MI

Sampling (Exercises)

DSAA

~~ex. 1.1~~

$x(t) \rightarrow$ real valued signal

$\omega_s = 10{,}000\pi$ rad/s. (Sampling rate)

Q: What values of ω is $X_r(\omega) = 0$ for sure?

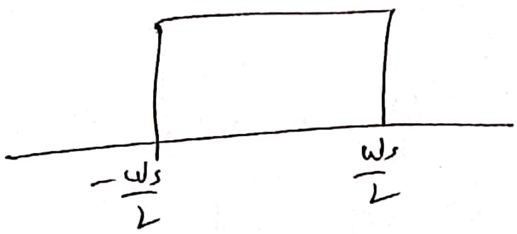
Sol: for $|\omega| > \frac{\omega_s}{2} = 5{,}000\pi$ rad/s.

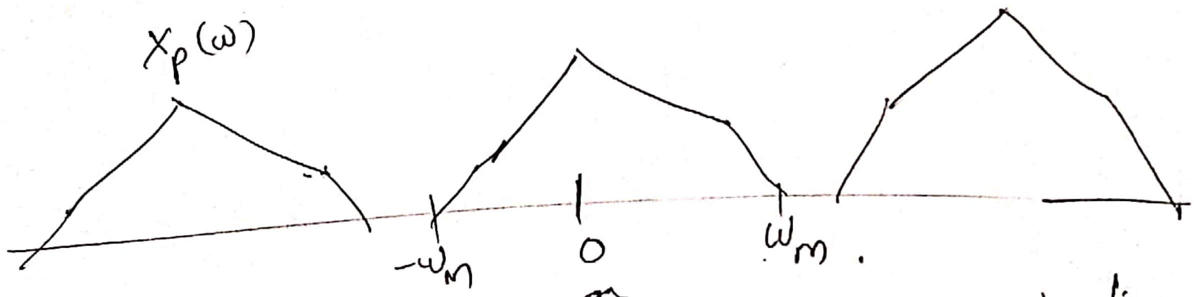
Details:

$$X_p(\omega) = H(\omega) X_p(\omega).$$

$\omega_M = \frac{\omega_s}{2}$.

H(ω)





reconstruction.

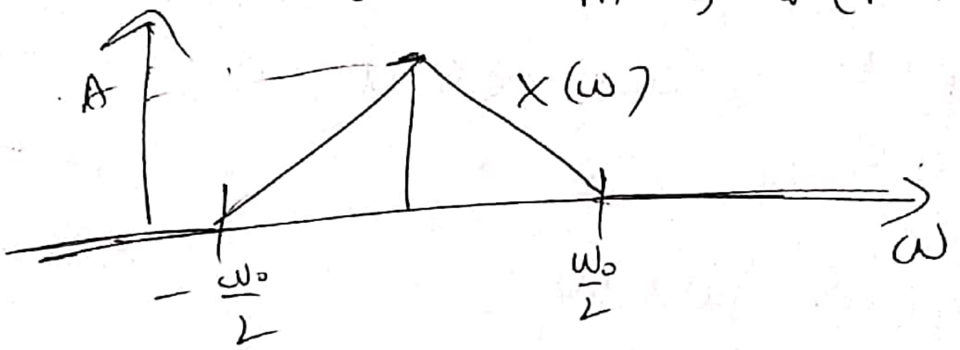
$$X_r(\omega) = \begin{cases} X(\omega) & |\omega| < 5{,}000 \text{ rad/s} \\ 0 & |\omega| > 5{,}000 \text{ rad/s.} \end{cases}$$

EP

Sampling Theorem.

Given $x(t)$ with Nyquist rate ω_0.

$\therefore \omega_0 = 2\omega_m \rightarrow \omega$ (max).



Find N.R. for

(i) $x(t) + x(t-1)$

Let $x_1(t) = x(t) + x(t-1)$

Based on time shifting property

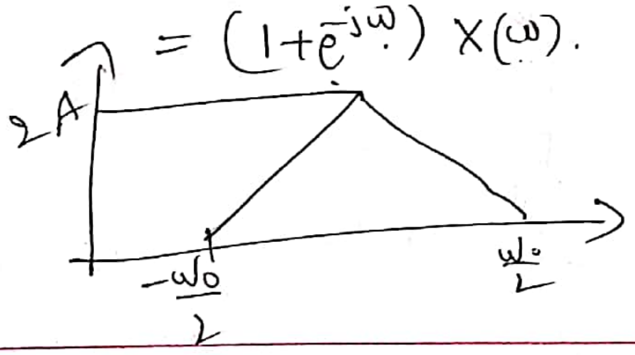
$x(t) \longleftrightarrow X(\omega)$

then $x(t-t_0) \longleftrightarrow e^{-j\omega t_0} X(\omega)$.

$X_1(\omega) = F\{x_1(t)\}$

$= X(\omega) + e^{-j\omega} \cdot X(\omega)$

$= (1+e^{-j\omega}) X(\omega)$.

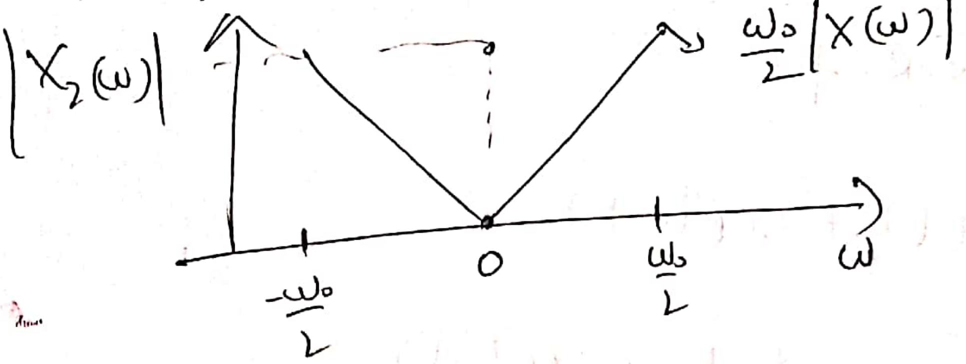


$\therefore \boxed{NR = \omega_0}$

(b) $x_2(t) = \frac{d x(t)}{dt}$.

$$\frac{d x(t)}{dt} \longleftrightarrow j\omega X(\omega)$$

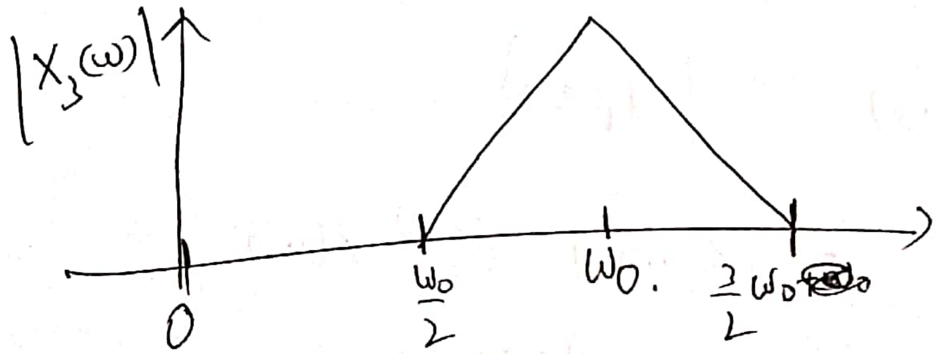
$\therefore X_2(\omega) = j\omega X(\omega)$



$\therefore \boxed{NR_2 = \omega_0}$

(c) $x_3(t) = x(t)\, e^{j\omega_0 t}$

$X_3(\omega) = X(\omega - \omega_0)$.



$\boxed{NR = \omega_0}$

Sampling (DSAA)

Given $s(t) = x_1(t) \cdot x_2(t)$

Find N.R. for $s(t)$.

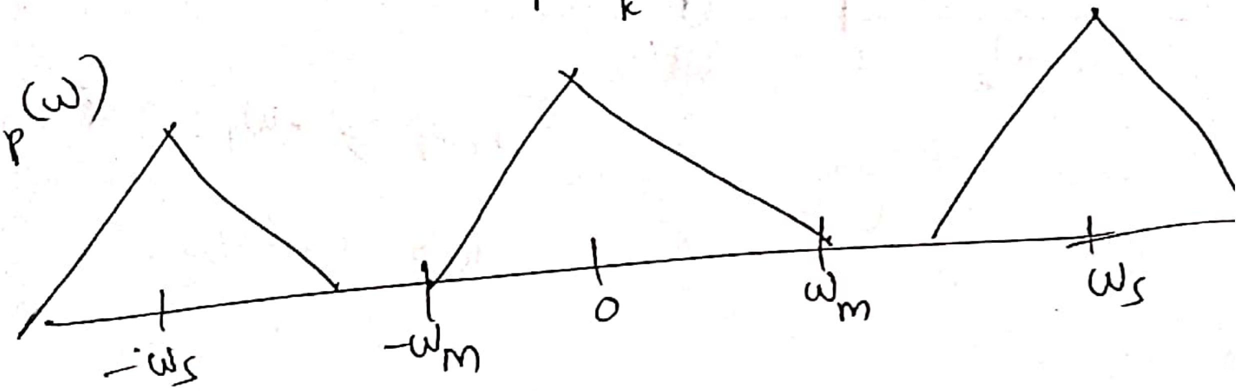
$$S_p(t) = p(t)\,s(t)$$

$$= p(t)\,\underbrace{\boxed{x_1(t)\,x_2(t)}}_{s(t)}$$

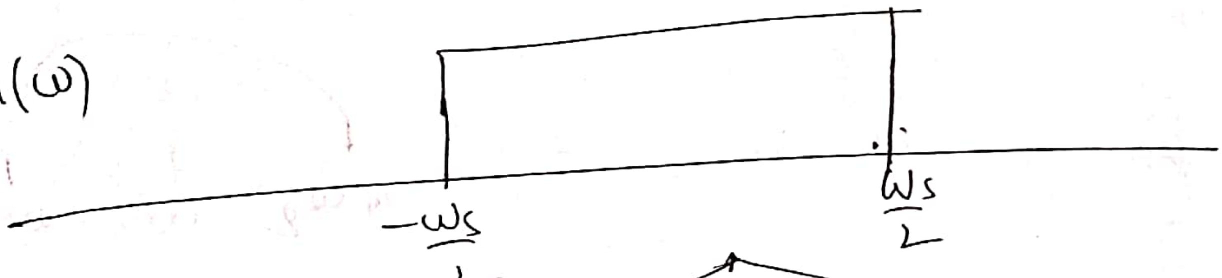
$$S_r(\omega) = H(\omega) \cdot S_p(\omega).$$

$$S_p(\omega) = \frac{1}{T} \sum_k S(\omega - k\omega_s).$$

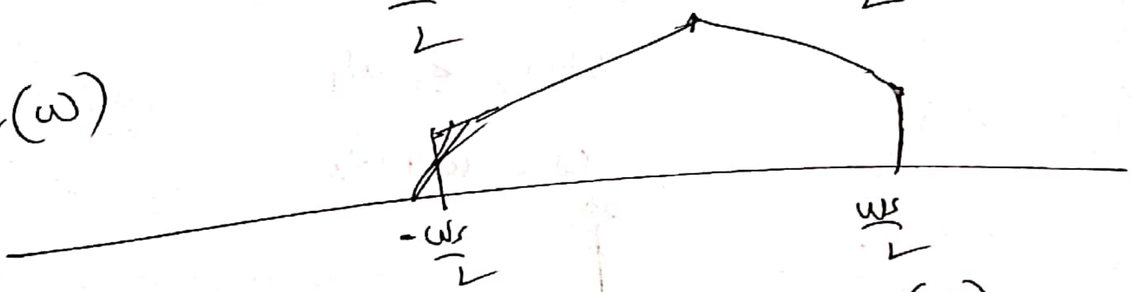
$S_p(\omega)$



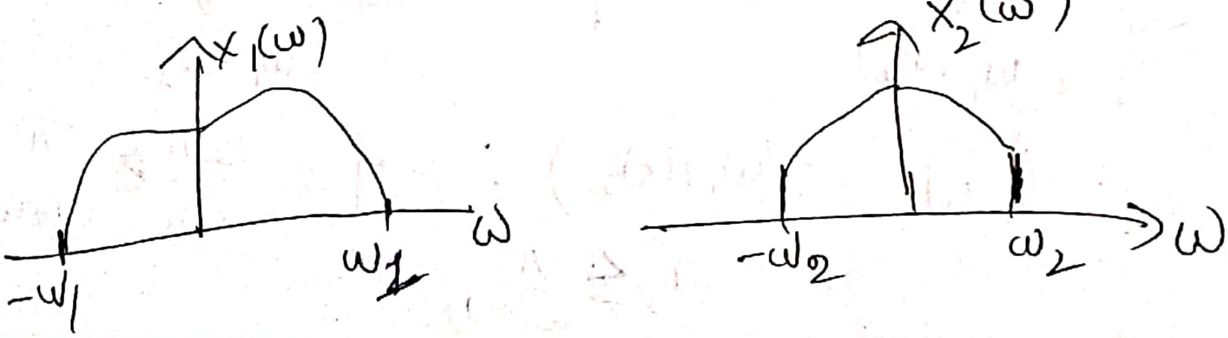
$H(\omega)$



$S_r(\omega)$



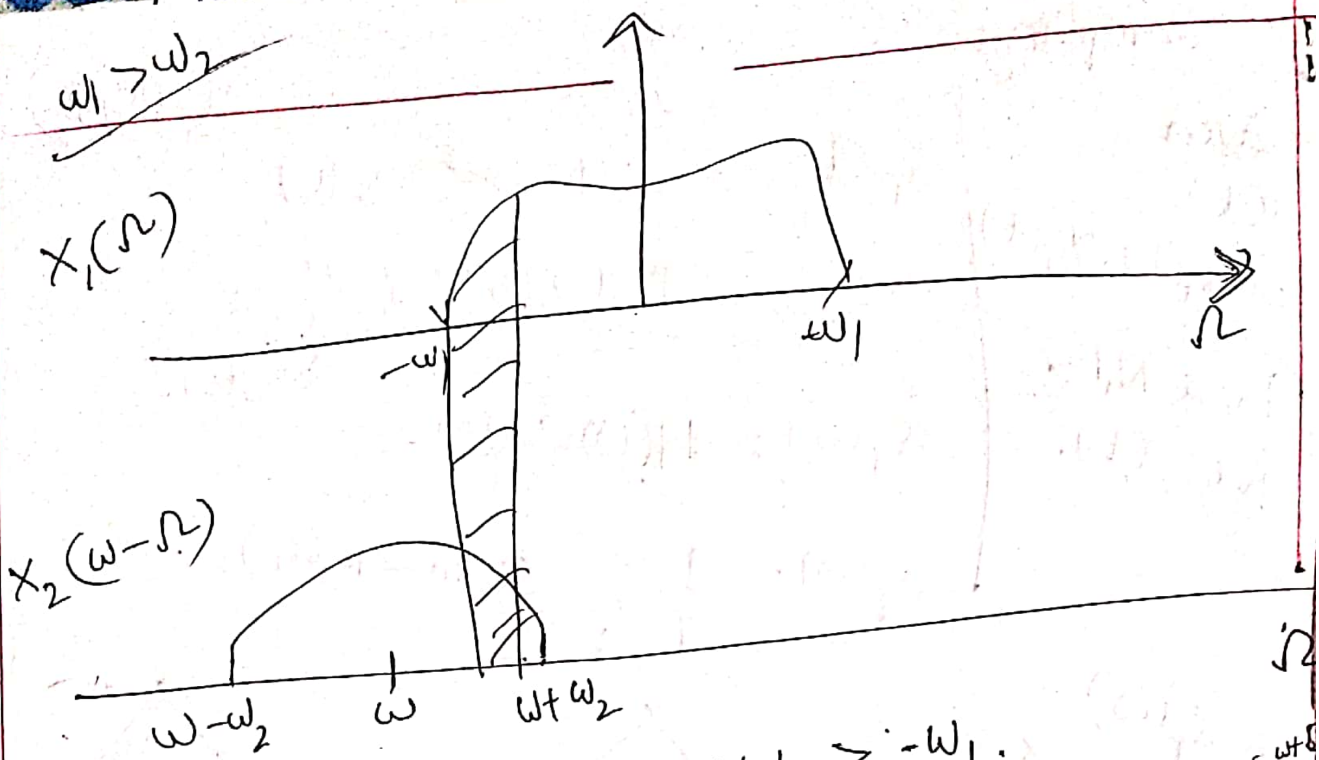
$$S(\omega) = X_1(\omega) * X_2(\omega).$$



EX | Text: A.O. (SxS) ex. 7.6 |

Simple eg.

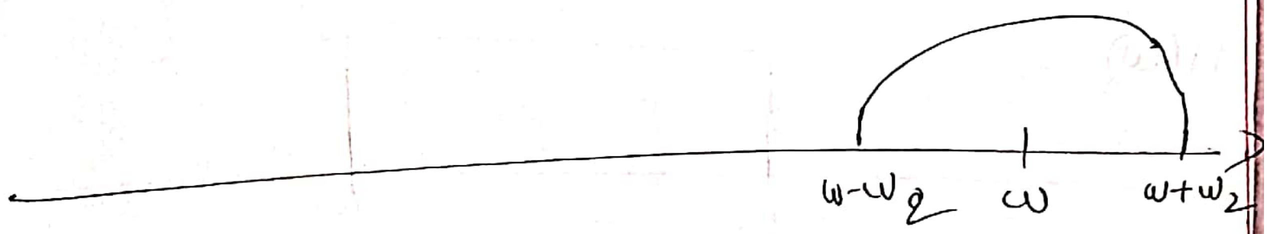
~~$\omega_1 > \omega_2$~~



$$\omega+\omega_2 \geq -\omega_1.$$

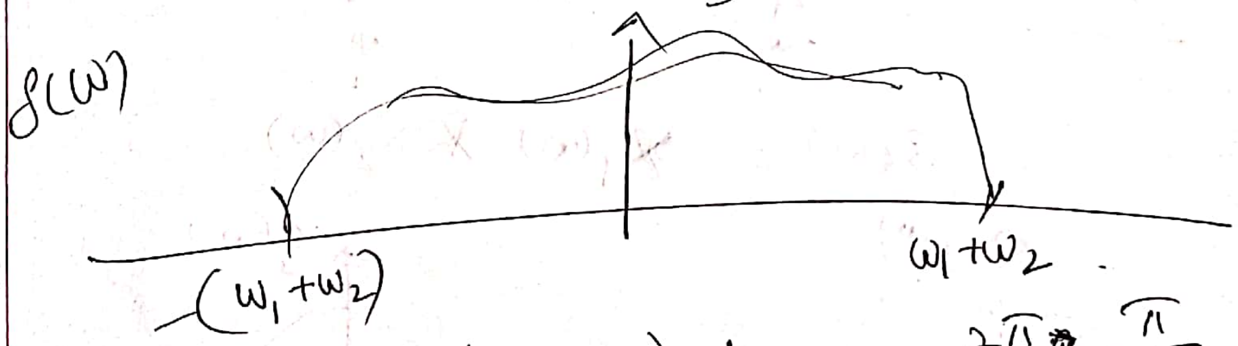
(1)

$$\omega \geq -\omega_1 - \omega_2$$



$$\omega-\omega_2 \leq \omega_1$$

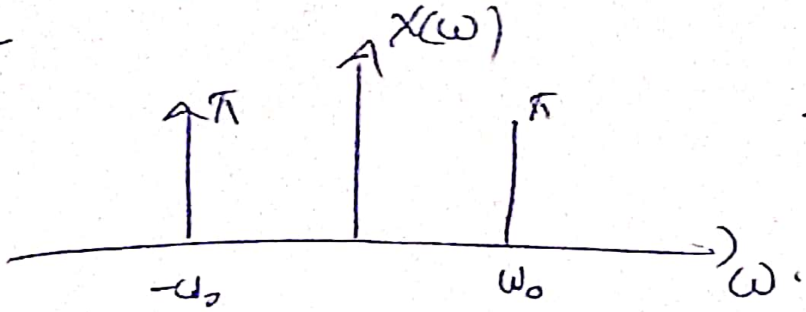
$$\omega \leq \omega_1+\omega_2$$



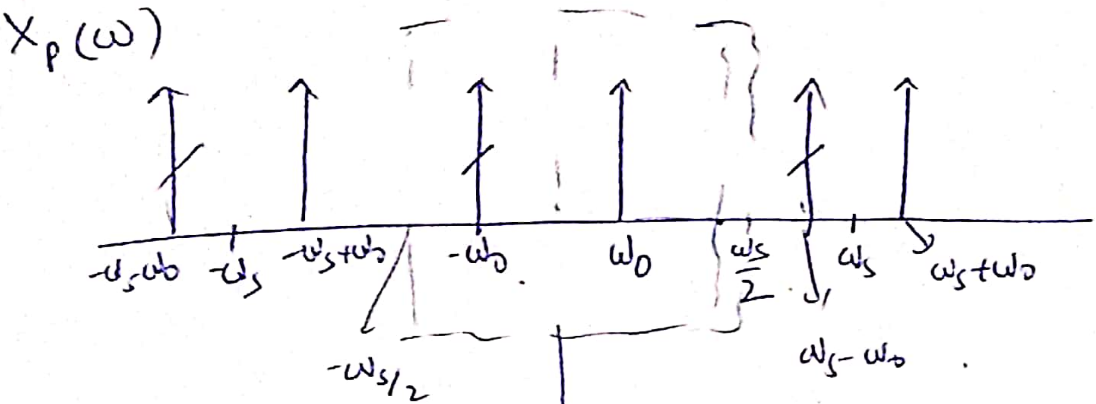
$$\therefore \omega_s = 2(\omega_1+\omega_2) \quad \therefore T = \frac{2\pi}{\omega_s} = \frac{\pi}{\omega_1+\omega_2}.$$

$$T_s \leq \frac{\pi}{\omega_1+\omega_2}$$

Simple eg.

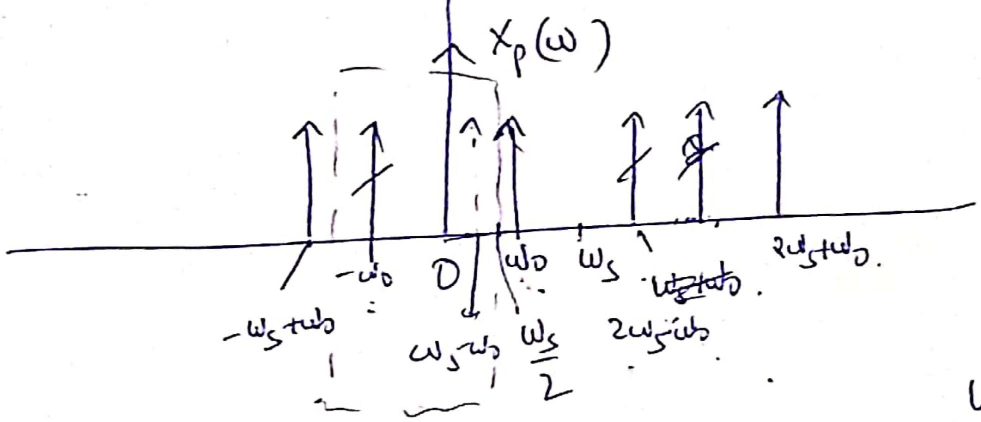


$\rightarrow \omega_s$ fn.



$\omega_s > 2\omega_0$

$\omega_s = 6\omega_0$



$\omega_s = \frac{3}{2}\omega_0$

$\frac{\omega_s}{2} = \frac{3}{4}\omega_0$

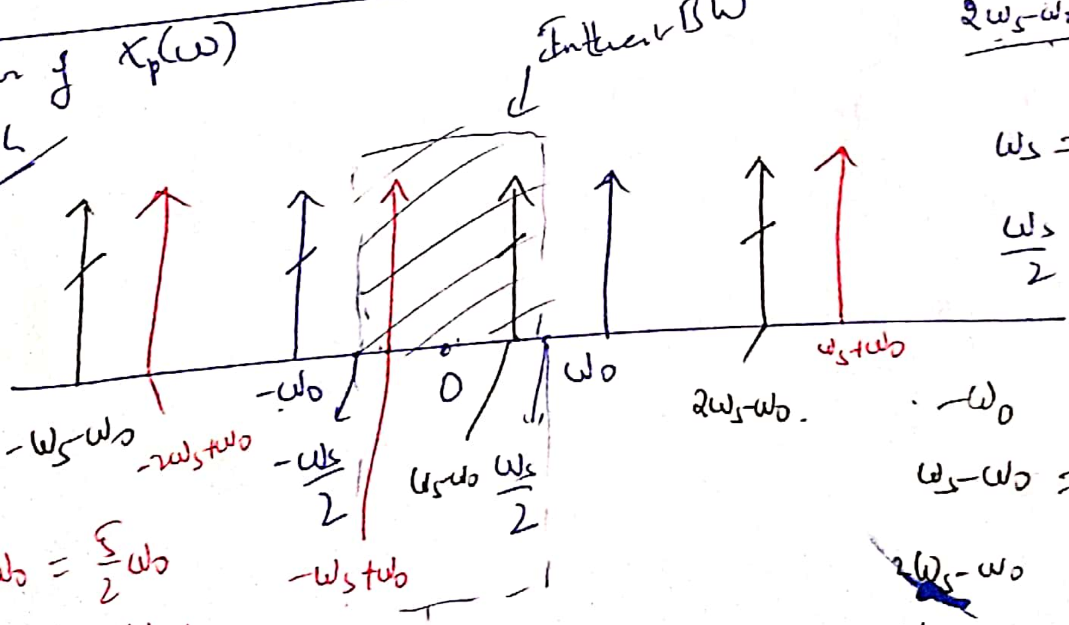
$\omega_s - \omega_0 = \frac{1}{2}\omega_0$

$\omega_s + \omega_0 = \frac{5}{2}\omega_0$

$2\omega_s - \omega_0 = 3\omega_0 - \omega_0 = 2\omega_0$

Construction of $X_p(\omega)$

ALIASING



$-\omega_0$

$\omega_s = \frac{3}{2}\omega_0$

$\frac{\omega_s}{2} = \frac{3}{4}\omega_0$

$\omega_s - \omega_0 = \frac{\omega_0}{2}$

$2\omega_s - \omega_0 = 2\omega_0$

$-\omega_s - \omega_0 = -\frac{5}{2}\omega_0$

$\omega_s + \omega_0 = \frac{5}{2}\omega_0$

$-\omega_s + \omega_0 = \frac{-1}{2}\omega_0$

$-2\omega_s + \omega_0 = -2\omega_0$

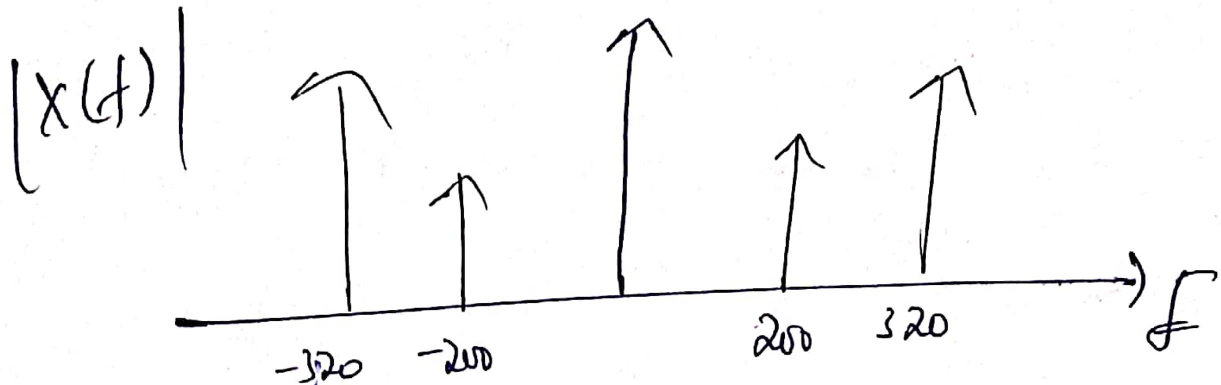
CW

TK

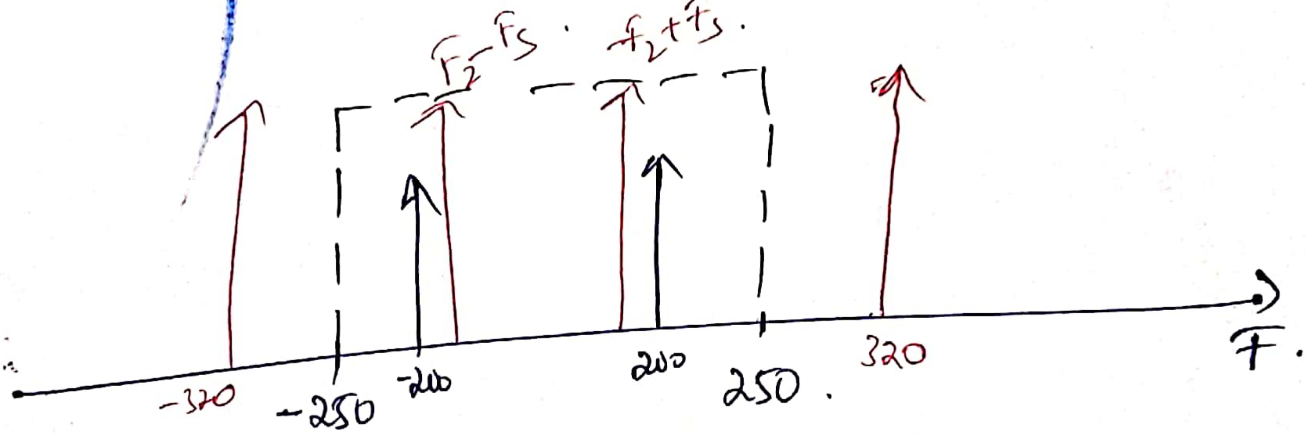
$x(t) = 2\cos 400\pi t + 6\cos 640\pi t$

$F_1 = 200$, $F_2 = 320$

Given $F_s = 500$



Sol: After sampling



Freqs: F_1 , $-F_1$; Correct

$F_2 - F_s$, $-F_2 + F_s$ Aliases.