ML Mid-1

1a) $X = (\text{Blue}, \text{True})^t$ $\quad \omega_1 = \text{yes}, \quad \omega_2 = \text{NO}$

$$P((\text{Blue}, \text{True})^t) = P(\text{Blue}/\text{yes})\, P(\text{True}/\text{yes})\, P(\text{yes}) + P(\text{Blue}/\text{No})\, P(\text{True}/\text{No})\, P(N)$$

$$= \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{1}{2} + \frac{2}{4} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{5}{32}$$

$$P(\omega_1/x) = \frac{P(x/\omega_1)\, P(\omega_1)}{P(x)} = \frac{P(x=\text{Blue}/\omega_1=\text{yes})\, P(x=\text{True}/\omega_1=\text{yes})\, P(\omega_1=\text{yes})}{P(x)}$$

$$= \frac{\frac{1}{4} \cdot \frac{3}{4} \cdot \frac{1}{2}}{5/32} = \frac{3}{5}$$

$$P(\omega_2/x) = \frac{P(x=\text{Blue}/\omega_2=\text{No}) \cdot P(x=\text{True}/\omega_2=\text{No})\, P(\omega_2=\text{No})}{P(x)}$$

$$= \frac{\frac{2}{4} \cdot \frac{1}{4} \cdot \frac{1}{2}}{5/32} = 2/5.$$

$\therefore$ since $P(\omega_1/x) > P(\omega_2/x)$, classify $X = (\text{Blue}, \text{True})^t$ to class $\omega_1 = \text{yes}$.

b) Similarly we can find $P(x)$, $P(\omega_1 = \text{yes}/x)$, $P(\omega_2 = \text{No}/x)$ for every x.

X	$P(X)$	$P(\omega_1 = yes/x)$	$P(\omega_2 = \cancel{yes}\ NO / x)$
R T	$\frac{4}{32}$	$3/4$	$1/4$
R F	$\frac{4}{32}$	$1/4$	$3/4$
G T	$7/32$	$6/7$	$1/7$
G F	$5/32$	$2/5$	$3/5$
B T	$5/32$	$3/5$	$2/5$
B F	$7/32$	$1/7$	$6/7$

$$P(error) = \sum P(error/x) \cdot P(x)$$

$$p(error|x) = \min\{P(\omega_1/x),\ P(\omega_2/x)\}$$

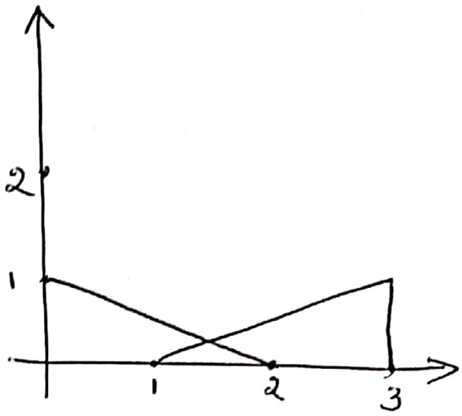
$$P(error) = \frac{1}{4} \cdot \frac{4}{32} + \frac{2}{5} \cdot \frac{5}{32} + \frac{1}{7} \cdot \frac{7}{32} + \frac{1}{7} \cdot \frac{7}{32} +$$

$$\cancel{\frac{1}{7} \cdot \frac{7}{32}} + \frac{1}{4} \cdot \frac{4}{32} + \frac{2}{8} \cdot \frac{8}{32} + \cancel{\frac{1}{7} \cdot \frac{7}{32}}$$

$$= \cancel{\frac{1}{8}} \quad \frac{8}{32} = \frac{1}{4} \ //$$

2 $p(x/\omega_1) = \begin{cases} 1 - x/2 & [0,2] \\ 0 & \end{cases}$

$p(x/\omega_2) = \begin{cases} \frac{x}{2} - \frac{1}{2} & [1,3] \\ 0 & \end{cases}$



$$\lambda_{ij} = \begin{bmatrix} 0 & 1 \\ 1.5 & 0 \\ 0.3 & 0.4 \end{bmatrix}$$

$$R(\alpha_1/x) = \lambda(\alpha_1/\omega_1)\,P(\omega_1/x) + \lambda(\alpha_1/\omega_2)\,P(\omega_2/x)$$

$$P(\omega_1/x) = \frac{P(x/\omega_1)\,P(\omega_1)}{P(x)}$$

$$P(x) = p(x/\omega_1)\,P(\omega_1) + p(x/\omega_2)\,P(\omega_2)$$

$$= \begin{cases} \left(1 - \frac{x}{2}\right)\frac{1}{2} & 0 \leq x \leq 1 \\ \frac{1}{2}(1 - x/2) + \frac{1}{2}\left(\frac{x}{2} - 1/2\right) & 1 \leq x \leq 2 \\ \frac{1}{2}\left(\frac{x}{2} - \frac{1}{2}\right) & 2 \leq x \leq 3 \end{cases}$$

$$p(x) = \begin{cases} \frac{1}{2} - \frac{x}{4} & 0 \leq x \leq 1 \\ \frac{1}{4} & 1 \leq x \leq 2 \\ \frac{x}{4} - \frac{1}{4} & 2 \leq x \leq 3 \end{cases}$$

check $\int p(x)\,dx = 1$

$$\left[\frac{x}{2} - \frac{x^2}{8}\right]_0^1 + \frac{1}{4}\left[x\right]_1^2 + \left[\frac{x^2}{8} - \frac{x}{4}\right]_2^3 = 1$$

$$\frac{1}{2} - \frac{1}{8} + \frac{1}{4} + \frac{9}{8} - \frac{3}{4} - \frac{4}{8} + \frac{2}{4} = 1$$

$$P(\omega_1/x) = \frac{\left(1-\frac{x}{2}\right)\frac{1}{2}}{\left(\frac{1}{2}-\frac{x}{4}\right)} = 1 \qquad 0 \le x \le 1$$

$$= \frac{(1-x/2)\,1/2}{1/4} = (2-x) \qquad 1 \le x \le 2$$

$$= 0 \qquad 2 \le x \le 3$$

$$P(\omega_2/x) = \begin{cases} 0 & x \in [0,1] \\ x-1 & x \in [1,2] \\ 1 & x \in [2,3] \end{cases}$$

$$\begin{bmatrix} R(\alpha_1/x) \\ R(\alpha_2/x) \\ R(\alpha_3/x) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1.5 & 0 \\ 0.3 & 0.4 \end{bmatrix} \begin{bmatrix} P(\omega_1/x) \\ P(\omega_2/x) \end{bmatrix}$$

$x \in [0,1]$

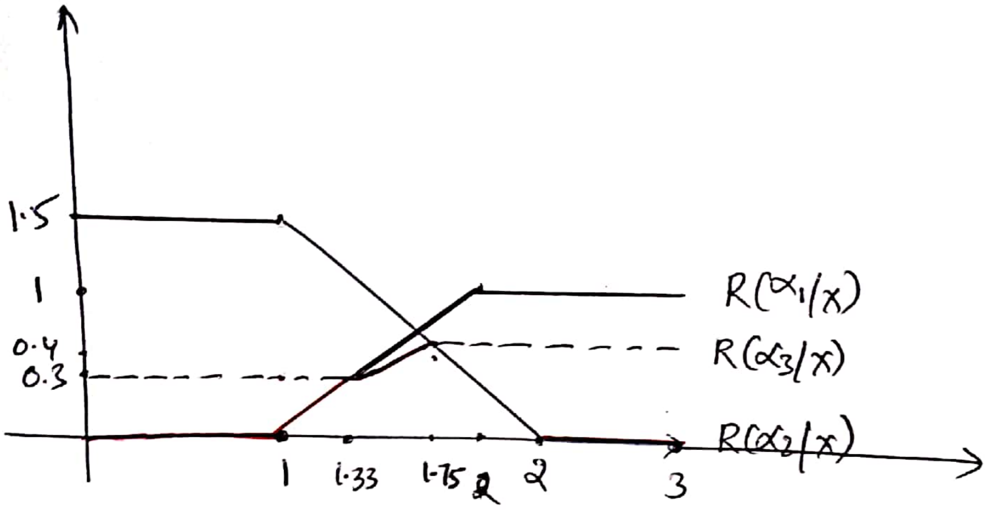
$$\begin{bmatrix} R(\alpha_1/x) \\ R(\alpha_2/x) \\ R(\alpha_3/x) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1.5 & 0 \\ 0.3 & 0.4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.5 \\ 0.3 \end{bmatrix}$$

$x \in [1,2]$

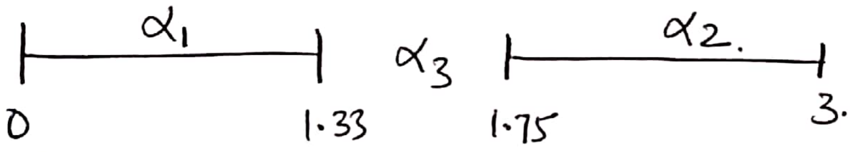
$$= \begin{bmatrix} 0 & 1 \\ 1.5 & 0 \\ 0.3 & 0.4 \end{bmatrix} \begin{bmatrix} 2-x \\ x-1 \end{bmatrix} = \begin{bmatrix} x-1 \\ 3-1.5x \\ 0.1x+0.2 \end{bmatrix}$$

$x \in [2,3]$

$$= \begin{bmatrix} 0 & 1 \\ 1.5 & 0 \\ 0.3 & 0.4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0.4 \end{bmatrix}$$



Decision boundaries.



3. Given $P(x) = \frac{1}{\theta^2} x e^{-x/\theta}$, $0 \leq x \leq \infty$

$$P(x_1, x_2, \dots x_n/\theta) = \frac{1}{\theta^2} x_1 e^{-x_1/\theta} \dots \frac{1}{\theta^2} x_n e^{-x_n/\theta}$$

Apply log on both sides, we get.

$$\log P(x/\theta) = -2n \log\theta + (\log x_1 + \log x_2 + \dots + \log x_n) - \left(\frac{x_1 + x_2 + \dots x_n}{\theta}\right) \rightarrow eq\ (1)$$

$$\frac{d}{d\theta} \log P(x/\theta) = d/d\theta. (-2n \log\theta + \frac{d}{d\theta}(\log x_1 + \log x_2 + \dots \log x_n) - \frac{d}{d\theta}\left[\frac{x_1 + x_2 + \dots + x_n}{\theta}\right]$$

$$= -\frac{2n}{\theta} + 0 + \frac{x_1 + x_2 + \dots + x_n}{\theta^2}$$

$$\frac{d}{d\theta} \log P(x/\theta) = 0$$

$$\Rightarrow \quad 2n\theta = x_1 + x_2 + \dots + x_n$$

$$\theta = \frac{x_1 + x_2 + \dots + x_n}{2n}$$

$$\theta = \frac{\sum_{k=1}^{n} x_k}{2n}$$