Differential Equations Mid Sem

Set-4

Max Marks = 21

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Section 1

1. $(x-y-2)\,dx = (2x-2y-3)\,dy$

$$\frac{dy}{dx} = \frac{x-y-2}{2x-2y-3}$$

Linear Non-homogeneous equation but can be reduced to homogeneous

Comparing $\frac{a}{a'}, \frac{b}{b'}$

$$\Rightarrow \quad \frac{1}{2} = \frac{-1}{-2} \Rightarrow \quad 1/2 = 1/2 \text{ Yes.}$$

Substitute $x-y=t$

$$1-\frac{dy}{dx} = \frac{dt}{dx}$$

Substitute in original eqn

$$-\frac{dt}{dx}+1 = \frac{t-2}{2t-3}$$

$$\frac{dt}{dx} = 1 - \frac{t-2}{2t-3}$$

$$\frac{dt}{dx} = \frac{2t-3-t+2}{2t-3}$$

$$\frac{dt}{dx} = \frac{t-1}{2t-3}$$

$$\int\left(\frac{2t-3}{t-1}\right)dt = \int dx$$

Substitute $t-1=u$
$dt = du$

Solving $\int\left(\frac{2t-3}{t-1}\right)dt$

$$= \int \frac{(2(u+1)-3)}{u}\,du$$

$$= \int \frac{(2u+2-3)}{u}\,du$$

$$= \int \left[\frac{2u}{u} + \frac{(-1)}{u}\right] du$$

$$= 2u - \ln u \quad (\text{Put } t \text{ back})$$

$$= 2t-2-\ln(t-1) \quad (\text{Put } x-y=t)$$

So

$$2(x-y) - \ln(x-y-1) = x + C$$

Ans.

Q-2

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$$\cos Px \cos y = P - \sin Px \sin y$$

$$\cos Px \cos y + \sin Px \sin y = P$$

$$\cos(Px - y) = P$$

$$Px - y = \cos^{-1}(P)$$

$$y = Px - \cos^{-1}(P) \quad \text{(Clairauts equation)}$$

diff. w.r.to x

$$\frac{dy}{dx} = P + x\frac{dP}{dx} + \frac{1}{\sqrt{1-p^2}}\frac{dP}{dx}$$

$$P = P + \frac{dP}{dx}\left(x + \frac{1}{\sqrt{1-p^2}}\right)$$

$$\frac{dP}{dx} = 0$$

$$\boxed{P = C}$$

General Solution

$$x + \frac{1}{\sqrt{1-p^2}} = 0$$

$$x\sqrt{1-p^2} = -1$$

$$1 - p^2 = \frac{1}{x^2}$$

$$p^2 = 1 - \frac{1}{x^2}$$

$$P = \pm\sqrt{\frac{x^2-1}{x^2}}$$

Solⁿ =

$$y = \pm\sqrt{\frac{x^2-1}{x^2}} \times x - \cos^{-1}\left(\pm\sqrt{\frac{x^2-1}{x^2}}\right)$$

$$y = \pm\sqrt{x^2-1} - \cos^{-1}\left(\pm\sqrt{\frac{x^2-1}{x^2}}\right)$$

$\boxed{\text{Singular Sol}^n}$

Q.3 $y = c_1 x^2 + e$ $\qquad x^2 + 2y^2 - y = c_2$

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Calculating both slopes

$y = c_1 x^2 + e$

$\frac{dy}{dx} = 2c_1 x$ (Put c_1 back)

$y = \frac{dy}{dx} \frac{x^2}{2x} + e$

$\frac{dy}{dx} = \frac{2(y-e)}{x}$

$x^2 + 2y^2 - y = c_2$

diff w.r.t. x

$2x + 4y \frac{dy}{dx} - \frac{dy}{dx} = 0$

~~$2x + 4y \frac{dy}{}$~~

$2x + \frac{dy}{dx}(4y-1) = 0$

$\frac{dy}{dx} = \frac{2x}{1-4y}$

Multiplying both slopes and product -1

$\frac{2(y-e)}{x} \frac{2x}{(1-4y)} = -1$

$4(y-e) = 4y-1$

$4y - 4e = 4y - 1$

$4e = 1$

$\boxed{e = 1/4}$ Ans

Section II

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Q-4 Solve

$$\frac{dy}{dx} = 2y\tan x + y^2\tan^2 x$$

$$\frac{dy}{dx} - 2y\tan x = y^2\tan^2 x$$

Bernoulli Equation $n=2$

Substitute $y^{1-2} \Rightarrow y^{-1} = t$

$$\frac{1}{1-n}\frac{dt}{dx} - 2\tan x\, t = \tan^2 x \qquad (n=2)$$

$$\frac{dt}{dx} + 2\tan x\, t = -\tan^2 x \qquad \text{(Linear Leibniz)}$$

$$If = e^{2\int \tan x\, dx} = e^{2(-\ln \cos x)} = \sec^2 x$$

So, answer

$$t \cdot \sec^2 x = -\int \tan^2 x \sec^2 x\, dx$$

$$\tan x = u$$

$$\sec^2 x\, dx = du$$

$$= -\int u^2\, du = -\frac{u^3}{3} + c$$

$$t\sec^2 x = -\frac{\tan^3 x}{3} + C$$

Substitute t back

$$\boxed{\frac{\sec^2 x}{y} = -\frac{\tan^3 x}{3} + C}$$ Ans

Q-5

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$$yp^2 + (x-y)p - x = 0$$

$$yp^2 + xp - yp - x = 0$$

$$yp^2 - yp + xp - x = 0$$

$$yp(p-1) + x(p-1) = 0$$

$$(yp + x) = 0 \quad \text{and} \quad p - 1 = 0$$

$$y\frac{dy}{dx} = -x$$

$$\int y\,dy = -\int x\,dx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + C$$

$$\frac{y^2}{2} + \frac{x^2}{2} + C = 0$$

$$\frac{dy}{dx} = 1$$

$$\int dy = \int dx$$

$$y = x + C$$

$$\cancel{y - x + C = 0}$$

$$\boxed{y - x + C = 0}$$

Multiplying together

final answer

$$\left(\frac{y^2}{2} + \frac{x^2}{2} + c\right)\left(y - x + c\right) = 0$$

Q-6 Next Page

Q-6 $y(1+xy)\,dx + (1-xy)x\,dy = 0$

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Check if equation is exact $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$1+2xy \neq 1-2xy$

This equation is of the form

$f(x,y)\,y\,dx + g(x,y)\,x\,dy = 0$ $\quad IF = \frac{1}{Mx - Ny}$

$$IF = \frac{1}{xy + x^2y^2 - xy + x^2y^2} = \frac{1}{2x^2y^2}$$

Reducing to exact

$$\underbrace{\left(\frac{1+xy}{2x^2y}\right)}_{M} dx + \underbrace{\left(\frac{1-xy}{2xy^2}\right)}_{N} dy = 0$$

Solving

$$\int \frac{1}{2x^2y}\,dx + \int \frac{1}{2x}\,dx - \int \frac{1}{2y}\,dy = 0$$

$$-\frac{1}{2xy} + \frac{1}{2}\ln x - \frac{1}{2}\ln y + C = 0$$

$$\boxed{-\frac{1}{xy} + \ln(x/y) + C = 0}$$ Ans