Name - Sayam Kumar
S20180010158
Page No 1

SDA Mid Sem Exam

Q-1

$x_A \sim N(5, 1)$

$x_B \sim N(4, 1.21)$

$n = 10{,}000$ samples

If x_A is normally distributed and x_B is normally distributed as well, then $\bar{x}_A - \bar{x}_B$ is also normally distributed with mean

$$E[\bar{x}_A - \bar{x}_B] = \mu_1 - \mu_2 = 5 - 4 = 1$$

and variance

$$Var(\bar{x}_A - \bar{x}_B) = Var(\bar{x}_A) + Var(\bar{x}_B) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{1}{10^4} + \frac{1.21}{10^4}$$

$$= \frac{2.21}{10^4}$$

So $(\bar{x}_A - \bar{x}_B) \sim N\left[1, \frac{2.21}{10^4}\right]$

Q-2 $\theta_1 = \dfrac{X_1 + kX_2 + X_3 + X_4}{8}$

Given θ_1 is unbiased estimator of μ

$\Rightarrow E(\theta_1) = \mu$

$\Rightarrow E\left(\dfrac{x_1 + kX_2 + X_3 + X_4}{8}\right) = \mu$

$\Rightarrow E(X_1) + k\,E(X_2) + E(X_3) + E(X_4) = 8\mu$

Sayam Kumar
S20180010158

$$\mu + k\mu + \mu + \mu = 8\mu$$
$$3\mu + k\mu = 8\mu$$
$$k\mu = 8\mu - 3\mu$$
$$\boxed{k=5}$$

Variance of unbiased estimator

$$Var(\theta_1) = E\left[(\theta_1-\theta)^2\right] \quad (\theta = \mu)$$
$$\cancel{E(\theta_1^2)}$$
$$= E\left[\theta_1^2 - 2\theta_1\mu + \mu^2\right]$$
$$= E\left[\theta_1^2\right] - 2\mu^2 + \mu^2$$
$$= E\left[\theta_1^2\right] - \mu^2$$
$$= \mu^2 - \mu^2 = 0$$

Variance of unbiased estimator is zero.

Q.3 $f(x) = x\theta\, e^{-\frac{1}{2}\theta x^2}, \quad x \geq 0$

① find MLE of θ

Sol for all x, product

$$x_1\theta\, e^{-\frac{1}{2}\theta x_1^2} \times x_2\theta\, e^{-\frac{1}{2}\theta x_2^2} \times x_3\, e^{-\frac{1}{2}\theta x_3^2} \cdots$$

Take log

$$LE = \ln(x_1\theta) + \ln(x_2\theta) + \dots \ln(x_n\theta) - \frac{1}{2}\theta\left(x_1^2 + x_2^2 + x_3^2 + \dots x_n^2\right)$$
$$LE = n\ln\theta + \ln\left(\prod_{i=1}^{n} x_i\right) - \frac{1}{2}\theta\left(\sum_{i=1}^{n} x_i^2\right)$$

Continued on Page 3

Sayam Kumar
S20180010158

likelihood = $n \ln\theta + \ln\left(\prod_{i=1}^{n} x_i\right) - \frac{1}{2}\theta\left(\sum_{i=1}^{n} x_i^2\right)$

differentiate with resp to θ and equate to 0 (zero)

$$\frac{n}{\theta} - \frac{1}{2}\left(\sum_{i=1}^{n} x_i^2\right) = 0$$

$$\frac{n}{\theta} = \frac{1}{2}\left(\sum_{i=1}^{n} x_i^2\right)$$

$$\boxed{\theta = \frac{2n}{\sum_{i=1}^{n}(x_i^2)}}$$

for data

$$\theta = \frac{2\times 5}{(1.25 + 4 + 0.81 + 0.64 + 1.5625)}$$

$$\theta = \frac{10}{9.2625} = 1.0796$$

Part 2 Method of ~~mom~~ moment estimator

Expectation of distribution = Mean of data

$$\sqrt{\frac{\pi}{2\theta}} = \frac{1.5 + 2 + 0.9 + 1.25 + 0.8}{5}$$

$$\sqrt{\frac{\pi}{2\theta}} = 1.29$$

$$\frac{\pi}{2\theta} = 1.6641$$

$$2\theta = \frac{\pi}{1.6641}$$

$$\theta_{MME} = \frac{\pi}{2(1.6641)}$$

$$\boxed{\theta_{MME} = 0.9434}$$

Ans

Q.4

Name - Sayam Kumar
S20180010158
Page No 4

(a) $\alpha = P(\text{type I error}) = P\left(\begin{array}{c}\text{reject } H_0 \\ \text{when } H_0 \\ \text{is true}\end{array}\right)$

$\sigma^2 = 0.0625$

$\sigma = 0.25$

$= P(4.84 < \text{sample mean} < 5.16)$ Converting to st.

$= \dfrac{(4.85-5)\times 3}{(0.5)} < \text{mean} < \dfrac{(5.16-5)\times 3}{0.5}$

$= (-0.9 < \text{mean} < 0.96)$

$= P(\text{mean} < 0.96) - P(\text{mean} < -0.9)$

$= 0.8315 - (1 - 0.8159)$

~~$= 0.0156$~~ $= 0.8315 - 0.1841$

$= 0.6474$

$\alpha = 1 - 0.6474$

$\boxed{\alpha = 0.3526}$ Ans

Part b On Page 5

(a) Sample mean = 5.15

$Z_0 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{5.15 - 5}{0.25/\sqrt{9}} = \frac{0.15 \times 3}{0.25} = \frac{15 \times 3}{25}$

$= \frac{9}{5} = 1.8$

$P(\text{value}) = 2\left[1 - z(1.8)\right] = 2\left[1 - 0.9641\right]$

$\boxed{= 0.0718}$

P value

Q-6 Continued on next page

Sayam Kumar
S20180010158

Q-5 $n = 25$ $\left(n < 30 \text{ follows } Z \text{ distribution} \right)$

Sayam Kumar
S20180010150
Page No 6

$\sigma = 0.8$, μ unknown

σ given

90% CI

$\alpha = 0.1$, $\Rightarrow$ $\frac{\alpha}{2} = 0.05$ | $\bar{x} = 89$

$$90\% \text{ CI} = \left(\bar{x} - Z_{0.05} \frac{\sigma}{\sqrt{n}} , \bar{x} + Z_{0.05} \frac{\sigma}{\sqrt{n}} \right)$$

$$= \left(89 - (-1.645) \frac{0.8}{5} , 89 + (-1.645) \frac{0.8}{5} \right)$$

$$= 89 - 0.2632 , 89 + 0.2632$$

$$= \left(88.7368 , 89.2632 \right) \text{ Ans}$$