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Q1 (20 %)	Q2 (20 %)	Q3 (20 %)	Q4 (20 %)	Q5 (20 %)	Total
20	20	10	10	20	80

1. Prove that:

$$n=1 \quad \left(\frac{1(1+1)}{2}\right)^2 = 1$$

$$n=k^3 \quad \left(\frac{k(k+1)}{2}\right)^2$$

$$n=(k+1)^3 \quad \left(\frac{(k+1)(k+2)}{2}\right)^2$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\frac{k^2(k+1)^2 + 4(k+1)^3}{4} = \frac{(k+1)^2(k+2)^2}{4} \quad (\text{prove it})$$

$$\frac{(k+1)^2(k^2 + 4(k+1))}{4} = \frac{(k+1)^2(k^2 + 4k + 4)}{4} = \frac{(k+1)^2(k+2)^2}{4}$$

2. Construct a truth table for the following logical statement

$$(p \rightarrow (q \vee r)) \wedge (\neg p \rightarrow (q \wedge \neg r))$$

p	q	r	$p \rightarrow (q \vee r)$	$\neg p \rightarrow (q \wedge \neg r)$	$(p \rightarrow (q \vee r)) \wedge (\neg p \rightarrow (q \wedge \neg r))$
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

Answer: 11100100

3. Construct a truth table to prove or disprove the following logical equivalence:

$$((p \wedge (q \vee r)) \rightarrow s) \equiv ((p \rightarrow s) \wedge (q \vee r \rightarrow s))$$

→
upward

4. Consider the following premises and conclusion:

- 1) All mammals have warm blood.
- 2) No reptiles have warm blood.
- 3) All reptiles lay eggs.
- 4) Therefore, no mammals lay eggs. (Conclusion)

- 1) mammals - $M(x)$ blood - $B(x)$ $\forall(x) B(x)$ or $\forall(x) (M(x) \rightarrow B(x))$ (domain is all animals)
- 2) $\neg \exists(x) (R(x) \wedge B(x))$
- 3) $\forall(x) L(x)$ or $\forall(x) (R(x) \rightarrow L(x))$ (domain is all animals)
- 4) $\neg \exists(x) (M(x) \wedge L(x))$

5. Write a sentence to describe this logical expression.

Logical Expression:

Meanings:

- p: You study for the final exam.
q: You pass the final exam.
r: You attend the review session.
s: You earn extra credit.

$$(p \rightarrow q) \wedge (r \rightarrow (q \vee s))$$

If you study for the final exam, then you pass the final exam, and if you attend the review session, then you will pass the exam or you will earn extra credit.

P	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \vee (q \vee r)$	s	$p \wedge (q \vee r) \rightarrow s$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	F
T	F	T	T	F	T	F	F
T	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	T	F	F	T	T
F	F	F	F	F	F	F	T

101010111111111

$(p \rightarrow s) \wedge (q \vee r \rightarrow s)$

p	s	$p \rightarrow s$	q	r	$q \vee r$
T	T	T	T	T	T
T	F	F	T	F	T
T	T	T	F	T	T
T	F	F	F	T	T
T	T	T	T	F	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	F	T
F	T	T	F	T	T
F	F	T	F	F	F
F	T	T	T	T	T
F	F	T	T	F	T
F	T	T	F	T	T
F	F	T	F	F	F
F	T	T	T	T	T

$q \vee r$	s
T	T
T	F
T	T
T	F
T	T
T	F
F	T
F	F
T	T
T	F
T	T
T	F
T	T
F	T

$q \vee r \rightarrow s$

1	T
2	F
3	T
4	F
5	T
6	F
7	T
8	F
9	T
10	F
11	T
12	F
13	T
14	F
15	T
16	T

$(p \rightarrow s) \wedge (q \vee r \rightarrow s)$

T
F
T
F
T
F
T
T
T
T
T
T
T
T
T
T

101010111111111

they are equivalent.

$$101010111111111 = 101010111111111$$