3/10/19

EE5601: Representation Learning

- Review MLP
- Autoencoder
- Sparse Autoencoder

$$R(\theta) = \sum_{i=1}^{N} R^{(i)}(\theta), \text{ where } \quad -①$$

$$R^{(i)}(\theta) = \sum_{k=1}^{K} \left(f_k(x^{(i)};\theta) - y_k^{(i)}\right)^2, \quad -②$$

$$\theta = \{\alpha_{ml}, \beta_{km}, \alpha_{10}, \ldots \alpha_{M0}, \beta_{10}, \ldots, \beta_{K0}\} \quad -③$$

$$\underbrace{1 \le l \le L}_{I/p}, \quad \underbrace{1 \le m \le M}_{\text{Hidden}}, \quad \underbrace{1 \le k \le K}_{O/p}$$

$$z_m^{(i)} = \sigma\left(\underline{x}^{(i)T} \cdot \underline{\alpha}_m + \alpha_{m0}\right); \quad -④ \qquad \underline{\alpha}_m = \begin{bmatrix} \alpha_{m1} \\ | \\ | \\ \alpha_{mL} \end{bmatrix}$$

$$\hat{y}_k^{(i)} = \sigma\left(\underline{z}^{(i)T} \cdot \underline{\beta}_k + \beta_{k0}\right) \quad -⑤$$

Now find $\frac{\partial}{\partial \beta_{km}} R^{(i)}(\theta)$.

$$\frac{\partial}{\partial \beta_{km}} R^{(i)}(\theta) = \frac{\partial}{\partial \beta_{km}} \left[\sum_{j=1}^{K} \left[f_j(x^{(i)};\theta) - y_j^{(i)}\right]^2\right]$$

Note that the term β_{km} appears only in the k^{th} output node.

$$\therefore \frac{\partial}{\partial \beta_{km}} R^{(i)}(\theta) = \frac{\partial}{\partial \beta_{km}} \left[f_k(x^{(i)};\theta) - y_k^{(i)} \right]^2$$

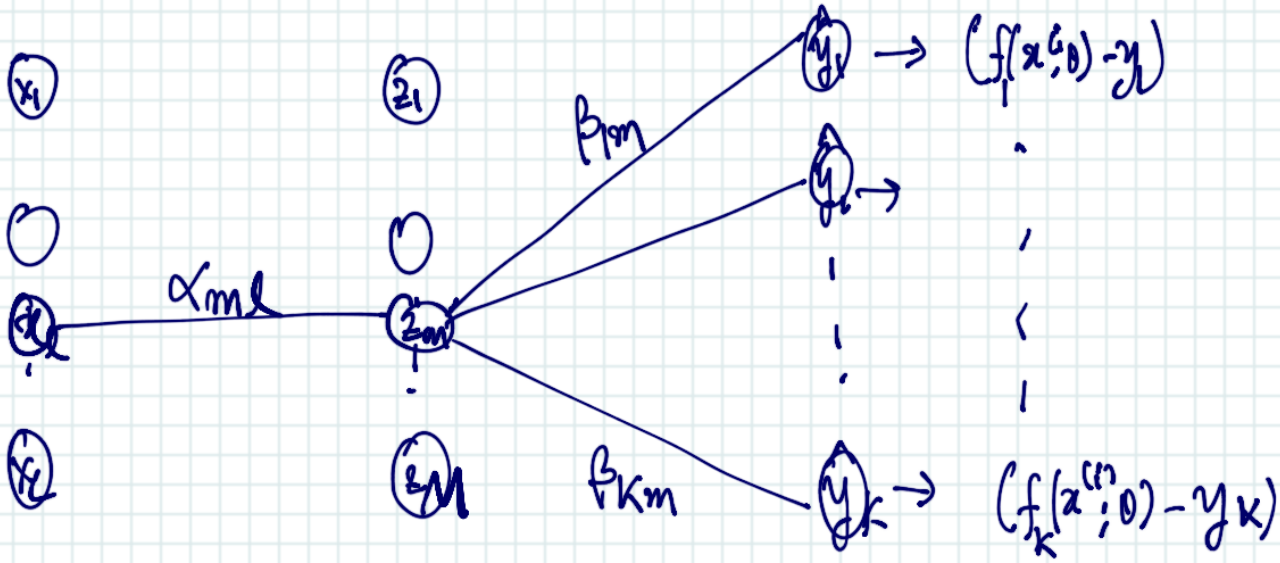
$$= 2 \cdot \left(f_k(x^{(i)};\theta) - y_k^{(i)} \right) \cdot f_k'(x^{(i)};\theta)$$

$$= 2 \cdot \left(f_k(x^{(i)};\theta) - y_k^{(i)} \right) \cdot \sigma'\left(\underline{z}^{(i)T} \cdot \beta_k + \beta_{k0} \right).$$

$$z_m \qquad -(6)$$

- Find $\dfrac{\partial R^{(i)}(\theta)}{\partial \alpha_{ml}}$ as H.W.
- Note that α_{ml} is going to appear in all the output node terms.
- Computing $\dfrac{\partial R^{(i)}(\theta)}{\partial \alpha_{ml}}$ would involve propagating $\left(f_k(x^{(i)};\theta) - y_k^{(i)} \right)$ (for all k) backwards. This leads to the back prop. algorithm.

$$\frac{\partial R^{(i)}(\theta)}{\partial \alpha_{ml}} = \sum_{k=1}^{K} 2 \cdot \left(f_k(x^{(i)};\theta) - y_k^{(i)} \right) \cdot \sigma'\left(\underline{z}^{(i)T} \beta_k + \beta_{k0} \right) \beta_{km} \, \sigma'\left(\underline{x}^{(i)T} \cdot \alpha_m + \alpha_{m0} \right) x_l . \qquad -(7)$$



Update expressions:

$$\beta_{km}^{(r)} = \beta_{km}^{(r-1)} - \gamma^{(r)} \cdot \sum_{i=1}^{N} \frac{\partial R^{(i)}(\theta)}{\partial \beta_{km}^{(r-1)}} \quad - 8$$

($\gamma^{(r)}$: learning rate ; r: iteration count)

$$\alpha_{ml}^{(r)} = \alpha_{ml}^{(r-1)} - \gamma^{(r)} \cdot \sum_{i=1}^{N} \frac{\partial R^{(i)}(\theta)}{\partial \alpha_{ml}^{(r-1)}} \quad - 9$$