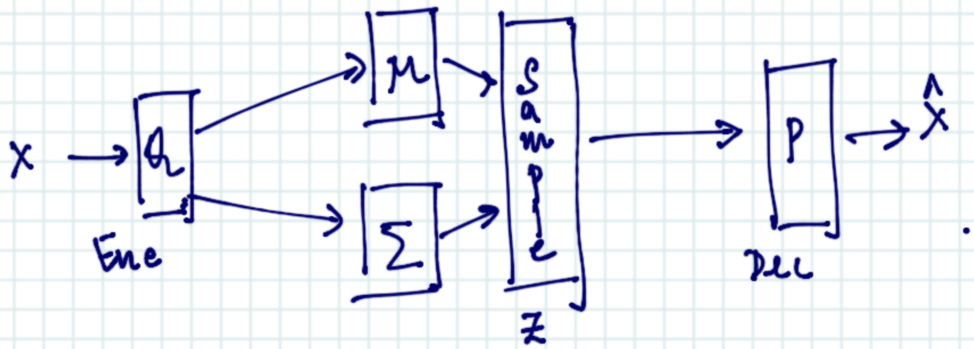
17/10/19

EE5601: Representation Learning

- Review
- The VAE cost function
- Reparameterization trick

Recall:



$$KL\left(Q(z)\,\|\,P(z|x)\right) = \sum_z Q(z)\cdot\log\left[\frac{Q(z)}{P(z|x)}\right]$$

$$= \sum_z Q(z)\left[\log Q(z) - \log P(z|x)\right]$$

$$= \sum_z Q(z)\cdot\left[\log Q(z) - \left[\log P(x|z) + \log P(z) - \log P(x)\right]\right]$$

$$= \sum_z Q(z)\cdot\left[\log Q(z) + \log P(x) - \log P(z) - \log P(x|z)\right]$$

$$KL\left(Q(z)\,\|\,P(z|x)\right) = KL\left(Q(z)\,\|\,P(z)\right) + \log P(x) - \sum_z Q(z)\cdot\log P(x|z)$$

$$\Rightarrow \log P(X) - KL\big(Q(z)\,||\,P(z|X)\big) = \sum_z Q(z) \log P(X|z) - KL\big(Q(z)\,||\,P(z)\big) \quad -①$$

In a VAE, the following assumptions are made:

1. $P(z) \sim \mathcal{N}(0, I)$ — ②

2. $P(x|z) \sim \mathcal{N}(f(z,\phi), \sigma^2 I)$ — ③

3. $Q(z)$ is replaced by $Q(z|X)$ — ④

4. $Q(z|x) \sim \mathcal{N}(\mu(x,\theta), \Sigma(x,\theta))$ — ⑤

Plug in ④ into ①

$$\log P(x) - KL\big(Q(z|x)\,||\,P(z|x)\big) = \sum_z Q(z|X) \cdot \log P(x|z) - KL\big(Q(z|x)\,||\,P(z)\big)$$

VAE finds the network parameters that maximize the above cost function.