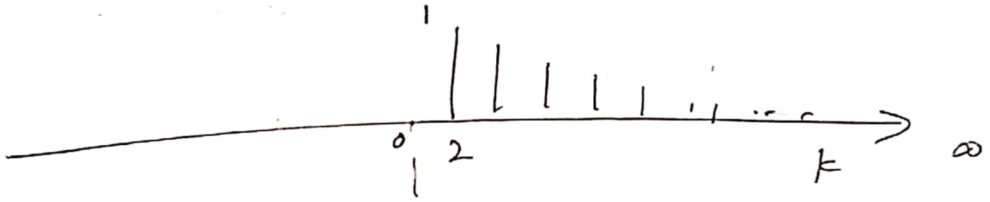
2.3 Exercises | S & S oppen. | 20/9/15

(M4) (DSAA)

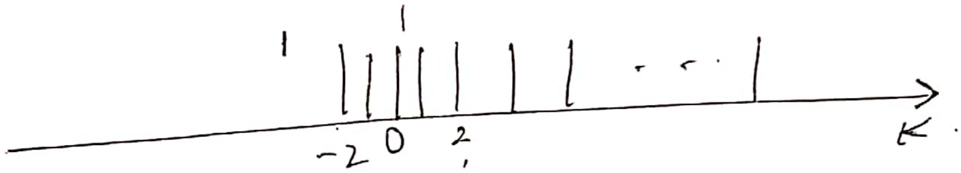
$x(n) = \left(\frac{1}{2}\right)^{n-2} u(n-2)$

$h(n) = u(n+2)$.

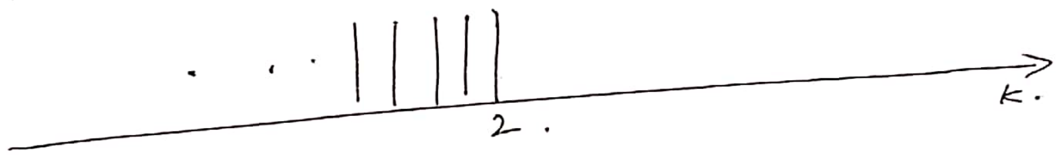
$x(k)$



$h(k)$



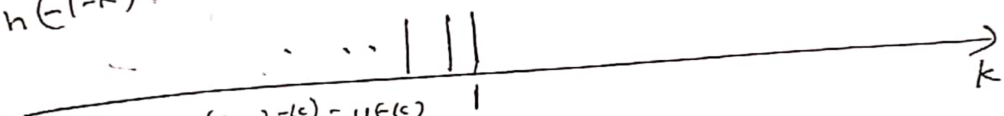
$h(-k)$



$h(-k) = u(2-k)$

$h(-1-k) = u(2-1-k) = u(1-k)$

$n < 0$.



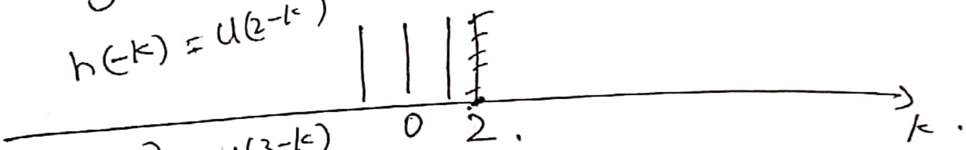
$h(n-k)$:

$h(-2-k) = u(2-2-k) = u(-k)$

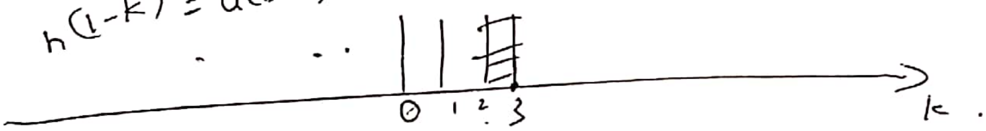
$\therefore n < 0 \quad y(n) = 0 \quad \because$ No-overlap.

$0 \leq n < 2$.

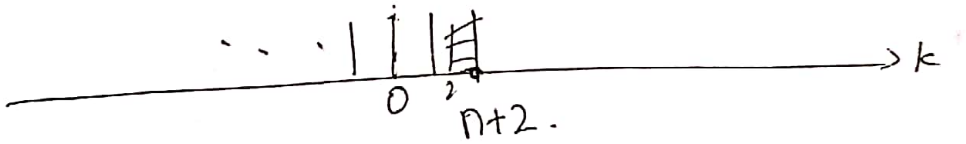
$h(-k) = u(2-k)$



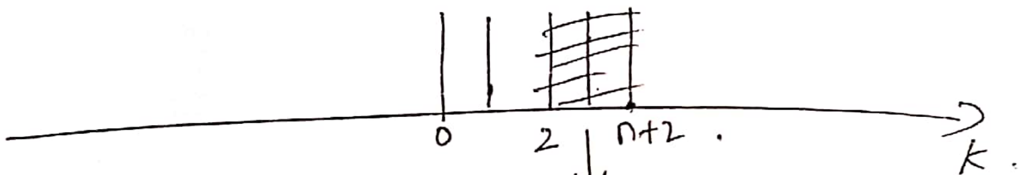
$h(1-k) = u(3-k)$



$h(n-k)$
$= h(n-(n+2))$
$= h(-2)$



$n \geq 2$



Overlap.

(Ex) $h(n-k)$
$= h(n-(n+2)) = h(-2)$
$= u(0)$.

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$n \geq 0$

$$y(n) = \sum_{k=2}^{n+2} \left(\frac{1}{2}\right)^{k-2}.$$

$k-2=m.$

$m \to 0.$

$n+\cancel{2}-\cancel{2}=m$

$$= \sum_{m=0}^{n} \left(\frac{1}{2}\right)^{m}.$$

$$= \frac{1-\left(\frac{1}{2}\right)^{n}}{1-\frac{1}{2}} = 2\left(1-\left(\frac{1}{2}\right)^{n}\right).$$

$$\boxed{\therefore y(n) = 2\left(1-\left(\frac{1}{2}\right)^{n}\right) u(n)}$$

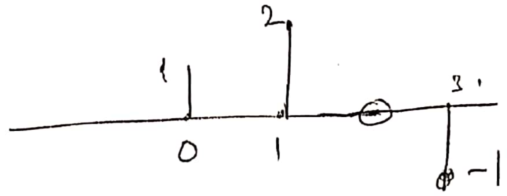
Exercise. 2.1. S & S. Oppenheim (Pg. 137). ①

2.1. Given $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$

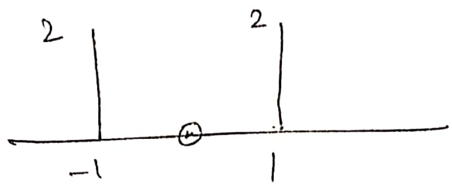
$h[n] = 2\delta[n+1] + 2\delta[n-1]$.

(a). $y_1(n) = x[n] * h(n)$.

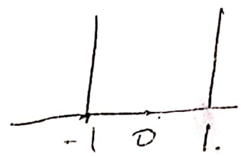
$x(k)$



$h(k)$



$\Rightarrow$ $h(-k)$



$n=0$. $y(0) = \sum x(k)\, h(-k)$

$= x(0)h(0) + x(1)h(1) + \cancel{x(3)(0)}$.

$= 0 + 2 \cdot 2 = 4$.

$\sum x(k)\, h(-1-k)$

$n=-1$ $y(-1) = x(0)h(-1) + x(1)(0)$.

$= 1 \cdot 2 = 2$.

$\sum x(k) h(-2-k)$

$n=-2$ $y(-2) = x(0)(0) + x(1)h(-3) = 0$.

$\sum x(k) h(1-k)$

$n=1$ $y(1) = x(0)h(1) + x(1)h(0) + x(3)h(-2)$

$= 1 \cdot 2 + 2 \cdot 0 = 2$.

$n=2$. $\quad \sum x(k)h(2-k)$

$y(2) = x(0)h(2) + x(1)h(1) + x(2)h(0)$
$+ x(3) \cdot h(-1)$

$= 0 + 2 \cdot 2 + 0 + (-1)2 = \underline{\underline{2}}.$

$n=3 \quad \sum x(k)h(3-k)$

$y(3) = x(0)h(3) + x(1)h(2)$
$+ x(2)h(1) + x(3)h(0)$
$= (-1) \cdot 0 = 0.$

$n=4 \quad \sum x(k)h(4-k)$

$y(4) = x(0)h(4) + \cdots + x(3)h(1)$
$= (-1)2 = -2.$

$n=5 \quad \sum x(k)h(5-k)$

$y(5) = x(3)h(2) = \underline{\underline{0}}.$

$\therefore y(n) = \{0, 2, \underset{\uparrow}{4}, 2, 2, 0, -2, 0\}$

3

3.

0

ex. 2.11

(a) $x(t) = u(t-3) - u(t-5).$

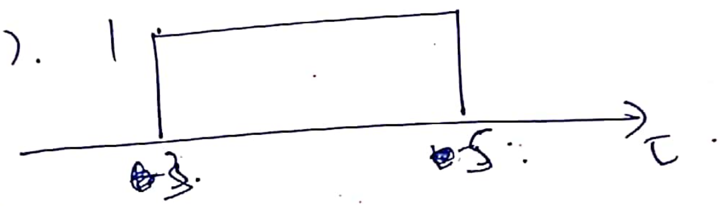
$h(t) = e^{-3t} u(t).$

$$y(t) = x(t) * h(t)$$

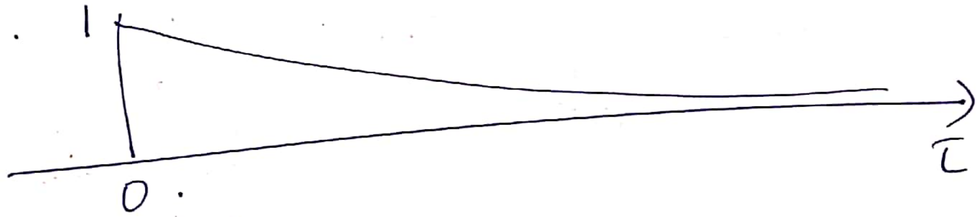
$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) \, d\tau$$

$$= \int_{-\infty}^{\infty} \big(u(\tau-3) - u(\tau-5)\big) \cdot e^{-3(t-\tau)} u(t-\tau) \, d\tau.$$

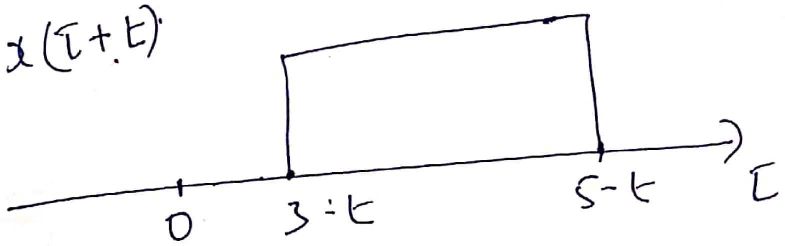
$x(\tau).$



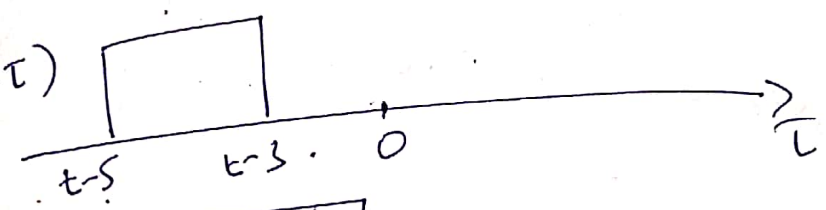
$h(\tau).$



$t<0.$ $x(\tau+t)$



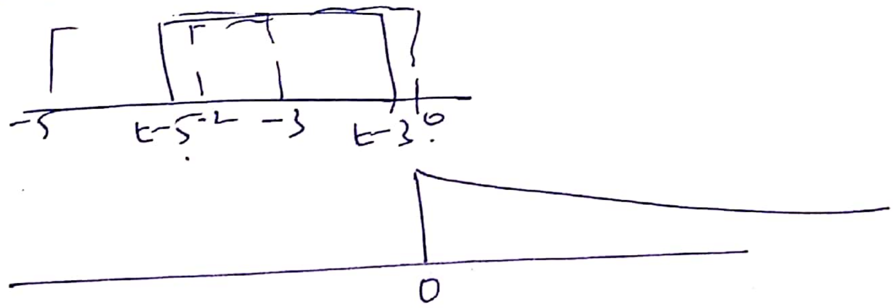
$x(t-\tau)$



$\therefore \boxed{y(t) = 0.}$ $t<0.$

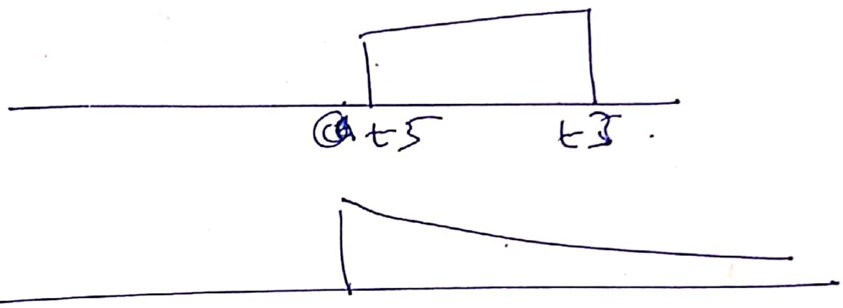
$0 < t < 3.$

$x(t-\tau)$



$$\boxed{y(t) = 0} \quad \text{for } 0 < t < 3.$$

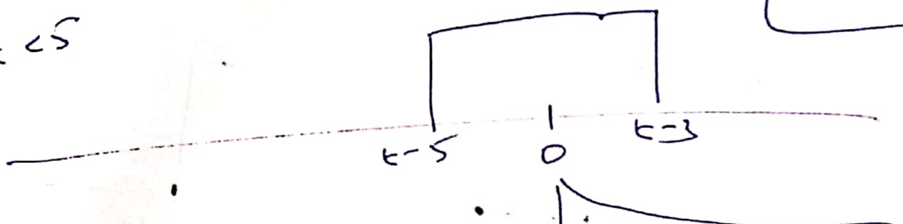
~~3 < t < 5~~ . $t > 5.$



$$y(t) = \int_{t-5}^{t-3} e^{-3\tau}\, d\tau = \boxed{\frac{e^{9-3t}}{3}(e^{6}-1)} \quad \boxed{t > 5}$$

~~t < 5~~

$3 < t < 5$



$$y(t) = \int_{0}^{t-3} e^{-3\tau}\, d\tau = \left.\frac{e^{-3\tau}}{-3}\right|_{0}^{t-3} = \frac{e^{-3(t-3)} - 1}{-3}.$$

$$\boxed{y(t) = \frac{1}{3}\left[1 - e^{9-3t}\right] \quad 3 < t < 5.}$$

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