

S.R.P

# Diffraction

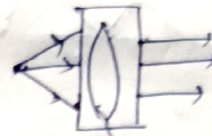
Fresnel vs Fraunhofer

Fresnel

Fraunhofer

use of convex lens

plane wave

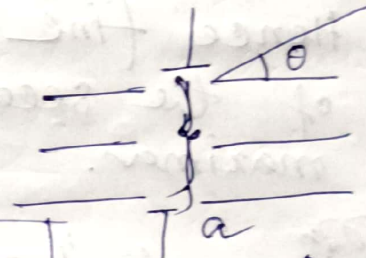


convex lens

point source of wave  
spherical/cylindrical wave  
any kind of wave

single slit Fraunhofer

Diffraction



$\theta$  = diffraction angle

$a$  = slit width

Wave function

$$y = ka \frac{\sin \alpha}{\alpha} \sin 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right)$$

$k$  = proportionality const.

$a$  = slit width

$$\alpha = \frac{\pi a \sin \theta}{\lambda}$$

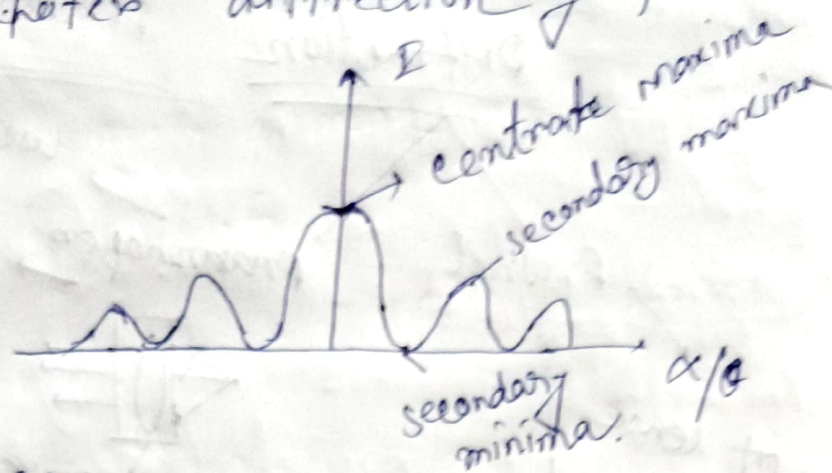
$$y = A \sin 2\pi (wt - \phi)$$

$$\therefore A = ka \frac{\sin \alpha}{\alpha}$$



$$\text{Intensity (I)} = A^2 = k^2 a^2 \frac{\sin^2 \alpha}{\alpha^2}$$

# Fraunhofer diffraction graph



## Question

- Write down an expression for the intensity due to single slit Fraunhofer diffraction
- Hence find the position and the condition of the secondary minima and secondary maxima.

Soln

① central maxima

② position of secondary minima and condition

③ position of ~~of~~ secondary maxima and condition of secondary maxima

$$I = k^2 a^2 \frac{\sin^2 \alpha}{\alpha^2}$$

$$\alpha \rightarrow 0 \Rightarrow \alpha \rightarrow 0$$

$$\therefore \lim_{\alpha \rightarrow 0} \frac{\sin \alpha}{\alpha} = 1$$

$$\therefore I = k^2 a^2 \cdot 1 = k^2 a^2 = I_0$$



S.B.L

$$f(E) = \frac{n(E)}{g(E)} \rightarrow \text{Number of quantum states}$$

$$E_F = \frac{h}{8m} \left( \frac{3N}{\pi V} \right)^{2/3}$$

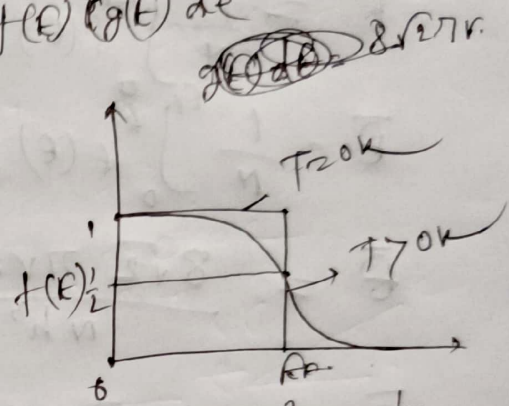
$$n(E) = f(E) \cdot g(E)$$

$$\int_0^{\infty} n(E) dE = \int_0^{\infty} f(E) \cdot g(E) dE$$

$$N = \int_0^{E_F} f(E) \cdot g(E) dE + \int_{E_F}^{\infty} f(E) \cdot g(E) dE$$

$$N = \int_0^{E_F} g(E) dE$$

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$$g(E) dE = \frac{8\sqrt{2} \pi V m^{3/2} E^{1/2} dE}{h^3}$$

$$N = \frac{8\sqrt{2} \pi V m^{3/2}}{h^3} \int_0^{E_F} E^{1/2} dE$$

$$N = \frac{16\sqrt{2} \pi V m^{3/2} E_F^{3/2}}{3 h^3}$$

multiplying the both side of the equation each term with the power of  $\frac{2}{3}$  then we get

$$E_F = \frac{h^2}{8m} \left( \frac{3N}{\pi V} \right)^{2/3}$$

$$\bar{E} = \frac{3}{5} E_F$$

⊗ ⊗ VI

↓ Average Energy or mean internal Energy

$$\bar{E} = \frac{1}{N} \int_0^{\infty} n(E) \cdot E \, dE$$

$$\bar{E} = \frac{1}{N} \int_0^{\infty} f(E) \cdot g(E) \cdot E \, dE$$

$$\bar{E} = \frac{1}{N} \int_0^{E_F} f(E) g(E) \cdot E \, dE + \frac{1}{N} \int_{E_F}^{\infty} f(E) g(E) \cdot E \, dE$$

$$\bar{E} = \frac{1}{N} \int_0^{E_F} f(E) g(E) \cdot E \, dE$$

$$\bar{E} = \frac{8\sqrt{2} \pi V m^{\frac{3}{2}}}{N h^3} \int_0^{E_F} E^{\frac{3}{2}} dE$$

$$\bar{E} = \frac{16\sqrt{2} \pi V m^{\frac{3}{2}}}{5 N h^3} \cdot E_F^{\frac{5}{2}}$$

$$= \frac{16\sqrt{2} \pi V m^{\frac{3}{2}}}{5 h^3} \cdot E_F^{\frac{5}{2}} \times \frac{3h^3}{16\sqrt{2} \pi V m^{\frac{3}{2}} E_F^{\frac{3}{2}}}$$

$$= \frac{3}{5} E_F$$