Name - Sayam Kumar
Roll No - S20180010158
Page No - 1

ML Mid Sem - 1

Q-1

+1 class data = (-1,-1), (0,0), (-1,0), (0,-1)

-1 class = (4,4), (5,5), (3,6), (6,3)

$\mu_1 = \frac{-2}{4}, \frac{-2}{4}$

$\mu_1 = (-1/2, -1/2)$

$\mu_2 = \left(\frac{18}{4}, \frac{18}{4}\right)$

$\mu_2 = \left(\frac{9}{2}, \frac{9}{2}\right)$

Covariance $\Sigma_1 \Rightarrow$

$(x-\mu) = -3/2, -3/2$
$-1/2, -1/2$
$-3/2, -1/2$
$-1/2, -3/2$

(a) $\begin{pmatrix} -3/2 \\ -3/2 \end{pmatrix} \begin{pmatrix} -3/2 & -3/2 \end{pmatrix}$

$\begin{pmatrix} 9/4 & 9/4 \\ 9/4 & 9/4 \end{pmatrix}$

(b) $\begin{pmatrix} -1/2 \\ -1/2 \end{pmatrix} \begin{pmatrix} -1/2 & -1/2 \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{pmatrix}$

(c) $\begin{pmatrix} -3/2 \\ -1/2 \end{pmatrix} \begin{pmatrix} -3/2, & -1/2 \end{pmatrix} = \begin{pmatrix} \frac{9}{4} & \frac{3}{4} \\ 3/4 & 1/4 \end{pmatrix}$

(d) $\begin{pmatrix} -1/2 \\ -3/2 \end{pmatrix} \begin{pmatrix} -1/2 & -3/2 \end{pmatrix} = \begin{pmatrix} 1/4 & 3/4 \\ 3/4 & 9/4 \end{pmatrix}$

Add

$\Sigma_1 = \frac{1}{4} \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$

$\Sigma_1 = \frac{1}{4} \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$

$(x-\mu) = (-1/2 \quad -1/2)$,
$(1/2, \ 1/2)$,
$(-3/2 \quad 3/2)$,
$(3/2, \ -3/2)$

(a) $\begin{pmatrix} -1/2 \\ -1/2 \end{pmatrix} \begin{pmatrix} -1/2 & -1/2 \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{pmatrix}$

(b) $\begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}^T = \begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{pmatrix}$

(c) $\begin{pmatrix} -3/2 \\ 3/2 \end{pmatrix} \begin{pmatrix} -3/2 & 3/2 \end{pmatrix} = \begin{pmatrix} 9/4 & -9/4 \\ -9/4 & 9/4 \end{pmatrix}$

(d) $\begin{pmatrix} 3/2 \\ -3/2 \end{pmatrix} \begin{pmatrix} 3/2 & -3/2 \end{pmatrix} = \begin{pmatrix} 9/4 & -9/4 \\ -9/4 & 9/4 \end{pmatrix}$

Add

$\Sigma = \frac{1}{4} \begin{pmatrix} 5, & -4 \\ -4, & 5 \end{pmatrix}$

$\Sigma_2 = \frac{1}{4} \begin{pmatrix} 5 & -4 \\ -4 & 5 \end{pmatrix}$

Part 1 Discriminant Part 1

Sayam Kumar
S20180010158
Page No 2

$$g\omega_1 = -\frac{1}{2}\left(x_1+\tfrac{1}{2},\ x_2+\tfrac{1}{2}\right)\frac{1}{4}\times\frac{1}{9}\begin{bmatrix}5 & -4\\ -4 & 5\end{bmatrix}\begin{pmatrix}x_1+1/2\\ x_2+1/2\end{pmatrix}$$

$$-\frac{2}{2}\ln 2\pi - \frac{1}{2}\ln 9 - \ln 1/3$$

$$g\omega_1 = -\frac{1}{72}\left(5x_1+5/2-4x_2-2,\ -4x_1-2+5x_2+5/2\right)\begin{pmatrix}x_1+1/2\\ x_2+1/2\end{pmatrix} - \ln 2\pi - \frac{1}{2}\ln 9 - \ln 1/3$$

$$g\omega_1 = \frac{-1}{72}\left(5x_1-4x_2+1/2,\ 5x_2-4x_1+1/2\right)\begin{pmatrix}x_1+1/2\\ x_2+1/2\end{pmatrix} - \ln 2\pi - \frac{1}{2}\ln 9 - \ln 1/3$$

$$g\omega_2 = \frac{-1}{2}\left(x_1-\frac{9}{2}\ \ x_2-\frac{9}{2}\right)\frac{1}{4}\times\frac{1}{9}\begin{bmatrix}5 & 4\\ 4 & 5\end{bmatrix}\begin{pmatrix}x_1-9/2\\ x_2-9/2\end{pmatrix} \cancel{2\pi} - \ln 2\pi - \frac{1}{2}\ln 9$$

$$-\ln 2/3$$

$$= \frac{-1}{72}\left(5x_1-\frac{45}{2}+4x_2-\frac{36}{2},\ 4x_1-\frac{36}{2}+5x_2-\frac{45}{2}\right)\begin{pmatrix}x_1-9/2\\ x_2-9/2\end{pmatrix} - \ln 2\pi - \frac{1}{2}\ln 9 - \ln\frac{2}{3}$$

$$= -1/72\left(5x_1+4x_2-40.5,\ 4x_1-5x_2-40.5\right)\begin{pmatrix}x_1-9/2\\ x_2-9/2\end{pmatrix} - \ln 2\pi - \frac{1}{2}\ln 9 - \ln 2/3$$

Part 2 Priors are equal

$$g\omega_1 = \frac{-1}{72}\left(5x_1-4x_2+1/2,\ 5x_2-4x_1+1/2\right)\begin{pmatrix}x_1+1/2\\ x_2+1/2\end{pmatrix} - \ln 2\pi - \frac{1}{2}\ln 9 - \ln 1/2$$

$$g\omega_2 = \frac{-1}{72}\left(5x_1+4x_2-40.5,\ 4x_1-5x_2-40.5\right)\begin{pmatrix}x_1-9/2\\ x_2+1/2\end{pmatrix} - \ln 2\pi - \frac{1}{2}\ln 9 - \ln 1/2$$

Ans

Sayam Kumar
S20180010158
Page No 3

Q-2 Assuming equal priors

$p(w_1) = p(w_2) = 1/2$

$p(x) = p(x|w_1)\,p(w_1) + p(x|w_2)\,p(w_2)$

$$
\begin{cases}
= \left(\frac{3-x}{2}\right) 1/2 & , 1 \le x \le 2 \\
= \left(\frac{3-x}{2}\right) 1/2 + \left(\frac{x-2}{2}\right) 1/2 = 1/4 & , 2 \le x \le 3 \\
= \frac{x-2}{4} & , 3 \le x \le 4
\end{cases}
$$

Now $p(w_1|x) = \frac{p(x|w_1)\,p(w_1)}{p(x)}$

$$
\begin{cases}
= \left(\frac{3-x}{4}\right) \Big/ \frac{3-x}{4} = 1 & \text{for } 1 \le x \le 2 \\
= \frac{\left(\frac{3-x}{2}\right) \times \frac{1}{2}}{} \times \frac{4}{1} = 3-x & \text{for } 2 \le x \le 3 \\
= 0 & 3 \le x \le 4
\end{cases}
$$

$p(w_2|x) =$

$$
\begin{aligned}
&0 && \text{for } 1 \le x \le 2 \\
&= \left(\frac{x-2}{2}\right) \times \frac{1}{2} \times \frac{4}{1} = x-2 && \text{for } 2 \le x \le 3 \\
&1 && \text{for } 3 \le x \le 4
\end{aligned}
$$

$$
p(w_1|x) = \begin{cases} 1 & [1,2] \\ 3-x & [2,3] \\ 0 & [3,4] \end{cases}
\qquad
p(w_2|x) = \begin{cases} 0 & [1,2] \\ x-2 & [2,3] \\ 1 & [3,4] \end{cases}
$$

~~x ∈ or~~ for $x \in [1,2]$

$$
\begin{matrix} R(\alpha_1|x) \\ R(\alpha_2|x) \\ R(\alpha_3|x) \end{matrix}
= \begin{bmatrix} 0 & 1.5 \\ 1 & 0 \\ 0.3 & 0.4 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 \\ 1 \\ 0.3 \end{bmatrix}
$$

for $x \in [3,4]$

$$
\begin{matrix} R(\alpha_1|x) \\ R(\alpha_2|x) \end{matrix}
= \begin{bmatrix} 0 & 1.5 \\ 1 & 0 \\ 0.3 & 0.4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}
$$

$$
R(\alpha_3|x) = \begin{bmatrix} 1.5 \\ 0 \\ 0.4 \end{bmatrix}
$$

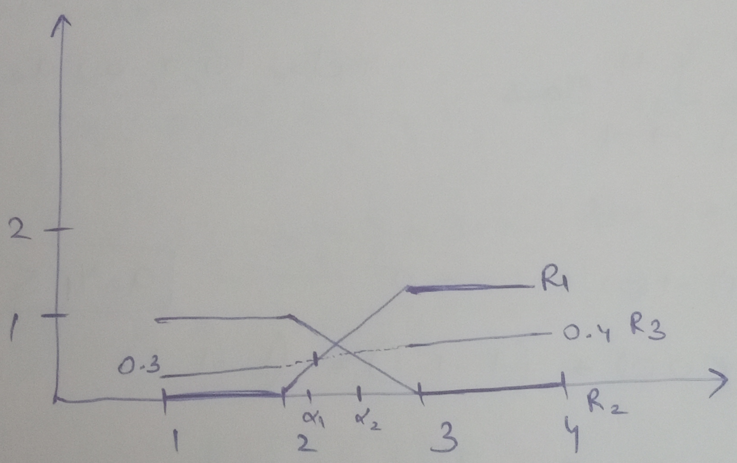
Sayam Kumar
S20180010158
Page No 4

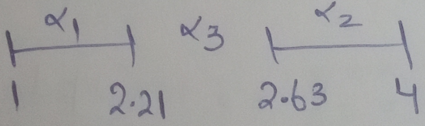
for $x \in [2,3]$

$$\begin{matrix} R(\alpha_1|x) \\ R(\alpha_2|x) \\ R(\alpha_3|x) \end{matrix} = \begin{bmatrix} 0 & 1.5 \\ 1 & 0 \\ 0.3 & 0.4 \end{bmatrix} \begin{bmatrix} 3-x \\ x-2 \end{bmatrix}$$

$$\cancel{= \frac{1.5x}{3}} \quad = \begin{bmatrix} 1.5x-3 \\ 3-x \\ 0.1x+0.1 \end{bmatrix}$$

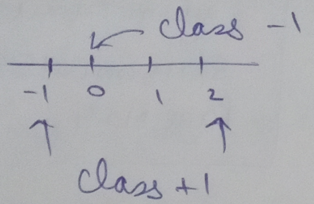
Plotting decision boundaries





Q-3 Next Page

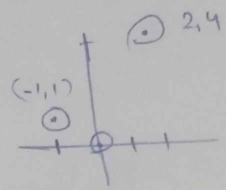
Q-3



Name - Sayam Kumar
Roll No - S20180010158
Page No - 5

Transforming space (x, x^2)

$-1, 1$ } class +1
$2, 4$
$0, 0$ → class -1



Initial Solution $= (0, 0, 0)$

Augmenting and normalizing

y_1 $\mathbb{I}_1$ = 1 -1 1 } +1 Class
y_2 $\mathbb{I}_2$ 1 2 4
y_3 $\mathbb{I}_3$ -1 0 0 → -1

Intercept

vector $(0, 0, 0)$ a_0

Calculate $= a_0 y_1 = 0(1) -1(0) +1(0) = 0$ $\boxed{a_0 y_1 \leq 0 \text{ update}}$

So $a_1 = (0\,0\,0) + (1\,-1\,1) = (1\,-1\,1)$

Calculate with y_2 $= 1 - 2 + 4 = 5$ $\boxed{\text{No update}}$

Calculate with y_3 $= -1(1) + 0 + 0 = -1 \leq 0$ update

$a_2 = (1\,-1\,1) + (-1\,0\,0) = (0\,-1\,1)$

$y_1 = a_2 y_1 = 1 + 1 = 2 \geq 0$ No update

$y_2 = a_2 y_2 = -2 + 4 = 2 \geq 0$ No update

~~$y_3 = a_2 y_3 = 0 \geq 0$ No update~~ $= 0$

~~Discriminant $-x + y + 0 = 0$~~

Sayam Kumar
S20180010158

$y_3 \Rightarrow a_2 y_3 = 0 \le 0$ update

$(0\ -1\ 1) + (-1\ 0\ 0)$

$a_3 = (-1\ -1\ 1)$

$y_1 \Rightarrow -1+1+1 = 1 \geq 0$

$y_2 \Rightarrow 4-2-1 = 1 \geq 0$

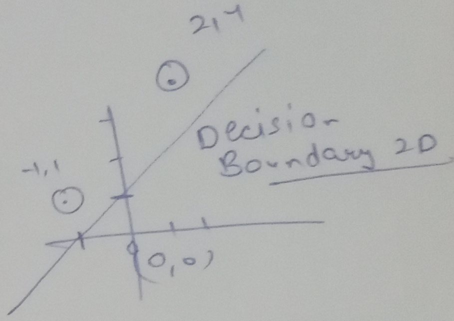
$y_3 = 1 \geq 0$

Intercept

$a_2 = (-1\ -1\ 1)$

Discriminant $-1 - x + y = 0$

$\boxed{y = x+1}$



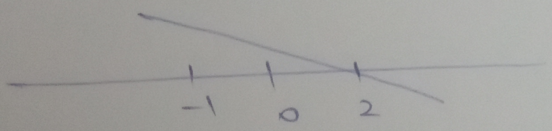
(4) Translate to 1 D

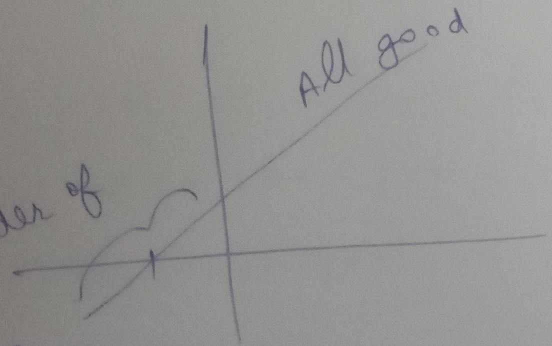
~~For 1R, y = 0~~

~~$x+1=0$~~

~~$x = -1$~~ ← Taking root

Taking underoot this complex plane





under of negative will give complex plane

Thanks