

# Problem set 3

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## Problem 2

Attach “Credits” data from R. Regress “balance” on (a) “gender” only

```
library(ISLR)
head(Credit)

##   ID  Income  Limit  Rating  Cards  Age  Education  Gender  Student  Married
## Ethnicity
## 1  1  14.891  3606    283     2   34          11   Male     No      Yes
## Caucasian
## 2  2 106.025  6645    483     3   82          15  Female    Yes      Yes
## Asian
## 3  3 104.593  7075    514     4   71          11   Male     No      No
## Asian
## 4  4 148.924  9504    681     3   36          11  Female    No      No
## Asian
## 5  5  55.882  4897    357     2   68          16   Male     No      Yes
## Caucasian
## 6  6  80.180  8047    569     4   77          10   Male     No      No
## Caucasian
##   Balance
## 1      333
## 2      903
## 3      580
## 4      964
## 5      331
## 6     1151

model1=lm(Balance~Gender,data=Credit)
model1

##
## Call:
## lm(formula = Balance ~ Gender, data = Credit)
##
## Coefficients:
## (Intercept)  GenderFemale
##      509.80         19.73
```

(b) “gender” and “ethnicity”

```
model2=lm(Balance~Gender+Ethnicity,data=Credit)
model2
```



```

-----
## GenderFemale          19.733          20.038          24.340
##                      (46.051)          (46.178)
##                      (40.963)
##
## EthnicityAsian          -19.371          1.637
##                      (65.107)
##                      (57.787)
##
## EthnicityCaucasian      -12.653          6.447
##                      (56.740)
##                      (50.363)
##
## Income
6.054***
##
##                      (0.582)
##
## Constant          509.803***          520.880***
230.029***
##                      (33.128)          (51.901)
##                      (53.857)
##
## -----
-----
## Observations          400          400          400
## R2          0.0005          0.001          0.216
## Adjusted R2          -0.002          -0.007          0.208
## Residual Std. Error 460.230 (df = 398) 461.337 (df = 396) 409.218 (df
= 395)
## F Statistic          0.184 (df = 1; 398) 0.092 (df = 3; 396) 27.161*** (df
= 4; 395)
##
=====
=====
## Note:                                     *p<0.1; **p<0.05;
***p<0.01

```

(e) Explain how gender affects “balance” in each of the models (a)- (c)

**Ans:**In each of the three models if the gender is female then the average balance is increased and the increase are approximately 20,20 and 24 units respectively.

(f) Compare the average credit card balance of a male African with a male Caucasian on the basis of model (b).

**Ans:**Based on model (b) we can say that for a male Caucasian the average credit card balance is approximately 13 units less than an African male.

- (g) Compare the average credit card balance of a male African with a male Caucasian when each earns 100,000 dollars. For comparison, use the model in (c).

**Ans:** When the income is fixed, on the basis of model (c) we can conclude that average credit card balance of a male Caucasian is approximately 7 units more than that of a male African.

- (h) Compare and comment on the answers in (f) and (g)

**Ans:** The two answers are contradictory. So we see that if we include “income” as another predictor and form a new model the interpretation with respect to the other predictor “ethnicity” also changes.

- (i) Based on the model in (c), predict the credit card balance of a female Asian whose income is 2000,000 dollars

**Ans:**

```
response=24.340+1.637+6.054*2000
response
## [1] 12133.98
```

- (j) Check the goodness of fit of the different models in (a)-(c) in terms of adjusted  $R^2$ . Which model would you prefer? **Ans:** Since the adjusted  $R$  squared value is greatest in case of the third model therefore the fit is most satisfactory in this case and we prefer the third model.

## Problem 4

**Step 1:** Generate  $x_{1i}$  from Normal(0,1) distribution,  $i = 1, 2, \dots, n$  **Step 2:** Generate  $x_{2i}$  from Bernoulli (0.3) distribution,  $i = 1, 2, \dots, n$  **Step 3:** Generate  $\epsilon_i$  from Normal(0,1) and hence generate the response  $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 (x_{1i} \times x_{2i}) + \epsilon_i$ ,  $i = 1, 2, \dots, n$ . **Step 4:** Run two separate multiple linear regressions (i) using the model in Step 3 and (ii) using the model in Step 3 without the interaction term. Repeat Steps 1-4,  $R = 1000$  times. At each simulation compute the MSE for the correct model (i.e. model with the interaction term) and the naive model (i.e. the model without the interaction term). Finally find the average MSE's for each model. From the output, demonstrate the impact of ignoring the interaction term. Carry out the analysis for  $n = 100$  and the following parametric configurations:  $(\beta_0, \beta_1, \beta_2, \beta_3) = (-2.5, 1.2, 2.3, 0.001)$ ,  $(-2.5, 1.2, 2.3, 3.1)$ . Set seed as 123

For the first choice of the parameters

```
set.seed(123)
for(i in 1:1000){
  x1=rnorm(100,0,1)
  x2=rbinom(100,1,0.3)
  e=rnorm(100,0,1)
```

```

y1= -2.5 + 1.2*x1 + 2.3*x2 + 0.001*(x1*x2) + e
model1=lm(y1~x1+x2+x1*x2)
model2=lm(y1~x1+x2)
mse1=mean((y1-predict(model1))^2)
avg_mse1=mse1/1000
mse2=mean((y1-predict(model2))^2)
avg_mse2=mse2/1000
}
avg_mse1

## [1] 0.001015287

avg_mse2

## [1] 0.001015437

```

**Conclusion:** So if we do not include the interaction term the average mse increases a little.

**For the second choice of the parameters**

```

set.seed(123)
for(i in 1:1000){
  x1=rnorm(100,0,1)
  x2=rbinom(100,1,0.3)
  e=rnorm(100,0,1)
  y1= -2.5 + 1.2*x1 + 2.3*x2 + 3.1*(x1*x2) + e
  model1=lm(y1~x1+x2+x1*x2)
  model2=lm(y1~x1+x2)
  mse1=mean((y1-predict(model1))^2)
  avg_mse1=mse1/1000
  mse2=mean((y1-predict(model2))^2)
  avg_mse2=mse2/1000
}
avg_mse1

## [1] 0.001015287

avg_mse2

## [1] 0.002816845

```

**Conclusion:** here we see that by not including the interaction term the average mse increases by quite some amount.