{1}{t}\right| dt$$

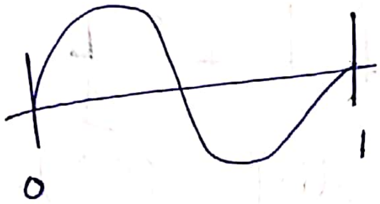
$$= \left.\frac{-1}{t^2}\right|_0^1 \Rightarrow \infty.$$

Violates Dirichlet Cond. 1.

Condition. 2

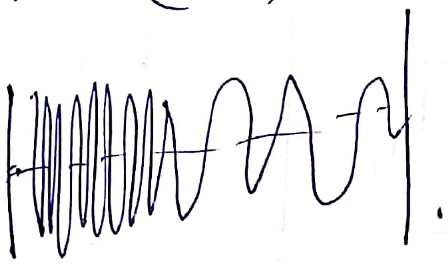
- No. of maxima and minima must be finite. - (within a single period).
- Means → bounded variation.

e.g. $x(t) = \sin(2\pi t)$



→ satisfies (i.e. C 2)

$x(t) = \sin\left(2\pi \frac{1}{t}\right)$ → does not satisfy (C2).



- Practical consideration.

$$C_k = \int x(t)\, e^{-jk\omega_0 t}\, dt. \quad \text{—— } \boxed{1}$$

for $x(t) = \sin\left(2\pi \frac{1}{t}\right) \Rightarrow \boxed{1}$ is intractable.

- Since it has too many max & min.

even numerical approx. is very difficult.

i.e., at low freq. part → no. of values ↑.

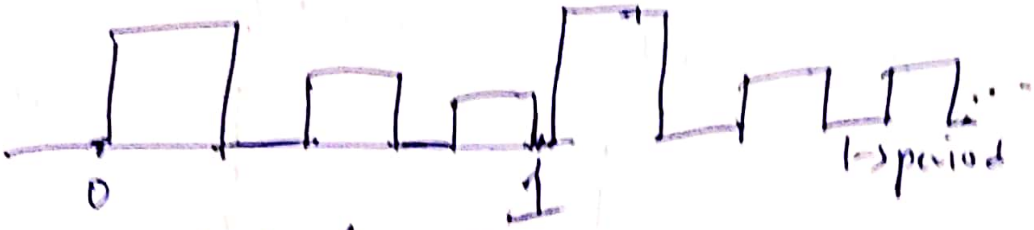
at very high freq. → " ↓.

→ Area → inaccurate.

Condition 3

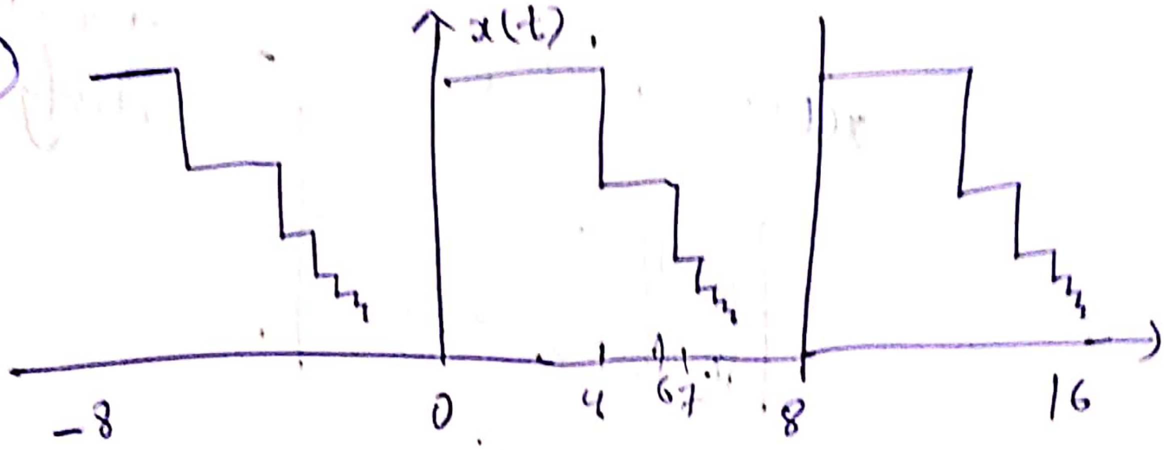
- In any finite interval no. of discontinuities should be finite.
- The values of discontinuities should be finite.

e.g. ①



- Satisfies (C3)

②



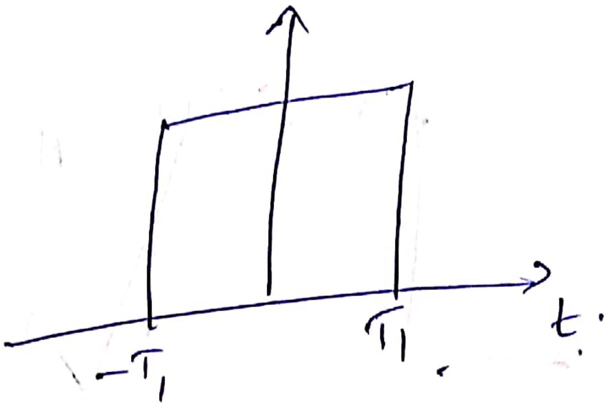
② → does not satisfy (C3) : part 1.

Practically → integration is intractable (in theory)

(✗ using numerical methods)

Gibbs phenomenon

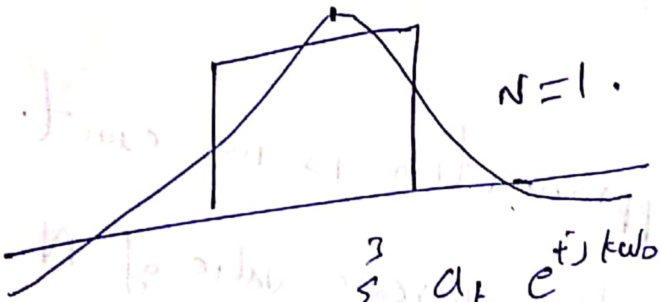
Consider the square wave



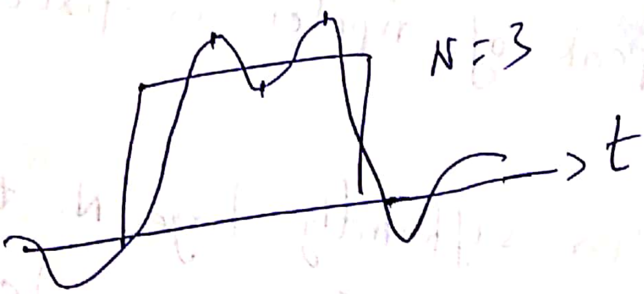
Approximation using fourier series.

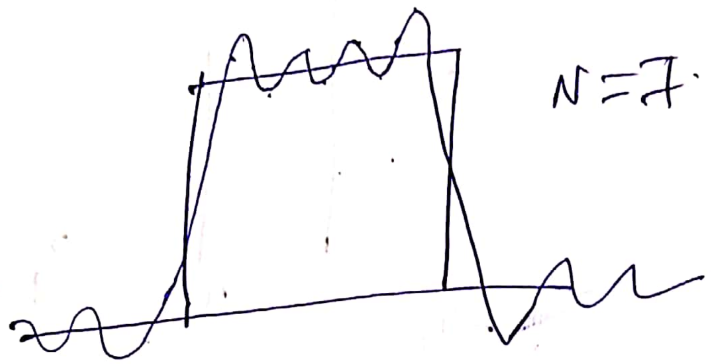
$$x_N(t) = \sum_{k=-N}^{N} a_k e^{jk\omega_0 t}.$$

$N=1.$ $\quad x_1(t) = a_{-1} e^{-j\omega_0 t} + a_1 e^{j\omega_0 t}.$

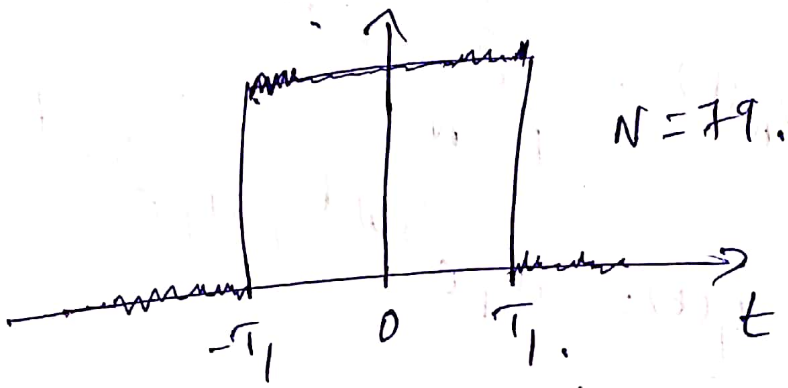


$N=3.$ $\quad x_3(t) = \sum_{k=-3}^{3} a_k e^{+jk\omega_0 t}$





⋮



Key points.

1. The approximation is not exact. $N < \infty$.
2. The ripples increase value of N. ($\propto N$).
3. peak of ripples is fixed for any N
   ($= 1.09$).
4. For sufficiently large N. the approximation is good (error $\to 0$) except at discontinuities.

Signal Reconstruction (synthesis)

Recall Fourier series decomposition:

$$x(t) = \sum_{k} c_k e^{jk\omega_0 t} \quad -(1)$$

$$c_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t} dt \quad -(2)$$

Recon:

$$x_M(t) = \sum_{k=-M}^{M} c_k e^{jk\omega_0 t}.$$

error

$$e_x(M) = \frac{1}{T'}\int_{T'} |x(t) - x_M(t)|^2 dt$$

$T' \to$ time support.

Converge

$$\lim_{M\to\infty} e_x(M) \to 0.$$

ie; $\lim_{M\to\infty} x_M(t) \longrightarrow x(t).$

Fourier Coefficients

(i) $x(t) = \sin \pi t \qquad 0 < t < 1.$

$T = 1. \qquad \omega_0 = \frac{2\pi}{1} = 2\pi.$

$$a_0 = \int_0^1 \sin \pi t \, dt = \frac{-\cos \pi t}{\pi}\Big|_0^1 = \frac{-(-1-1)}{\pi}$$

$$\therefore a_0 = \frac{2}{\pi}.$$

$$a_k = \frac{1}{1}\int_0^1 \sin \pi t \, e^{-jk2\pi.t} \, dt$$

$$= \int_0^1 \frac{(e^{j\pi t} - e^{-j\pi t})}{2j} . e^{-j2\pi k t} \, dt$$

$$= \frac{1}{2j}\left\{ \frac{e^{-j\pi(2k-1)t}}{-j\pi(2k-1)}\Big|_0^1 - \frac{e^{-j\pi(2k+1)t}}{-j\pi(2k+1)}\Big|_0^1 \right\}$$

$$= \frac{1}{2j} . \left[\frac{e^{-j\pi(2k-1)} - 1}{-j\pi(2k-1)} - \frac{e^{-j\pi(2k+1)} - 1}{-j\pi(2k+1)} \right]$$

$$= \frac{1}{2j} . \frac{2}{j\pi} . \left[\frac{1}{2k-1} - \frac{1}{2k+1} \right] = \frac{-2}{\pi(4k^2-1)}.$$

$$\therefore a_k = \frac{2}{\pi(1-4k^2)}.$$

Ex 2.45.

$y(t) = x(t) * h(t)$. LTI ✓.

(a) ~~Find~~ $y_1(t) = x'(t) * h(t)$.

(~~a~~). show that $y_1(t) = y'(t)$.

Sol① $y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$.

$$\frac{d}{dt} y(t) = \int_{-\infty}^{\infty} h(\tau) \frac{d}{dt} x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x'(t-\tau)\, d\tau.$$

$$= h(t) * x'(t).$$

Sol②

$$x'(t) = \lim_{h \to 0} \frac{x(t) - x(t-h)}{h}.$$

$$x'(t) * h(t) = \lim_{h \to 0} \frac{x(t) * h(t) - x(t-h) * h(t)}{h}$$

$$= \lim_{h \to 0} \frac{y(t) - y(t-h)}{h}.$$

$$= \frac{d}{dt} y(t).$$

MQ QP. T3033. S & S Anna Univ (2009). (DSAA)

Q. 11(b). Determine if the following systems are linear or not.

(i) $\frac{dy}{dt} + 3t\, y(t) = t^2 x(t)$. — ①

Consider two inputs $x_1(t)$, $x_2(t)$.

Consider two inputs $x_1(t)$, $x_2(t)$ with corresponding outputs $y_1(t)$ & $y_2(t)$.

$$\frac{dy_1(t)}{dt} + 3t\, y_1(t) = t^2 x_1(t) \quad \text{— ②}$$

$$\frac{dy_2(t)}{dt} + 3t\, y_2(t) = t^2 x_2(t) \quad \text{— ③}$$

Let $x(t) = a x_1(t) + b x_2(t)$.

& $y(t) = a y_1(t) + b y_2(t)$.

plugin in ①

$$\frac{d}{dt}[a y_1(t) + b y_2(t)] + 3t\,(a y_1(t) + b y_2(t)) = t^2 (a x_1(t) + b x_2(t))$$

$$\Rightarrow a\left\{\frac{d}{dt} y_1(t) + 3t\, y_1(t) - t^2 x_1(t)\right\} + b\left\{\frac{d}{dt} y_2(t) + 3t\, y_2(t) - t^2 x_2(t)\right\} = 0. \quad \text{— ④}$$

From ②, ③ & ④ ⇒ ① is a linear system.

(EX)

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(ii) $y(n) = 2x(n) + \frac{1}{x(n-1)}$ — system. —②.

Let $x^{(1)}(n) = a\,x_1(n) + b\,x_2(n)$

$x(n) - \boxed{H(\cdot)} - y(n)$

$$H(x^{(1)}(n)) = 2(a x_1(n) + b x_2(n)) + \frac{1}{a x_1(n-1) + b x_2(n-1)}$$

Let $y^{(1)}(n) = a\,y_1(n) + b\,y_2(n)$

$$= a\left(2x_1(n) + \frac{1}{x_1(n-1)}\right) + b\left(2x_2(n) + \frac{1}{x_2(n-1)}\right)$$

$\therefore$ $$y^{(1)}(n) = 2x^{(1)}(n) + \frac{a}{x_1(n-1)} + \frac{b}{x_2(n-1)}$$

$\therefore$ $y^{(1)}(n) \neq H(x^{(1)}(n))$

$\therefore$ ① is non-linear

(b) / Anna Univ / S&S / T3033.1 24/04/2017

(DSAA)

Time-invariant property

- Verification.

system (a) $y(t) = t\,x(t)$

$x(t-t_0) \longrightarrow y(t-t_0)$. property. (desired)

step ① $x(t) \xrightarrow{\text{replace}} x(t-t_0)$

$y_1(t) = t\,x(t-t_0)$.

② $y(t) \xrightarrow{\text{replace}} y(t-t_0)$

$y(t-t_0) = (t-t_0)\,x(t-t_0)$.

$\therefore y_1(t) \neq y(t-t_0) \Rightarrow$ Time varying system

(b) $y(n) = x(2n)$ → upsampler.

step①. $x(n) \xrightarrow{\text{replace}} x(n-n_0)$.

ie; a delay of n_0.

$x(2n) \longrightarrow x(2n-n_0) = y_1(n)$

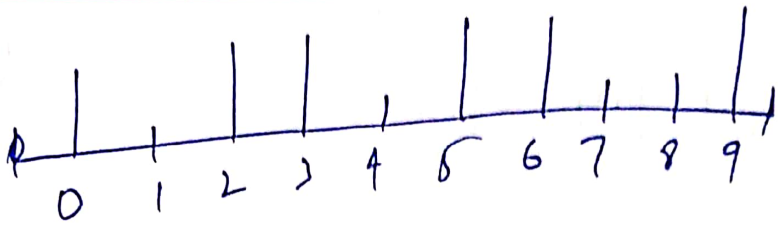
② $y(n)$ replace → $y(n-n_0)$

$y(n-n_0) = x(2(n-n_0))$

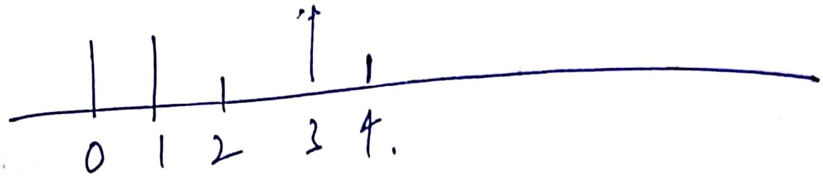
$\therefore y_1(n) \neq y(n-n_0)$ → Time Varying

(Ex)

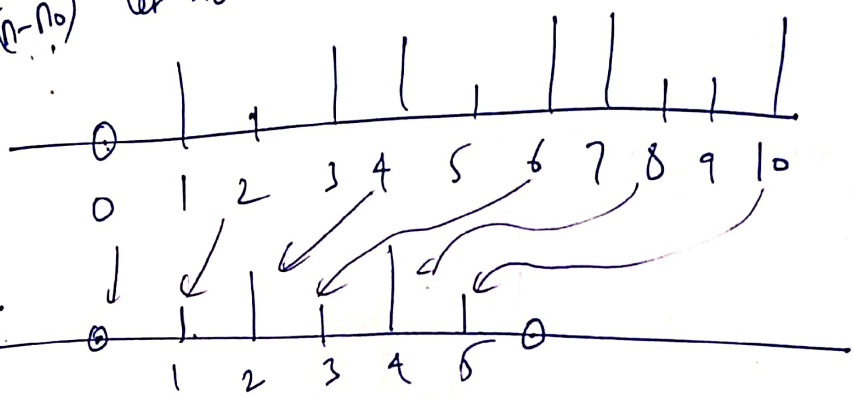
$x(n)$.



$y(n)$

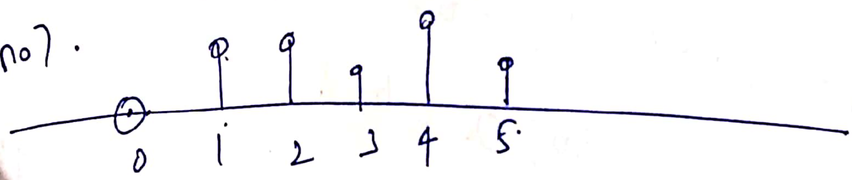


$x(n-n_0)$ let $n_0 = 1$



$y_1(n)$.

$y(n-n_0)$.



$\therefore y(n-n_0) \neq y_1(n)$. $\Rightarrow$ Time Variance.

(b) Determine whether the following LTI systems with the given impulse responses are stable or not.

(i) $h(n) = a^n u(n)$

sol ÷ The necessary and sufficient condition for stability is $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$.

Given $h(n) = a^n u(n)$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |a^n u(n)|$$

$$= \sum_{n=0}^{\infty} a^n$$

If $a \geq 1$, the summation $= \infty$, then LTI system is unstable.

If $a < 1$, the summation $= \frac{1}{1-a}$, a finite value, so given ~~given~~ LTI system is stable.

(4M)

(ii) $h(n) = b^n u(-n)$ $b > 1$.

sol: $$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |b^n u(-n)|$$

$$= \sum_{n=-\infty}^{0} b^n = \sum_{n=0}^{\infty} b^{-n} = \sum_{n=0} \left(\frac{1}{b}\right)^n$$

$$= \frac{1}{1-\left(\frac{1}{b}\right)}$$

If $b > 1$, the summation is a finite value.

∴ The given LTI system is stable.

(4M

(EX)

3 (a) Determine whether the following LTI systems with the given impulse responses are stable or not.

(i) $h(n) = e^{-6|n|}$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |e^{-6|n|}|.$$

$e^{-6|n|} > 0 \ \forall n.$

$$= \sum_{n=-\infty}^{\infty} e^{-6|n|} = \sum_{n=-\infty}^{-1} e^{6n} + \sum_{n=0}^{\infty} e^{-6n}.$$

$\Rightarrow$ Convergent series.

ie: $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

$\therefore$ $\boxed{\text{System is stable}}$

(3M)

(ii) $h(n) = 3^n u(-n)$

Sol :-

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |3^n u(-n)|.$$

$$= \sum_{n=-\infty}^{0} 3^n = \sum_{n=0}^{\infty} 3^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1}{1-\left(\frac{1}{3}\right)}$$

The summation is a finite value.

So given LTI system is stable.

(3M)

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check linearity, Time invariance & Causality.

(i) $y(n) = n^2 x(n)$.

$x_1(n) \rightarrow y_1(n)$ & $x_3(n) = a x_1(n) + b x_2(n)$

$x_2(n) \rightarrow y_2(n)$

$y_3(n) = a y_1(n) + b y_2(n)$..

$= a n^2 x_1(n) + b n^2 x_2(n)$.

$y_3'(n) = n^2 (a x_1(n) + b x_2(n))$

$\therefore y_3'(n) = y_3(n)$. → Linear.

$n \rightarrow n - n_0$

$y(n-n_0) = (n-n_0)^2 \cdot x(n-n_0)$

$y'(n) = n^2 x(n-n_0)$

$\therefore y'(n) \neq y(n-n_0)$ → Time varying

$y(n) = n^2 x(n)$.

o/p y(n) depends only on current input.

- Causal.

(3M)

(ii) $y(t) = A x(t) + B$.

$x_1(t) \rightarrow y_1(t)$, $x_2(t) \rightarrow y_2(t)$

$x_3(t) = a x_1(t) + b x_2(t)$. —①

~~$y_3(t) = A x_3(t) + B$.~~

$y_3(t) = a y_1(t) + b y_2(t)$ —②.

$= a(A x_1(t) + B) + b(A x_2(t) + B)$

$= A(a x_1(t) + b x_2(t)) + (a+b) B$.

$= A x_3(t) + (a+b) B$.

$y_3'(t) = A(x_3(t)) + B$

$\therefore y_3'(t) \neq y_3(t)$ — Not linear

$y(t-t_0) = A x(t-t_0) + B$.

$y'(t) = A x(t-t_0) + B$.

$\therefore y'(t) = y(t-t_0)$ $\rightarrow$ Time invariant

$y(t) = A x(t) + B$.

at $t<0$ $y(t) = A x(t) + B$.

even for $x(t) = 0$ $t<0$

$y(t) = B \neq 0$. (3M)

Hence not Causal.

$y(t) = x(t^2)$.

$x_1(t) \rightarrow y_1(t)$, $x_2(t) \rightarrow y_2(t)$

$x_3(t) = a x_1(t) + b x_2(t)$.

$y_3'(t) = x_3(t^2) = a x_1(t^2) + b x_2(t^2)$

$y_3(t) = a y_1(t) + b y_2(t)$.

$\therefore y_3'(t) = y_3(t) \rightarrow$ linear

$y(t-t_0) = x[(t-t_0)^2]$ as. $y'(t) \neq y(t-t_0)$

$y_1'(t) = \cancel{x((t-t_0)^2)}$ $x(t^2 - t_0)$. $\rightarrow$ Time ~~invariant~~ Varying !

$y_1(t) \neq y(t-t_0)$.

$y(t) = x(t^2)$ ..

does Not depends only on Current i/p $x(t)$

~~and $y(t)=0$ for $x(t)$~~

ie; eg: $t = -3$. , and say $x(9) \neq 0$.

$y(-3) = x(9)$ depends on future input

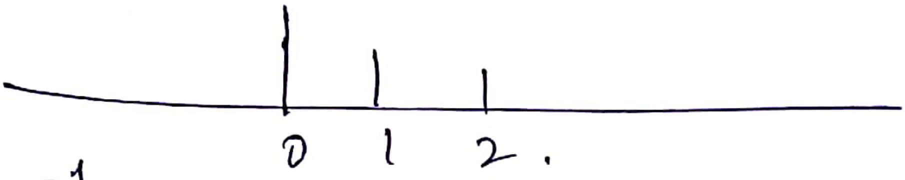
- Non-causal

(3M)

$y(t) = x(t^2)$.

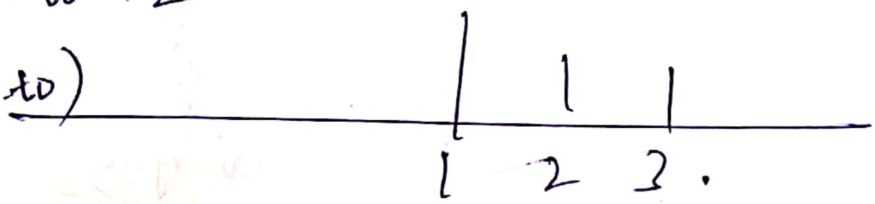
delay i/p t_0. $x(t)$ - replace $x(t - t_0)$.

$x(t)$

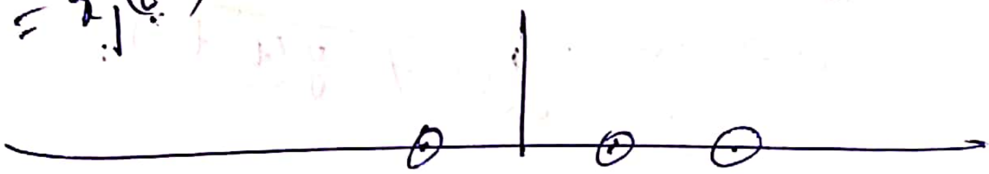


$t_0 = 1$

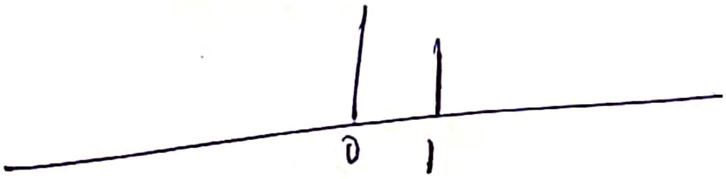
$x_1(t) = x(t - t_0)$



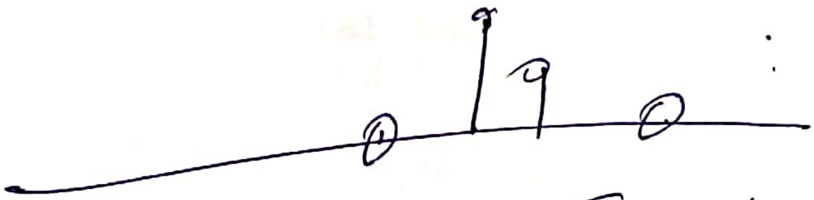
$y_1(t) = x_1(t^2)$



$y(t) =$



$y(t-1)$



$\therefore y(t) = x(t^2) \rightarrow$ Time Varying

2.14.

given. $x[n] = \alpha^n u[n]$.

Consider
$g[n] = x[n] - \alpha\, x[n-1]$.

$\therefore g(n) = \alpha^n u(n) - \alpha \cdot \alpha^{n-1} u[n-1]$

$g[n] = \alpha^n \cdot \delta[n]$.

Not needed.

again. given $\alpha = \frac{1}{2}$.

$y[n] \leftarrow x[n] * h[n] = \left(\frac{1}{2}\right)^n \left(u[n+2] - u[n-2]\right)$

Find $h[n]$.

$$y[n] = \left(\frac{1}{2}\right)^{n+2} u[n+2] \left(\frac{1}{2}\right)^{-2}$$

$$- \left(\frac{1}{2}\right)^{n-2} u[n-2] \left(\frac{1}{2}\right)^{2}.$$

$$y[n] = 4 \cdot x[n+2] - \frac{1}{4} x[n-2].$$

$$\therefore h[n] = 4\,\delta[n+2] - \frac{1}{4}\,\delta[n-2].$$

TH

M9

Properties of Linear Time-invariant Systems

(DSAA)

- def.

discrete: $x[n] \longrightarrow \boxed{h(n)} \longrightarrow y[n]$.

$$y[n] = \sum_k x(k)\, h[n-k] = x[n] * h[n].$$

Continuous: $x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = x(t) * h(t)$$

Point 1.

Only LTI systems can be completely characterized by an Impulse response.

e.g. S. $h[n] = \begin{cases} 1 & n = 0,1 \\ 0 & \text{otherwise} \end{cases}$.

If S is an LTI system then $h[n]$ is sufficient to characterize the i/p to o/p relation.

$$y(n) = \sum x(k)\, h(n-k) = \sum h(k)\, x(n-k)$$

$$= \sum_{k=0}^{1} x[n-k]$$

$\therefore\ y[n] = x(n) + x[n-1]$. $\longrightarrow$ Sys. eq.

TH + Ex

P(1) | date 04/09/2015 | LTI sys. prop | S & S Oppenheim | A.0 | 1103 | 929

(1)

Consider a non-linear system, that has same $h[n]$.

eg. (1)

$$y[n] = (x[n] + x[n-1])^2.$$

Put $x[n] = \delta[n]$.

$$h[n] = (\delta[n] + \delta[n-1])^2 = \delta[n] + \delta[n-1].$$

eg. (2) $y(n) = \max(x[n], x[n-1])$

$$x[n] = \delta[n]$$

$$h[n] = \max(\delta[n], \delta[n-1])$$

$$h[0] = \delta[0]$$

$$h[1] = \delta[1].$$

$$\therefore h[n] = \delta[n] + \delta[n-1].$$

$\therefore$ both S_1, S_2 have same $h[n]$;

hence $h[n]$ is insufficient to characterize a non-linear system.

2.3.1 Commutative Property

$$x[n] * h[n] = h[n] * x[n] = \sum h[k]\, x[n-k].$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

Let $n-k = m$. $k = n-m$.

$k = -\infty$, $m = \infty$

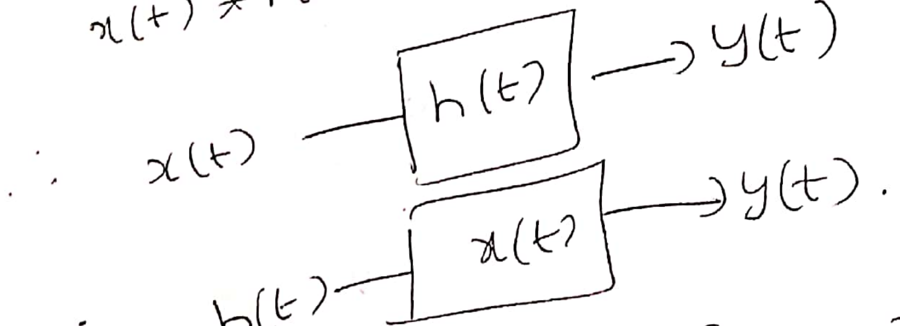
$k = +\infty$, $m = -\infty$

$$= \sum_{m=-\infty}^{\infty} x[n-m]\, h[m].$$

$$= h[n] * x[n].$$

Similarly

$$x(t) * h(t) = h(t) * x(t).$$



also

$$\int x(\tau)\, h(t-\tau)\, d\tau = -\int_{\infty}^{-\infty} x(t-s)\, h(s)\, ds.$$

$t - \tau = s.$

$\tau = s + t$

$$= \int_{-\infty}^{\infty} x(t-s)\, h(s)\, ds.$$

$$= h(t) * x(t).$$

3.3

Associative property

$$x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$$

L

LHS → $\sum_k x(k)\,(h_1(n-k) * h_2(n-k))$.

$h_1(m) * h_2(m)$

$= h_1(l)\, h_2(m-l)$.

$= \sum_k x(k) \sum_m h_1(m)\, h_2(n-k-m)$.

$n-k-m = l$.

$m = n-k-l$

$= \sum_k x(k) \sum_l h_1(n-k-l)\, h_2(l)$

$= \sum_l \underline{\sum_k x(k)\, h_1(n-l-k)} \cdot h_2(l)$.

$= \sum_l (x(n-l) * h_1(n-l))\, h_2(l)$ → $\sum w(n-l)\, h_2(l)$

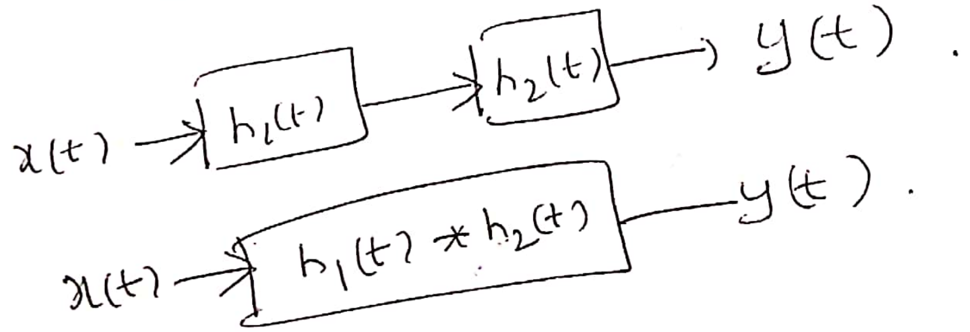
$w(n) * h_2(n)$

$= \underline{(x(n) * h_1(n)) * h_2(n)}$.

where

$w(n-l) = x(n-l) * h_1(n-l)$

$|||^{ly}$ $x(t) * (h_1(t) * h_2(t)) = (x(t) * h_1(t)) * h_2(t)$.



Since $x(n) * h(n) = h(n) * x(n)$

we have

$$x(n) * (h_1(n) * h_2(n)) = x(n) * (h_2(n) * h_1(n))$$

$$x[n] \longrightarrow \boxed{h_2[n] * h_1[n]} \longrightarrow y[n].$$

Illy $x(n) * (h_2(n) * h_1(n)) = (x(n) * h_2(n)) * h_1(n)$

$$x[n] \longrightarrow \boxed{h_2(n)} \longrightarrow \boxed{h(n)} \longrightarrow y(n).$$

Note: Order doesn't matter for $h_1(n)$, $h_2(n)$

Non-linear system.

$$x[n] \longrightarrow \boxed{(\cdot)^2} \longrightarrow \boxed{(\cdot)\times 4} \longrightarrow y[n]$$

$$\therefore y[n] = 4x^2[n]$$

$$x(n) \longrightarrow \boxed{\times 2} \longrightarrow \boxed{(\cdot)^2} \longrightarrow y[n]$$

$$x(n) \longrightarrow \boxed{(\ )^2} \longrightarrow \boxed{\times 2} \longrightarrow y_1[n]$$

$$y_1[n] = 2x^2[n].$$

Not Commutative.

S&S | A.O | P 108 | S 2.3.3 |

2.3.2

Distributive Property.

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

LHS $\rightarrow \sum x(k)(h_1(n-k) + h_2(n-k))$.

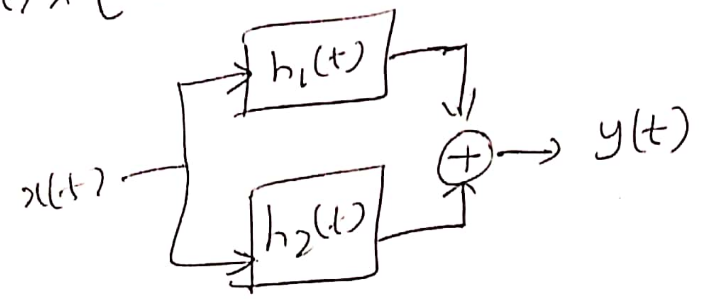
$= \sum x(k) h_1(n-k) + \sum x(k) h_2(n-k)$

$= x[n] * h_1[n] + x[n] * h_2[n]$.

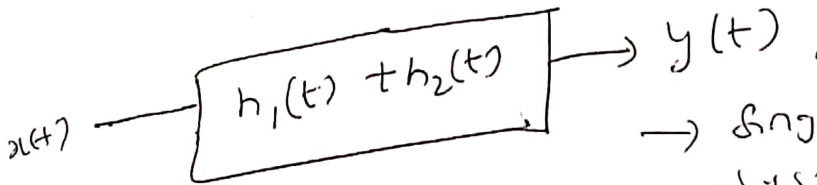
Similarly in continuous case

$$x(t) * (h_1(t) + h_2(t)) = x(t) * h_1(t) + x(t) * h_2(t)$$

$\therefore$



Parallel system

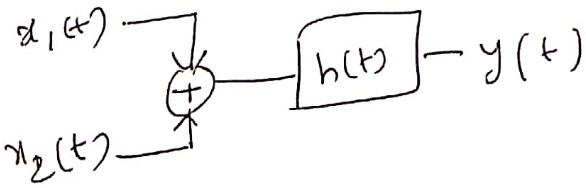


$\rightarrow$ Single system.

$$h(t) = h_1(t) + h_2(t).$$

Corollory $\rightarrow$

$$(x_1(t) + x_2(t)) * h(t) = x_1(t) * h(t) + x_2(t) * h(t).$$



10-Sept-2015 2.3.4. (4)

LTI systems with & without memory.

LTI system is memoryless.

if and only if $h[n] = 0$ for $n \neq 0$.

$\therefore$ $h[n] = A\,\delta[n]$. → impulse response.

for $n = 0$ $h[0] = A$.

Now apply convolution.

$$y[n] = \sum h(k)\, x(n-k) = \sum x(k)\, h(n-k)$$

$$= \sum_{k} x(k) \cdot A\,\delta(n-k).$$

$\therefore y[n] = A \cdot x[n]$ →

Now when $A = 1$.

$y[n] = x[n]$ → Identity system.

$$x[n] \to \boxed{\delta[n]} \to x[n].$$

ie; $x[n] * \delta[n] = x[n]$.

$$x[n] = \sum x(k)\, \delta(n-k)$$

↓

sifting property

S KS | A.0 | P108-109 | S 2.3.4

eg. 2.10

$$x[n] = \left(\tfrac{1}{2}\right)^n u[n] + 2^n u[-n].$$

$$h[n] = u[n].$$

Goal: Use distributive property to find $y(n)$.

Distribution.

$$x(n) * (h_1(n) + h_2(n)) = x(n) * h_1(n) + x(n) * h_2(n)$$

2nd prop

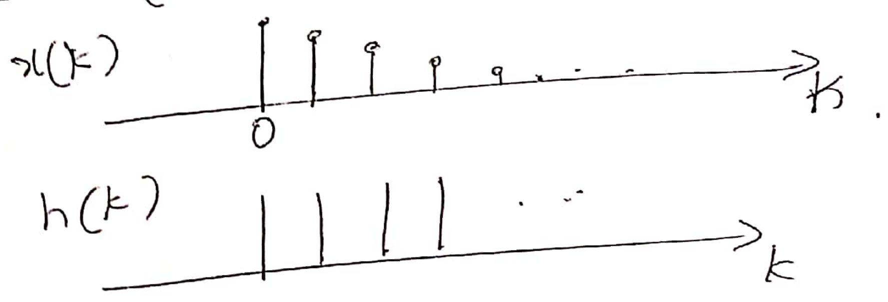
$$(x_1(n) + x_2(n)) * h(n) = x_1(n) * h(n) + x_2(n) * h(n).$$

Solution: $x(n) * h(n)$

$$= \left[\left(\tfrac{1}{2}\right)^n u(n) + 2^n u(-n)\right] * u[n].$$

$$= \left(\tfrac{1}{2}\right)^n u(n) * u(n) + 2^n u(-n) * u[n].$$

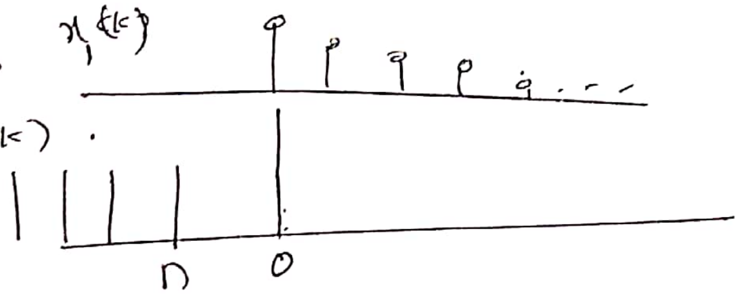
Term (1) $\left(\tfrac{1}{2}\right)^n u(n) * u(n).$



Ex 2.10.

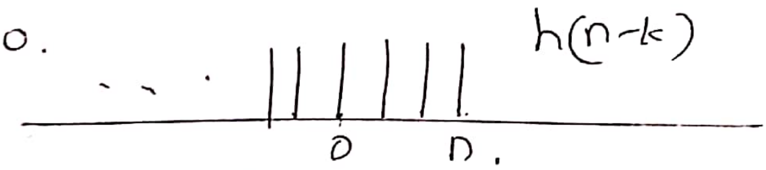
let $y_1(n) = \left(\frac{1}{2}\right)^n u(n) * u(n)$

$n < 0.$



$\therefore y_1(n) = 0 \quad n < 0$

$n \geq 0.$



$$\therefore y(n) = \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k = \frac{\left(\frac{1}{2}\right)^{n+1} - 1}{\frac{1}{2} - 1} = \frac{1 - \left(\frac{1}{2}\right)^{n+1}}{\frac{1}{2}}$$

$$= 2\left(1 - 2^{-1-n}\right)$$

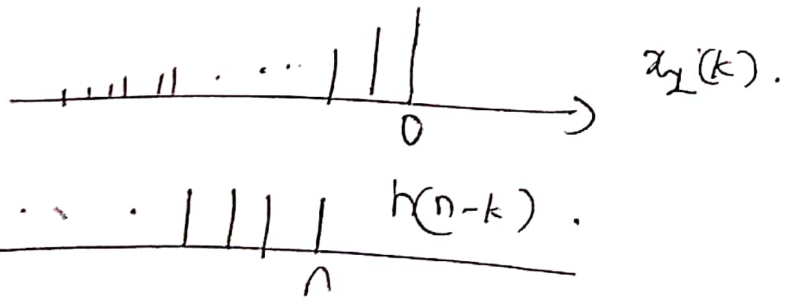
$$= 2 - 2^{-n}.$$

$$\therefore y_1(n) = \left(2 - 2^{-n}\right) u(n). \quad \text{— } (1)$$

Now

$y_2(n) = 2^n . u(-n) * u(n)$

$n < 0.$



S x5| A0| P106| eg 2.10 (2)|

[redacted]

Ex. 2.10

$$y_2(n) = \sum_{k=-\infty}^{n} 2^k$$

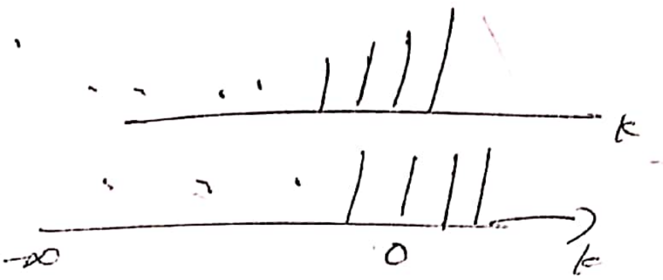
$$= \sum_{m=-\infty}^{0} 2^{m+n} = 2^k \cdot$$

$k - n = m$

$$= 2^n \cdot \sum_{l=0}^{\infty} 2^{-l} = 2^n \cdot \frac{1}{1-\frac{1}{2}} = 2^{n+1} \quad \rightarrow n \leq 0$$

$n \geq 0$

$h(n-k)$



$$y_2(n) = \sum_{k=-\infty}^{0} 2^k = 2.$$

$$\therefore y(n) = (2 - 2^{-n})\, u(n) + 2^{n+1} u(-n-1) + 2u(n)$$

$$= \cancel{4u(n)}$$

$$= (4 - 2^{-n})\, u(n) + 2^{n+1} u(-n-1)$$

2.3 Exercises | S x S oppen. |

1(2)
20/9/15

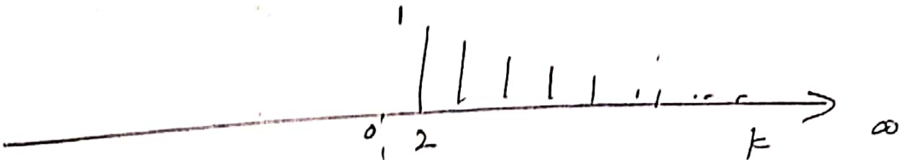
(DSAA)

(M4)

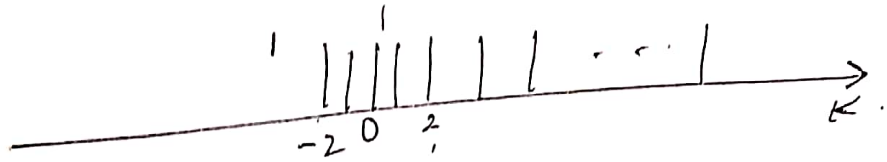
$x(n) = \left(\frac{1}{2}\right)^{n-2} u(n-2)$

$h(n) = u(n+2)$.

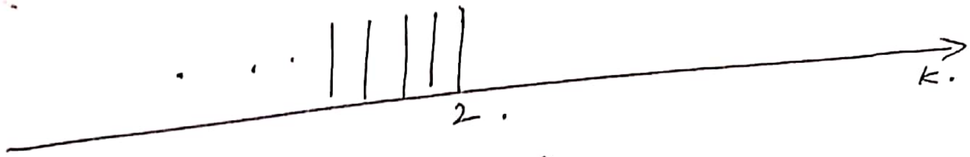
$x(k)$



$h(k)$



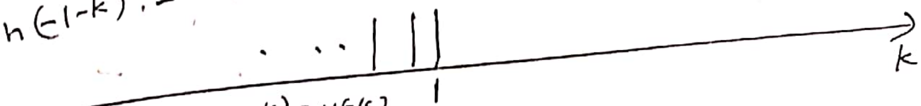
$h(-k)$



$h(-k) = u(2-k)$

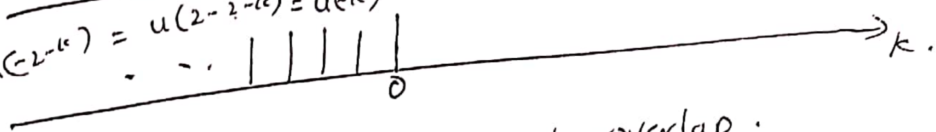
$h(-1-k) = u(2-1-k) = u(1-k)$

$n < 0$.



$h(n-k)$.

$h(-2-k) = u(2-2-k) = u(-k)$

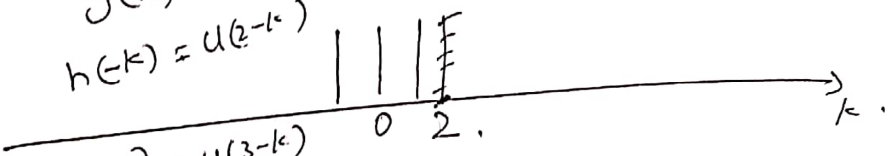


as No-overlap.

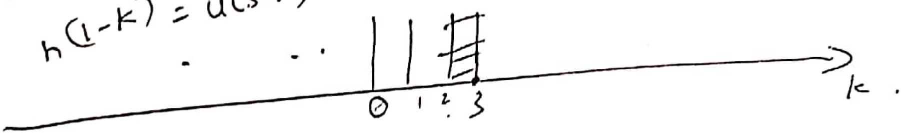
$\therefore n < 0 \quad y(n) = 0$

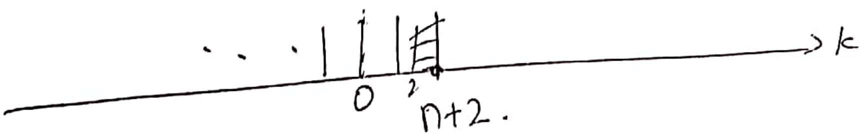
$0 \le n < 2$.

$h(-k) = u(2-k)$



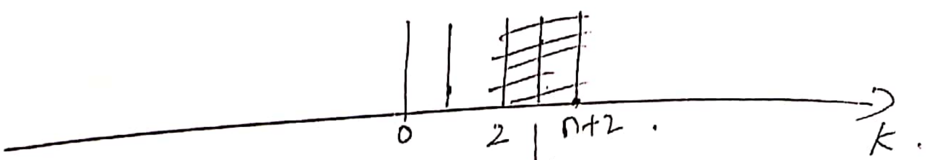
$h(1-k) = u(3-k)$





$h(n-k)$
$= h(n-(n+2))$
$= h(-2)$

$n \ge 2$



Overlap.

$h(n-k)$
$= h(n-(n+2)) = h(-2)$
$= u(0)$.

(Ex)

S x S | A.O | P138 | ex 2.3 |

$n \geq 0$

$$y(n) = \sum_{k=2}^{n+2} \left(\frac{1}{2}\right)^{k-2}.$$

$k-2 = m.$

$m \to 0.$

$n+2-2 = m$

$$= \sum_{m=0}^{n} \left(\frac{1}{2}\right)^{m}.$$

$$= \frac{1-\left(\frac{1}{2}\right)^{n}}{1-\frac{1}{2}} = 2\left(1-\left(\frac{1}{2}\right)^{n}\right).$$

$$\boxed{\therefore\ y(n) = 2\left(1-\left(\frac{1}{2}\right)^{n}\right) u(n)}$$

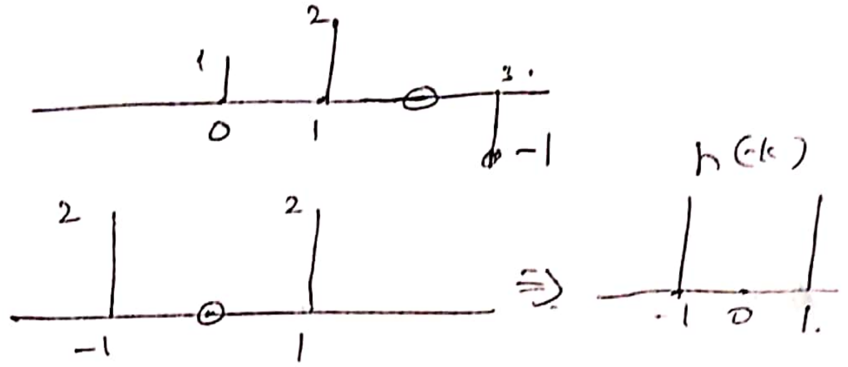
2.1. Given $x(n) = \delta[n] + 2\delta[n-1] - \delta[n-3]$

$$h[n] = 2\delta[n+1] + 2\delta[n-1].$$

(a). $y_1(n) = x(n) * h(n)$.

$x(k)$

$h(k)$



$n=0$. $y(0) = \sum x(k)\, h(-k)$

$= x(0)\,h(0) + x(1)\,h(-1) + \cancel{x(3)\,(0)}$.

$= 0 + 2 \cdot 2 = 4$.

$\sum x(k)\, h(-1-k)$

$n=-1$ $\quad y(-1) = x(0)\,h(-1) + x(1)\,(0)$.

$= 1 \cdot 2 = 2$.

$\sum x(k)\, h(-2-k)$

$n=-2$ $\quad y(-2) = x(0)\,(0) + x(1)\,h(-3) = 0$.

$\sum x(k)\, h(1-k)$

$n=1$ $\quad y(1) = x(0)\,h(1) + x(1)\,h(0) + x(3)\,h(-2)$

$= 1 \cdot 2 + 2 \cdot 0 = 2$.

$n = 2.$ $\quad \sum x(k)\, h(2-k)$

$y(2) = x(0)\, h(2) + x(1)\, h(1) + x(2)\, h(0)$
$\quad + x(3) \cdot h(-1)$

$= 0 + 2 \cdot 2 + 0 + (-1)\, 2 = \underline{\underline{2}}.$

$n = 3$ $\quad \sum x(k)\, h(3-k)$

$y(3) = x(0)\, h(3) + x(1)\, h(2)$
$\quad + x(2)\, h(1) + x(3)\, h(0)$
$\quad = (-1) \cdot 0 = 0.$

$n = 4$ $\quad \sum x(k)\, h(4-k)$

$y(4) = x(0)\, h(4) + \cdots + x(3)\, h(1)$
$\quad = (-1)\, 2 = -2.$

$n = 5$ $\quad \sum x(k)\, h(5-k)$

$y(5) = x(3)\, h(2) = \underline{\underline{0}}.$

$\therefore y(n) = \{ 0, 2, \underset{\uparrow}{4}, 2, 2\ 0\ -2, 0 \}$

M9 **Unit step response of LTI :**

$u(n) \to \boxed{LTI} \to s(n)$ | $u(t) \to \boxed{h(t)\ LTI} \to s(t)$.

$s(n) = u(n) * h(n)$.

$\therefore s(n) = \sum_{k=-\infty}^{\infty} u(k)\, h(n-k) = \sum_{k=0}^{\infty} h(n-k)$. $n-k = m$

$\therefore \boxed{s(n) = \sum_{m=-\infty}^{n} h(m)}$. — ①. $\to$ Unit step resp.

$s(n) \to$ ~~o/p of~~ accumulator of $h(n)$ (upto to n).

or. $s(n) = u(n) * h(n) = h(n) * u(n)$.

$h(n) \to \boxed{u(n)} \to s(n)$.

$s(n) = \sum_{k=-\infty}^{\infty} h(k)\, u(n-k) = \sum_{k=-\infty}^{n} h(k)$.

$\longrightarrow$ as $n \geq k$.

$\therefore s(n) \to$ o/p of system with Impulse resp $\to u(n)$
i/p $= h(n)$ ✓

Recover $h(n)$ from $s(n)$.

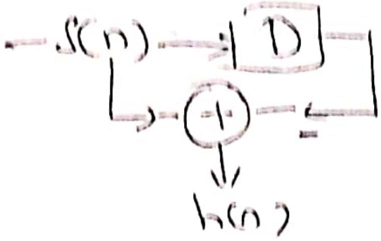
from ① $s(n) = \sum_{m=-\infty}^{n} h(m)$

$= \sum_{m=-\infty}^{n-1} h(m) + h(n)$.

from ① $s(n-1) = \sum_{m=-\infty}^{n-1} h(m)$

$\therefore \; h(n) = s(n) - s(n-1)$ → first diff.

ie, $h(n)$ → o/p of diff. between $s(n)$ & $s(n-1)$
(step resp at n & $n-1$.)



In continuous time.

$$u(t) \longrightarrow \boxed{h(t)} \longrightarrow s(t)$$

$$s(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau = \int_{-\infty}^{t} h(\tau)\, d\tau$$

→ Integrator

& $h(t) = \frac{d}{dt} s(t) = s'(t)$ → differentiator.

eg. 2.13

Consider a time shift. $h(n) \to \delta(n-n_0)$.

$$x(n) \to \boxed{D_{3/n_0}} \to x(n-n_0)$$

Stability condition. $\sum_{n} |h(n)| = \sum_{n} |\delta(n-n_0)| = 1 < \infty$

$\therefore$ $\boxed{\text{delay}}$ $\to$ Stable sys.

$$x(n) \to \boxed{+n_0} \to x(n+n_0)$$

advance

$\therefore \sum_{n} |h(n)| = \sum_{n=-\infty}^{\infty} |\delta(n+n_0)| = 1 < \infty$

$\boxed{\text{advance}}$ $\to$ Stable

Accumulator.

$$x(n) \to \boxed{u(n)} \to y(n)$$

$\hookrightarrow$ accum.

$h(n) = u(n)$

$$\sum_{n} |h(n)| = \sum_{n} |u(n)| = \sum_{n=0}^{\infty} (1) = \infty.$$

$\therefore$ $\boxed{\text{accum.}}$ $\to$ not stable.

|||ly $h(t) = u(t)$

$$\int |h(t)| dt = \int |u(t)| dt = \infty$$

Integrator $\to$ unstable.

$$y(t) = \int_{-\infty}^{t} x(\tau) d\tau$$

$x(\tau) \to \delta(\tau)$

$\therefore h(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$

$\therefore$ $h(t) = u(t)$

S x S | A.O | P114 | eg 2.13 |

LTI → with $h(n) = u(n)$.

o/p $y(n) = \sum h(k)\, x(n-k) = \sum x(k)\, h(n-k)$.

$= \sum_{-\infty}^{\infty} u(k)\, x(n-k)$

$y(n) = \sum_{k=0}^{\infty} x(n-k)$. $n-k=m \Rightarrow k=0 \to m=n$, $k=\infty$ $m=-\infty$

$y(n) = \sum_{m=-\infty}^{n} x(m) \longrightarrow$ accumulator.

Inverse. $y(n) = \sum_{m=-\infty}^{n-1} x(m) + x(n)$

$\therefore x(n) = y(n) - y(n-1)$.

i.e. $w(n) = y(n) - y(n-1)$

$\therefore h_1(n) = \delta(n) - \delta(n-1) \longrightarrow$ difference sys.

$h(n) = u(n)$, $h_1(n) = \delta(n) - \delta(n-1)$

$\therefore h(n) * h_1(n) = u(n) * (\delta(n) - \delta(n-1))$

$= u(n) * \delta(n) - u(n) * \delta(n-1)$

$\sum \delta(k)\, u(n-k)$
$= u(n)$.

$u(n-1)$

$\sum \delta(k-1)\, u(n-k)$
$= u(n-1)$.

$\sum u(k)\, \delta(n-1-k)$
$= u(n-1)$.

$\therefore h(n) * h_1(n) = u(n) - u(n-1)$
$= \delta(n)$

2.3.6 | Causality in LTI | S & S - Oppenheim | 14/9/2015 | pg. 112 | Theory

Systems → Causal only if $y(n)$ depends on $x(n), x(n-1), \ldots$ Not on $x(n+1)$ etc.

For discrete LTI systems we have Causality iff

$$h(n) = 0 \quad \text{for } n < 0.$$

$$y(n) = x(n) * h(n) = h(n) * x(n)$$

$$= \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=0}^{\infty} h(k)\, x(n-k).$$

$n-k=m$

$k=n-m.$

$$= \sum_{m=-\infty}^{n} x(m)\, h(n-m).$$

Causal.

o/p $y(n)$ depends only on i/ps upto $m=n$.

eg:- $y(n) = 2x(n) + 3.$

(i) $x(n) = 0 \Rightarrow y(n) = 3 \neq 0.$ Violates Initial Rest

zero in → zero out prop

∴ Not linear.

(ii) depends only on $x(n)$ → Causal.

exercise. $y(t) = \int_{-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau \rightarrow \overset{S.T}{=} \int_{0}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$

Classroom exercise.

S&S | A.0 | P112 | S2.3.6 |

$x(n) \rightarrow \boxed{h(n)} \rightarrow y(n)$

$|x(n)| < B$ & $|y(n)| < B' < \infty$ — stable. $\forall n$.

For LTI   if $|x(n)| < B$

$$y(n) = \sum_k x(k)\, h(n-k).$$

$$|y(n)| = \left| \sum_k x(k)\, h(n-k) \right|$$

$$\leq \sum_k |x(k)\, h(n-k)| \qquad |z_1 z_2| = |z_1||z_2|$$

$$= \sum_k |x(k)| \cdot |h(n-k)| \qquad \text{but } x(k) < B$$

$$|y(n)| \leq B \cdot \sum_k |h(n-k)|.$$

$\therefore |y(n)| \leq B'$ if $\sum_k |h(n-k)| = \sum_m |h(m)| < \infty$

$\therefore$ LTI is stable iff $\{h(m)\}$ is absolutely summable.

$$\therefore \boxed{\sum_k |h(k)| < \infty} \rightarrow \text{stable discrete LTI system.}$$

Simply in continuous time

$$\boxed{\int h(t)\, dt < \infty} \rightarrow \text{Stable Continuous LTI system}$$