Bachelor of Engineering in Mechanical Engineering Examination, 2018

(1st Year, 1st Semester)

MATHEMATICS - I

Use a separate Answer Script for each Group
(Symbols/ Notations have their usual meanings)

Group – A (50 marks)

Answer any five questions

1.(a) If $y = e^{m\sin^{-1}x}$ then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$.
Hence find y_n at $x = 0$.

(b) State Roll's theorem. Examine the applicability of Rolle's theorem in [-1, 1] for the function $f(x) = |x|$.

(4+2)+4

2.(a) State and prove Cauchy's Mean Value theorem.

(b) Use Lagrange Mean Value Theorem to prove that when $x > 0$,

$$0 < \frac{1}{x}\log\frac{e^x - 1}{x} < 1.$$

(2+4)+4

3.(a) Expand $(\sin^{-1}x)^2$ in series by Maclaurin's theorem.

(b) If α and β be the roots of $ax^2 + bx + c = 0$ then evaluate

$$\underset{x\to\alpha}{Lt}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}.$$

(c) Show that the minimum value of $\dfrac{(2x-1)(x-8)}{x^2-5x+4}$ is greater than its maximum value.

4+2+4

4 (a) Find the radius of curvature at the point (x,y) on the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

(b) Find the asymptotes of the curve $y^3 - x^2y + 2y^2 + 4y + 1 = 0$.

(c) Find the range of values of x for which the curve $y = x^4 - 16x^3 + 42x^2 + 12x + 1$ is concave or convex with respect to x-axis and identify the points of inflexion, if any.

3+3+4

5(a) Show that the function $f(x,y)=\begin{cases} \dfrac{x^3+y^3}{x-y} \text{ when } x \neq y \\ 0, \text{ when } x=y \end{cases}$

is not continuous at (0,0) but the partial derivatives f_x and f_y exist at (0,0).

(b) If $u=f(x,y)$ be a homogeneous function in two independent variables x, y of degree n, then show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu$$

5+5

6(a) Consider the function f defined by

$$f(x,y)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2} \text{ when } x^2+y^2 \neq 0 \\ 0, \text{ when } x^2+y^2=0 \end{cases}$$

Show that $f_{xy} \neq f_{yx}$ at (0,0).

(b) If $u=\log(x^3+y^3+z^3-3xyz)$ then show that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=-\frac{9}{(x+y+z)^2}$$

5+5

7(a) Find the volume of the greatest rectangular parallelopiped inscribed in the ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$.

(b) Test the convergence of the series $\sum u_n$ where $u_n=\dfrac{3.6.9.....3n}{7.10.13.....(3n+4)}x^n, \quad x>0$.

5+5

Ref. No. : EX/ME/MATH/T/111/2018

BACHELOR OF ENGINEERING IN MECHANICAL ENGINEERING EXAMINATION, 2018

(1st Year, 1st Semester)

Mathematics-I

Time: Three hours | Full Marks: 100

(50 marks for each Group)

(Symbols and notations have their usual meanings)

Use a separate Answer-Script for each Group

GROUP-B (50 Marks)

Answer ***Q. No. 8*** and any ***three*** from the rest.

8. Evaluate $\int_0^{\pi/2} \sin^4 x \, dx$. 5

9. a) Use first mean value theorem to show that

$$\frac{\pi}{6} \le \int_0^{\frac{1}{2}} \frac{dx}{\sqrt{(1-x^2)(1-\theta^2 x^2)}} \le \frac{\pi}{6}\frac{1}{\sqrt{1-\frac{\theta^2}{4}}},\ \ \theta^2 < 1.$$ 7

b) Evaluate $\iiint \frac{dxdydz}{\sqrt{1-x^2-y^2}}$, the field of integration being the positive octant of the sphere $x^2 + y^2 + z^2 = 1$. 8

10. a) Evaluate $\iint_D \{2a^2 - 2a(x+y) - (x^2+y^2)\}\, dxdy$, where the region D bounded by the circle $x^2 + y^2 + 2a(x+y) = 2a^2$. 8

b) Find the volume of the solid obtained by revolving one arch of the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ about x-axis. 7

11. a) State and proved fundamental theorem of integral calculus. 8

b) Using a double integral, prove that $B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)};\ a, b > 0$. 7

12. Examine the convergence of the following improper integrals:

a) $\int_3^{\infty} \frac{1}{\sqrt{x(x-1)(x-2)}}$ b) $\int_2^{\infty} \frac{dx}{\log x}$ c) $\int_0^1 \frac{\cos\frac{1}{x}}{\sqrt{x}}\, dx$ 15

13. a) A function f defined by $f(x,y) = \begin{cases} -\frac{1}{x^2} & if \quad 0 < y < x < 1 \\ \frac{1}{y^2} & if \quad 0 < x < y < 1 \\ 0 & elsewhere \end{cases}$

Find $\int_0^1 \left\{\int_0^1 f(x,y)\, dy\right\} dx$, $\int_0^1 \left\{\int_0^1 f(x,y)\, dx\right\} dy$.

Does the integral of $\iint_R f(x,y)\ dx\, dy$ exists over R=[0, 1; 0, 1]? 7
Justify your answer.

b) Calculate the value of $\int_0^1 \frac{x}{x+1}\, dx$ using (i) Trapezoidal rule, (ii) Simpson's $\frac{1}{3}$ rule by taking six intervals. 8
