LECTURE 6(4-9-2020)

Inexact equations reducible to exact form

- Some times a differential equation which is not exact, can be reduced to exact equation by multiplying with a suitable factor(function in terms of x and y) called as integrating factor.
- Although a differential equation of type $Mdx+Ndy=0$ always has an integrating factor, there is no general method of finding them. Here we shall explain some of the methods for finding the integrating factors.

Method I :Integrating factor found by Inspection

- In some cases , the integrating factor can be found after regrouping the terms of the equation and recognizing each group as being a part of an exact differential

List of exact differential equations

(a) $d\left(\frac{x}{y}\right)=\frac{ydx-xdy}{y^2}$

(b) $d\left(\frac{y}{x}\right)=\frac{xdy-ydx}{x^2}$

(c) $d(xy) = xdy + ydx$

(d) $d\left(\frac{x^2}{y}\right)=\frac{2yxdx-x^2dy}{y^2}$

(e) $d\left(\frac{y^2}{x}\right)=\frac{2xydy-y^2dx}{x^2}$

(f) $d\left(\frac{x^2}{y^2}\right)=\frac{2xy^2dx-2x^2ydy}{y^4}$

(g) $d\left(\frac{y^2}{x^2}\right)=\frac{2x^2ydy-2xy^2dx}{x^4}$

(h) $d\left(\frac{1}{xy}\right)=-\frac{xdy+ydx}{x^2y^2}$

(i) $d\left(\log\frac{y}{x}\right)=\frac{xdy-ydx}{xy}$

(j) $d\left(\log\frac{x}{y}\right)=\frac{ydx-xdy}{xy}$

(k) $d\left(\tan^{-1}\frac{x}{y}\right)=\frac{ydx-xdy}{x^2+y^2}$

(l) $d\left(\tan^{-1}\frac{y}{x}\right)=\frac{xdy-ydx}{x^2+y^2}$

(m) $d\left(\frac{e^x}{y}\right)=\frac{ye^xdx-e^xdy}{y^2}$

(n) $d\left(\frac{e^y}{x}\right)=\frac{xe^ydy-e^ydx}{x^2}$

(o) $d\left(-\frac{1}{xy}\right)=\frac{xdy+ydx}{x^2y^2}$

(p) $d\left[\frac{1}{2}\log(x^2+y^2)\right]=\frac{xdx+ydy}{x^2+y^2}$

Example:

- (a) $y(2xy+e^{x})dx = e^{x}dy$(*refer to notes for the solution)* lecture 6 notes(4-9-2020).pdf
- *(b)* $xdy-ydx=a(x^{2}+y^{2})$

Method II: Integrating factor of homogeneous equation

- If the differential equation Mdx+Ndy=0 is homogeneous and Mx+Ny≠0 the $\frac{1}{Mx+Ny}$ is the integrating factor.
- Example:
- $(x^2y - 2xy^2)$dx-$(x^3 - 3x^2y)$dy=0
- Refer to notes for the solution
- lecture 6 notes(4-9-2020).pdf

Method III:IF of the equation of the type f(xy)ydx+g(xy)xdy=0

- If the equation Mdx+Ndy=0 be of this form

f(xy)ydx+g(xy)xdy=0 then the integrating factor is $\frac{1}{Mx-Ny}$ where $Mx - Ny \neq 0$

Example: (1+xy)ydx+(1-xy)xdy=0

Refer to notes for the solution

[lecture 6 notes(4-9-2020).pdf]