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Q1 (20 %)	Q2 (20 %)	Q3 (20 %)	Q4 (20 %)	Q5 (20 %)	Total
20	20	20	10	20	90

1. Prove that:

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

2. Construct a truth table for the following logical statement

$$(p \to (q \vee r)) \wedge (\neg p \to (q \wedge \neg r))$$

3. Construct a truth table to prove or disprove the following logical equivalence:

$$\big((p \wedge (q \vee r)) \to s\big) \equiv \big((p \to s) \wedge (q \vee r \to s)\big)$$

4. Consider the following premises and conclusion:
   1) All mammals have warm blood.
   2) No reptiles have warm blood.
   3) All reptiles lay eggs.
   4) Therefore, no mammals lay eggs. (Conclusion)

M(x) is mammals
R(x) is reptiles
B(x) have warm bloods
E(x) lay eggs

1) $\forall x(M(x) \to B(x))$
2) $\forall x \neg R(x) \to B(x)$ or $\neg\exists x(R(x) \wedge B(x))$
3) $\forall x(R(x) \to E(x))$
4) $\neg\exists x(M(x) \wedge E(x))$

5. Write a sentence to describe this logical expression.

Logical Expression:

$$(p \to q) \wedge (r \to (q \vee s))$$

Meanings:
p: You study for the final exam.
q: You pass the final exam.
r: You attend the review session.
s: You earn extra credit.

If you study for the final exam then you pass the final exam and If you attend the review session then you either pass the final exam or you earn extra credit.

1. $1^3+2^3+3^3+\dots+n^3=\left(\frac{(n+1)n}{2}\right)^2$

$n=1$ $\quad 1^3=\left(\frac{1\cdot(1+1)}{2}\right)^2 \Rightarrow 1^3=\left(\frac{2}{2}\right)^2 \quad 1=1$ ✓

$n=k$ $\quad 1^3+2^3+3^3+\dots+k^3=\left(\frac{k(k+1)}{2}\right)^2$ ✓

$n=k+1$ $\quad 1^3+2^3+3^3+\dots+k^3+(k+1)^3=\left(\frac{(k+1)(k+2)}{2}\right)^2$

$$\left(\frac{k(k+1)}{2}\right)^2+(k+1)^3=\left(\frac{(k+1)(k+2)}{2}\right)^2$$

$$\left(\frac{(k+1)(k+2)}{2}\right)^2=\left(\frac{(k+1)(k+2)}{2}\right)^2$$ ✓

2. $(p\to(q\vee r))\wedge(\neg p\to(q\wedge\neg r))$ $\quad 2^3=8$ rows.

p	q	r	$q\vee r$	$p\to(q\vee r)$	$\neg r$	$q\wedge\neg r$	$\neg p$	$\neg p\to(q\wedge\neg r)$	$(p\to(q\vee r))\wedge(\neg p\to(q\wedge\neg r))$
T	T	T	T	T	F	F	F	T	T
T	T	F	T	T	T	T	F	T	T
T	F	F	F	F	T	F	F	T	F
T	F	T	T	T	F	F	F	T	T
F	F	F	F	T	T	F	T	F	F
F	T	F	T	T	T	T	T	T	T
F	F	T	T	T	F	F	T	F	F
F	T	T	T	T	F	F	T	F	F

3. $((p\wedge(q\vee r))\to s)\equiv((p\to s)\wedge(q\vee r\to s))$ $\quad 2^4=16$.

p	q	r	s	$q\vee r$	$p\wedge(q\vee r)$	$(p\wedge(q\vee r))\to s$	$p\to s$	$q\vee r\to s$	$((p\to s)\wedge(q\vee r\to s))$
T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	T	F	F	F	F
T	T	F	F	T	T	F	F	F	F
T	F	F	F	F	F	T	F	T	F
T	F	T	F	T	T	F	F	F	F
T	T	F	T	T	T	T	T	T	T
T	F	T	T	T	T	T	T	T	T
T	F	F	T	F	F	T	T	T	T
F	F	F	F	F	F	T	T	T	T
F	F	F	T	F	F	T	T	T	T
F	F	T	T	T	F	T	T	T	T
F	T	T	T	T	F	T	T	T	T
F	T	F	T	T	F	T	T	T	T
F	T	F	F	T	F	T	T	F	F
F	T	T	F	T	F	T	T	F	F
F	F	T	F	T	F	T	T	F	F

not logically equivalent ✗