Ex 2.45.

$$y(t) = x(t) * h(t). \qquad LTI \checkmark.$$

(a) ~~Find~~ $y_1(t) = x'(t) * h(t)$.

(ii). show that $y_1(t) = y'(t)$.

Sol ① $$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau.$$

$$\frac{d}{dt} y(t) = \int_{-\infty}^{\infty} h(\tau) \frac{d}{dt} x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x'(t-\tau)\, d\tau.$$

$$= h(t) * x'(t).$$

Sol ② $$x'(t) = \lim_{h \to 0} \frac{x(t) - x(t-h)}{h}.$$

$$x'(t) * h(t) = \lim_{h \to 0} \frac{x(t) * h(t) - x(t-h) * h(t)}{h}$$

$$= \lim_{h \to 0} \frac{y(t) - y(t-h)}{h}.$$

$$= \frac{d}{dt} y(t).$$

M.Q QP. T3033. S & S Anna Univ (2009). (DSAA)

Q. 11(b). Determine if the following systems are linear or not.

(i) $\frac{dy}{dt} + 3t\,y(t) = t^2 x(t)$. — ①

Consider two inputs $x_1(t)$, $x_2(t)$.
with corresponding outputs $y_1(t)$ & $y_2(t)$.

$$\frac{dy_1(t)}{dt} + 3t\,y_1(t) = t^2 x_1(t) \quad — ②$$

$$\frac{dy_2(t)}{dt} + 3t\,y_2(t) = t^2 x_2(t) \quad — ③$$

let $x(t) = a x_1(t) + b x_2(t)$.
& $y(t) = a y_1(t) + b y_2(t)$.

plugin in ①

$$\frac{d}{dt}[a y_1(t) + b y_2(t)] + 3t(a y_1(t) + b y_2(t)) = t^2(a x_1(t) + b x_2(t))$$

$$\Rightarrow a\left\{\frac{d}{dt}y_1(t) + 3t\,y_1(t) - t^2 x_1(t)\right\} + b\left\{\frac{d}{dt}y_2(t) + 3t\,y_2(t) - t^2 x_2(t)\right\} = 0. \quad — ④$$

From ②, ③ & ④ ⇒ ① is a linear system.

(EX)

(ii) $y(n) = 2x(n) + \dfrac{1}{x(n-1)}$ — system. — (2).

Let $x^{(1)}(n) = a\,x_1(n) + b\,x_2(n)$

$$x(n) \to \boxed{H(\cdot)} \to y(n)$$

$$H(x^{(1)}(n)) = 2(a x_1(n) + b x_2(n)) + \frac{1}{a x_1(n-1) + b x_2(n-1)}$$

Let $y^{(1)}(n) = a\,y_1(n) + b\,y_2(n)$

$$= a\left(2x_1(n) + \frac{1}{x_1(n-1)}\right) + b\left(2x_2(n) + \frac{1}{x_2(n-1)}\right)$$

$$\therefore y^{(1)}(n) = 2x^{(1)}(n) + \frac{a}{x_1(n-1)} + \frac{b}{x_2(n-1)}$$

$$\therefore y^{(1)}(n) \neq H(x^{(1)}(n))$$

$\therefore$ (1) is non-linear

(DSAA)

Time-invariant property

- Verification.

system (a) $y(t) = t\,x(t)$

$x(t-t_0) \longrightarrow y(t-t_0)$. property (desired)

step ① $x(t) \xrightarrow{\text{replace}} x(t-t_0)$

$y_1(t) = t\,x(t-t_0)$.

② $y(t) \xrightarrow{\text{replace}} y(t-t_0)$

$y(t-t_0) = (t-t_0)\,x(t-t_0)$.

$\therefore y_1(t) \neq y(t-t_0) \Rightarrow$ Time varying system

(b) $y(n) = x(2n) \rightarrow$ upsampler.

step ①. $x(n) \xrightarrow{\text{replace}} x(n-n_0)$.

ie; a delay of n_0.

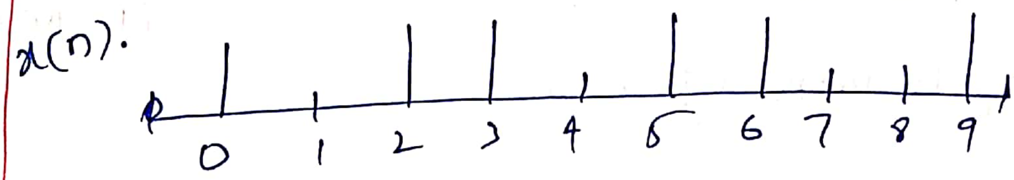
$x(2n) \longrightarrow x(2n-n_0) = y_1(n)$

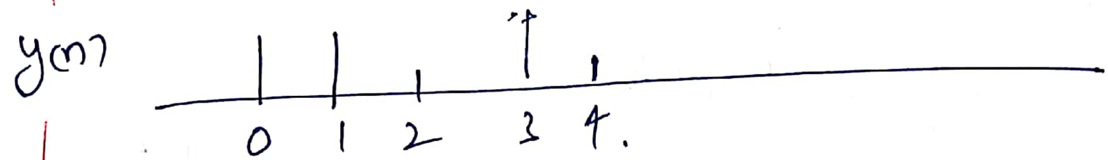
② $y(n)$ replace $\rightarrow y(n-n_0)$

$y(n-n_0) = x(2(n-n_0))$

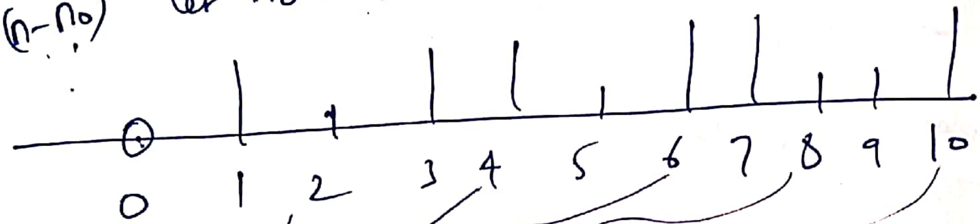
$\therefore y_1(n) \neq y(n-n_0)$ $\rightarrow$ Time Varying.

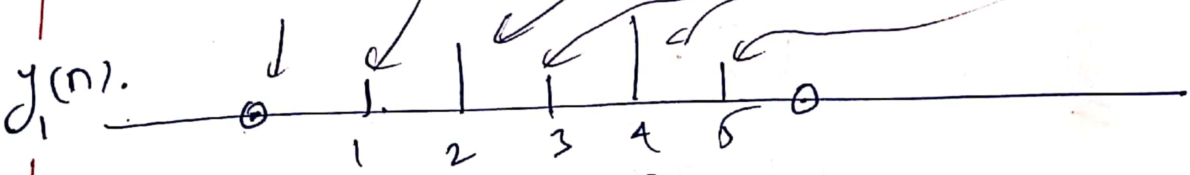
(Ex)

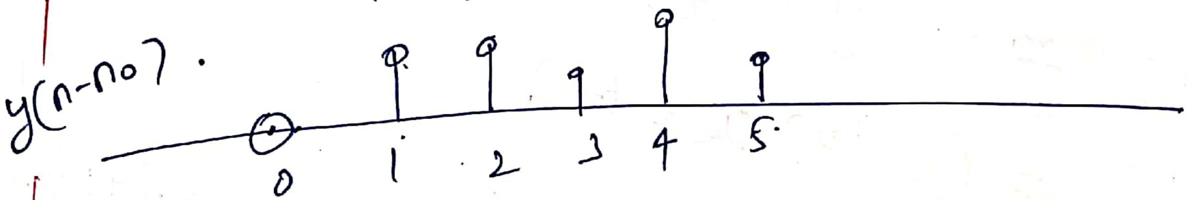




$x(n-n_0)$ let $n_0 = 1$







$\therefore y(n-n_0) \neq y_1(n)$, $\Rightarrow$ Time Variance.

6) Determine whether the following LTI systems with the given impulse responses are stable or not. (3)

(i) $h(n) = a^n u(n)$

sol ÷ The necessary and sufficient condition for stability is

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty.$$

Given $h(n) = a^n u(n)$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |a^n u(n)|$$

$$= \sum_{n=0}^{\infty} a^n$$

If $a \geq 1$, the summation $= \infty$, then LTI system is unstable.

If $a < 1$, the summation $= \frac{1}{1-a}$, a finite value, so given ~~given~~ LTI system is stable.

(4M)

(ii) $h(n) = b^n u(-n)$ $b > 1$.

sol: $$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |b^n u(-n)|$$

$$= \sum_{n=-\infty}^{0} b^n = \sum_{n=0}^{\infty} b^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{b}\right)^n$$

$$= \frac{1}{1-\left(\frac{1}{b}\right)}$$

If $b > 1$, the summation is a finite value.

∴ The given LTI system is stable.

(4M)

3 (a) Determine whether the following LTI systems with the given impulse responses are stable or not.

(i) $h(n) = e^{-6|n|}$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |e^{-6|n|}|. \qquad e^{-6|n|} > 0 \ \forall n.$$

$$= \sum_{n=-\infty}^{\infty} e^{-6|n|}. = \sum_{n=-\infty}^{-1} e^{6n} + \sum_{n=0}^{\infty} e^{-6n}.$$

$\Rightarrow$ Convergent series.

ie: $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

$\therefore$ $\boxed{\text{System is stable}}$

(3M)

(ii) $h(n) = 3^{n} u(-n)$

sol:- $$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |3^{n} u(-n)|.$$

$$= \sum_{n=-\infty}^{0} 3^{n} = \sum_{n=0}^{\infty} 3^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\tfrac{1}{3}\right)^{n} = \frac{1}{1-\left(\frac{1}{3}\right)}$$

The summation is a finite value.

So given LTI system is stable.

(3M)

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