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SDA Paper

Q-1 $\vec{\mu} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix}$

$$Cov((X_1 - X_2), X_2) = Cov(X_1, X_2) + Cov(X_2, X_2)$$

$$= \Sigma + \sigma^2 I$$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 8 \end{bmatrix}$$

Ans

Q-2 $y = 11.506 + 12.208x$

$$R^2 = 1 - \frac{RSS}{TSS}$$

$$RSS = \sum_{i=1}^{n} (y_i - \bar{y}_i)^2 = 1 + 12^2 + 25^2 + 23^2 + 30^2$$

$$= 2199$$

$$TSS = \sum_{i=1}^{n} (y_i - \bar{y})^2 = 5739.5$$

$$R^2 = 1 - \frac{2199}{5739.5} = 1 - 0.383 = 0.616$$

$$\text{Adjusted } R^2 = 1 - \frac{\frac{RSS}{(n-p-1)}}{\frac{TSS}{n-1}} = 1 - 0.383 \times \frac{4}{3}$$

$$= 0.4893 \text{ Ans}$$

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Q-3 Factor loadings

Factor 1	Factor 2
-.97	0.05
0.85	0.22
0.06	0.87
0.92	0.15
-0.05	0.99

Communalities

$$h_i^2 = \sum_{j=1}^{m} \hat{l}_{ij}^2$$

Variable 1 0.943 = $(-0.97)^2 + (0.05)^2$

Var 2 0.7709 = $(0.85)^2 + (0.22)^2$

Var 3 0.7605 = $(0.06)^2 + (0.22)^2$

Var 4 0.8689 = $(0.92)^2 + (0.15)^2$

Var 5 0.982599 = $(-0.05)^2 + (0.99)^2$

Communalities

Q-4 $n = 50$

$$S = \begin{bmatrix} 2 & 0.8 \\ 0.8 & 0.6 \end{bmatrix}$$

Calculating eigen values

$$|S - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 0.8 \\ 0.8 & 0.6-\lambda \end{vmatrix} = 0$$

$$1.2 + \lambda^2 - 2.6\lambda + 0.64 = 0$$

$$\lambda^2 - 2.6\lambda + 0.56 = 0$$

Eigen values

roots are 2.36301 and 0.23698

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Calculating eigen vectors

(a) $\lambda = 2.363$

$$S\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 2.363\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{pmatrix} 2x_1 + 0.8x_2 \\ 0.8x_1 + 0.6x_2 \end{pmatrix} = \begin{pmatrix} 2.636x_1 \\ 2.363x_2 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0.9106 \\ -0.413 \end{pmatrix}$$

(b) $\lambda = 0.236$

$$S\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0.236\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{pmatrix} 2x_1 + 0.8x_2 \\ 0.8x_1 + 0.6x_2 \end{pmatrix} = \begin{pmatrix} 0.236x_1 \\ 0.263x_2 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0.4192 \\ 0.9106 \end{pmatrix}$$

Principal Components

(1) $0.91x_1 - 0.413x_2$

(2) $0.413x_1 + 0.91x_2$

(a) $0.91x_1 - 0.413x_2$

(b) $0.413x_1 + 0.91x_2$

Ans

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Q-5

	1	2	3	4
1	0			
2	①	0		
3	11	2	0	
4	5	3	4	0

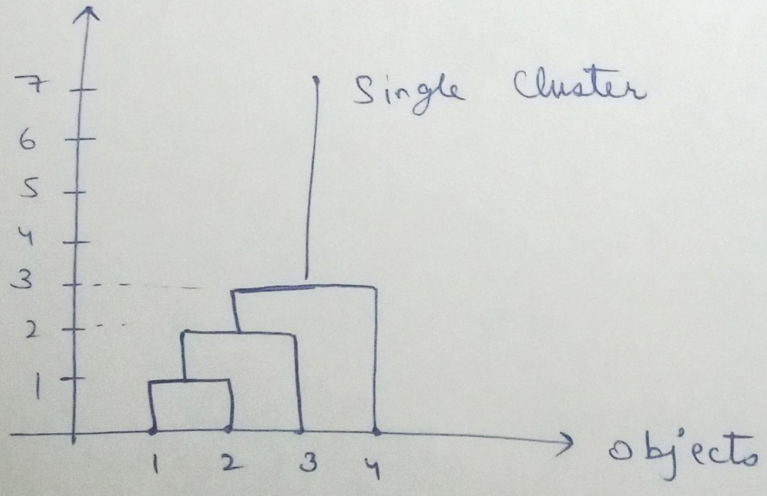
Group 1 and 2

	(1,2)	3	4
(1,2)	0		
3	②	0	
4	3	4	0

Group {(1,2), 3}

	((1,2),3)	4
((1,2),3)	0	
4	3	0

for clustering, take $\min(d(A,B), d(C,B))$ where A and C are merged and cell ((A,C),B) needs to be filled in



Dendogram

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Q.6 99% Confidence Interval

$$x = \begin{bmatrix} 2 & 12 \\ 8 & 9 \\ 6 & 9 \\ 8 & 10 \end{bmatrix}$$

$$\mu = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

Bonferroni's approach

$1-\alpha = 0.99$

$\alpha = 0.01$

$p = 2$

Confidence regions

$$x_1 = \bar{x_1} \pm t_{n-1}\left(\frac{\alpha}{2p}\right)\sqrt{\frac{s_{11}}{n}}$$

$$x_2 = \bar{x_2} \pm t_{n-1}\left(\frac{\alpha}{2p}\right)\sqrt{\frac{s_{22}}{n}}$$

$$S = \frac{1}{3}(x-\mu)^T(x-\mu)$$

$$= \frac{1}{3}\begin{bmatrix} -4 & 2 & 0 & 2 \\ 2 & -1 & -1 & 0 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 2 & -1 \\ 0 & -1 \\ 2 & 0 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 24 & -10 \\ -10 & 6 \end{bmatrix}$$

$s_{11} = 8$ $\quad s_{22} = 6/3 = 2$

$$x_1 = 6 \pm t_{(3,0.0025)}\sqrt{\frac{8}{4}}$$

$$= 6 \pm (7.64)\sqrt{2} = 6 \pm 10.802$$

$$x_2 = 10 \pm t_{(3,0.0025)}\sqrt{\frac{2}{4}}$$

$$= 10 \pm 5.403$$

99% confidence intervals

$x_1 \in (-4.802, 16.802)$

$x_2 \in (4.597, 15.403)$

Ans

$x_1 \in (-4.802, 16.802)$

$x_2 \in (4.597, 15.403)$

Ans