ML End Sem

Sayam Kumar
S20180010158

Type 2 Question Paper

Q-1 $x_1 = (0,2)$ $x_2 = (1,0)$ $\rightarrow$ +1 class

$x_3 = (4,4)$ $x_4 = (5,5)$ $\rightarrow$ -1 class

Also $\alpha_4 = 0$

Using KKT conditions,

(i) $W = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \sum_{i=1}^{4} \alpha_i y_i x_i = \alpha_1 \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 4\alpha_3 + \alpha_2 \\ 4\alpha_3 + 2\alpha_1 \end{pmatrix}$

(ii) $\alpha_i \geq 0 \ \forall \ i(1,4)$

(iii) $\sum \alpha_i y_i = 0 \Rightarrow \alpha_1 + \alpha_2 - \alpha_3 - \alpha_4 = 0$ $(\alpha_4 = 0)$

$\Rightarrow \boxed{\alpha_1 + \alpha_2 = \alpha_3}$ — 4

(iv) $1 - (1) \begin{pmatrix} 4\alpha_3 + \alpha_2 \\ 4\alpha_3 + 2\alpha_1 \end{pmatrix}^T \begin{pmatrix} 0 \\ 2 \end{pmatrix} + b = 0$

$1 - [2(4\alpha_3 + 2\alpha_1) + b] = 0$ — 1

$1 - [(1) (4\alpha_3 + \alpha_2, 4\alpha_3 + 2\alpha_1) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b] = 0$ - 2

$1 - 4\alpha_3 - \alpha_2 - b = 0$ — 2

$1 - (-1) [(4\alpha_3 + \alpha_2, 4\alpha_3 + 2\alpha_1) \begin{pmatrix} 4 \\ 4 \end{pmatrix} + b] = 0$

$1 + 32\alpha_3 + 4\alpha_2 + 8\alpha_1 + b = 0$ — 3

Solving eq

$\alpha_1 = 0.0429$ $b = 1.4$

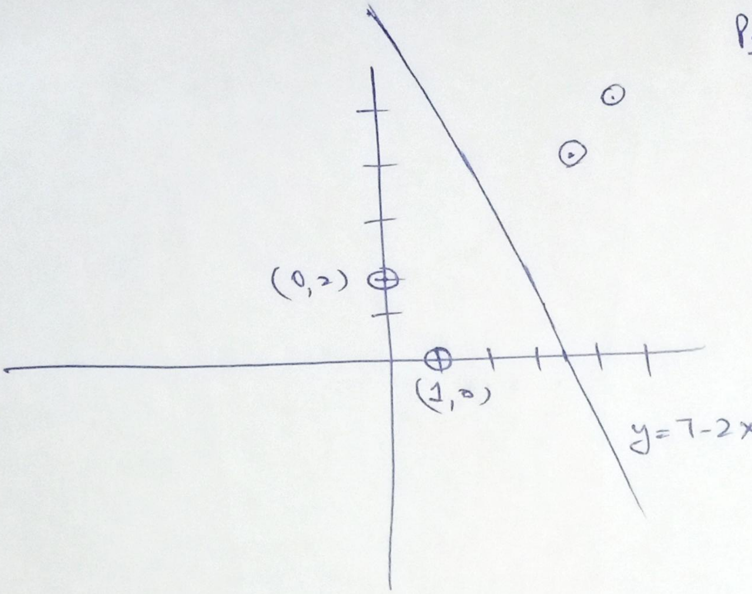
$\alpha_2 = -0.1143$

$\alpha_3 = -0.0714$

Hard Linear SVM

$\Rightarrow -0.399 x_1 - 0.1998 x_2 + 1.4 = 0$

Sayam
Kumar
S20180010158



Q.2 $\{1, 2, 6, 8\}$ $\quad P(w_1) = P(w_2) = 0.5$

$(\mu_1, \sigma_1^2) = (1,1) \quad (\mu_2, \sigma_2^2) = (7,1)$

a = class 1
b = class 2

Step 1 Apply E step,

$$P(a|x_1) = \frac{P(x_1|a)\,P(a)}{P(x_1)} = \frac{P(x_1|a)\,P(a)}{P(x_1|a)\,P(a) + P(x_2|a)\,P(a)}$$

formula $\quad P(x_i|a) = \frac{1}{\sqrt{2\pi\sigma_a^2}}\, e\left(-\frac{(x_i-\mu_a)^2}{2\sigma_a^2}\right)$

~~$a_1 = 0.3989$~~ ~~$b_1 = 0.001$~~

$a_1 = 0.999$ | $a_2 = 0.999$ | $a_3 = 0.001$
$b_1 = 0.001$ | $b_2 = 0.001$ | $b_3 = 0.999$
$a_4 = 0.001$, $b_4 = 0.999$

update $P(w_1) = \frac{\Sigma a}{n} = \frac{2}{4} = 1/2 \qquad P(w_2) = \frac{\Sigma b}{n} = 1/2$

$\mu_1 = \frac{\Sigma a_i x_i}{\Sigma a_i} = \frac{3}{2} \qquad \mu_2 = \frac{\Sigma b_i x_i}{\Sigma b} = \frac{6+8}{2} = 7$

$\sigma_1^2 = \frac{\Sigma a_i (x_i - \mu_1)^2}{\Sigma a_i} = 1/2 \qquad \sigma_2^2 = \frac{\Sigma b_i (x_i - \mu_2)^2}{\Sigma b_i} = \frac{2}{2} = 1$

Sayam Kumar
Page 3
S20180010158

Q-3 Given data

$$D = \begin{pmatrix}-1\\-1\end{pmatrix}, \begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}3\\3\end{pmatrix}$$

$$\text{Mean} = \begin{pmatrix}0.75\\0.75\end{pmatrix}$$

$$\text{Mean shifted data} = \begin{pmatrix}-1.75\\-1.75\end{pmatrix}, \begin{pmatrix}-0.75\\-0.75\end{pmatrix}, \begin{pmatrix}0.25\\0.25\end{pmatrix}, \begin{pmatrix}2.25\\2.25\end{pmatrix}$$

Covariance Matrix of data $= \begin{bmatrix}2.1875 & 2.1875\\ 2.1875 & 2.1875\end{bmatrix}$

~~Calcup~~ Computing eigen vectors using PCA constraints

$$e_1 = \begin{pmatrix}0.707\\0.707\end{pmatrix} \qquad e_2 = \begin{pmatrix}-0.707\\0.707\end{pmatrix}$$

eigen values (4.375) $\qquad (0)$

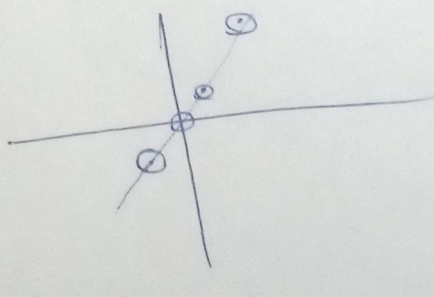
The Projection Matrix

$$P = \begin{pmatrix}0.707\\0.707\end{pmatrix}$$

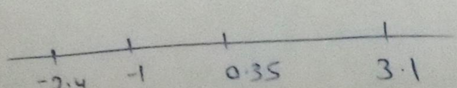
Transformed data

$(-2.4, -1, 0.35, 3.1)$

The representation is loss less. One eigen value is 0.



This can be represented in straight line



Ans

Sayam Kumar
S20180010158

Q-4

$f(t) = 2t + 3$

for node 3

$net_3 = x_1 - x_2$

$f(n_3) = 2(x_1 - x_2) + 3$

for node 4

$net_4 = x_2 - x_1$

$f(n_4) = 2(x_2 - x_1) + 3$

$f(5) = f(n_3) + f(n_4)$

$= 2(x_1 - x_2) + 3 + 2(x_2 - x_1) + 3$

$= 6$ [independent of x_1 and x_2] (net 5)

$\Rightarrow f(5) = 2(6) + 3$

$= 12 + 3 = 15$

$f(5) = 15$ (output is independent of x_1 and x_2)

So for circuit (b),

$w_1 = 0$, $w_2 = 0$, $b = 15$ Ans

Q-5 $\begin{matrix} x_1 = \\ x_2 \end{matrix}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

Page 5

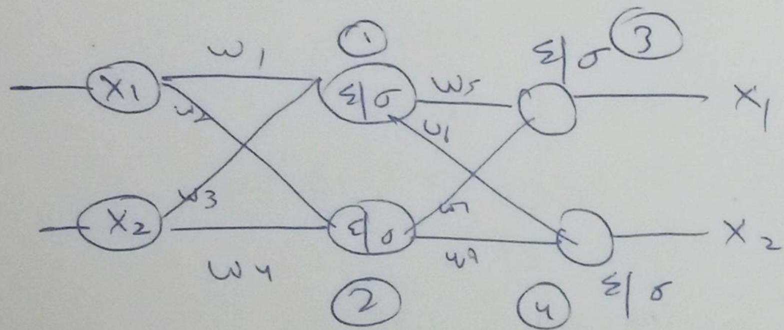
Sayam Kumar
S20180010158

weights = $w_1, w_2, w_3, w_4, w_5, w_6,$
$w_7, w_8 = (0.1, 0.2, 0.3, 0.4, -0.5, 0.6, -0.7, 0.8)$

learning rate = 0.1

$(t1, t2) = (1, 0)$

$J = \frac{1}{2}\left[(t2 - z2)^2 + (t1 - z1)^2\right]$



Node1 = 0.331

Node2 = 0.31

Node 3 =

Node = 4 =

=