Homework 0

Submitted by,

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1) Given,

$$\mu_s(x_s) = \sum_{x_1, x_2, \dots, x_{s-1}, x_{s+1}, \dots, x_n} p(x_1, x_2, \dots, x_s, \dots, x_n)$$

prove that,

$$\mu_s(x_s) = K \psi_s \prod_{t \in N(s)} M_{ts}(x_s)$$

where,

$$M_{ts}(x_s) = \sum_{x_{V_t}} \psi_{st}(x_s, x_t)\, p(x_{V_t}; T_t)$$

Ans:

We have,

$$G = (V, E(G)), \quad |V| = n$$

$$p(x_1, x_2, \dots, x_n) = \frac{1}{Z} \prod_{u \in V} \psi_u(x_u) \prod_{(s,t) \in E} \psi_{s,t}(x_s, x_t) \quad \text{—(1)}$$

define,

neighbourhood, $N(s) = \{u \in V \mid \exists e \in E(G), e = (u, s)\}$

$x_{V_t} = \{x_u \mid u \in V_t\}$, V_t = vertex set of a subtree

Note that,

$$\text{(i)} \quad V = \{s\} \cup \left\{\bigcup_{t \in N(s)} V_t\right\} \quad \text{—(2)}$$

$$\text{(ii)} \quad E = \left\{\bigcup_{t \in N(s)} (s, t)\right\} \cup \left\{\bigcup_{t \in N(s)} E_t\right\} \quad \text{—(3)}$$

Now,

(1) $\Rightarrow$

$$p(x_1, \dots, x_n) = \frac{1}{Z} \prod_{u \in V} \psi_u(x_u) \prod_{(u,v) \in E} \psi_{u,v}(x_u, x_v)$$

$$= \frac{1}{Z} \psi_s(x_s) \prod_{u \in V \setminus s} \psi_u(x_u) \prod_{(u,v) \in E} \psi_{uv}(x_u, x_v) \quad \text{(from 2)}$$

$$= \frac{1}{Z} \psi_s(x_s) \prod_{u \in \bigcup_{t \in N(s)} V_t} \psi_u(x_u) \prod_{u,v \in \bigcup_{t \in N(s)} E_t} \psi_{uv}(x_u, x_v) \prod_{(s,t),\, t \in N(s)} \psi_{st}(x_s, x_t)$$

(from 3)

Now,

$$\mu_s(x_s) = \sum_{x_1,\dots,x_{s-1},x_{s+1},\dots,x_n} p(x_1,\dots,x_s,\dots,x_n)$$

$$= \sum_{x_1,\dots,x_{s-1},x_{s+1},\dots,x_n} \frac{1}{Z}\,\psi_s(x_s) \prod_{u\in \cup_{t\in N(s)} V_t} \psi_u(x_u) \prod_{(u,v)\in \cup_{t\in N(s)} E_t} \psi_{uv}(x_u,x_v) \prod_{(s,t),\, t\in N(s)} \psi_{st}(x_s,x_t)$$

$$= \sum_{x_1,\dots,x_{s-1},x_{s+1},\dots,x_n} 1/Z\;\psi_s(x_s) \prod_{u\in V\setminus s} \psi_u(x_u) \prod_{(u,v)\in \cup_{t\in N(s)} E_t} \psi_{uv}(x_u,x_v) \prod_{t\in N(s)} \psi_{st}(x_s,x_t) \qquad \text{—— (4)}$$

Note that,

$$p(x_{V_t};T_t) \propto \prod_{u\in V_t} \psi_u(x_u) \prod_{(u,v)\in E_t} \psi_{uv}(x_u,x_v)$$

then,

(4) $\Rightarrow$

$$\mu_s(x_s) = K\,\psi_s(x_s) \sum_{x_1,\dots,x_{s-1},x_{s+1},\dots,x_n} \prod_{t\in N(s)} p(x_{V_t};T_t) \prod_{t\in N(s)} \psi_{st}(x_s,x_t)$$

$$= K\,\psi_s(x_s) \sum_{x_1,\dots,x_{s-1},x_{s+1},\dots,x_n} \prod_{t\in N(s)} \psi_{st}(x_s,x_t)\; p(x_{V_t};T_t)$$

$$= K\,\psi_s(x_s) \sum_{x_{V_{t_1}}} \sum_{x_{V_{t_2}}} \cdots \sum_{x_{V_{t_n}}} \psi_{st_1}(x_s,x_{t_1})\, p(x_{V_{t_1}};T_{t_1}) \cdots \psi_{st_n}(x_s,x_{t_n})\, p(x_{V_{t_n}};T_{t_n})$$

$$= K\,\psi_s(x_s) \sum_{x_{V_{t_1}}} \psi_{st_1}(x_s,x_{t_1})\, p(x_{V_{t_1}};T_{t_1}) \cdots \sum \psi_{st_n}(x_s,x_{t_n})\, p(x_{V_{t_n}};T_{t_n})$$

$$= K\,\psi_s(x_s) \prod_{t\in N(s)} \sum_{x_{V_t}} \psi_{st}(x_s,x_t)\, p(x_{V_t};T_t)$$

$$= K\,\psi_s(x_s) \prod_{t\in N(s)} M_{ts}(x_s,x_t)$$

Where,

$$M_{ts}(x_s) = \sum_{x_{V_t}} \psi_{st}(x_s, x_t)\, \beta(x_{V_t}; T_t)$$

Ans: 2

The name of the file is

EE18MTECH11030_HW0_Question2.ipynb.

The white pixels represent large disparity.

The black pixels represent small disparity.

The codes takes approximately 2.5 minutes to run.