Chapter 3: Finite MDP

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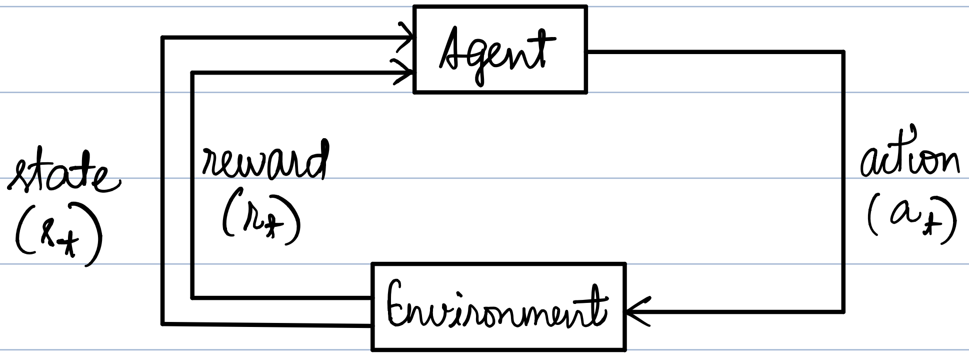
3.1. Agent-Environment Interface

- agent ← learner, decision maker
- environment ← everything outside agent



Thus, resulting trajectory: $S_0, A_0, R_0, S_1, A_1, R_1 \ldots.$

- S, A, R are finite in a finite MDP

$$p(s', r \mid s, a) \doteq \Pr\{S_t = s', R_t = r \mid S_{t-1} = s, A_{t-1} = a\}$$

where $s, s' \in S$, $r \in R$, $a \in A(s)$

↑ defines the dynamics of the MDP

↳ $p : S \times R \times S \times A \longrightarrow [0,1]$

$$\sum_{s' \in S} \sum_{r \in R} p(s', r \mid s, a) = 1 \quad \forall\, s \in S,\ a \in A(s)$$

In a MDP, p characterizes the environment's dynamics

↳ Probability of each possible value for S_t and R_t depends only on the immediate preceeding state and action, S_{t-1} and A_{t-1}

state transition probability :

$$p(s'|s,a) \doteq Pr\{S_t = s' \mid S_{t-1} = s, A_{t-1} = a\}$$

$$= \sum_{r \in R} p(s', r \mid s, a)$$

$$\hookrightarrow p : S \times S \times A \rightarrow [0,1]$$

expected reward :

$$r(s,a) \doteq E[R_t \mid S_{t-1} = s, A_{t-1} = a]$$

$$= \sum_{r \in R} r \sum_{s' \in S} p(s' \mid s, a)$$

$$\hookrightarrow r : S \times A \rightarrow \mathbb{R}$$

3.2. Goals and Rewards

- reward hypothesis
  - ↳ all goals can be expressed as expected reward maximisation of a specific scalar signal

3.3. Returns and Episodes

- expected return (for episodic tasks)

$$G_t \doteq R_{t+1} + R_{t+2} + \cdots\cdots + R_T$$

$T \leftarrow$ final time step (terminal state)

- discounted return (for continuing tasks)

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots\cdots$$

$0 \leq \gamma \leq 1 \leftarrow$ discount rate

- $G_t \doteq R_{t+1} + \gamma G_{t+1}$

3.5. Policy and Value functions

- policy → $\pi(a|s)$
- value function (state value function)
  $\hookrightarrow v_\pi(s) \doteq E_\pi[G_t | S_t = s]$
- action value function
  $\hookrightarrow q_\pi(a,s) \doteq E_\pi[G_t | S_t = s, A_t = a]$

- value function is a measure of the "goodness" of a state

3.12 : $v_\pi(s) = \sum_{a \in A} \pi(a|s) \cdot q_\pi(a,s)$

3.13 :
$$
\begin{aligned}
q_\pi(s) &= E[G_t | s, a] \\
&= E[R_{t+1} + \gamma G_{t+1} | s, a] \\
&= \sum_{a \in A} p(s'|s,a) \cdot [R_{t+1} + \gamma V(s')]
\end{aligned}
$$

Bellman Equation :

$$
\begin{aligned}
v_\pi(s) &= E[G_t | S_t = s] \\
&= E[R_{t+1} + \gamma G_{t+1} | S_t = s] \\
&= \sum_{a \in A} \pi(a|s) \, E[q_\pi(a,s) | S_t = s] \\
&= \sum_{a \in A} \pi(a|s) \sum_{s'} \sum_{r} p(s', r | s, a) \cdot [r + \gamma v_\pi(s')]
\end{aligned}
$$

3.6. Optimal policy and Value functions

$$\pi^* \leftarrow v_{\pi^*}(s) \geq v_\pi(s) \quad \forall s \in S$$

$$q_*(s,a) = \max_{\pi} q_\pi(s,a)$$