s.e

Eigen values & vectors

1. Finding Eigen values
2) Whether a given square matrix satisfies Caley Hamilton theory
3. Finding Eigen vectors
4. Diagonalisation of a matrix

Whether a matrix is diagonlisable or not

The diagonalisable form

Finding a matrix which diagonalises the given matrix

To check
Algebraic multiplicity = geomatric multiplicity

→ Find the Algebraic multiplicity and geometric multiplicity of an Eigen value of the matrix

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$ and show geometric multiplicity cannot be > a algebraic multiplicity.

Solu:

The characterstics equ. is $|A - \lambda I| = 0$

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$A = \begin{bmatrix}3&10&5\\-2&-3&4\\3&5&7\end{bmatrix}$

$= \begin{bmatrix}3&10&5\\-2&-3&4\\3&5&7\end{bmatrix} - \begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}$

$= \begin{bmatrix}3-\lambda&10&5\\-2&-3-\lambda&4\\3&5&7-\lambda\end{bmatrix}$

$(\lambda^2-4\lambda-1)$

$= \cancel{(3-\lambda)\{(-3-\lambda)(7-\lambda)\} - 10}$

$= (3-\lambda)\left[-21+3\lambda-7\lambda+\lambda^2+20\right) - 10(-14+2\lambda$

$+5(-10+9+3\lambda)$

$= \cancel{3\lambda - \lambda^3 + \dots}$

$= (3-\lambda)(\lambda^2-4\lambda-1) + 5(3-\lambda)$

$= \cancel{(3-\lambda)(\lambda^3 - \dots\lambda - 5)}$

$(3-\lambda)(\lambda^2-4\lambda+4)$

$= (3-\lambda)(\lambda-2)^2$

$= 3-\lambda = 0$ | $\lambda - 2 = 0$

$\lambda = 3$ | $\lambda = 2$

$\lambda = 2, 2, 3$

The Algebric multiplicity of the eigen valle 2 is '2'

$A\alpha = \lambda\alpha$ where α is a null vector.

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 2 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\Rightarrow 3x + 10y + 5z - 2x = 0$

$\Rightarrow x + 10y + 5z = 0$

$\therefore -2x - 3y - 4z - 2y = 0$

$\Rightarrow -2x - 5y - 4z = 0 \Rightarrow 2x + 5y + 4z = 0.$

$\therefore 3x + 5y + 7z - 2z = 0$

$\Rightarrow 3x + 5y + 5z = 0.$

The coefficiant matrix is $\begin{bmatrix} 1 & 10 & 5 \\ 2 & 5 & 4 \\ 3 & 5 & 5 \end{bmatrix}$

$R_3' = R_3 - R_2$

$\begin{bmatrix} 1 & 10 & 5 \\ 2 & 5 & 4 \\ 1 & 0 & 1 \end{bmatrix}$ $R_1' = R_1 - R_3$

$= \begin{bmatrix} 0 & 10 & 4 \\ 2 & 5 & 4 \\ 1 & 0 & 1 \end{bmatrix}$ $R_2' \to R_2 - 2R_3$

$= \begin{bmatrix} 0 & 10 & 4 \\ 0 & 5 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ $R_1' \to R_1 - 2R_2$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ $R_1 \to R_2$

The Ran of the $[A] = 2$ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 5 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

geometric multipliaty =

number of – Rank of matrix