

# CHAPTER 5

## Formal Relational Query Languages

# Chapter Outline

- ▶ Relational Algebra
- ▶ Tuple Relational Calculus
- ▶ Domain Relational Calculus

# Relational Algebra Overview

- ▶ Relational algebra is the basic set of operations for the relational model
- ▶ These operations enable a user to specify **basic retrieval requests (or queries)**
- ▶ The result of an operation is a *new relation*, which may have been formed from one or more *input* relations
  - ▶ This property makes the algebra “closed” (all objects in relational algebra are relations)

# Relational Algebra Overview (continued)

- ▶ The **algebra operations** thus produce new relations
  - ▶ These can be further manipulated using operations of the same algebra
- ▶ A sequence of relational algebra operations forms a **relational algebra expression**
  - ▶ The result of a relational algebra expression is also a relation that represents the result of a database query (or retrieval request)

# Relational Algebra Overview

- ▶ Relational Algebra consists of several groups of operations
  - ▶ Unary Relational Operations
    - ▶ SELECT (symbol:  $\sigma$  (sigma))
    - ▶ PROJECT (symbol:  $\pi$  (pi))
    - ▶ RENAME (symbol:  $\rho$  (rho))
  - ▶ Relational Algebra Operations From Set Theory
    - ▶ UNION (  $\cup$  ), INTERSECTION (  $\cap$  ), DIFFERENCE (or MINUS, - )
    - ▶ CARTESIAN PRODUCT (  $\times$  )
  - ▶ Binary Relational Operations
    - ▶ JOIN (several variations of JOIN exist)
    - ▶ DIVISION
  - ▶ Additional Relational Operations
    - ▶ OUTER JOINS, OUTER UNION
    - ▶ AGGREGATE FUNCTIONS (These compute summary of information: for example, SUM, COUNT, AVG, MIN, MAX)

# Unary Relational Operations: SELECT

- ▶ The SELECT operation (denoted by  $\sigma$  (sigma)) is used to select a *subset* of the tuples from a relation based on a **selection condition**.
  - ▶ The selection condition acts as a **filter**
  - ▶ Keeps only those tuples that satisfy the qualifying condition
  - ▶ Tuples satisfying the condition are *selected* whereas the other tuples are discarded (*filtered out*)
- ▶ Examples:
  - ▶ Select the EMPLOYEE tuples whose department number is 4:

$\sigma_{DNO = 4} (EMPLOYEE)$

- ▶ Select the employee tuples whose salary is greater than \$30,000:

$\sigma_{SALARY > 30,000} (EMPLOYEE)$

# Unary Relational Operations: SELECT

► In general, the *select* operation is denoted by  $\sigma_{\langle \text{selection condition} \rangle}(R)$  where

- the symbol  $\sigma$  (sigma) is used to denote the *select* operator
- the selection condition is a Boolean (conditional) expression specified on the attributes of relation R
- tuples that make the condition **true** are selected
  - appear in the result of the operation
- tuples that make the condition **false** are filtered out
  - discarded from the result of the operation

# Unary Relational Operations: SELECT (continued)

## ▶ SELECT Operation Properties

- ▶ The SELECT operation  $\sigma_{\langle \text{selection condition} \rangle}(R)$  produces a relation S that has the same schema (same attributes) as R
- ▶ SELECT  $\sigma$  is commutative:
  - ▶  $\sigma_{\langle \text{condition1} \rangle}(\sigma_{\langle \text{condition2} \rangle}(R)) = \sigma_{\langle \text{condition2} \rangle}(\sigma_{\langle \text{condition1} \rangle}(R))$
- ▶ Because of commutativity property, a cascade (sequence) of SELECT operations may be applied in any order:
  - ▶  $\sigma_{\langle \text{cond1} \rangle}(\sigma_{\langle \text{cond2} \rangle}(\sigma_{\langle \text{cond3} \rangle}(R))) = \sigma_{\langle \text{cond2} \rangle}(\sigma_{\langle \text{cond3} \rangle}(\sigma_{\langle \text{cond1} \rangle}(R)))$
- ▶ A cascade of SELECT operations may be replaced by a single selection with a conjunction of all the conditions:
  - ▶  $\sigma_{\langle \text{cond1} \rangle}(\sigma_{\langle \text{cond2} \rangle}(\sigma_{\langle \text{cond3} \rangle}(R))) = \sigma_{\langle \text{cond1} \rangle \text{ AND } \langle \text{cond2} \rangle \text{ AND } \langle \text{cond3} \rangle}(R)$
- ▶ The number of tuples in the result of a SELECT is less than (or equal to) the number of tuples in the input relation R



# The following query results refer to this database state

**Figure 5.6**

One possible database state for the COMPANY relational database schema.

## EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

## DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

## DEPT\_LOCATIONS

Dnumber	Dlocation
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

## WORKS\_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

## PROJECT

Pname	Pnumber	Plocation	Dnum
ProductX	1	Bellaire	5
ProductY	2	Sugarland	5
ProductZ	3	Houston	5
Computerization	10	Stafford	4
Reorganization	20	Houston	1
Newbenefits	30	Stafford	4

## DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

# Unary Relational Operations: PROJECT

- ▶ PROJECT Operation is denoted by  $\pi$  (pi)
- ▶ This operation keeps certain *columns* (attributes) from a relation and discards the other columns.
  - ▶ PROJECT creates a vertical partitioning
    - ▶ The list of specified columns (attributes) is kept in each tuple
    - ▶ The other attributes in each tuple are discarded
- ▶ Example: To list each employee's first and last name and salary, the following is used:

$\pi_{\text{LNAME, FNAME, SALARY}}(\text{EMPLOYEE})$

# Unary Relational Operations: PROJECT (cont.)

- ▶ The general form of the *project* operation is:

$$\pi_{\langle \text{attribute list} \rangle}(R)$$

- ▶  $\pi$  (pi) is the symbol used to represent the *project* operation
  - ▶  $\langle \text{attribute list} \rangle$  is the desired list of attributes from relation R.
- ▶ The project operation *removes any duplicate tuples*
  - ▶ This is because the result of the *project* operation must be a *set of tuples*
    - ▶ Mathematical sets *do not allow* duplicate elements.

# Unary Relational Operations: PROJECT (contd.)

## ► PROJECT Operation Properties

- The number of tuples in the result of projection  $\pi_{\langle \text{list} \rangle}(R)$  is always less or equal to the number of tuples in  $R$ 
  - If the list of attributes includes a *key* of  $R$ , then the number of tuples in the result of PROJECT is *equal* to the number of tuples in  $R$
- PROJECT is *not* commutative
  - $\pi_{\langle \text{list1} \rangle} (\pi_{\langle \text{list2} \rangle} (R)) = \pi_{\langle \text{list1} \rangle} (R)$  as long as  $\langle \text{list2} \rangle$  contains the attributes in  $\langle \text{list1} \rangle$

# Examples of applying SELECT and PROJECT operations

**Figure 8.1** Results of SELECT and PROJECT operations. (a)  $\sigma_{(Dno=4 \text{ AND Salary} > 25000) \text{ OR } (Dno=3 \text{ AND Salary} > 30000)}(EMPLOYEE)$ . (b)  $\pi_{Lname, Fname, Salary}(EMPLOYEE)$ . (c)  $\pi_{Sex, Salary}(EMPLOYEE)$ .

(a)

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888865555	5
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888865555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5

(b)

Lname	Fname	Salary
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmed	25000
Borg	James	55000

(c)

Sex	Salary
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

# Relational Algebra Expressions

- ▶ We may want to apply several relational algebra operations one after the other
  - ▶ Either we can write the operations as a single **relational algebra expression** by nesting the operations, or
  - ▶ We can apply one operation at a time and create **intermediate result relations**.
- ▶ In the latter case, we must give names to the relations that hold the intermediate results.

# Single expression versus sequence of relational operations (Example)

- ▶ To retrieve the first name, last name, and salary of all employees who work in department number 5, we must apply a select and a project operation
- ▶ We can write a *single relational algebra expression* as follows:
  - ▶  $\pi_{\text{FNAME, LNAME, SALARY}}(\sigma_{\text{DNO}=5}(\text{EMPLOYEE}))$
- ▶ OR We can explicitly show the *sequence of operations*, giving a name to each intermediate relation:
  - ▶  $\text{DEP5\_EMPS} \leftarrow \sigma_{\text{DNO}=5}(\text{EMPLOYEE})$
  - ▶  $\text{RESULT} \leftarrow \pi_{\text{FNAME, LNAME, SALARY}}(\text{DEP5\_EMPS})$

# Unary Relational Operations: RENAME

- ▶ The RENAME operator is denoted by  $\rho$  (rho)
- ▶ In some cases, we may want to *rename* the attributes of a relation or the relation name or both
  - ▶ Useful when a query requires multiple operations
  - ▶ Necessary in some cases (see JOIN operation later)



# Unary Relational Operations: RENAME (continued)

- ▶ The general RENAME operation  $\rho$  can be expressed by any of the following forms:
  - ▶  $\rho_S(B_1, B_2, \dots, B_n)(R)$  changes both:
    - ▶ the relation name to  $S$ , *and*
    - ▶ the column (attribute) names to  $B_1, B_1, \dots, B_n$
  - ▶  $\rho_S(R)$  changes:
    - ▶ the *relation name* only to  $S$
  - ▶  $\rho_{(B_1, B_2, \dots, B_n)}(R)$  changes:
    - ▶ the *column (attribute) names* only to  $B_1, B_1, \dots, B_n$

# Unary Relational Operations: RENAME (continued)

- ▶ For convenience, we also use a *shorthand* for renaming attributes in an intermediate relation:
  - ▶ If we write:
    - $\text{RESULT} \leftarrow \pi_{\text{FNAME, LNAME, SALARY}}(\text{DEP5\_EMPS})$
    - RESULT will have the *same attribute names* as DEP5\_EMPS (same attributes as EMPLOYEE)
  - If we write:
    - $\text{RESULT}(\text{F, M, L, S, B, A, SX, SAL, SU, DNO}) \leftarrow \pi_{\text{RESULT(F.M.L.S.B,A,SX,SAL,SU,DNO)}}(\text{DEP5\_EMPS})$
    - The 10 attributes of DEP5\_EMPS are *renamed* to F, M, L, S, B, A, SX, SAL, SU, DNO, respectively

Note: the  $\leftarrow$  symbol is an assignment operator

# Example of applying multiple operations and RENAME

**Figure 8.2** Results of a sequence of operations. (a)  $\pi_{\text{Fname, Lname, Salary}}(\sigma_{\text{Dno}=3}(\text{EMPLOYEE}))$ .  
(b) Using intermediate relations and renaming of attributes.

(a)

Fname	Lname	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

(b)

TEMP

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1985-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

# Relational Algebra Operations from Set Theory: UNION

- ▶ UNION Operation
  - ▶ Binary operation, denoted by  $\cup$
  - ▶ The result of  $R \cup S$ , is a relation that includes all tuples that are either in R or in S or in both R and S
  - ▶ Duplicate tuples are eliminated
  - ▶ The two operand relations R and S must be “type compatible” (or UNION compatible)
    - ▶ R and S must have same number of attributes
    - ▶ Each pair of corresponding attributes must be type compatible (have same or compatible domains)

# Relational Algebra Operations from Set Theory: UNION

## ► Example:

- To retrieve the social security numbers of all employees who either *work in department 5* (RESULT1 below) or *directly supervise an employee who works in department 5* (RESULT2 below)

- We can use the UNION operation as follows:

$\text{DEP5\_EMPS} \leftarrow \sigma_{\text{DNO}=5} (\text{EMPLOYEE})$

$\text{RESULT1} \leftarrow \pi_{\text{SSN}}(\text{DEP5\_EMPS})$

$\text{RESULT2}(\text{SSN}) \leftarrow \pi_{\text{SUPERSSN}}(\text{DEP5\_EMPS})$

$\text{RESULT} \leftarrow \text{RESULT1} \cup \text{RESULT2}$

- The union operation produces the tuples that are in either RESULT1 or RESULT2 or both

**Figure 8.3** Result of the UNION operation  $\text{RESULT} \leftarrow \text{RESULT1} \cup \text{RESULT2}$ .

**RESULT1**

Ssn
123456789
333445555
666884444
453453453

**RESULT2**

Ssn
333445555
888665555

**RESULT**

Ssn
123456789
333445555
666884444
453453453
888665555

# Relational Algebra Operations from Set Theory

- ▶ Type Compatibility of operands is required for the binary set operation UNION  $\cup$ , (also for INTERSECTION  $\cap$ , and SET DIFFERENCE  $-$ , see next slides)
- ▶  $R1(A1, A2, \dots, An)$  and  $R2(B1, B2, \dots, Bn)$  are type compatible if:
  - ▶ they have the same number of attributes, and
  - ▶ the domains of corresponding attributes are type compatible (i.e.  $\text{dom}(Ai) = \text{dom}(Bi)$  for  $i=1, 2, \dots, n$ ).
- ▶ The resulting relation for  $R1 \cup R2$  (also for  $R1 \cap R2$ , or  $R1 - R2$ , see next slides) has the same attribute names as the *first* operand relation  $R1$  (by convention)

# Relational Algebra Operations from Set Theory: INTERSECTION

- ▶ INTERSECTION is denoted by  $\cap$
- ▶ The result of the operation  $R \cap S$ , is a relation that includes all tuples that are in both R and S
  - ▶ The attribute names in the result will be the same as the attribute names in R
- ▶ The two operand relations R and S must be “type compatible”



# Relational Algebra Operations from Set Theory: SET DIFFERENCE (cont.)

- ▶ SET DIFFERENCE (also called MINUS or EXCEPT) is denoted by -
- ▶ The result of  $R - S$ , is a relation that includes all tuples that are in  $R$  but not in  $S$ 
  - ▶ The attribute names in the result will be the same as the attribute names in  $R$
- ▶ The two operand relations  $R$  and  $S$  must be “type compatible”

# Example to illustrate the result of UNION, INTERSECT, and DIFFERENCE

**Figure 8.4** The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b)  $\text{STUDENT} \cup \text{INSTRUCTOR}$ . (c)  $\text{STUDENT} \cap \text{INSTRUCTOR}$ . (d)  $\text{STUDENT} - \text{INSTRUCTOR}$ . (e)  $\text{INSTRUCTOR} - \text{STUDENT}$ .

(a) STUDENT

F <sub>n</sub>	L <sub>n</sub>
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

F <sub>name</sub>	L <sub>name</sub>
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(b)

F <sub>n</sub>	L <sub>n</sub>
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

(c)

F <sub>n</sub>	L <sub>n</sub>
Susan	Yao
Ramesh	Shah

(d)

F <sub>n</sub>	L <sub>n</sub>
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

(e)

F <sub>name</sub>	L <sub>name</sub>
John	Smith
Ricardo	Browne
Francis	Johnson

# Some properties of UNION, INTERSECT, and DIFFERENCE

- ▶ Notice that both union and intersection are *commutative* operations; that is
  - ▶  $R \cup S = S \cup R$ , and  $R \cap S = S \cap R$
- ▶ Both union and intersection can be treated as n-ary operations applicable to any number of relations as both are *associative* operations; that is
  - ▶  $R \cup (S \cup T) = (R \cup S) \cup T$
  - ▶  $(R \cap S) \cap T = R \cap (S \cap T)$
- ▶ The minus operation is not commutative; that is, in general
  - ▶  $R - S \neq S - R$

# Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT

- ▶ CARTESIAN (or CROSS) PRODUCT Operation
  - ▶ This operation is used to combine tuples from two relations in a combinatorial fashion.
  - ▶ Denoted by  $R(A_1, A_2, \dots, A_n) \times S(B_1, B_2, \dots, B_m)$
  - ▶ Result is a relation  $Q$  with degree  $n + m$  attributes:
    - ▶  $Q(A_1, A_2, \dots, A_n, B_1, B_2, \dots, B_m)$ , in that order.
  - ▶ The resulting relation state has one tuple for each combination of tuples—one from  $R$  and one from  $S$ .
  - ▶ Hence, if  $R$  has  $n_R$  tuples (denoted as  $|R| = n_R$ ), and  $S$  has  $n_S$  tuples, then  $R \times S$  will have  $n_R * n_S$  tuples.
  - ▶ The two operands do NOT have to be "type compatible"

# Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT (cont.)

- ▶ Generally, CROSS PRODUCT is not a meaningful operation
  - ▶ Can become meaningful when followed by other operations
- ▶ Example (not meaningful):
  - ▶  $\text{FEMALE\_EMPS} \leftarrow \sigma_{\text{SEX}='F'}(\text{EMPLOYEE})$
  - ▶  $\text{EMP\_NAMES} \leftarrow \pi_{\text{FNAME}, \text{LNAME}, \text{SSN}}(\text{FEMALE\_EMPS})$
  - ▶  $\text{EMP\_DEPENDENTS} \leftarrow \text{EMP\_NAMES} \times \text{DEPENDENT}$
- ▶ EMP\_DEPENDENTS will contain every combination of EMP\_NAMES and DEPENDENT
  - ▶ whether or not they are actually related

# Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT (cont.)

- ▶ To keep only combinations where the DEPENDENT is related to the EMPLOYEE, we add a SELECT operation as follows
- ▶ Example (meaningful):
  - ▶  $\text{FEMALE\_EMPS} \leftarrow \sigma_{\text{SEX}='F'}(\text{EMPLOYEE})$
  - ▶  $\text{EMP\_NAMES} \leftarrow \pi_{\text{FNAME}, \text{LNAME}, \text{SSN}}(\text{FEMALE\_EMPS})$
  - ▶  $\text{EMP\_DEPENDENTS} \leftarrow \text{EMP\_NAMES} \times \text{DEPENDENT}$
  - ▶  $\text{ACTUAL\_DEPS} \leftarrow \sigma_{\text{SSN}=\text{ESSN}}(\text{EMP\_DEPENDENTS})$
  - ▶  $\text{RESULT} \leftarrow \pi_{\text{FNAME}, \text{LNAME}, \text{DEPENDENT\_NAME}}(\text{ACTUAL\_DEPS})$
- ▶ RESULT will now contain the name of female employees and their dependents

## Figure 8.5 The CARTESIAN PRODUCT (CROSS PRODUCT) operation.

### FEMALE\_EMPS

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Alicia	J	Zelaya	999887777	1968-07-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

### EMPNames

Fname	Lname	Ssn
Alicia	Zelaya	999887777
Jennifer	Wallace	987654321
Joyce	English	453453453

*continued on next slide*

## Figure 8.5 (continued) The CARTESIAN PRODUCT (CROSS PRODUCT) operation.

EMP\_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	...
Alicia	Zelaya	999887777	333445555	Alice	F	1986-04-05	...
Alicia	Zelaya	999887777	333445555	Theodore	M	1983-10-25	...
Alicia	Zelaya	999887777	333445555	Joy	F	1958-05-03	...
Alicia	Zelaya	999887777	987654321	Abner	M	1942-02-28	...
Alicia	Zelaya	999887777	123456789	Michael	M	1988-01-04	...
Alicia	Zelaya	999887777	123456789	Alice	F	1988-12-30	...
Alicia	Zelaya	999887777	123456789	Elizabeth	F	1967-05-05	...
Jennifer	Wallace	987654321	333445555	Alice	F	1986-04-05	...
Jennifer	Wallace	987654321	333445555	Theodore	M	1983-10-25	...
Jennifer	Wallace	987654321	333445555	Joy	F	1958-05-03	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...
Jennifer	Wallace	987654321	123456789	Michael	M	1988-01-04	...
Jennifer	Wallace	987654321	123456789	Alice	F	1988-12-30	...
Jennifer	Wallace	987654321	123456789	Elizabeth	F	1967-05-05	...
Joyce	English	453453453	333445555	Alice	F	1986-04-05	...
Joyce	English	453453453	333445555	Theodore	M	1983-10-25	...
Joyce	English	453453453	333445555	Joy	F	1958-05-03	...
Joyce	English	453453453	987654321	Abner	M	1942-02-28	...
Joyce	English	453453453	123456789	Michael	M	1988-01-04	...
Joyce	English	453453453	123456789	Alice	F	1988-12-30	...
Joyce	English	453453453	123456789	Elizabeth	F	1967-05-05	...

*continued on next slide*



## Figure 8.5 (continued) The CARTESIAN PRODUCT (CROSS PRODUCT) operation.

### ACTUAL\_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...

### RESULT

Fname	Lname	Dependent_name
Jennifer	Wallace	Abner

# Binary Relational Operations: JOIN

- ▶ JOIN Operation (denoted by  $\bowtie$  )
  - ▶ The sequence of CARTESIAN PRODECT followed by SELECT is used quite commonly to identify and select related tuples from two relations
  - ▶ A special operation, called JOIN combines this sequence into a single operation
  - ▶ This operation is very important for any relational database with more than a single relation, because it allows us *combine related tuples* from various relations
  - ▶ The general form of a join operation on two relations  $R(A_1, A_2, \dots, A_n)$  and  $S(B_1, B_2, \dots, B_m)$  is:
$$R \bowtie_{\langle \text{join condition} \rangle} S$$
  - ▶ where R and S can be any relations that result from general *relational algebra expressions*.

# Binary Relational Operations:

## JOIN (cont.)

- ▶ Example: Suppose that we want to retrieve the name of the manager of each department.
  - ▶ To get the manager's name, we need to combine each DEPARTMENT tuple with the EMPLOYEE tuple whose SSN value matches the MGRSSN value in the department tuple.
  - ▶ We do this by using the join  $\bowtie$  operation.
- ▶  $\text{DEPT\_MGR} \leftarrow \text{DEPARTMENT} \bowtie_{\text{MGRSSN}=\text{SSN}} \text{EMPLOYEE}$
- ▶ MGRSSN=SSN is the join condition
  - ▶ Combines each department record with the employee who manages the department
  - ▶ The join condition can also be specified as  $\text{DEPARTMENT.MGRSSN} = \text{EMPLOYEE.SSN}$

## Figure 8.6 Result of the JOIN operation

$\text{DEPT\_MGR} \leftarrow \text{DEPARTMENT} \bowtie_{\text{Mgr\_ssn}=\text{Ssn}} \text{EMPLOYEE.}$

**DEPT\_MGR**

Dname	Dnumber	Mgr_ssn	...	Fname	Minit	Lname	Ssn	...
Research	5	333445555	...	Franklin	T	Wong	333445555	...
Administration	4	987654321	...	Jennifer	S	Wallace	987654321	...
Headquarters	1	888665555	...	James	E	Borg	888665555	...

# Some properties of JOIN

- ▶ Consider the following JOIN operation:

$$\begin{array}{ccc} \text{▶ } R(A_1, A_2, \dots, A_n) & \bowtie & S(B_1, B_2, \dots, B_m) \\ & R.A_i = S.B_j & \end{array}$$

- ▶ Result is a relation Q with degree  $n + m$  attributes:
  - ▶  $Q(A_1, A_2, \dots, A_n, B_1, B_2, \dots, B_m)$ , in that order.
- ▶ The resulting relation state has one tuple for each combination of tuples— $r$  from  $R$  and  $s$  from  $S$ , but *only if they satisfy the join condition*  $r[A_i] = s[B_j]$
- ▶ Hence, if  $R$  has  $n_R$  tuples, and  $S$  has  $n_S$  tuples, then the join result will generally have *less than*  $n_R * n_S$  tuples.
- ▶ Only related tuples (based on the join condition) will appear in the result

# Some properties of JOIN

- ▶ The general case of JOIN operation is called a Theta-join:  $R \bowtie_{\theta} S$   
*theta*
- ▶ The join condition is called *theta*
- ▶ *Theta* can be any general boolean expression on the attributes of R and S; for example:
  - ▶  $R.A_i < S.B_j \text{ AND } (R.A_k = S.B_l \text{ OR } R.A_p < S.B_q)$
- ▶ Most join conditions involve one or more equality conditions “AND”ed together; for example:
  - ▶  $R.A_i = S.B_j \text{ AND } R.A_k = S.B_l \text{ AND } R.A_p = S.B_q$

# Binary Relational Operations: EQUIJOIN

- ▶ EQUIJOIN Operation
- ▶ The most common use of join involves join conditions with *equality comparisons* only
- ▶ Such a join, where the only comparison operator used is  $=$ , is called an EQUIJOIN.
  - ▶ In the result of an EQUIJOIN we always have one or more pairs of attributes (whose names need not be identical) that have identical values in every tuple.
  - ▶ The JOIN seen in the previous example was an EQUIJOIN.

# Binary Relational Operations: NATURAL JOIN Operation

## ► NATURAL JOIN Operation

- Another variation of JOIN called NATURAL JOIN — denoted by  $*$  — was created to get rid of the second (superfluous) attribute in an EQUIJOIN condition.
  - because one of each pair of attributes with identical values is superfluous
- The standard definition of natural join requires that the two join attributes, or each pair of corresponding join attributes, *have the same name* in both relations
- If this is not the case, a renaming operation is applied first.



# Binary Relational Operations

## NATURAL JOIN (continued)

- ▶ Example: To apply a natural join on the DNUMBER attributes of DEPARTMENT and DEPT\_LOCATIONS, it is sufficient to write:
  - ▶  $\text{DEPT\_LOCS} \leftarrow \text{DEPARTMENT} * \text{DEPT\_LOCATIONS}$
- ▶ Only attribute with the same name is DNUMBER
- ▶ An implicit join condition is created based on this attribute:  
 $\text{DEPARTMENT.DNUMBER} = \text{DEPT\_LOCATIONS.DNUMBER}$
- ▶ Another example:  $Q \leftarrow R(A,B,C,D) * S(C,D,E)$ 
  - ▶ The implicit join condition includes *each pair* of attributes with the same name, “AND”ed together:
    - ▶  $R.C = S.C \text{ AND } R.D = S.D$
  - ▶ Result keeps only one attribute of each such pair:
    - ▶  $Q(A,B,C,D,E)$

# Example of NATURAL JOIN operation

**Figure 8.7** Results of two natural join operations. (a)  $\text{proj\_dept} \leftarrow \text{project} \bowtie \text{dept}$ . (b)  $\text{dept\_locs} \leftarrow \text{department} \bowtie \text{dept\_locations}$ .

(a)

**PROJ\_DEPT**

Pname	Pnumber	Plocation	Dnum	Dname	Mgr_ssn	Mgr_start_date
ProductX	1	Bellaire	5	Research	333445555	1988-05-22
ProductY	2	Sugarland	5	Research	333445555	1988-05-22
ProductZ	3	Houston	5	Research	333445555	1988-05-22
Computerization	10	Stafford	4	Administration	987654321	1995-01-01
Reorganization	20	Houston	1	Headquarters	888665555	1981-06-19
Newbenefits	30	Stafford	4	Administration	987654321	1995-01-01

(b)

**DEPT\_LOCS**

Dname	Dnumber	Mgr_ssn	Mgr_start_date	Location
Headquarters	1	888665555	1981-06-19	Houston
Administration	4	987654321	1995-01-01	Stafford
Research	5	333445555	1988-05-22	Bellaire
Research	5	333445555	1988-05-22	Sugarland
Research	5	333445555	1988-05-22	Houston

# Complete Set of Relational Operations

- ▶ The set of operations including SELECT  $\sigma$ , PROJECT  $\pi$ , UNION  $\cup$ , DIFFERENCE  $-$ , RENAME  $\rho$ , and CARTESIAN PRODUCT  $\times$  is called a *complete set* because any other relational algebra expression can be expressed by a combination of these five operations.
- ▶ For example:
  - ▶  $R \cap S = (R \cup S) - ((R - S) \cup (S - R))$
  - ▶  $R \bowtie_{\langle \text{join condition} \rangle} S = \sigma_{\langle \text{join condition} \rangle} (R \times S)$

# Binary Relational Operations: DIVISION

## ► DIVISION Operation

- The division operation is applied to two relations
- $R(Z) \div S(X)$ , where  $X \subset Z$ . Let  $Y = Z - X$  (and hence  $Z = X \cup Y$ ); that is, let  $Y$  be the set of attributes of  $R$  that are not attributes of  $S$ .
- The result of DIVISION is a relation  $T(Y)$  that includes a tuple  $t$  if tuples  $t_R$  appear in  $R$  with  $t_R[Y] = t$ , and with
  - $t_R[X] = t_s$  for every tuple  $t_s$  in  $S$ .
- For a tuple  $t$  to appear in the result  $T$  of the DIVISION, the values in  $t$  must appear in  $R$  in combination with every tuple in  $S$ .

# Example of DIVISION

**Figure 8.8** The DIVISION operation. (a) Dividing SSN\_PNOS by SMITH\_PNOS. (b)  $T \leftarrow R \div S$ .

(a)

**SSN\_PNOS**

Essn	Pno
123456789	1
123456789	2
666884444	3
453453453	1
453453453	2
333445555	2
333445555	3
333445555	10
333445555	20
999887777	30
999887777	10
987987987	10
987987987	30
987654321	30
987654321	20
888665555	20

**SMITH\_PNOS**

Pno
1
2

**SSNS**

Sen
123456789
453453453

(b)

**R**

A	B
a1	b1
a2	b1
a3	b1
a4	b1
a1	b2
a3	b2
a2	b3
a3	b3
a4	b3
a1	b4
a2	b4
a3	b4

**S**

A
a1
a2
a3

**T**

B
b1
b4

# Table 8.1 Operations of Relational Algebra

**Table 8.1** Operations of Relational Algebra

OPERATION	PURPOSE	NOTATION
SELECT	Selects all tuples that satisfy the selection condition from a relation $R$ .	$\sigma_{\langle \text{selection condition} \rangle}(R)$
PROJECT	Produces a new relation with only some of the attributes of $R$ , and removes duplicate tuples.	$\pi_{\langle \text{attribute list} \rangle}(R)$
THETA JOIN	Produces all combinations of tuples from $R_1$ and $R_2$ that satisfy the join condition.	$R_1 \bowtie_{\langle \text{join condition} \rangle} R_2$
EQUIJOIN	Produces all the combinations of tuples from $R_1$ and $R_2$ that satisfy a join condition with only equality comparisons.	$R_1 \bowtie_{\langle \text{join condition} \rangle} R_2$ , OR $R_1 \bowtie_{(\langle \text{join attributes 1} \rangle), (\langle \text{join attributes 2} \rangle)} R_2$
NATURAL JOIN	Same as EQUIJOIN except that the join attributes of $R_2$ are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R_1 \star_{\langle \text{join condition} \rangle} R_2$ , OR $R_1 \star_{(\langle \text{join attributes 1} \rangle), (\langle \text{join attributes 2} \rangle)} R_2$ OR $R_1 \star R_2$

*continued on next slide*

# Table 8.1 Operations of Relational Algebra (continued)

**Table 8.1** Operations of Relational Algebra

OPERATION	PURPOSE	NOTATION
UNION	Produces a relation that includes all the tuples in $R_1$ or $R_2$ or both $R_1$ and $R_2$ ; $R_1$ and $R_2$ must be union compatible.	$R_1 \cup R_2$
INTERSECTION	Produces a relation that includes all the tuples in both $R_1$ and $R_2$ ; $R_1$ and $R_2$ must be union compatible.	$R_1 \cap R_2$
DIFFERENCE	Produces a relation that includes all the tuples in $R_1$ that are not in $R_2$ ; $R_1$ and $R_2$ must be union compatible.	$R_1 - R_2$
CARTESIAN PRODUCT	Produces a relation that has the attributes of $R_1$ and $R_2$ and includes as tuples all possible combinations of tuples from $R_1$ and $R_2$ .	$R_1 \times R_2$
DIVISION	Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in $R_1$ in combination with every tuple from $R_2(Y)$ , where $Z = X \cup Y$ .	$R_1(Z) \div R_2(Y)$

# Query Tree Notation

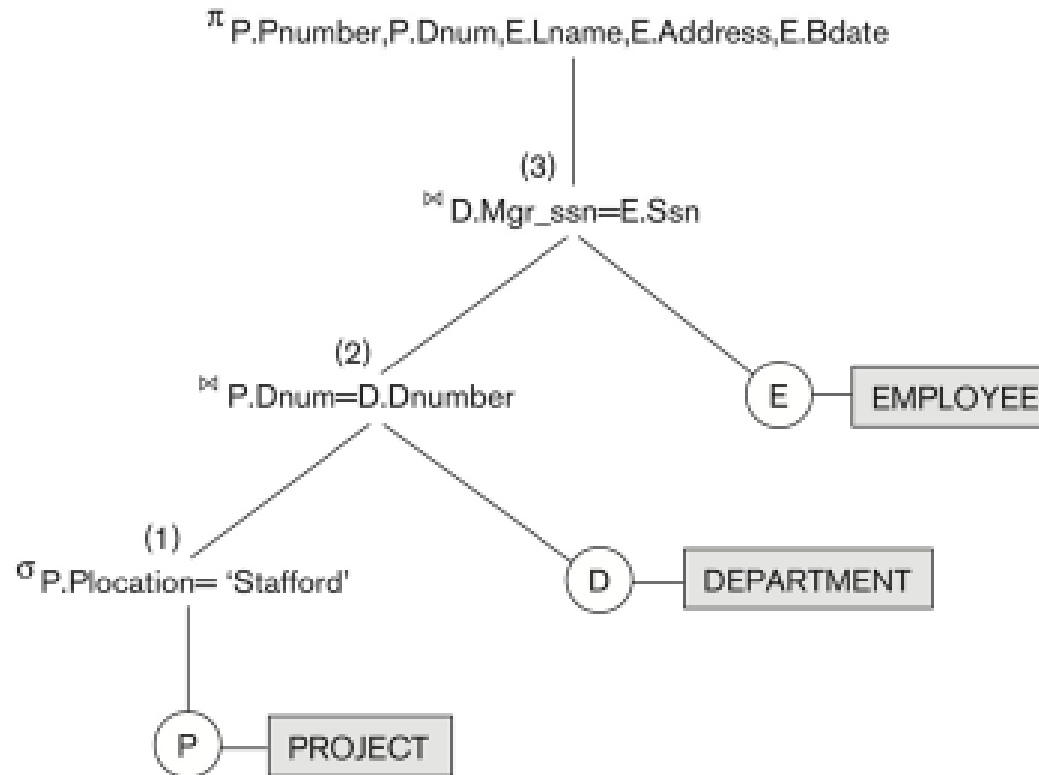
## ► Query Tree

- An internal data structure to represent a query
- Standard technique for estimating the work involved in executing the query, the generation of intermediate results, and the optimization of execution
- Nodes stand for operations like selection, projection, join, renaming, division, ....
- Leaf nodes represent base relations
- A tree gives a good visual feel of the complexity of the query and the operations involved
- Algebraic Query Optimization consists of rewriting the query or modifying the query tree into an equivalent tree.



# Example of Query Tree

Figure 8.9 Query tree corresponding to the relational algebra expression for Q2.



# Additional Relational Operations: Aggregate Functions and Grouping

- ▶ A type of request that cannot be expressed in the basic relational algebra is to specify mathematical **aggregate functions** on collections of values from the database.
- ▶ Examples of such functions include retrieving the average or total salary of all employees or the total number of employee tuples.
  - ▶ These functions are used in simple statistical queries that summarize information from the database tuples.
- ▶ Common functions applied to collections of numeric values include
  - ▶ SUM, AVERAGE, MAXIMUM, and MINIMUM.
- ▶ The COUNT function is used for counting tuples or values.

# Aggregate Function Operation

- ▶ Use of the Aggregate Functional operation  $\mathcal{F}$ 
  - ▶  $\mathcal{F}_{\text{MAX Salary}}(\text{EMPLOYEE})$  retrieves the maximum salary value from the EMPLOYEE relation
  - ▶  $\mathcal{F}_{\text{MIN Salary}}(\text{EMPLOYEE})$  retrieves the minimum Salary value from the EMPLOYEE relation
  - ▶  $\mathcal{F}_{\text{SUM Salary}}(\text{EMPLOYEE})$  retrieves the sum of the Salary from the EMPLOYEE relation
  - ▶  $\mathcal{F}_{\text{COUNT SSN, AVERAGE Salary}}(\text{EMPLOYEE})$  computes the count (number) of employees and their average salary
    - ▶ Note: count just counts the number of rows, without removing duplicates

# Using Grouping with Aggregation

- ▶ The previous examples all summarized one or more attributes for a set of tuples
  - ▶ Maximum Salary or Count (number of) Ssn
- ▶ Grouping can be combined with Aggregate Functions
- ▶ Example: For each department, retrieve the DNO, COUNT SSN, and AVERAGE SALARY
- ▶ A variation of aggregate operation  $\mathcal{F}$  allows this:
  - ▶ Grouping attribute placed to left of symbol
  - ▶ Aggregate functions to right of symbol
  - ▶  $\text{DNO } \mathcal{F} \text{ COUNT SSN, AVERAGE Salary (EMPLOYEE)}$
- ▶ Above operation groups employees by DNO (department number) and computes the count of employees and average salary per department

# Figure 8.10 The aggregate function operation.

- $\rho_{R(Dno, No\_of\_employees, Average\_sal)}(Dno \bowtie COUNT Ssn, AVERAGE Salary (EMPLOYEE)).$
- $Dno \bowtie salary (EMPLOYEE).$
- $\bowtie COUNT Ssn, AVERAGE Salary (EMPLOYEE).$

R

(a)

Dno	No_of_employees	Average_sal
5	4	33250
4	3	31000
1	1	55000

(b)

Dno	Count_ssn	Average_salary
5	4	33250
4	3	31000
1	1	55000

(c)

Count_ssn	Average_salary
8	35125

## Figure 7.1a Results of GROUP BY and HAVING (in SQL). Q24.

Fname	Minit	Lname	<u>Ssn</u>	...	Salary	Super_ssn	Dno
John	B	Smith	123456789		30000	333445555	5
Franklin	T	Wong	333445555		40000	888665555	5
Ramesh	K	Narayan	666884444		38000	333445555	5
Joyce	A	English	453453453	...	25000	333445555	5
Alicia	J	Zelaya	999887777		25000	987654321	4
Jennifer	S	Wallace	987654321		43000	888665555	4
Ahmad	V	Jabbar	987987987		25000	987654321	4
James	E	Bong	888665555		55000	NULL	1

Dno	Count (*)	Avg (Salary)
5	4	33250
4	3	31000
1	1	55000

Result of Q24

Grouping EMPLOYEE tuples by the value of Dno

*continued on next slide*

# Additional Relational Operations (continued)

- ▶ The OUTER JOIN Operation
  - ▶ In NATURAL JOIN and EQUIJOIN, tuples without a *matching* (or *related*) tuple are eliminated from the join result
    - ▶ Tuples with null in the join attributes are also eliminated
    - ▶ This amounts to loss of information.
  - ▶ A set of operations, called OUTER joins, can be used when we want to keep all the tuples in R, or all those in S, or all those in both relations in the result of the join, regardless of whether or not they have matching tuples in the other relation.

# Additional Relational Operations (continued)

- ▶ The left outer join operation keeps every tuple in the first or left relation  $R$  in  $R \bowtie_{\text{left}} S$ ; if no matching tuple is found in  $S$ , then the attributes of  $S$  in the join result are filled or “padded” with null values.
- ▶ A similar operation, right outer join, keeps every tuple in the second or right relation  $S$  in the result of  $R \bowtie_{\text{right}} S$ .
- ▶ A third operation, full outer join, denoted by  $\bowtie_{\text{full}}$ , keeps all tuples in both the left and the right relations when no matching tuples are found, padding them with null values as needed.



## Figure 8.12 The result of a LEFT OUTER JOIN operation.

### RESULT

Fname	Minit	Lname	Dname
John	B	Smith	NULL
Franklin	T	Wong	Research
Alicia	J	Zelaya	NULL
Jennifer	S	Wallace	Administration
Ramesh	K	Narayan	NULL
Joyce	A	English	NULL
Ahmad	V	Jabbar	NULL
James	E	Borg	Headquarters

# Additional Relational Operations (continued)

## ► OUTER UNION Operations

- The outer union operation was developed to take the union of tuples from two relations if the relations are *not type compatible*.
- This operation will take the union of tuples in two relations  $R(X, Y)$  and  $S(X, Z)$  that are **partially compatible**, meaning that only some of their attributes, say  $X$ , are type compatible.
- The attributes that are type compatible are represented only once in the result, and those attributes that are not type compatible from either relation are also kept in the result relation  $T(X, Y, Z)$ .

# Relational Model Concepts

- ▶ A Relation is a mathematical concept based on the ideas of sets
- ▶ The model was first proposed by Dr. E.F. Codd of IBM Research in 1970 in the following paper:
  - ▶ "A Relational Model for Large Shared Data Banks,"  
Communications of the ACM, June 1970
- ▶ The above paper caused a major revolution in the field of database management and earned Dr. Codd the coveted ACM Turing Award

# Informal Definitions

- ▶ Informally, a **relation** looks like a **table** of values.
- ▶ A relation typically contains a **set of rows**.
- ▶ The data elements in each **row** represent certain facts that correspond to a real-world **entity** or **relationship**
  - ▶ In the formal model, rows are called **tuples**
- ▶ Each **column** has a column header that gives an indication of the meaning of the data items in that column
  - ▶ In the formal model, the column header is called an **attribute name** (or just **attribute**)

# Example of a Relation

Relation Name

**STUDENT**

Attributes

Tuples

Name	Ssn	Home_phone	Address	Office_phone	Age	Gpa
Benjamin Bayer	305-61-2435	373-1616	2918 Bluebonnet Lane	NULL	19	3.21
Chung-cha Kim	381-62-1245	375-4409	125 Kirby Road	NULL	18	2.89
Dick Davidson	422-11-2320	NULL	3452 Elgin Road	749-1253	25	3.53
Rohan Panchal	489-22-1100	376-9821	265 Lark Lane	749-6492	28	3.93
Barbara Benson	533-69-1238	839-8461	7384 Fontana Lane	NULL	19	3.25

**Figure 5.1**

The attributes and tuples of a relation STUDENT.

# Informal Definitions

- ▶ Key of a Relation:
  - ▶ Each row has a value of a data item (or set of items) that uniquely identifies that row in the table
    - ▶ Called the *key*
  - ▶ In the STUDENT table, SSN is the key
- ▶ Sometimes row-ids or sequential numbers are assigned as keys to identify the rows in a table
  - ▶ Called *artificial key* or *surrogate key*

# Formal Definitions - Schema

- ▶ The **Schema** (or description) of a Relation:
  - ▶ Denoted by  $R(A_1, A_2, \dots, A_n)$
  - ▶  $R$  is the **name** of the relation
  - ▶ The **attributes** of the relation are  $A_1, A_2, \dots, A_n$
- ▶ Example:  
CUSTOMER (Cust-id, Cust-name, Address, Phone#)
  - ▶ CUSTOMER is the relation name
  - ▶ Defined over the four attributes: Cust-id, Cust-name, Address, Phone#
- ▶ Each attribute has a **domain** or a set of valid values.
  - ▶ For example, the domain of Cust-id is 6 digit numbers.

# Formal Definitions - Tuple

- ▶ A **tuple** is an ordered set of values (enclosed in angled brackets ' $\langle \dots \rangle$ ')
  - ▶ Each value is derived from an appropriate *domain*.
- ▶ A row in the CUSTOMER relation is a 4-tuple and would consist of four values, for example:
  - ▶  $\langle 632895, \text{"John Smith"}, \text{"101 Main St. Atlanta, GA 30332"}, \text{"(404) 894-2000"} \rangle$
  - ▶ This is called a 4-tuple as it has 4 values
  - ▶ A tuple (row) in the CUSTOMER relation.
- ▶ A relation is a **set** of such tuples (rows)



# Formal Definitions - Domain

- ▶ A **domain** has a logical definition:
  - ▶ Example: “USA\_phone\_numbers” are the set of 10 digit phone numbers valid in the U.S.
- ▶ A domain also has a data-type or a format defined for it.
  - ▶ The USA\_phone\_numbers may have a format: (ddd)ddd-dddd where each d is a decimal digit.
  - ▶ Dates have various formats such as year, month, date formatted as yyyy-mm-dd, or as dd mm,yyyy etc.
- ▶ The attribute name designates the role played by a domain in a relation:
  - ▶ Used to interpret the meaning of the data elements corresponding to that attribute
  - ▶ Example: The domain Date may be used to define two attributes named “Invoice-date” and “Payment-date” with different meanings

# Formal Definitions - State

- ▶ The **relation state** is a subset of the Cartesian product of the domains of its attributes
  - ▶ each domain contains the set of all possible values the attribute can take.
- ▶ Example: attribute Cust-name is defined over the domain of character strings of maximum length 25
  - ▶  $\text{dom}(\text{Cust-name})$  is `varchar(25)`
- ▶ The role these strings play in the CUSTOMER relation is that of the *name of a customer*.

# Formal Definitions - Summary

- ▶ Formally,
  - ▶ Given  $R(A_1, A_2, \dots, A_n)$
  - ▶  $r(R) \subset \text{dom}(A_1) \times \text{dom}(A_2) \times \dots \times \text{dom}(A_n)$
- ▶  $R(A_1, A_2, \dots, A_n)$  is the **schema** of the relation
- ▶  $R$  is the **name** of the relation
- ▶  $A_1, A_2, \dots, A_n$  are the **attributes** of the relation
- ▶  $r(R)$ : a specific **state** (or "value" or "population") of relation  $R$  - this is a *set of tuples* (rows)
  - ▶  $r(R) = \{t_1, t_2, \dots, t_n\}$  where each  $t_i$  is an  $n$ -tuple
  - ▶  $t_i = \langle v_1, v_2, \dots, v_n \rangle$  where each  $v_j$  *element-of*  $\text{dom}(A_j)$

# Formal Definitions - Example

- ▶ Let  $R(A1, A2)$  be a relation schema:
  - ▶ Let  $\text{dom}(A1) = \{0,1\}$
  - ▶ Let  $\text{dom}(A2) = \{a,b,c\}$
- ▶ Then:  $\text{dom}(A1) \times \text{dom}(A2)$  is all possible combinations:  
 $\{ \langle 0,a \rangle, \langle 0,b \rangle, \langle 0,c \rangle, \langle 1,a \rangle, \langle 1,b \rangle, \langle 1,c \rangle \}$
- ▶ The relation state  $r(R) \subset \text{dom}(A1) \times \text{dom}(A2)$
- ▶ For example:  $r(R)$  could be  $\{ \langle 0,a \rangle, \langle 0,b \rangle, \langle 1,c \rangle \}$ 
  - ▶ this is one possible state (or “population” or “extension”)  $r$  of the relation  $R$ , defined over  $A1$  and  $A2$ .
  - ▶ It has three 2-tuples:  $\langle 0,a \rangle, \langle 0,b \rangle, \langle 1,c \rangle$

# Definition Summary

<u>Informal Terms</u>		<u>Formal Terms</u>
Table		Relation
Column Header		Attribute
All possible Column Values		Domain
Row		Tuple
Table Definition		Schema of a Relation
Populated Table		State of the Relation

# Example - A relation STUDENT

Relation Name

**STUDENT**

Attributes

Name	Ssn	Home_phone	Address	Office_phone	Age	Gpa
Benjamin Bayer	305-61-2435	373-1616	2918 Bluebonnet Lane	NULL	19	3.21
Chung-cha Kim	381-62-1245	375-4409	125 Kirby Road	NULL	18	2.89
Dick Davidson	422-11-2320	NULL	3452 Elgin Road	749-1253	25	3.53
Rohan Panchal	489-22-1100	376-9821	265 Lark Lane	749-6492	28	3.93
Barbara Benson	533-69-1238	839-8461	7384 Fontana Lane	NULL	19	3.25

Tuples

**Figure 5.1**

The attributes and tuples of a relation STUDENT.

# Characteristics Of Relations

- ▶ Ordering of tuples in a relation  $r(R)$ :
  - ▶ The tuples are *not considered to be ordered*, even though they appear to be in the tabular form.
- ▶ Ordering of attributes in a relation schema  $R$  (and of values within each tuple):
  - ▶ We will consider the attributes in  $R(A_1, A_2, \dots, A_n)$  and the values in  $t = \langle v_1, v_2, \dots, v_n \rangle$  to be ordered .
    - ▶ (However, a more general alternative definition of relation does not require this ordering. It includes both the name and the value for each of the attributes ).
    - ▶ Example:  $t = \{ \langle \text{name}, \text{"John"} \rangle, \langle \text{SSN}, 123456789 \rangle \}$
    - ▶ This representation may be called as “self-describing”.

# Same state as previous Figure (but with different order of tuples)

**Figure 5.2**

The relation STUDENT from Figure 5.1 with a different order of tuples.

## STUDENT

Name	Ssn	Home_phone	Address	Office_phone	Age	Gpa
Dick Davidson	422-11-2320	NULL	3452 Elgin Road	749-1253	25	3.53
Barbara Benson	533-69-1238	839-8461	7384 Fontana Lane	NULL	19	3.25
Rohan Panchal	489-22-1100	376-9821	265 Lark Lane	749-6492	28	3.93
Chung-cha Kim	381-62-1245	375-4409	125 Kirby Road	NULL	18	2.89
Benjamin Bayer	305-61-2435	373-1616	2918 Bluebonnet Lane	NULL	19	3.21



# Characteristics Of Relations

- ▶ Values in a tuple:
  - ▶ All values are considered atomic (indivisible).
  - ▶ Each value in a tuple must be from the domain of the attribute for that column
    - ▶ If tuple  $t = \langle v_1, v_2, \dots, v_n \rangle$  is a tuple (row) in the relation state  $r$  of  $R(A_1, A_2, \dots, A_n)$
    - ▶ Then each  $v_i$  must be a value from  $dom(A_i)$
- ▶ A special **null** value is used to represent values that are unknown or not available or inapplicable in certain tuples.

# Characteristics Of Relations

- ▶ Notation:
  - ▶ We refer to **component values** of a tuple  $t$  by:
    - ▶  $t[A_i]$  or  $t.A_i$
    - ▶ This is the value  $v_i$  of attribute  $A_i$  for tuple  $t$
  - ▶ Similarly,  $t[A_u, A_v, \dots, A_w]$  refers to the subtuple of  $t$  containing the values of attributes  $A_u, A_v, \dots, A_w$ , respectively in  $t$

# Tuple Relational Calculus

# Tuple Relational Calculus

- ▶ A nonprocedural query language, where each query is of the form

$$\{t \mid P(t)\}$$

- ▶ It is the set of all tuples  $t$  such that predicate  $P$  is true for  $t$
- ▶  $t$  is a *tuple variable*,  $t[A]$  denotes the value of tuple  $t$  on attribute  $A$
- ▶  $t \in r$  denotes that tuple  $t$  is in relation  $r$
- ▶  $P$  is a *formula* similar to that of the predicate calculus

# Predicate Calculus Formula

1. Set of attributes and constants
2. Set of comparison operators: (e.g.,  $<$ ,  $\leq$ ,  $=$ ,  $\neq$ ,  $>$ ,  $\geq$ )
3. Set of connectives: and ( $\wedge$ ), or ( $\vee$ ), not ( $\neg$ )
4. Implication ( $\Rightarrow$ ):  $x \Rightarrow y$ , if  $x$  is true, then  $y$  is true

$$x \Rightarrow y \equiv \neg x \vee y$$

5. Set of quantifiers:

- ▶  $\exists t \in r (Q(t)) \equiv$  "there exists" a tuple  $t$  in relation  $r$  such that predicate  $Q(t)$  is true
- ▶  $\forall t \in r (Q(t)) \equiv Q$  is true "for all" tuples  $t$  in relation  $r$

# Example Queries

- Find the *ID*, *name*, *dept\_name*, *salary* for instructors whose salary is greater than \$80,000

$$\{t \mid t \in \text{instructor} \wedge t[\text{salary}] > 80000\}$$

Notice that a relation on schema (*ID*, *name*, *dept\_name*, *salary*) is implicitly defined by the query

- As in the previous query, but output only the *ID* attribute value

$$\{t \mid \exists s \in \text{instructor} (t[ID] = s[ID] \wedge s[\text{salary}] > 80000)\}$$

Notice that a relation on schema (*ID*) is implicitly defined by the query

# Example Queries

- Find the names of all instructors whose department is in the Watson building

$$\{t \mid \exists s \in \text{instructor } (t[\text{name}] = s[\text{name}] \wedge \exists u \in \text{department } (u[\text{dept\_name}] = s[\text{dept\_name}] \wedge u[\text{building}] = \text{"Watson"}))\}$$

- Find the set of all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or both

$$\{t \mid \exists s \in \text{section } (t[\text{course\_id}] = s[\text{course\_id}] \wedge s[\text{semester}] = \text{"Fall"} \wedge s[\text{year}] = 2009 \vee \exists u \in \text{section } (t[\text{course\_id}] = u[\text{course\_id}] \wedge u[\text{semester}] = \text{"Spring"} \wedge u[\text{year}] = 2010))\}$$

# Example Queries

- Find the set of all courses taught in the Fall 2009 semester, and in the Spring 2010 semester

$$\{t \mid \exists s \in \text{section } (t[\text{course\_id}] = s[\text{course\_id}] \wedge s[\text{semester}] = \text{"Fall"} \wedge s[\text{year}] = 2009 \wedge \exists u \in \text{section } (t[\text{course\_id}] = u[\text{course\_id}] \wedge u[\text{semester}] = \text{"Spring"} \wedge u[\text{year}] = 2010))\}$$

- Find the set of all courses taught in the Fall 2009 semester, but not in the Spring 2010 semester

$$\{t \mid \exists s \in \text{section } (t[\text{course\_id}] = s[\text{course\_id}] \wedge s[\text{semester}] = \text{"Fall"} \wedge s[\text{year}] = 2009 \wedge \neg \exists u \in \text{section } (t[\text{course\_id}] = u[\text{course\_id}] \wedge u[\text{semester}] = \text{"Spring"} \wedge u[\text{year}] = 2010))\}$$



# Universal Quantification

- ▶ Find all students who have taken all courses offered in the Biology department
  - ▶  $\{t \mid \exists r \in \text{student } (t[ID] = r[ID]) \wedge$   
 $(\forall u \in \text{course } (u[\text{dept\_name}] = \text{"Biology"} \Rightarrow$   
 $\exists s \in \text{takes } (t[ID] = s[ID] \wedge$   
 $s[\text{course\_id}] = u[\text{course\_id}]))\}$

# Domain Relational Calculus

# Domain Relational Calculus

- ▶ A nonprocedural query language equivalent in power to the tuple relational calculus
- ▶ Each query is an expression of the form:

$$\{ \langle x_1, x_2, \dots, x_n \rangle \mid P(x_1, x_2, \dots, x_n) \}$$

- ▶  $x_1, x_2, \dots, x_n$  represent domain variables
- ▶  $P$  represents a formula similar to that of the predicate calculus

# Example Queries

- ▶ Find the *ID*, *name*, *dept\_name*, *salary* for instructors whose salary is greater than \$80,000
  - ▶  $\{ \langle i, n, d, s \rangle \mid \langle i, n, d, s \rangle \in instructor \wedge s > 80000 \}$
- ▶ As in the previous query, but output only the *ID* attribute value
  - ▶  $\{ \langle i \rangle \mid \langle i, n, d, s \rangle \in instructor \wedge s > 80000 \}$
- ▶ Find the names of all instructors whose department is in the Watson building
  - ▶  $\{ \langle n \rangle \mid \exists i, d, s (\langle i, n, d, s \rangle \in instructor \wedge \exists b, a (\langle d, b, a \rangle \in department \wedge b = \text{“Watson”})) \}$

# Example Queries

- Find the set of all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or both

$$\{ \langle c \rangle \mid \exists a, s, y, b, r, t ( \langle c, a, s, y, b, r, t \rangle \in \text{section} \wedge s = \text{"Fall"} \wedge y = 2009 ) \vee \exists a, s, y, b, r, t ( \langle c, a, s, y, b, r, t \rangle \in \text{section} ] \wedge s = \text{"Spring"} \wedge y = 2010 ) \}$$

This case can also be written as

$$\{ \langle c \rangle \mid \exists a, s, y, b, r, t ( \langle c, a, s, y, b, r, t \rangle \in \text{section} \wedge ( (s = \text{"Fall"} \wedge y = 2009) \vee (s = \text{"Spring"} \wedge y = 2010) ) ) \}$$

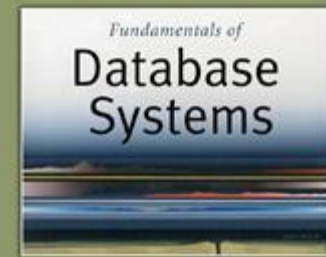
- Find the set of all courses taught in the Fall 2009 semester, and in the Spring 2010 semester

$$\{ \langle c \rangle \mid \exists a, s, y, b, r, t ( \langle c, a, s, y, b, r, t \rangle \in \text{section} \wedge s = \text{"Fall"} \wedge y = 2009 ) \wedge \exists a, s, y, b, r, t ( \langle c, a, s, y, b, r, t \rangle \in \text{section} ] \wedge s = \text{"Spring"} \wedge y = 2010 ) \}$$

# Universal Quantification

- ▶ Find all students who have taken all courses offered in the Biology department
  - ▶  $\{ \langle i \rangle \mid \exists n, d, tc ( \langle i, n, d, tc \rangle \in student \wedge$   
 $(\forall ci, ti, dn, cr ( \langle ci, ti, dn, cr \rangle \in course \wedge dn = \text{“Biology”}$   
 $\Rightarrow \exists si, se, y, g ( \langle i, ci, si, se, y, g \rangle \in takes )) ) \}$
  - ▶ Note that without the existential quantification on student, the above query would be unsafe if the Biology department has not offered any courses.

# End of Chapter 5



5th Edition

Elmasri / Navathe