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UG3

Q-1 $(3x^2 - 9x^2y^2 + 2xy)dx + (6y^2 + ax^3y^2 + x^2)\,dy = 0$

$Mdx + Ndy = 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$-18x^2y + \cancel{2x} = 3ax^2y^2 + \cancel{2x}$

$-18\cancel{x^2}\cancel{y} = 3a\cancel{x^2}y\cancel{y}$

$3ay = -18$

$a = -\frac{6}{y}$

Solution ~~∫Mdy + ∫ ∫N~~

$\int M dx + \int N dy = c$

$\downarrow$ y constant $\downarrow$ eliminating x terms

$x^3 - \frac{9}{3}x^3y^2 + x^2y + \frac{6}{3}y^3 = c$

$\boxed{x^3 - 3x^3y^2 + x^2y + 2y^3 = c}$ Ans

Q.2 Solve

$(D^2 - 3D + 2)y = \sin e^{-x}$

$(D^2 - 3D + 2)y = 0$ finding y_{CF}

$(D-2)(D-1)y = 0$

$y_{cf} = c_1 e^{2x} + c_2 e^{x}$

Finding particular solution

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Using variation of parameters method

$y_1 = e^{2x}$

$y_2 = e^{x}$

$$W = \begin{vmatrix} e^{2x} & e^{x} \\ 2e^{2x} & e^{x} \end{vmatrix} = e^{3x} - 2e^{3x} = -e^{3x}$$

$$y_{parti} = \int -\frac{\sin(e^{-x})\, e^{2x}}{e^{3x}}\,dx + \int \frac{\sin(e^{-x})\, e^{x}}{e^{3x}}\,dx$$

$$= -\cos(e^{-x}) + \left[e^{-x}\cos(e^{-x}) - \sin(e^{-x})\right]$$

$$= -e^{2x}\sin(e^{-x})$$

$$y\ complete = -e^{2x}\sin(-x) + c_1 e^{2x} + c_2 e^{2x}$$

Ans

Q-3 $W = 32\ lb$

$m = \frac{w}{g} = \frac{32}{32} = 1\ slug$

$k = 4\ lb/ft$ $\qquad f(t) = \cos^2(t)$

$\beta = 2\frac{dx}{dt}$

Eq

$$m\frac{d^2x}{dt^2} = -kx - \beta\frac{dx}{dt} + f(t)$$

$$\frac{d^2x}{dt^2} + 4x + 2\left(\frac{dx}{dt}\right)^2 = \cos^2 t$$

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 4x = \cos^2 t$$

$$(D^2 + 2D + 4)x = \cos^2 t$$

$y_{cf} = D^2 + 2D + 4$

$D = \frac{-2 \pm \sqrt{4-16}}{2} = -1 \pm i\sqrt{3}$

$y_{cf} = e^{-x}\left[c_1\cos(\sqrt{3}x) + c_2\sin(\sqrt{3}x)\right]$

$x_{CF} = e^{-t}\left[c_1\cos(\sqrt{3}t) + c_2\sin(\sqrt{3}t)\right]$

Imaginary Roots $\Rightarrow$ Under-damped motion

$$x_p = \frac{\cos^2 t}{(D^2+2D+4)} = \frac{\cos 2t - 1}{D^2+2D+4} = \frac{\cos 2t}{D^2+2D+4} - \frac{1}{D^2+2D+4}$$

$\underline{D^2 = -4}$

$$= \frac{\cos 2t}{2D} - \frac{1}{D^2+2D+4}$$

$$= \frac{1}{2}\frac{\sin 2t}{2} - \left(1 + (D^2+2D+3)\right)^{-1} 1.$$

$$\underline{x_p} = \frac{\sin 2t}{4} - (1 - D^2 - 2D - 3)\,1 = \frac{\sin 2t}{4} + 2$$

$$x_{complete} = \frac{\sin 2t}{4} + 2 + e^{-t}\left[c_1\cos\sqrt{3}t + c_2\sin(\sqrt{3}t)\right]$$

At $t=0, x=1 \Rightarrow C_1 = -\frac{e}{e} = -1$

At $\frac{dx}{dt} = 0, t = 0 \Rightarrow \sqrt{3}c_2 + \frac{3}{2} = 0$

$c_2 = \frac{-3}{2\sqrt{3}} = \frac{-\sqrt{3}}{2}$

Ans

Q.4 (a) $y_n = a2^n + b(-2)^n$ | Page 4 | S20180010158

$y_{n+1} = 2a2^n + (-2)b(-2)^n$

$y_{n+2} = 4a2^n + 4b(-2)^n$

$y_{n+2} = 4[a2^n + b(-2)^n] \Rightarrow \boxed{y_{n+2} = 4y_n}$ Ans

(ii)

$y_{n+2} - 4y_{n+1} + 4y_n = n^2 2^n$

$E^2 y_n - 4E y_n + 4y_n = n^2 2^n$

$(E^2 - 4E + 4) y_n = n^2 2^n$

$y_{CF} \Rightarrow (E^2 - 4E + 4) = 0$

$(E-2)^2 = 0$

$E = 2, 2$

$y_{cf} = (c_1 + c_2 n)2^n$

Trial for $y_{particular}$

$(c_0 + c_1 n + c_2 n^2)2^n$

Repeating twice, multiply by n^2

$y_{Trial} = (c_0 + c_1 n + c_2 n^2) n^2 2^n$

Satisfy $\left(c_0 + c_1(n+2) + c_2(n+2)^2\right)(n+2)^2 2^{(n+2)} - 4\left[c_0 + c_1(n+1) + c_2(n+1)^2\right](n+1)^2 2^{(n+1)}$

$+ 4(c_0 + c_1 n + c_2 n^2) n^2 2^n = n^2 2^n$

$y(n) = \frac{1}{3} 2^{n-4}\left(48c_2 n + 48c_1 + (n-2)n(n-1)^2\right)$

Ans

$y_{Particular} = \frac{(n-1)^2 n (n-2)}{3} 2^{n-4}$ $y_{CF} = (c_1 + c_2 n)2^n$

Q-5 $\frac{dy}{dx} = 2x-y$ $y(1)=3$ (Page-5)

$x_0=1,\ y_0=3$ S20180010158

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\,dx$$

$$= 3 + \int_{x_0}^{x} (2x-3)\,dx = 3 + (x^2-3x)_1^x = 3 + (x^2-3x) - (1-3)$$

$$= x^2 - 3x + 5$$

Second approximation

$$y_2 = y_0 + \int_{x_0}^{x} f(x, y_1)\,dx$$

$$= 3 + \int_1^x 2x - (x^2-3x+5)\,dx = 3 + \int_1^x 2x - x^2 + 3x - 5\,dx$$

$$= 3 + \int_1^x -x^2 + 5x - 5\,dx$$

$$= 3 + \left(\frac{-x^3}{3} + \frac{5}{2}x^2 - 5x\right)_1^x$$

$$= 3 + \left[\left(-\frac{x^3}{3} + \frac{5}{2}x^2 - 5x\right) - \left(-\frac{1}{3} + \frac{5}{2} - 5\right)\right]$$

$$= \frac{5x^2}{2} - \frac{x^3}{3} - 5x + \frac{35}{6}$$

At $x = 1.2$

$y = 2.857$ Ans

Q-6 $\quad y' = 3x + y^2 \qquad y(1) = 1.2$

$x_0 = 1$

$y_0 = 1.2$ $\qquad$ R-K 4th Order

$h = 0.2$

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$k_1 = h f(x_0, y_0)$

$= 0.2 \left[3 + (1.2)^2\right] = 0.888$

$k_2 = h f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_1\right)$

$= 0.2 f(1.1, 1.644) = 1.2005$

$k_3 = h f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_2\right)$

$= 0.2 f(1.1, 1.8) = 1.308$

$k_4 = h f(x_0 + h, y_0 + k_3)$

$= 0.2 f(1.2, 2.508) = 1.978$

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 1.2 + \frac{1}{6}(7.883)$

$\boxed{= 2.5138}$ Ans

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Q-7 $x+y+z = f(x^2+y^2+z^2)$

Partial with r to x (y const)

$$1+\frac{\partial z}{\partial x} = f'(x^2+y^2+z^2)(2x) \quad -(1)$$

Partial diff w r to y

$$1+\frac{\partial z}{dy} = f'(x^2+y^2+z^2)\,2y \quad -(2)$$

Divide (1) and (2)

$$\frac{1+\frac{\partial z}{\partial x}}{1+\frac{\partial z}{\partial y}} = \frac{2x}{2y}$$

$$\boxed{y+y\frac{\partial z}{\partial x} = x+x\frac{\partial z}{\partial y}}$$ Ans

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Q-8 $$z\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right) = z^2+(x^2+y^2)$$

$P = z, \quad Q = -z$

$$\frac{dx}{z} = \frac{dy}{-z}$$

$$\boxed{-x+C=y}$$

Make $PP_1+QQ_1+RR_1=0$

$P_1 = -\left(z+\frac{x^2}{z}\right), \quad Q_1 = \frac{-y^2}{z}, \quad R_1 = 1$

$\Rightarrow -(z^2+x^2)-y^2+x^2+y^2+z^2=0$

$0=0$ ✓

Solution $\int P_1\,dx+\int Q_1\,dy+\int R_1\,dz=0$

$$-zx+\left(-\frac{x^3}{3z}\right)+\frac{-y^3}{3z}+z=0 \rightarrow$$ Ans

Complete Ans

$$F\left(x+y,\ -zx+\left(-\frac{x^3}{3z}\right)-\frac{y^3}{3z}+z\right)=0$$

$$\left(x+y,\ -zx-\frac{x^3}{3z}-\frac{y^3}{3z}+z\right)$$