DE-ASSIGNMENT :- 4 & 5

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1) Given,

$$\frac{dy}{dx} = \frac{x^2}{y^2+1}, \quad y(0) = 0 \qquad \text{at } x = 0.5 \quad y = ? \qquad (\text{upto } y_2)$$

W.K.T,

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\,dx$$

$$= 0 + \int_0^x \frac{x^2}{0+1}\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^x$$

$$\therefore y_1 = \frac{x^3}{3}$$

Now,

$$y_2 = y_0 + \int_{x_0}^{x} f(x, y_1)\,dx$$

$$= 0 + \int_0^x \frac{x^2}{\left(\frac{x^3}{3}\right)^2 + 1}\,dx$$

$$= \int_0^x \frac{9x^2}{x^6+9}\,dx.$$

let, $x^3 = t$

$3x^2 dx = dt$ $\Rightarrow$ $y_2 = \int_0^x \frac{3dt}{t^2+9}$

$$= 3\int_0^x \frac{dt}{t^2+3^2}$$

W.K.T, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

Now,

$$y_2 = \cancel{3} \times \frac{1}{\cancel{3}} \tan^{-1}\left(\frac{t}{3}\right)\Big]_0^x$$

$$= \tan^{-1}\left(\frac{t}{3}\right)\Big]_0^x$$

$$= \tan^{-1}\left(\frac{x^3}{3}\right)\Big]_0^x$$

$$\therefore \boxed{y_2 = \tan^{-1}\left(\frac{x^3}{3}\right)} \qquad \text{at, } x = 0.5 \qquad \Rightarrow \boxed{\tan y = \frac{x^3}{3}}$$

$$y_2 = \tan^{-1}\left(\frac{1}{24}\right) = 2.38594.$$

Exact Solution:-

$$\frac{dy}{dx} = \frac{x^2}{y^2+1}$$

by variable separable method,

$$(y^2+1)\,dy = x^2\,dx.$$

$$y^2dy + dy = x^2dx.$$

by doing integration on both sides.

$$\int y^2dy + \int dy = \int x^2dx.$$

$$\frac{y^3}{3} + y = \frac{x^3}{3} + c.$$

Now, at $x=0$, $y=0$

$$c = 0.$$

$$\Rightarrow \boxed{\frac{y^3}{3} + y = \frac{x^3}{3}}$$

We got $\tan y = \frac{x^3}{3}$ from Picard's method.

Now, consider $\tan y$ expansion,

$$\tan y = y + \frac{y^3}{3} + \frac{2y^5}{15} + \cdots$$

Picard's method:-

$$y + \frac{y^3}{3} + \frac{2y^5}{15} + \cdots = \frac{x^3}{3}.$$

Exact solution:

$$y + \frac{y^3}{3} = \frac{x^3}{3}.$$

$\because$ If we approziamate $\tan y$ expansion to $y + \frac{y^3}{3}$ we got exact soln.

so, both are similar and we got very very less difference in 'y' value.

We got almost same 'y' value in both methods.

2) Given,

$$\frac{dy}{dx} = 1-2xy, \quad y(0)=0$$

at $x=0.4$, $y=?$ Taylor's series (upto 4th derivative)

W.K.T,

$$y_1 = y_0 + \frac{h}{1!}y'(0) + \frac{h^2}{2!}y''(0) + \frac{h^3}{3!}y'''(0) + \frac{h^4}{4!}y''''(0)$$

NOW,

$y_0=0$, $h = x-x_0 = 0.4-0 = 0.4$, $y'(x) = 1-2xy$, $y'(0)=1$, $y''(x) = -2y-2xy'$, $y''(0)=0$, $\cancel{y'''(0)=0.04266}$ $\cancel{y'''(0)=0.4266}$

$y''' = -4y'-2xy'' = -4$

Substitute these values in above expansion,

$$y_1 = 0 + \frac{0.4}{1!}(1) + 0 + \frac{(0.4)^3}{3!} \times (-4)$$

$\cancel{\boxed{y = 0.4}}$ at $x=0.4 \Longrightarrow y = 0.4 - 0.04266$

$$\boxed{y = 0.35734}$$

here, we have considered expansion upto 4th derivative.

W.K.T,

truncating error $\propto h^{n+1}$

$\boxed{\text{truncating error} \propto h^5}$

Exact solution:-

$$\frac{dy}{dx} = 1-2xy$$

$$\frac{dy}{dx} + y(2x) = 1.$$

$$y \cdot e^{\int 2x\,dx} = \int 1 \cdot e^{\int 2x\,dx}\,dx.$$

$$y \cdot e^{x^2} = \int e^{x^2}dx \quad (\because \text{Consider } e^{x^2} \text{ exp upto 4 terms})$$

$$= \int \left(1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!}\right)dx.$$

$$y \cdot e^{x^2} = x + \frac{x^3}{3.1!} + \frac{x^5}{5.2!} + \frac{x^7}{7.3!} + c.$$

$\because y(0)=0, \quad c=0.$

$$y \cdot e^{x^2} = x + \frac{x^3}{3.1!} + \frac{x^5}{5.2!} + \frac{x^7}{7.3!} \quad (\because \text{at } x=0.4)$$

$$y \cdot e^{0.16} = 0.4 + \frac{(0.4)^3}{3.1!} + \frac{(0.4)^5}{5.2!} + \frac{(0.4)^7}{7.3!}$$

$y.e^{0.16} = 0.4 + 0.0213 + 0.00102 + 0.00039$

$= 0.42269 + 0.00002$

$= 0.42271.$

Taylor's method:

$\boxed{y = 0.95734}$

$y = \frac{0.42271}{e^{0.16}} = \frac{0.42271}{1.1735}$

$\boxed{y = 0.36021}$

We got similar values of 'y' by doing these two methods.

3) Given,

$\frac{dy}{dx} = x+y+xy \quad y(0)=1 \quad y(0.1)=?$ Euler's method

$h=0.025$

W.K.T,

$y_1 = y_0 + hf(x_0, y_0)$

$y_1 = 1 + 0.025[0+1+0]$

$\boxed{y_1 = 1.025}$ at $x = 0.$

$y_2 = y_1 + hf(x_0+h, y_1)$

$= 1.025 + 0.025[f(0.025, 1.025)]$

$= 1.025 + 0.025[1.025 \times 0.025 + 0.025 + 1.025]$

$= 1.025 + 0.025[1.075625]$

$= 1.025 + 0.02689$

$\boxed{y_2 = 1.05189}$ at $x = 0.025$

$y_3 = y_2 + hf(x_0+2h, y_2)$

$= 1.05189 + 0.025\, f(0.05, 1.05189)$

$= 1.05189 + 0.025[0.05 \times 1.05189 + 0.05 + 1.05189]$

$= 1.05189 + 0.025[1.15448]$

$= 1.05189 + 0.028862$

$\boxed{y_3 = 1.080751.}$ at $x = 0.05$

$$y_4 = y_3 + hf(x_0+3h, y_3)$$
$$= 1.080751 + 0.025(f(0.075, 1.080751))$$
$$= 1.080751 + 0.025[0.075 + 1.080751 + 0.075 \times 1.080751]$$
$$= 1.080751 + 0.025[1.236807]$$
$$= 1.080751 + 0.0309201$$
$$\boxed{y_4 = 1.11167118} \text{ at } x = 0.075$$

$$y_5 = y_4 + hf(x_0+4h, y_4)$$
$$= 1.11167118 + 0.025 f(0.1, 1.11167118)$$
$$= 1.11167118 + 0.025[1.322838]$$
$$= 1.11167118 + 0.033070$$
$$\boxed{y_5 = 1.14474213} \text{ at } x = 0.1$$

Exact solution:-

$$\frac{dy}{dx} = x + y + xy$$
$$\frac{dy}{dx} = y(1+x) + x.$$
$$\frac{dy}{dx} - y(1+x) = x. \implies \text{Leibnitz eq}$$
$$y.e^{\int(1-x)dx} = \int e^{\int(1-x)dx}.x.dx$$
$$y.e^{-x-\frac{x^2}{2}} = \int e^{\left(-x-\frac{x^2}{2}\right)}.x.dx$$
$$= \int\left(1 + \frac{\left(-x-\frac{x^2}{2}\right)}{1!} + \frac{\left(-x-\frac{x^2}{2}\right)^2}{2!} + \ldots\right)x\,dx$$
$$= \int x + \left(-x^2-\frac{x^3}{2}\right) + \frac{\left(-x^2+\frac{x^4}{4}+x^3\right)x}{2}.dx$$
$$= \int\left(x + \left(-x^2-\frac{x^3}{2}\right) + \frac{x^3}{2} + \frac{x^5}{8} + \frac{x^4}{2}\right)dx$$
$$= \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^6}{48} + \frac{x^5}{10} + c.$$
$$\text{at } x=0, y=1 \implies \boxed{1=c}$$

$$y \cdot e^{-x-\frac{x^2}{2}} = \frac{(0.1)^2}{2} - \frac{(0.1)^3}{3} + \frac{(0.1)^6}{48} + \frac{(0.1)^5}{10} + 1$$

$= 0.005 - 0.000333 + 0.0000000283 + 0.000001 + 1$

$= 1.005 - 0.000333$

$= 1.004667$

$y \cdot e^{-0.1-0.005} = 1.004667$

$y \cdot e^{-0.105} = 1.004667$

$$y = \frac{1.004667}{e^{-0.105}}$$

$$= \frac{1.004667}{0.90032}$$

$\boxed{y = 1.115889}$

Euler's method:-

$\boxed{y = 1.14474213}$

$\therefore$ We got similar values of 'y' from these two methods.

4) Given $y' = x + \sin y$, $y(0) = 1$, $y(0.4) = ?$ Modified euler's method

$h = 0.2$ (correct upto 4 decimal)

x	y	$\frac{dy}{dx} = f(x,y) = x + \sin y$	Mean slope	$y_n = y_{n-1} + h(\text{mean slope})$
0	1	$\sin 1 = 0.8414$	-	$y_1 = y_0 + h f(x_0, y_0)$ $= 1 + 0.2 \times 0.8414$ $= 1.1682$
(x_0+h) 0.2	$1 + 0.2 \sin 1$ $= 1.1682$	$0.2 + \sin(1.1682)$ $= 0.92 + 0.2$ $= 1.12$	$\frac{1}{2}(0.8414 + 1.12)$ $= 0.9807$	$y_1^{(1)} = y_0 + h(\text{mean})$ $= 1 + 0.2 \times 0.9807$ $= 1.1961$.
0.2	1.1961	$0.2 + \sin(1.1961)$ $= 1.1306$	$\frac{1}{2}(0.8414 + 1.1306)$ $= 0.9860$	$y_1^{(2)} = 1 + 0.2(\text{mean})$ $= 1 + 0.2 \times 0.9860$ $= 1.1972$.
0.2	1.1972	$0.2 + \sin(1.1972)$ $= 1.1310$	$\frac{1}{2}(0.8414 + 1.131)$ $= 0.9862$	$y_1^{(3)} = 1 + h(\text{mean})$ $= 1 + 0.2 \times 0.9862$ $= 1.1972$ (same as $y_1^{(2)}$)

x	y	$\frac{dy}{dx}=f(\ )$	mean slope	$y_n = y_{n-1} + h(\text{mean})$ ⑦
0.2	1.1972	1.1310	–	$y_2 = y_1 + h f(x_1, y_1)$ $= 1.1972 + 0.2(1.1310)$ $= 1.4234$
$(x+2h)$ 0.4	1.4234	$0.4 + 0.98915$ $= 1.3891$	$\frac{1}{2}[1.1310 + 1.3891]$ $= 1.2600$	$y_2^{(1)} = y_1 + h(\text{mean})$ $= 1.1972 + 0.2(1.26)$ $= 1.4492$
0.4	1.4492	$0.4 + 0.9926$ $= 1.3926$	$\frac{1}{2}[1.131 + 1.3926]$ $= 1.2618$	$y_2^{(2)} = y_1 + h(\text{mean})$ $= 1.1972 + 0.2(1.2618)$ $= 1.44956$
0.4	1.44956	$0.4 + 0.992659$ $= 1.39265$	$\frac{1}{2}[1.131 + 1.39265]$ $= 1.2618$	$y_2^{(3)} = y_1 + h(\text{mean})$ $= 1.1972 + 0.2(1.2618)$ $= 1.44956$ (same as $y_2^{(2)}$)

$\therefore y_2 = 1.44956$ is the value of y at $x = 0.4$.

5) Given,

$$y' + y = 0, \quad y(0) = 1 \qquad y(0.2) = ?$$
$$h = 0.1$$

W.K.T, for third order R.K method

$$k_1 = hf(x_0, y_0),$$
$$k_2 = hf(x_0 + \tfrac{1}{2}h,\ y_0 + \tfrac{1}{2}k_1)$$
$$k_3 = hf(x_0 + h,\ y_0 + 2k_2 - k_1)$$
$$y_1 = y_0 + \frac{1}{6}(k_1 + 4k_2 + k_3)$$

Now,

$$k_1 = 0.1(-1) = -0.1$$
$$k_2 = 0.1\, f(0.05, 0.95)$$
$$= 0.1(-0.95)$$
$$= -0.095$$
$$k_3 = h f(0.1,\ 1 - 0.19 + 0.1)$$
$$= 0.1(-0.91)$$
$$= -0.091.$$

$$y_1 = 1 + \frac{1}{6}(-0.1 - 0.38 - 0.091)$$
$$= 1 + \frac{1}{6}(-0.571)$$
$$= 1 - 0.095166$$

$\boxed{y_1 = 0.90483}$ at $x = 0.1$

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Now, consider y_1 as y_0, x_1 as x_0

$$k_1 = hf(x_1, y_1)$$
$$= 0.1\, f(0.1, 0.90483)$$
$$= -0.090483$$

$$k_2 = hf(x_1+\tfrac{1}{2}h,\, y_1+\tfrac{1}{2}k_1)$$
$$= 0.1\, f(0.15, 0.8595)$$
$$= -0.08595$$

$$k_3 = hf(x_1+h,\, y_1+2k_2-k_1)$$
$$= 0.1\, f(0.2,\, 0.90483-0.17190+0.09048)$$
$$= 0.1\, f(0.2,\, 0.82341)$$
$$= -0.082341.$$

Now,
$$y_2 = y_1 + \cancel{hf}\left(\tfrac{1}{6}(k_1+4k_2+k_3)\right)$$
$$= 0.90483 + \tfrac{1}{6}(-0.090483 - 0.34380 - 0.082341)$$
$$= 0.90483 + \tfrac{1}{6}(-0.516624)$$
$$= 0.90483 - 0.086104$$
$$\therefore \boxed{y_2 = 0.818760} \longrightarrow \text{at } x = 0.2.$$

<u>Exact form</u>:

$$\frac{dy}{dx} = -y$$
$$\int \frac{dy}{y} = \int -dx$$
$$\ln y = -x + c$$

at $x=0$, $y=1$.

$$\ln 1 = 0 + c \implies \boxed{c=0}$$
$$\therefore \boxed{y = e^{-x}}$$

at $x = 0.2$, $y = e^{-0.2} = 0.818730$

$$\therefore \boxed{y = 0.818730}$$

<u>R.K method</u>:

$$\boxed{y = 0.818760}$$

$\because$ We got similar values of 'y' from these two methods.

6) Given,

$$\frac{dy}{dx} = \sqrt{x+y}, \quad y(0.4) = 0.41$$
$$h = 0.4$$

4th order R.K method
$y(0.8) = ?$

W.K.T,

$k_1 = hf(x_0, y_0)$
$= 0.4\, f(0.4, 0.41)$
$= 0.4 \times 0.9$
$= 0.36$

$k_2 = hf(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_1)$
$= 0.4\, f(0.4+0.2, 0.41+0.18)$
$= 0.4\, f(0.6, 0.59)$
$= 0.4 \times 1.0908$
$= 0.4363$

$k_3 = hf(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_2)$
$= 0.4\, f(0.4+0.2, 0.41+0.21815)$
$= 0.4\, f(0.6, 0.62815)$
$= 0.4 \times 1.1081$
$= 0.4432$

$k_4 = hf(x_0 + h, y_0 + k_3)$
$= 0.4\, f(0.8, 0.41+0.4432)$
$= 0.4\, f(0.8, 0.8532)$
$= 0.4 \times 1.28576$
$= 0.5143$

Now,

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$
$= 0.41 + \frac{1}{6}(0.36 + 0.8726 + 0.8864 + 0.5143)$
$= 0.41 + \frac{1}{6}(2.6333)$
$= 0.41 + 0.43888$

$\therefore \boxed{y_1 = 0.84888} \longrightarrow$ at $x = 0.8$

Exact Solution:-

$\frac{dy}{dx} = \sqrt{x+y}$ (let us consider $(x+y)$ as t)

$\frac{dy}{dx} = \sqrt{x+y}$.

$x + y = t$

$1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{dt}{dx} - 1. \implies \frac{dt}{dx} - 1 = \sqrt{t}$

$$\frac{dt}{dx} = \frac{t+1}{2t}$$

$$\frac{(2t)\,dt}{t+1} = dx.$$

$$\frac{(2t+2)dt - 2dt}{t+1} = dx.$$

$$\int 2dt - \int \frac{2}{t+1} dt = \int dx$$

$$2t - 2\log(t+1) = x + c$$

$$2(\sqrt{x+y} - \log(\sqrt{x+y}+1)) = x+c.$$

at $x=0.4$, $y=0.41$.

$$2(\sqrt{0.81} - \log(1+\sqrt{0.81})$$

$$\frac{dt}{dx} = 1+\sqrt{t}$$

$$\frac{dt}{1+\sqrt{t}} = dx.$$

consider, $(1+\sqrt{t}) = p$

$$\frac{dt}{2\sqrt{t}} = dp.$$

$$dt = 2\sqrt{t}\,dp.$$

$$\Rightarrow \frac{2\sqrt{t}\,dp}{p} = dx. \quad (\because \sqrt{t} = p-1)$$

$$\frac{2(p-1)}{p}dp = dx.$$

$$\int 2\left(dp - \frac{dp}{p}\right) = \int dx$$

$$2p - 2\log p = x + c.$$

$$2[(1+\sqrt{t}) - \log(1+\sqrt{t})] = x+c$$

$$\Rightarrow 2[(1+\sqrt{x+y}) - \log(1+\sqrt{x+y})] = x+c$$

at $x=0.4$, $y=0.41$

$$\Rightarrow 2[1+\sqrt{0.81} - \log(1+\sqrt{0.81})] = 0.4+c$$

$$2[1.9 - 0.2787] = 0.4+c.$$

$$2[1.6213] = 0.4+c.$$

$$3.2426 = 0.4+c$$

$$\boxed{c = 2.8426}$$

Now, at $x=0.8$, $y=?$

$$2[(1+\sqrt{x+y}) - \log(1+\sqrt{x+y})] = x+2.8426.$$

$$2\left[1+\sqrt{x+y} - \left[(\sqrt{x+y}) - \frac{(\sqrt{x+y})^2}{2} + \frac{(\sqrt{x+y})^3}{3} - \cdots\right]\right] = x + 2.8426.$$

$\because$ W.K.T,

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

but, in this case we have taken upto two terms
(∵ to make problem easy)

$$\Rightarrow 2\left[1+\cancel{\sqrt{x+y}}-\cancel{\sqrt{x+y}}+\left(\frac{x+y}{2}\right)\right] = x + 2.8426.$$

$$\Rightarrow 2+\cancel{x}+y = \cancel{x}+2.8426$$

$$y = 2.8426-2$$

$$\boxed{y = 0.8426}$$

<u>4th order R-K method</u> :-

$$\boxed{y = 0.84888}$$

∴ We got the similar values of 'y' in both methods.

7) Given,

$$z = (x^2+a^2)(y^2+b^2) \quad \text{——— (1)}$$

$$z = x^2y^2 + x^2b^2 + a^2y^2 + a^2b^2$$

Now, Partial derivative w.r.t 'x'

$$\frac{\partial z}{\partial x} = 2xy^2 + 2xb^2$$

$$\frac{\partial z}{\partial x} = 2x(y^2+b^2) \quad \text{——— (2)}$$

Now, Partial derivative w.r.t 'y'

$$\frac{\partial z}{\partial y} = 2yx^2 + 2ya^2$$

$$\frac{\partial z}{\partial y} = 2y(x^2+a^2) \quad \text{——— (3)}$$

Multiply (2) & (3).

$$\frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y} = 4xy(x^2+a^2)(y^2+b^2)$$

$$\therefore \boxed{\frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y} = 4xyz} \quad (\because \text{from (1)})$$

order of PDE is 1 ($\because$ only $\frac{\partial z}{\partial x}$ terms)
degree of PDE is 2 ($\because \frac{\partial z}{\partial x} \cdot \frac{\partial z}{\partial y}$ is there)

Linearity:-

It is non-linear because coefficients of derivatives are neither functions of independent variables nor dependent variable. and degree is 2 in this case.

Homogenity:-

It is homogeneous because every term has a dependent variable 'z'.

8) Given,

$x+y+z = f(x^2+y^2+z^2)$ f is the arbitary function

Form the PDE by eliminating arbitrary function.

NOW, Partial derivative w.r.t x.

$$1+\frac{\partial z}{\partial x} = f'(x^2+y^2+z^2) \cdot 2x \quad \text{---} (1)$$

NOW, Partial derivative w.r.t y.

$$1+\frac{\partial z}{\partial y} = f'(x^2+y^2+z^2) \cdot 2y \quad \text{---} (2)$$

Consider, (2) $\div$ (1).

$$\frac{\left(1+\frac{\partial z}{\partial y}\right)}{\left(1+\frac{\partial z}{\partial x}\right)} = \frac{\cancel{f'(x^2+y^2+z^2)}}{f'(x^2+y^2+z^2)} \cdot \frac{\cancel{2}y}{\cancel{2}x}$$

$$\Rightarrow y + y\frac{\partial z}{\partial y} = x + x\frac{\partial z}{\partial y}$$

$$\Rightarrow \boxed{x\frac{\partial z}{\partial y} - y\frac{\partial z}{\partial x} = y-x} \longrightarrow$$ required Partial diff Eq by eliminating arbitrary function.

9) i) Given, $\frac{\partial z}{\partial x}+\frac{\partial^2 z}{\partial y^2}=zy$

→ this PDE is linear because dependent variable & its partial derivatives appear in degree one only and also not multiplied together.

→ this PDE is homogeneous because every term has dependent variable or its partial derivatives $(z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y})$.

ii) Given, $\sqrt{x}\frac{\partial z}{\partial x}+\sqrt{y}\frac{\partial z}{\partial y}=\sqrt{z}$

→ this PDE is semi linear because coefficients of derivatives are functions of the independent variables alone.

→ this PDE is homogeneous because every term has dependent variable or its partial derivatives.

iii) Given, $(x^2+y^2)\frac{\partial z}{\partial x}+z\frac{\partial z}{\partial y}=z^3x+y$.

→ this PDE is Quasi linear because derivative with coefficient that depends on independent variables and derivatives of unknown function

→ this PDE is non-homogeneous because last term 'y' is independent on dependent variable 'z'.

iv) Given, $\frac{\partial^2 z}{\partial x^2}+\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)=zy$

→ this PDE is nonlinear because degree of derivatives is 2 and coefficients do not depend on either dependent variables nor independent variables.

→ this PDE is homogeneous because every term has dependent variable or its partial derivatives.

v) Given, $\frac{\partial^2 z}{\partial x^2}+\left(\frac{\partial^2 z}{\partial y^2}\right)^3=z+2y$

→ this PDE is non linear because degree of derivatives is more than 1 and coefficients do not depend on either dependent variables or independent variables.

→ this PDE is non homogeneous because last term '2y' is independent on dependent variable 'z'.

10) Given, $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ $\quad \frac{\partial z}{\partial y} = -2\sin y$ when $x=0, z=0$ y is odd $\frac{\pi}{2}$.

Now we have to integrate w.r.t x.

i.e.,

$$\frac{\partial z}{\partial y} = -\cos x . \sin y + f(y) \quad (\because f(y) \text{ is fn of } y)$$

$$-2\sin y = -\cos 0 . \sin y + f(y) \quad (\because \text{given})$$

$$f(y) = -\sin y \quad (\because y \text{ is odd multiple of } \frac{\pi}{2})$$

Now,

$$\frac{\partial z}{\partial y} = -\cos x \sin y - \sin y .$$

We have to integrate w.r.t y.

$$z = -\cos x . -\cos y + \cos y . + g(x)$$

$$z = \cos x \cos y + \cos y + g(x).$$

$$0 = \cos 0 . \cos\left(\frac{(2n+1)\pi}{2}\right) + \cos\left(\frac{(2n+1)\pi}{2}\right) + g(x) \quad (\because \text{given})$$

$$0 = 1.0 + 0 + g(x) \quad (\because \cos\left(\frac{(2n+1)\pi}{2}\right) = 0)$$

$$\therefore g(x) = 0.$$

$$\therefore z = \cos x . \cos y + \cos y$$

$$\boxed{z = \cos y(\cos x + 1)}$$

11) Given,

$$\frac{y^2 z}{x} \frac{\partial z}{\partial x} + xz \frac{\partial z}{\partial y} = y^2$$

If the eq of form $p . \frac{\partial z}{\partial x} + q . \frac{\partial z}{\partial y} = r$ then the auxillary equation will be

$$\frac{\partial x}{p} = \frac{\partial y}{q} = \frac{\partial z}{r}$$

here,

$$p = \frac{y^2 z}{x}, \quad q = xz, \quad r = y^2$$

auxillary equation will be :-

$$\frac{\partial x}{y^2z/x} = \frac{\partial y}{xz} = \frac{\partial z}{y^2}$$

Now, consider ∂x & ∂z equality :- (all variables are separable)

$$\frac{x.\partial x}{y^2 z} = \frac{\partial z}{y^2}$$

$$\int x dx = \int z dz$$

$$\frac{x^2}{2} = \frac{z^2}{2} + c'$$

$$\boxed{x^2 - z^2 = C} \text{ — (1)}$$

Now consider ∂x & ∂y equality :-

$$\frac{x \partial x}{y^2 \cancel{z}} = \frac{\partial y}{x \cancel{z}}$$

$$\int x^2 dx = \int y^2 \partial y$$

$$\frac{x^3}{3} = \frac{y^3}{3} + c'$$

$$\boxed{x^3 - y^3 = C} \text{ — (2).}$$

$\therefore$ final solution is $F(x^2 - z^2, x^3 - y^3) = 0$ ($\because$ from (1) & (2)

12) Given,

$$z(z^2 + xy)(Px - qy) = x^4 \quad \text{where } \frac{\partial z}{\partial x} = P, \ \frac{\partial z}{\partial y} = q.$$

$$(z^3 + xyz)[Px - qy] = x^4.$$

$$(xz^3 + x^2yz)\frac{\partial z}{\partial x} - (yz^3 + xy^2z)\frac{\partial z}{\partial y} = x^4.$$

here, auxillary equation will be :

$$\frac{\partial x}{x(z^3 + xyz)} = \frac{\partial y}{-y(z^3 + xyz)} = \frac{\partial z}{x^4}$$

Now, consider ∂x & ∂y equality : (Two variables are separable)

$$\frac{\partial x}{x(z^3+xyz)} = \frac{\partial y}{-y(z^3+xyz)}$$

$$\int \frac{\partial x}{x} = \int \frac{\partial y}{-y}$$

$$\ln x = -\ln y + c.$$

$$\ln xy = c.$$

$$\boxed{xy = c_1} \text{ --- (1).}$$

NOW, consider ∂x & ∂z equality:

$$\frac{\partial x}{x(z^3+xyz)} = \frac{\partial z}{x^4}$$

$$\frac{\partial x}{x(z^3+c_1 z)} = \frac{\partial z}{x^4} \quad (\because \text{from}(1),\ xy = c_1)$$

$$\frac{\partial x}{z^3+c_1 z} = \frac{\partial z}{x^3}.$$

$$\int x^3 \partial x = \int (z^3+c_1 z)\partial z$$

$$\frac{x^4}{4} = \frac{z^4}{4} + c_1\frac{z^2}{2} + c_2'. \quad (\because \text{from}(1),\ c_1 = xy).$$

$$\frac{x^4}{4} = \frac{z^4}{4} + \frac{xyz^2}{2} + c_2'$$

$$\boxed{x^4 - z^4 - 2xyz^2 = c_2} \text{ --- (2).}$$

$\therefore$ final solution of PDE from (1) & (2) is $F(xy, (x^4-z^4-2xyz^2)) = 0$

13) Given,

$$x(y^2-z^2)\frac{\partial z}{\partial x} + y(z^2-x^2)\frac{\partial z}{\partial y} = z(x^2-y^2).$$

here,

auxillary equation will be :-

$$\frac{\partial x}{x(y^2-z^2)} = \frac{\partial y}{y(z^2-x^2)} = \frac{\partial z}{z(x^2-y^2)}$$

Since, all variables are not separable. We will consider multipliers.

i.e., $\frac{dx}{p} = \frac{dy}{q} = \frac{dz}{r} = \frac{P_1 dx + q_1 dy + R_1 dz}{pP_1 + qq_1 + rr_1}$

P_1, q_1, r_1 are functions of x, y, z or constants.

Such that $pP_1 + qq_1 + rr_1 = 0 \Rightarrow \int P_1 dx + \int Q_1 dy + \int r_1 dz = c$.

here, $p = x(y^2 - z^2)$, $q = y(z^2 - x^2)$, $r = z(x^2 - y^2)$.

Multipliers Set-1:- Clearly W.K.T, $P_1 = x$, $q_1 = y$, $r_1 = z$

$\therefore \int P_1 dx + \int q_1 dy + \int r_1 dz = c.$

$\int x\,dx + \int y\,dy + \int z\,dz = c.$

$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = c.$

$\boxed{x^2 + y^2 + z^2 = c_1}$ —— (1)

Multipliers Set-2:- another set is $P_1 = yz$, $q_1 = xz$, $r_1 = xy$.

$\therefore \int P_1 dx + \int q_1 dy + \int r_1 dz = c.$

$\int yz\,dx + \int xz\,dy + \int xy\,dz = c.$

$xyz + xyz + xyz = c.$

$3xyz = c.$

$\boxed{xyz = c_2}$ —— (2).

from (1) & (2),

final solution of PDE is $F(x^2 + y^2 + z^2, xyz) = 0.$