Assignment (CA2)

i) Let $v = \left\{ f : [0,1] \to \mathbb{R} \mid f \text{ is integrade and } \int_0^1 f(x)dx = 0 \right\}$

Given

$f, g \in v$

$(f \oplus g)(x) = f(x) + g(x), \quad x \in [0,1]$

$\alpha \in \mathbb{R}$

$(\alpha \cdot f)(x) = \alpha f(x), \quad x \in [0,1]$

Prove that $(v, \oplus, \cdot)$ is a vector space

$f(x) = x$
$g(x) = \cos x$

$(f \oplus g)(x)$

$= x + \cos x$

$\frac{1}{3} + \frac{2}{3}$

② Verify whether the set

$$S = \left\{ (x,y) \in \mathbb{R}^2 \mid x^2 + 3y \leq 1 \right\},$$ with usual

vector sum and scalar multiplication is

a vector space or not.

$(x_1, y_1), (x_2, y_2) \subset \mathbb{R}^2$

$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2).$

$\alpha \in \mathbb{R} \quad \alpha(x_1, y_1) = (\alpha \cdot x_1, \alpha \cdot y_1)$

③ Let $W = \left\{ (x,y) \in \mathbb{R}^2 \mid 5x + 10y = 0 \right\}$

Prove that W is a vector space with usual
vector sum and usual scaler multiplication.