18/9/19

EE 5601: Representation Learning

- Review
- ML Estimation
  - Gaussian RV
- Recall: $L(x;\underline{\theta}) \equiv \prod_{i=1}^{N} f_X(x_i;\underline{\theta})$ (likelihood function)

$$\log(L(X;\theta)) \equiv \log\left[\prod_{i=1}^{N} f_X(x_i;\underline{\theta})\right]$$

$$= \sum_{i=1}^{N} \log f_X(x_i;\underline{\theta})$$

Ex: Gaussian RV: $f_X(x;\underline{\theta}) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$; $\underline{\theta} = [\mu,\sigma]^T$

Find μ_{MLE}, σ^2_{MLE}; given $\{x_1, \ldots x_N\}$ i.i.d. samples.

<u>Soln:</u> Recall, $\underline{\theta}_{MLE} = \underset{\underline{\theta}}{\arg\max}\ L(X;\underline{\theta})$

$$= \underset{\underline{\theta}}{\arg\max}\ \log[L(X;\underline{\theta})].$$

$$\log[L(x;\underline{\theta})] = \sum_{i=1}^{N} \log f_X(x_i;\underline{\theta})$$

$$= \sum_{i=1}^{N} \log\left[\frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x_i-\mu)^2}{2\sigma^2}\right]\right]$$

$$= \sum_{i=1}^{N} \left[\log \left[\frac{1}{\sqrt{2\pi}\sigma} \right] - \frac{1}{2\sigma^2} (x_i - \mu)^2 \right]$$

$$\mu_{MLE} = \underset{\mu}{\arg\max} \log [L(x; \underline{\theta})]$$

Set $\frac{\partial}{\partial \mu} \log [L(x; \underline{\theta})] = 0$

This gives us $\boxed{\mu_{MLE} = \frac{\sum_{i=1}^{N} x_i}{N}}$

Similarly, $\boxed{\sigma^2_{MLE} = \frac{\sum_{i=1}^{N} (x_i - \mu_{MLE})^2}{N}}$