Exploring probability distributions for bivariate temporal granularities

Abstract

Recent advances in technology greatly facilitates recording and storing data at much finer temporal scales than was previously possible. As the frequency of time-oriented data increases, the number of questions about the observed variable that need to be addressed by visual representation also increases. We propose some new tools to explore this type of data, which deconstruct time in many different ways. There are several classes of time deconstructions including linear time granularities, circular time granularities and aperiodic calendar categorizations. Linear time granularities respect the linear progression of time such as hours, days, weeks and months. Circular time granularities accommodate periodicities in time such as hour of the day, and day of the week. Aperiodic calendar categorizations are neither linear nor circular, such as day of the month or public holidays.

The hierarchical structure of linear granularities creates a natural nested ordering resulting in single-order-up and multiple-order-up granularities. For example, hour of the week and second of the hour are both multiple-order-up, while hour of the day and second of the minute are single-order-up.

Visualizing data across granularities which are either single-order-up or multiple-order-up or periodic/aperiodic helps us to understand periodicities, pattern and anomalies in the data. Because of the large volume of data available, using displays of probability distributions conditional on one or more granularities is a potentially useful approach. This work provides tools for creating granularities and exploring the associated time series within the tidy workflow, so that probability distributions can be examined using the range of graphics available in (Wickham 2016).

Contents

1 Introduction

Temporal data are available at various resolution depending on the context. Social and economic data like GDP are often collected and reported at coarser temporal scales like monthly, quarterly or annually. With recent advancement in technology, more and more data are recorded and stored at much finer temporal scales. Energy consumption is collected every half an hour, while energy supply is collected every minute and web search data might be collected at every second. As the frequency of data increases, the number of questions about periodicity of the observed variable also increases. For example, data collected at an hourly scale can be analyzed using coarser temporal scales like days, months or quarters. This approach requires deconstructing time in various possible ways. Calendar-based graphics[] can unpack the temporal variable, at different resolutions, to digest multiple seasonalities, and special events. They mainly pick out patterns in the weekly and monthly structure well and are capable of checking the weekends or special days. Any sub-daily resolution temporal data can also be displayed using this type of faceting [reference trellis plot] with days of the week, month of the year and another sub-daily deconstruction of time.

But calendar effects are not restricted to conventional day-of-week, month-of-year ways of deconstructing time. A temporal granularity which results from such a deconstruction may be intuitively described as a sequence of time granules, each one consisting of a set of time instants[]. There can be several classes of time deconstructions including linear time granularities, circular time granularities and aperiodic calendar categorizations. Linear time granularities respect the linear progression of time such as hours, days, weeks and months. Circular time granularities accommodate periodicities in time such as hour of the day, and day of the week. Aperiodic calendar categorizations are neither linear nor circular, such as day of the month or public holidays. Also, the hierarchical structure of time creates a natural nested ordering. For example, hours are nested within days, days within weeks, weeks within months, and so on. Hence, we can construct single-order-up granularities like second of the minute or multiple-order-up granularities like second of the hour.

It is important to be able to navigate through all of these temporal granularities to have multiple perspectives on the observed data. This idea aligns with the notion of EDA (Tukey 1977) which emphasizes the use of multiple perspectives on data to help formulate hypotheses before proceeding to hypothesis testing.

The motivation for this work comes from the desire to provide methods to better understand large quantities of measurements on energy usage reported by smart meters in household across Australia, and indeed many parts of the world. Smart meters currently provide half-hourly use in kWh for each household, from the time that they were installed, some as early as 2012. Households are distributed geographically, and have different demographic properties such as the existence of solar panels, central heating or air conditioning. The behavioral patterns in households vary substantially, for example, some families use a dryer for their clothes while others hang them on a line, and some households might consist of night owls, while others are morning larks.

It is common to see aggregates of usage across households, total kWh used each half hour by state, for example, because energy companies need to understand maximum loads that they will have to plan ahead to accommodate. But studying overall energy use hides the distributions of usage at finer scales, and making it more difficult to find solutions to improve energy efficiency.

We propose that the analysis of probability distributions of smart meter data at finer or coarser scales can be benefited from the approach of Exploratory Data Analysis (EDA). EDA calls for utilizing visualization and transformation to explore data systematically. It is a process of generating hypothesis, testing them and consequently refining them through investigations.

This paper utilizes the nestedness of time granularities to obtain multiple-order-up granularities from single-order-up ones.

Finally, visualizing data across single/multiple order-up granularities help us to understand periodicities, pattern and anomalies in the data. Because of the large volume of data available, using displays of probability distributions conditional on one or more granularities is a potentially useful approach. However, this approach can lead to a myriad of choices all of which are not useful. Analysts are expected to iteratively visualize these choices for exploring possible patterns in the data. But too many choices might leave him bewildered.

This work provides tools for systematically exploring bivariate granularities within the tidy workflow through proper study of what can be considered a prospective graphic for exploration. Pairs of granularities are categorized as either a *harmony* or *clash*, where harmonies are pairs of granularities that aid exploratory data analysis, and clashes are pairs that are incompatible with each other for exploratory analysis. Probability distributions can be examined using the range of graphics available in the ggplot2 package.

In particular, this work provides the following tools.

- Functions to create multiple-order-up time granularities. This is an extension to the lubridate package, which allows for the creation of some calendar categorizations, usually single-order-up.
- Checks on the feasibility of creating plots or drawing inferences from two granularities together. Pairs of granularities can be categorized as either a *harmony* or *clash*, where harmonies are pairs of granularities that aid exploratory data analysis, and clashes are pairs that are incompatible with each other for exploratory analysis.

2 Definitions of time granularities

Often we partition time into months, weeks, days and so on in a hierarchical manner to relate it to data. Such discrete abstractions of time can be thought of as time granularities (Aigner et al. 2011). Examples of time abstractions may also include day-of-week, time-of-day, week-of-year, day-of-month, month-of-year, working day/non-working day, etc which are useful to represent different periodicities in the data. Let us call all of these different abstractions of time as "calendar categorizations".

These calendar categorizations can be linear, circular or aperiodic. The calendar categorizations as "linear" if they respect the linear progression of time. We call these **linear time granularities**. Examples include hours, days, weeks and months. **Circular time granularities** accommodate periodicities in time such as hour of the day, and day of the week. **Aperiodic time granularities** are neither linear nor circular, such as day of the month or public holidays.

Providing a formalism to these abstractions is important to model a time series across differently grained temporal domains.

2.1 Linear

There has been several attempts to provide the framework for formally characterizing time-granularities and identifying their structural properties, relationships and symbolic representations. One of the first attempts occur in (Bettini et al. 1998) with the help of the following definitions:

Definition: A **time domain** is a pair $(T; \leq)$ where T is a non-empty set of time instants and $\leq$ is a total order on T.

A time domain can be **discrete** (if there is unique predecessor and successor for every element except for the first and last one in the time domain), or it can be **dense** (if it is an infinite set). A time domain is assumed to be discrete for the purpose of our discussion.

Definition: A linear **granularity** is a mapping G from the integers (the index set) to subsets of the time domain such that:

(C1) if $i < j$ and $G(i)$ and $G(j)$ are non-empty, then each element of $G(i)$ is less than all elements of $G(j)$, and
(C2) if $i < k < j$ and $G(i)$ and $G(j)$ are non-empty, then $G(k)$ is non-empty.

Definition: Each non-empty subset $G(i)$ is called a **granule**, where i is one of the indexes and G is a linear granularity.

The first condition implies that the granules in a linear granularity are non-overlapping and their index order is same as time order. Figure 1 shows the implication of this condition. If we consider the bottom linear

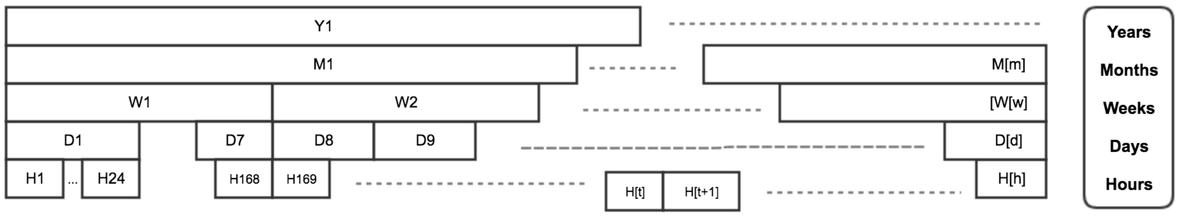


Figure 1: The time domain distributed as linear granularities

granularity (Aigner et al. 2011) as hourly and the entire horizon has T hours then it will have $\lfloor T/24 \rfloor$ days, $\lfloor T/(24*7) \rfloor$ weeks and so on.

The definitions and rules for linear granularities are inadequate to reflect periodicities in time, like weekly, monthly or yearly seasonality. Hence, there is a need to define circular time granularities in a different approach.

2.2 Circular

Periodicity is very common in all kinds of data and circular granularities can be construed to reflect such periodicities. Circular granularities can be constructed by putting to use two linear granularities. However, mappings between linear granularities can be regular or irregular. For example, a regular mapping exists between minutes and hours, where 60 minutes always add up to 1 hour. On contrary, an irregular mapping exists between days and months, since one might can have days ranging from 28 to 31. Hence, circular granularities can also be "periodic" or "aperiodic" depending on

Suppose we have a tsibble with a time index in one column and keys and variables in other columns.

A time domain, as defined by Bettini, is essentially a mapping of row numbers (the index set) to the time index.

A linear granularity is a mapping of row numbers to subsets of the time domain. For example, if the time index is days, then a linear granularity might be weeks, months or years. Circular granularity, joining two linear granularities, therefore is a mapping between these two subsets of time domain. We use modular arithmetic to define circular granularity and use the following definitions:

Definition: Equivalence class Let $m \in N \backslash 0$. For any $a \in Z$ (set of integers), $[a]$ is defined as the equivalence class to which a belongs if $[a] = \{b \in Z | a \equiv (b \mod m)\}$.

The set of all equivalence classes of the integers for a modulus m is called the ring of integers modulo m, denoted by Z_m. Thus $Z_m = \{[0], [1], ..., [m-1]\}$. However, we often write $Z_m = \{0, 1, ..., (m-1)\}$, which is the set of integers modulo m.

Definition: A **circular granularity** C, joining two linear granularity with a modular period m is defined to be a mapping from the integers Z (Index Set) to Z_m, such that

$C(s) = (s \mod m)$ for $s \in Z$.

For example, suppose C is a circular granularity denoting hour-of-day and we have hourly data for 100 hours. The modular period $m = 24$, since each day consists of 24 hours and C is a mapping from $1, 2, \ldots, 100$ to $0, 1, 2, \ldots, 23$ such that $C(s) = s \mod 24$ for $s \in 1, 2, \ldots, 100$.

Definition: A **cycle** is defined as the progression of each circular granularity with modular period m through $\{1, 2, \ldots, (m-1), 0\}$ once.

Definition: A **circular granule** represents an equivalence class inside each cycle.

2.3 Aperiodic

Definition: An **Aperiodic circular granularity** can not be defined using modular arithmetic in a similar fashion. The modulus for these type of calendar categorizations are not constant due to unequal length of some linear granularities. For example, please refer to the table below:

HOM:	$C_3(s) = s \mod 720$ (approximately)	$n_3 = 744$
HOY:	$C_4(s) = s \mod 8760$ (except for leap years)	$n_4 = 8784$
DOM:	$C_6(s) = \lfloor s/24 \rfloor \mod 30$ (approximately)	$n_6 = 31$
DOY:	$C_7(s) = \lfloor s/24 \rfloor \mod 365$ (except for leap years)	$n_7 = 366$
WOM:	$C_8(s) = \lfloor s/168 \rfloor \mod 4$ (approximately)	$n_8 = 5$
WOY:	$C_9(s) = \lfloor s/168 \rfloor \mod 52$ (approximately)	$n_9 = 53$
MOY:	$C_{10}(s) = \lfloor s/720 \rfloor \mod 12$ (approximately)	$n_{10} = 12$

Table 1: Illustrative aperiodic circular granularities with time index in hours

Identifying repeating (periodic/aperiodic) patterns are necessary in revealing patterns and future trends of a temporal data. Often there is a need for periodicity detection to find whether and how frequent a periodic/aperiodic pattern is repeated within the series. To consider the exhaustive set of temporal regularities that might exist in the data, we can categorize the set of temporal granularities to single or multiple order up granularities.

3 Computation of time granularities

3.1 Definitions of time granularities based on order

The hierarchical structure of time creates a natural nested ordering which can produce **single-order-up** or **multiple-order-up** granularities. We shall use the notion of a hierarchy table to define them.

Consider a **hierarchy table** to be consisting of two columns:

- The first column represents the (temporal) units in ascending order of hierarchy. Any two units can join together to form a granularity.
- The second column represents the relationship between subsequent (temporal) units. For periodic granularities, this could be represented by a constant, whereas, for a aperiodic one, there is no single constant which can be used to represent their relationship.

and, **order**, is defined as the position of the units in the hierarchical table.

We refer to granularities which are nested within multiple levels as **multiple-order-up** granularities and those concering a single level as **single-order-up** granularities. Let us look at few calendars to see examples of single and multiple order-up granularities.

So far we have used the Gregorian calendar as it is the most widely used calendar. But it is far from being the only one. All calendars fall under three types - solar, lunar or lunisolar/solilunar but the day is the basic unit of time underlying all calendars. Various calendars, however, use different conventions to structure days into larger units: weeks, months, years and cycle of years. Any civil day is divided into 24 hours and each hour into 60 minutes and each minute into 60 seconds. There are exceptions where in London, for example, length of each "hour" varies from about 39 minutes in December to 83 minutes in June. The French revolutionary calendar divided each day into 10 "hours", each "hour" into 100 "minutes" and each "minute" into 100 "seconds". Nevertheless, for any calendar a hierarchy can be defined. For example, in Mayan calendar, one day was referred to as 1 kin and the calendar was structured as follows:

- 1 kin = 1 day
- 1 uinal = 20 kin
- 1 tun = 18 uinal
- 1 katun = 20 tun

- 1 baktun = 20 katun

Thus, the hierarchy table for the Mayan calendar would look like the following:

Units	Conversion factor
kin	20
uinal	18
tun	20
katun	20
baktun	1

Examples of multiple-order-up granularities can be kin of the tun or kin of the baktun whereas examples of single-order-up granularities may include kin of the uinal, uinal of the tun etc.

In the next section, we discuss the computation of any single-order-up and multiple-order-up granularities in a periodic and aperiodic set up.

3.1.1 Single-order-up granularities

Suppose, z denotes the index of the tsibble, x,y are two units in the hierarchy table with $order(x) < order(y)$. Let $f(x, y)$ denotes the accessor function for computing the single-order-up granularity relating x and y and $c(x, y)$ is a constant which relates x and y.

Then $f(x, y)$ can be computed using modular arithmetic as follows:

$$f(x,y) = \lfloor z/c(z,x) \rfloor \mod c(x,y)$$

where $y = x + 1$

See table[] for an illustration of computing single order up granularities for Mayan calendar.

Single-order-up granularities $kin\_uinal := z \mod 20$
$uinal\_tun := \lfloor z/20 \rfloor \mod 18$
$tun\_katun := \lfloor z/20 * 18 \rfloor \mod 20$
$katun\_baktun := \lfloor z/20 * 18 * 20 \rfloor \mod 20$

Table 3: Illustrative single-order-up granularities for Mayan calendar with kin as the index

3.1.1.1 Aperiodic single-order-up granularities

Aperiodic single-order-up granularities refers to the granularities which are formed with units that does not repeat its values in regular intervals or periods. In Gregorian calendar, examples may include days of the month where each month may consists of 28, 29, 30 or 31 days. So there is no single number that can be used for converting days to months. We cannot compute the aperiodic single-order-up granularities using the index of the tsibble and modular arithmetic like in the periodic case using just the hierarchy table and relationships of units.

3.1.2 Computation of multiple order-up granularities

Computation of multiple-order-up granularities will differ if the units in the hierarchy table are periodic or aperiodic. We split the method of computation into two cases - one concerining all periodic single-order-up granularities and the second with mixed single-order-up granularities.

3.1.2.1 All single-order-up granularities are periodic

z is the index set of the tsibble and x, y are two linear granularities with $order(x) < order(y)$. Also, $f(x, y)$ denotes the accessor function for computing circular granularity which relates x and y and $c(x, y)$ is a constant which relates x and y. It is easy to see that for $order(x + 1) = order(y)$, the function is same as the single-order-up granularities.

Then, the accessor function f can be used recursively to obtain any multiple-order-up granularities as follows:

$$
\begin{aligned}
f(x, y) &= f(x, x + 1) + c(x, x + 1)(f(x + 1, y) - 1) \\
&= f(x, x + 1) + c(x, x + 1)[f(x + 1, x + 2) + c(x + 1, x + 2)(f(x + 2, y) - 1) - 1] \\
&= f(x, x + 1) + c(x, x + 1)(f(x + 1, x + 2) - 1) + c(x, x + 1)c(x + 1, x + 2)(f(x + 2, y) - 1) \\
&= f(x, x + 1) + c(x, x + 1)(f(x + 1, x + 2) - 1) + c(x, x + 2)(f(x + 2, y) - 1) \\
&\vdots \\
&= \sum_{i=0}^{order(y)-order(x)-1} c(x, x + i)(f(x + i, x + i + 1) - 1)
\end{aligned} \tag{1}
$$

Let us use the equation to compute the multiple-order-up granularity uinal_katun for Mayan calendar, which is periodic.

From Equation(1), we have

$$
\begin{aligned}
f(uinal, baktun) &= f(uinal, tun) + c(uinal, tun)f(tun, katun) + c(uinal, katun)f(katun, baktun) \\
&= \lfloor z/20 \rfloor \mod 18 + 20 * \lfloor z/20 * 18 \rfloor \mod 20 + 20 * 18 * 20 \lfloor z/20 * 18 * 20 \rfloor \mod 20
\end{aligned} \tag{2}
$$

(using equations 3)

3.1.2.2 Mixed single-order-up granularities - circular or aperiodic

Let us turn to Gregorian calendar for addressing this case.

Suppose we have a hierarchy table for Gregorian calendar as **??**. Since months consists of unequal number of days, any temporal unit which is higher in order than months will also have unequal number of days. This is an example of a hierarchy table which has both periodic and aperiodic single-order-up granularities. The single-order-up granularity day_month is aperiodic. Any single-order-up granularities which are formed by units below days are periodic. Similarly, all single-order-up granularities which are formed using units whose orders are higher than months are also periodic.

linear granularities	Conversion factor	format
minute	60	markdown
hour	24	markdown
day	aperiodic	markdown
month	3	markdown
quarter	4	markdown
year	1	markdown

There can be three scenarios for obtaining calendar categorization here: - granularities consisting of two units whose orders are less than day
- granularities consisting of two units whose orders are more than month
- granularities consisting of one unit with order at most day and another with order at least month

The calendar categorization resulting from the first two cases are periodic and has been handled in the earlier section.

The calendar categorization resulting from the last case are aperiodic. Examples might include day of the quarter or hour of the month. In this section, we will see how to obtain aperiodic circular granularities of these types.

$$f_{(hour,month)}(z) = f_{(hour,day)}(z) + c(hour,day)f_{(day,month)}(z) \tag{3}$$

Here, the first part of the equation is a single-order-up granularity which can be obtained using last section. The second part is not single-order-up and can not be broken down further since each month consists of different number of days. In this case, it is important for us to know which month of the year and if the year is a leap year to obtain day of the month.

4 Interaction

Figure 2 (a) shows the letter value plot of electricity consumption of Victoria across days of the month for few months like January, February, April and December. Letter value plots convey detailed information about tails of the distribution and outliers are unexpected observations rather than extreme observations. M, F, E, D and C represents 50%, 25%, 12.5%, 6.25% and 3.13% of the tail area respectively. Some observations that can be made from these letter-value plots include a) Right tails for most days in January lying above the median is more extended than the left tails contrary to days in April which has longer left tails. b) Days in mid January and February are characterized by high variation in consumption level, but days in January have longer right tails. c) Last five days in December (Christmas holidays) shows low variation in consumption with longer left tails implying people typically consume less electricity. We can conclude that Month-of-Year and Day-of-Month are harmonies.

Figure 2 (c) shows box plot of electricity consumption of Victoria from 2012 to 2014 across days of the year by the 1st, 15th, 29th and 31st days of the month. The box plot is a a very compact distributional summary, displaying median, quartile boundaries, hinges, whiskers and outliers. All facets do not contain data across same x-axis levels leading to difficulty in comparison across facets. Hence, Day-of-Month and Day-of-Year are clashes.

Take another example Figure 2 (d) showing violin plot across days of the month faceted by week of the month. Violin plots are a combination of box plot and density plot. Here, the first week of the month correspond to certain days of the month which are different for different week of the month. These kinds of graphics can hinder our comprehension across different weeks of the month and can be categorized as a incompatible combination to plot.

In Figure 2 (e), variations across week of the year conditional on week of the month can be observed through a ridge plot. The y-axis represents week of the year and the x-axis represents electricity consumption. Ridge plots are density plots all aligned to the same horizontal scale and presented with a slight overlap. They are designed to bring out changes in distributions over time. The data is distributed unequally for different levels of weeks of the year for different facets, which hinders comparison across facets. So, Week-of-Month and Week-of-Year are clashes.

Figure 2 (f) shows decile plots of consumption across day of the year and month of the year. Decile plots are useful in displaying distribution through deciles without much clutter. This plot, however, seem ineffective for comparison as levels of x-axis for which observed data is plotted are completely disjoint across facets. So Month-of-Year and Day-of-Year are clashes as well.

From the above examples, we can see that the choices of the circular granularities are harmonies while some are clashes. Also since some choices work and others don't, it must be the attributes of the circular granularities or their relationships which are in play in deciding if the resulting plot would be a good candidate for exploratory analysis.

When we plot some observations with two circular or aperiodic granularities used as aesthetics or facets, problems might arise when we have empty combinations. It is important to check how these two granularities

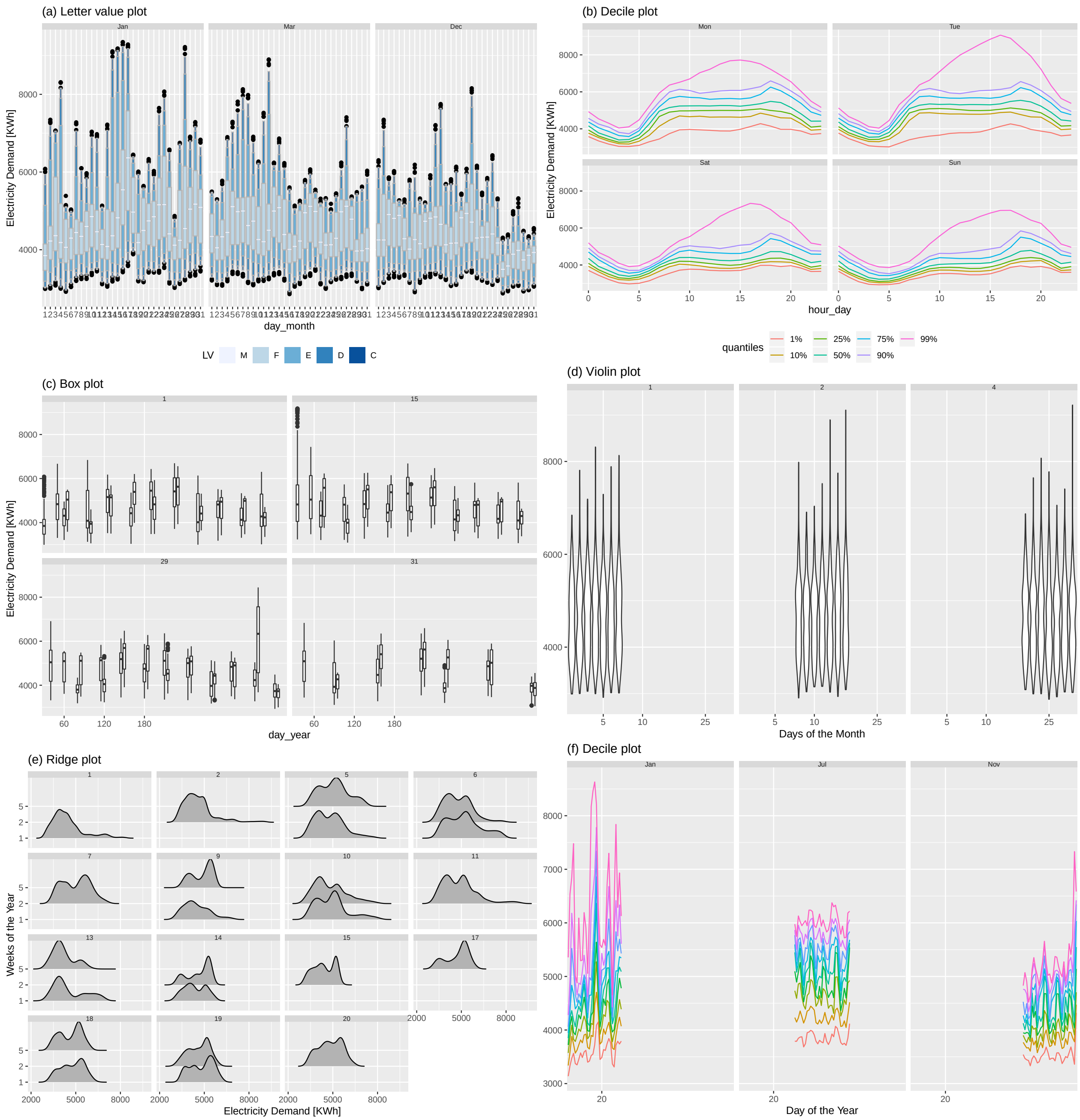


Figure 2: Various probability distribution plots of electricity consumption data of Victoria from 2012 to 2014. (a) Letter value plot by DoM and MoY, (b) Decile plot by HoD and DoW (c) Box plot by DoY and DoM, (d) Violin plot of DoM and WoM, (e) Ridge plot by WoM and WoY, (f) Decile plot by DoY and MoY. Only plots (a) and (b) show harmonised time variables.

interact with each other, before we expect to gain insights by plotting them together.

For example, suppose we have two circular or aperiodic granularities C_1 and C_2, such that C_1 maps row numbers to a set $\{A, B, C, D\}$, and C_2 maps row numbers to a set $\{X, Y, Z\}$. That is, let S_{ij} be the set of row numbers such that for all $s \in S_{ij}$, $C_1(s) = i$ and $C_2(s) = j$. In this example, we have 12 such sets S_{ij} because i can take 4 values and j can take 3 values. The graphs that don't work are those where many of these 12 sets are empty. Now, many situations can lead to any of these sets being empty. Let us discuss the following cases, where one or more of these sets can be empty.

Firstly, empty combinations can arise due to the structure of the calendar or hierarchy. These are called "structurally" empty combinations. Let us take a specific example, where C_1 maps row numbers to Day-of-Month and C_2 maps row numbers to Week-of-Month. Here C_1 can take 31 values while C_2 can take 5 values. There will be $31 \times 5 = 155$ sets S_{ij} corresponding to the possible combinations of WOM and DOM. Many of these are empty. For example $S_{1,5}$, $S_{21,2}$, etc.This is also intuitive since the first day of the month can never correspond to fifth week of the month. In fact, most of these 155 sets will be empty, making the combination of C_1 and C_2 in a graph unhelpful. These are structurally empty sets in that it is impossible for them to have any observations.

Secondly, empty combinations can turn up due to differences in event location or duration in a calendar. These are called "event-driven" empty combinations. Again, let us consider a specific example to illustrate this. Let C_1 be DOW and C_2 be WorkingDay/NonWorkingDay. Here C_1 can take 7 values while C_2 can take 2 values. So there are 14 sets S_{ij} corresponding to the possible combinations of DOW and WD/NWD. While potentially all of these can be non-empty (it is possible to have a public holiday on any DOW), in practice many of these combinations will probably have very few observations. For example, there are few if any public holidays on Wednesdays or Thursdays in any given year in Melbourne.

Thirdly, empty combinations can be a result of how granularities are constructed. Let C_1 maps row numbers to "Business days", which are days from Monday to Friday except holidays and C_2 is Day-of-Month. Then the weekends in Days-of-Month would not correspond to any Business days and would have missing observations due to the way the granularities are constructed. This is different from the structurally empty combinations because structure of the calendar does not lead to these missing combinations, but the construction of the granularity does. Hence, they are referred to as "build-based" empty combinations.

An example when there will be no empty combinations could be where C_1 maps row numbers to Day-of-Week and C_2 maps row numbers to Month-of-Year. Here C_1 can take 7 values while C_2 can take 12 values. So there are $12 \times 7 = 84$ sets S_{ij} corresponding to the possible combinations of DOW and MOY. All of these are non-empty because every DOW can occur in every month. So graphics involving C_1 and C_2 are potentially useful.

The combinations of circular/aperiodic granularities which lead to structurally, event-driven or build-based empty-combinations are referred to as to as **clashes**. And the ones that ones that do not lead to any missing combinations are called **harmonies** as they promote the exploratory analysis through visualization.

Even with harmonies, problems might occur due to rarely occuring categories.

suppose we have T hourly observations, and C_1 is HOD and C_2 is DOW. Each element of C_1 occurs approximately T/n_1 times while each element of C_2 occurs approximately T/n_2 times. There are no structurally empty combinations, and each combination will occur on average an equal number of times as $T \to \infty$, so the average number of observations per combination is $T/(n_1 n_2)$. If we require at least k observations to create a meaningful panel, then provided $T \geq n_1 n_2 k$, the visualization will be acceptable. The value of k will depend on what type of visualization we are producing. For scatterplots, even $k = 3$ may be acceptable, but for density estimates, we probably need $k \geq 30$.

The above discussion shows that problems will occur when $n_1 n_2 > T/k$, even without structurally empty combinations.

Problems will also occur when we have rarely occurring categories such as the 366th day of the year, or the 31st day of the month.

4.1 Harmony and Clashes

In Figure 2 (c), we have empty combinations when we plot observations with "Day-of-Month" and "Day-of-Year". Here, the 1st day of the month can never correspond to 2nd, 3rd or 4th day of the year. Hence, "Day-of-Month" and "Day-of-Year" are clashes due to the way they map to calendar.

In Figure 2 (a), we will not have any empty combinations because every DoM can occur in all MoY. Here, mapping from days to months is irregular in the sense that one month can consist of 28, 29, 30 or 31 days. There is no denying that the 29th, 30th or 31st day of the month needs to be analysed with caution due to the irregular mapping, however, the first 28 days of the month will occur in all months of the year.

More generally, if we have two circular granularities C1 and C2 which has [A, B, C, D] and [X, Y, Z] categories/levels. When C1 and C2 are used as aesthetics or facets, problems arise when we encounter empty combinations. Let $S_{i,j}$ be the set of combination of the levels of C1 and C2. In this example, we have 12 such sets $S_{i,j}$ because i can take 4 values and j can take 3 values. The graphs that don't work are those where many of these 12 sets are empty. In other words, if there are levels of the x-axis which are not spanned by levels of the faceted variable or vice versa we will have structurally empty sets leading to potential ineffective graphs.

Thus, the common link that differentiates harmonies from clashes are:

a) There should not be any levels of the faceted variable which is empty for one or more levels of the factor plotted across x-axis.

b) There should not be any level of the factor plotted across x-axis which doesn't have values for all levels of factors plotted across facets.

5 Visualization

Analysts often want to fit their data to statistical models, either to test hypotheses or predict future values. However, improper choice of models can lead to wrong predictions. One important use of visualization is exploratory data analysis, which is gaining insight into how data is distributed to inform data transformation and modeling decisions.

But with huge amount of data being available, sometimes mean, median or any one summary statistic is not enough to understand a data set. Soon enough following questions become more interesting:

- Are values clustered around mean/median or mostly around tails? In other words, what is the combined weight of tails relative to the rest of the distribution?
- Does values rise very quickly between 25th percentile and median but not as quickly between median and 75th percentile? More generally, how the variation is the data set changes across different percentiles/deciles?
- Is the tail on the left hand side longer than that on the right side? Or are they equally balanced around mean/median?

This is when displaying a probability distribution becomes a potentially useful approach.

The entire distribution can be visualized or some contextual summary statistics can be visualized to emphasize certain properties of the distribution. These properties can throw light on central tendency, skewness, kurtosis, variation of the distribution and can also be useful in detecting extreme behavior or anomalies in the data set.

5.1 Statistical distribution plots

Most commonly used techniques to display distribution of data include the histogram (Karl Pearson), which shows the prevalence of values grouped into bins and the box-and-whisker plots (Tukey 1977) which convey statistical features such as the median, quartile boundaries, hinges, whiskers and extreme outliers. The box plot is a compact distributional summary, displaying less detail than a histogram. Due to wide spread popularity and simplicity in implementation, a number of variations are proposed to the original one which provides alternate definitions of quantiles, whiskers, fences and outliers. Notched box plots (Mcgill, Tukey, and Larsen 1978, 1978) has box widths proportional to the number of points in the group and display confidence interval around medians aims to overcome some drawbacks of box plots.

The vase plot (Benjamini 1988, 1988) was a major revision from the concept of box plots where the width of box at each point is proportional to estimated density. Violin plots (Hintze and Nelson 1998, 1998) display the density for all data points and not only the box. The summary plot (Potter et al. 2010, 2010) combines a minimal box plot with glyphs representing the first five moments (mean, standard deviation, skewness, kurtosis and tailings), and a sectioned density plot crossed with a violin plot (both color and width are mapped to estimated density), and an overlay of a reference distribution. The highest density region (HDR) box plot proposed by (Hyndman 1996) displays a probability density region that contains points of relatively highest density. The probabilities for which the summarization is required can be chosen based on the requirement. These regions do not need to be contiguous and help identify multi-modality. The letter-value box plot (Hofmann, Wickham, and Kafadar 2017, 2006) was designed to adjust for number of outliers proportional to the data size and display more reliable estimates of tail. Because this display just adds extra letter values, it suffers from the same problems as the original box plot, and multimodality is almost impossible to spot(Wickham and Stryjewski, n.d.).

Moreover, much like the quartiles divide the data set equally into four equal parts, extensions might include dividing the data set even further. The deciles plots consist of 9 values that split the data set into ten parts and the percentile plot consists of 99 values that split the data set into hundred parts. A large data set is required before the extreme percentiles can be estimated with any accuracy.

Finally, a density plot which uses a kernel density estimate to show the probability density function of the variable can show the entire distribution. Also, a Ridge line plot (sometimes called Joy plot) shows the distribution of a numeric value for several groups. Distribution can be represented using histograms or density plots, all aligned to the same horizontal scale and presented with a slight overlap.

The time series variable can be plotted against many time granularities to get more understanding of the underlying periodicty, however, we will restrict ourselves to see the distribution of the time series across bivariate temporal granularities. That neccessiates plotting one temporal granularity along the x-axis and the other one across facets.

Now, due to the hierarchical arrangement of the granularities, there are certain granularities which when plotted together do not give us the layout to do exploration, for example, structurally empty combinations (clashes) are not recommended to plot together. The harmonies when plotted together can help exploration. But still the question remains that which distribution plot should be chosen to bring out the best of exploratory data analysis. This is a function of which features of the distribution we are interested to look at, how much display space is available to us and also if the number of observations are enough for that distribution plot.

5.2 Advice algorithm for exploring conditional probability distributions

Recommendations for distribution plots depend on the levels(very high/high/medium/low) of the two granularities plotted. They will vary depending on which granularity is placed on the x-axis and which one across facets. Assumptions are made to ensure display is not too cluttered by the space occupied by various kinds of distribution plots. Moreover, the recommendation system ensures that there are just enough observations before choosing a distribution plot.

Levels are categorized as very high/high/medium/low each for the facet variable and the x-axis variable. Default values for these levels are based on levels of common temporal granularities like day of the month, day of a fortnight or day of a week. For example, any levels above 31 is considered as very high, any levels between 14 to 31 are taken as high and that between 7 to 14 is taken as medium and below 7 is low. 31, 14 and 7 are the levels of days-of-month, days-of-fortnight and days-of week respectively. These default values are decided based on usual cognitive power while comparing across facets and display size available to us. Let us consider case by case and see which plots are better suitable in which scenarios.

- very high facet and x-axis levels

X-axis variable is treated as a categorical variable and hence any plots which

6 Case studies

6.1 Smart meter data of Australia

Smart meters provide large quantities of measurements on energy usage for households across Australia, and indeed many parts of the world. Households are distributed geographically and have different demographic properties such as the existence of solar panels, central heating or air conditioning. The behavioral patterns in households vary substantially, for example, some families use a dryer for their clothes while others hang them on a line, and some households might consist of night owls, while others are morning larks.

It is common to see aggregates of usage across households, total kwh used each half-hour by state, for example, because energy companies need to understand maximum loads that they will have to plan ahead to accommodate. But studying overall energy use hides the distributions of usage at finer scales, and making it more difficult to find solutions to improve energy efficiency.

One of the customer trial (Department of the Environment and Energy 2018) conducted as part of the Smart Grid Smart City (SGSC) project (2010-2014) in Newcastle, New South Wales and some parts of Sydney provides customer wise data on half-hourly energy usage and detailed information on appliance use, climate, retail and distributor product offers, and other related factors. It would be interesting to explore the energy consumption distribution for these customers and gain more insights on their energy behavior which are otherwise lost either due to aggregation or looking only at coarser temporal units. The idea here is to show how looking at the time across different granularities together can help identify different behavioral patterns. We look at the behavior of typical and extreme behaviors of 50 households from that trial.

Figure **??** shows the overlay quantile plot of energy consumption of 50 households. The black line is the median, whereas the pink band covers 25th to 75th percentile, the orange band covers 10th to 90th percentile and the green band covers 1st to 99th percentile. Each facet represents a month and energy consumption across each hours of the day is shown inside each facet. It can be observed that the median is very close (and hence not distinctly visible) to the lower boundaries of all the other bands implying energy consumption for these households are extremely left skewed.75% of the households has distinct higher evening peaks compared to morning peaks, especially in the winter months. The top 5% households behave very differently than even the top 10% households in terms of total energy consumption across hours of the day.

```
library(gravitas)
library(tsibble)
load("data/sm_cust50.RData")


library(ggplot2)
sm_cust50 %>%
  prob_plot("month_year", "hour_day",
    response = "general_supply_kwh",
    plot_type = "quantile",
    quantile_prob = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99)
  ) +
  ggtitle("")
```

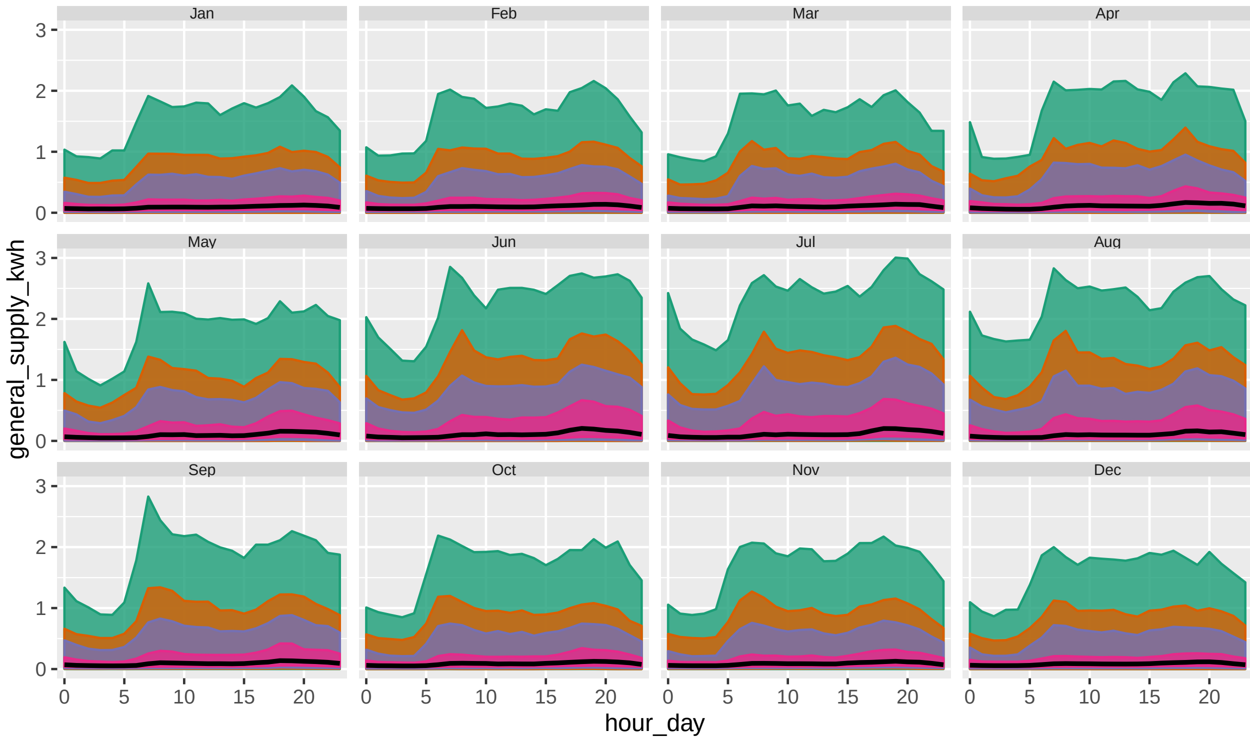


Figure **??** shows that on morning peaks for Saturday and Sunday are higher and occurs in a later hour of the day compared to weekdays. Thsi is true for both top 50% and 75% of the customers. Hence, this is indicative of the fact that at least 50% of the households in this subset has traditional work days (Mon-Fri) and weekends(Sat-Sun). All the percentiles peaks up in the morning hours and then drops in the afternoon, before again peaking up in the evening.

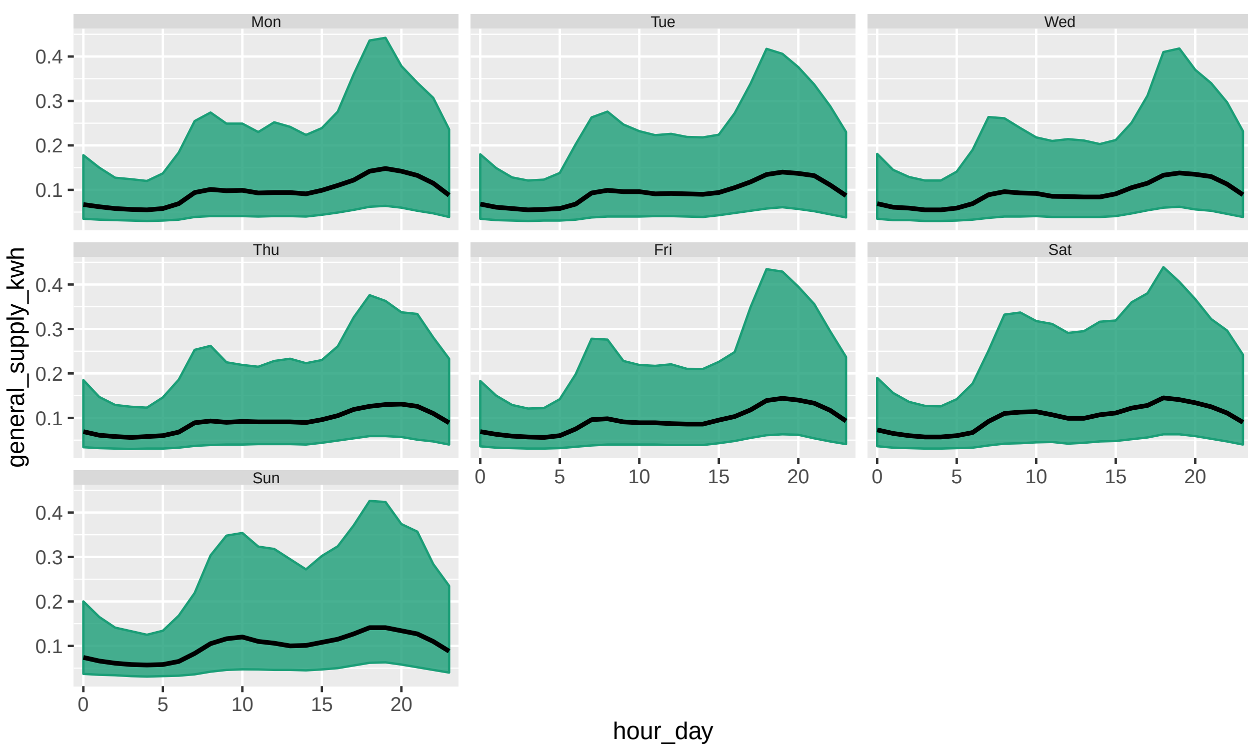
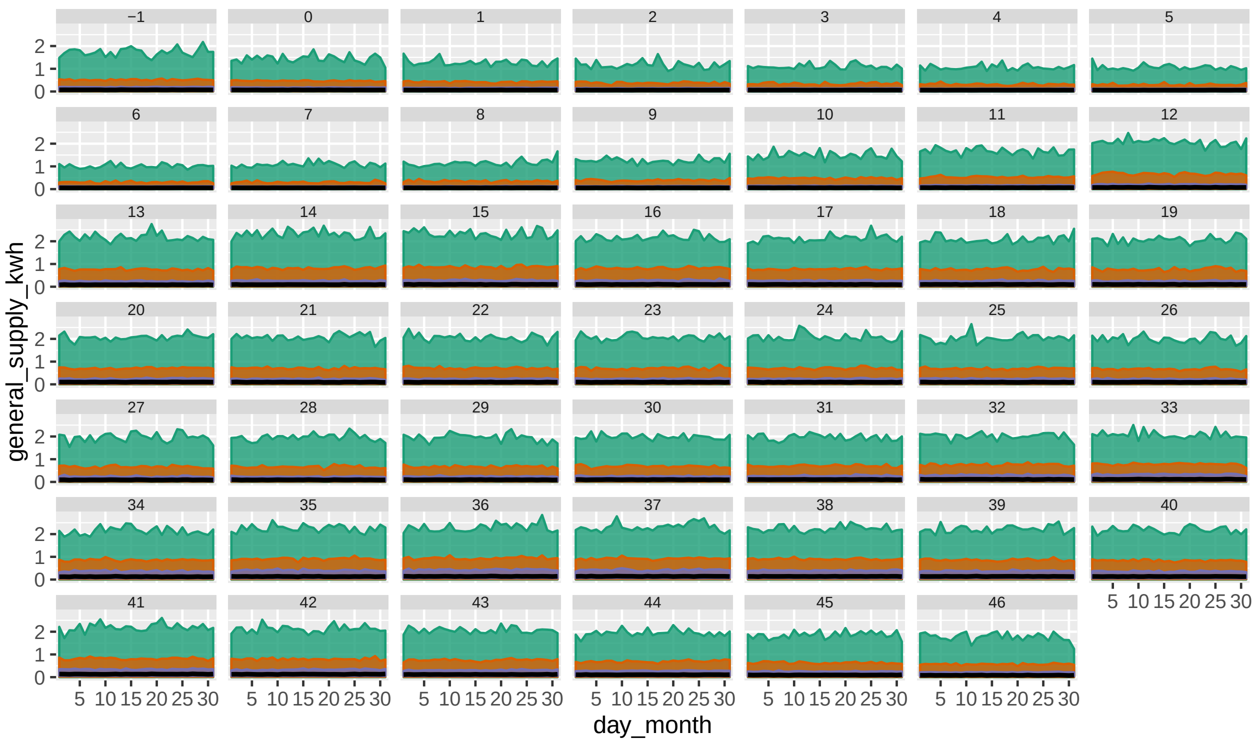


Table 4: A hierarchy table for T20 cricket

units	convert_fct
index	1
ball	6
over	20
inning	2
match	1

Figure **??** shows distribution of energy consumption across hour hour of the day across different days of the month. We can't see much variation across days of week implying that the energy behavior doesn't change depending on which day of the month it is.



6.2 T20 cricket data of Indian Premiere League

Th application is not only restricted to temporal data. We provide an example of cricket to illustrate how this can be generalised in other applications. The Indian Premier League (IPL) is a professional Twenty20 cricket league in India contested by eight teams representing eight different cities in India. With eight teams, each team plays each other twice in a home-and-away round-robin format in the league phase. In a Twenty20 game the two teams have a single innings each, which is restricted to a maximum of 20 overs. Hence, in this format of cricket, a match will consist of 2 innings, an innings will consist of 20 overs, an over will consist of 6 balls. We have sourced the the ball by ball data for IPL season 2008 to 2016 from Kaggle. The dataset contains the information on batting team, bowling team, balls of the over, over of the innings, innings of the match and total runs per ball for 577 matches spanning over 9 seasons (2008 to 2016). It also has information on which team won, winning margin and several other useful information.

A hierarchy like table 4 can be construed for this game format, where index here would refer to ball as the data set contains ball-by-ball data.

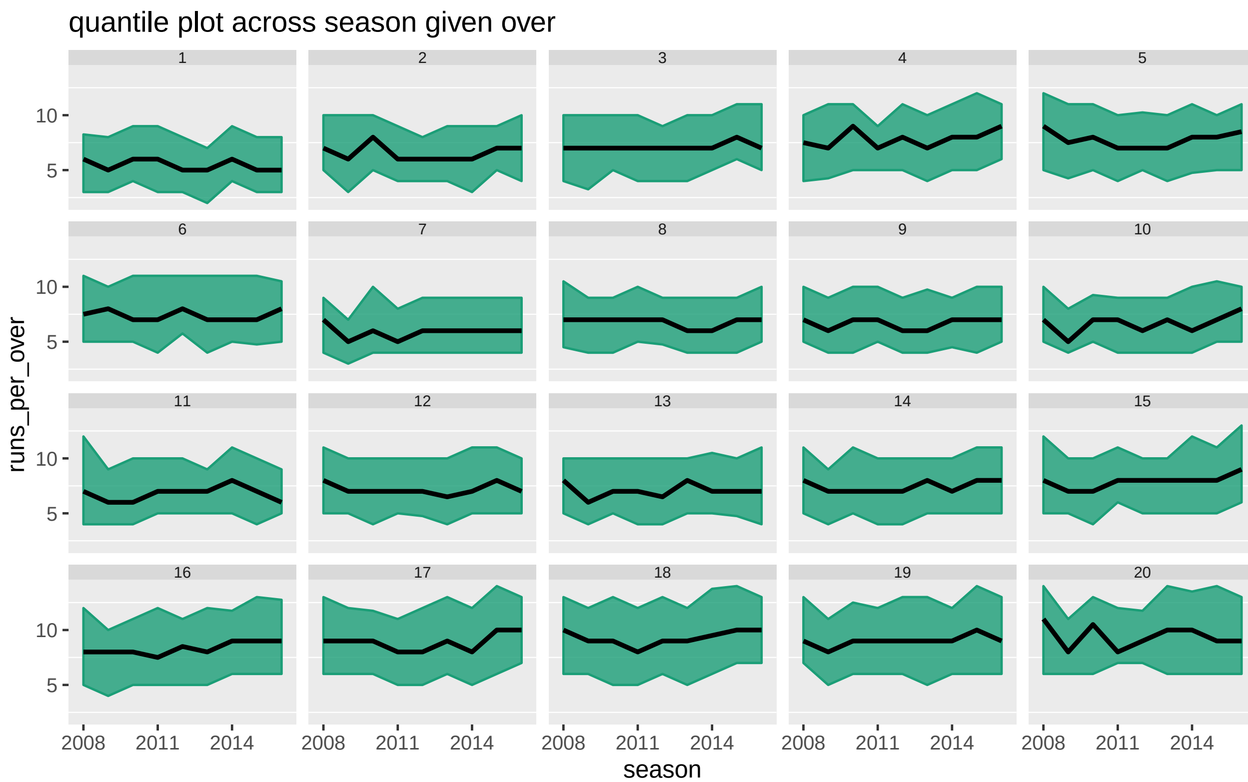


Figure 3: Quantile plot of runs per over across overs of different seasons.

There are many interesting questions that can possibly be answered with such a data set, however, we will explore a few and understand how the proposed approach in the paper can help answer some of the questions.

First, we look at the distribution of runs per over across over of the innings and seasons in 3. The distribution of runs per over has not significantly changed from 2008 to 2016. There is no clear pattern/trend that runs per over is increasing or decreasing across seasons. Hence, we work with subsets of seasons and try to see how the strategies of the winning teams differ across seasons or how in a particular season the strategy was different for the winning team and the ones who did not qualify for the playoffs.

Mumbai Indians(MI) and Chennai Super kings(CSK) are considered one of the best teams in IPL with multiple winning titles and always appearing in final 4 from 2010 to 2015. It would be interesting to see the difference in their strategies throughout all matches in the two seasons. The following two questions might help us partially understand their strategies.

- Q1: How their run rates vary depending on if they bat first or 2nd? Is there a chance that they are more likely to win if they bat first?
- Q2: Which team is more consistent in their approach in terms of run rate across different overs of the innings?

```
#> # A tibble: 2 x 3
#>   inning   `0`   `1`
#>   <fct>  <dbl> <dbl>
#> 1 1        240   460
#> 2 2        229   447
```

```
#> # A tibble: 2 x 3
#>   inning   `0`   `1`
#>   <fct>  <dbl> <dbl>
```

```
#> 1 1        248   440
#> 2 2        237   429
```

```
#>   win_by_runns
#>  Min.   : 2.00
#>  1st Qu.:18.00
#>  Median :39.00
#>  Mean   :39.95
#>  3rd Qu.:50.00
#>  Max.   :87.00
```

```
#>   win_by_runs
#>  Min.   : 9.00
#>  1st Qu.:22.00
#>  Median :24.00
#>  Mean   :28.02
#>  3rd Qu.:33.00
#>  Max.   :60.00
```

Figure 4 shows that for CSK, irrepective of the fact they won or lost, distribution of run rate in case they are batting in the 2nd innings is always skewed on the right side, implying that they always manage to give a good fight to win even if they are losing. Run rate can go up to 6 runs per over depending on the situation. For MI, this is not true. When they won, their run rates in the 2nd innings went up to 6 and led to their win, but stayed within 4 whenever they lost. This implies CSK is more of a fighter than MI when it comes to defending in the 2nd innings. However, we can delve deeper and try to see which overs are the most volatile for CSK and if it is different to that of MI?

In Figure 5, we can observe the letter value plot describing the distribution of run rate per over across over of the innings and innings of the match. It can be observed that for both innings, the run rate for Mumbai Indians rise faster across overs of the innings. There is an upward shift of the distribution in the second part of the innings (after 10th over) and more variability (longer and distinct letter values) as well. CSK has been more uniform in their approach maintaining consistent scores per over. Since CSK's last over in the 2nd innings really get volatile, after a consistent performance in the other overs - this behavior is exhibited as an outlier in Figure 4.

–>

over	1	2
1	55	44
2	55	44
3	55	44
4	55	44
5	55	44
6	55	44
7	55	44
8	55	44
9	55	44
10	55	44
11	55	44
12	55	44
13	55	44
14	55	43
15	55	42
16	55	41
17	55	41
18	55	39

over	1	2
19	55	39
20	55	29

over	1	2
1	39	52
2	39	52
3	39	52
4	39	52
5	39	52
6	39	52
7	39	52
8	39	52
9	39	52
10	39	52
11	38	51
12	38	51
13	37	50
14	36	50
15	36	49
16	36	48
17	36	48
18	36	44
19	36	38
20	36	27

```
#> # A tibble: 13 x 1
#>    batting_team
#>    <chr>
#>  1 Kolkata Knight Riders
#>  2 Royal Challengers Bangalore
#>  3 Chennai Super Kings
#>  4 Kings XI Punjab
#>  5 Delhi Daredevils
#>  6 Rajasthan Royals
#>  7 Mumbai Indians
#>  8 Deccan Chargers
#>  9 Kochi Tuskers Kerala
#> 10 Pune Warriors
#> 11 Sunrisers Hyderabad
#> 12 Rising Pune Supergiants
#> 13 Gujarat Lions
```

Q3: Is run rate set to reduce in overs where fielding is good?

For establishing that the fielder fielded well in a particular over, we can see how many catches and run outs were made in that particular over. If a batsman is bowled out, it does not necessarily signify good fielding. So we only include catches and run out as a measure of fielding. Difference in run rates should be negative if fielding is good. Let us see if this fact is true.Thus, Figure 6 shows the difference between run rate between two subsequent overs are negative when good fielding leads to one or two dismissals in an over.

```
#> # A tibble: 4 x 21
#>   fielding_wckts   `1`   `2`   `3`   `4`   `5`   `6`   `7`   `8`   `9`
```

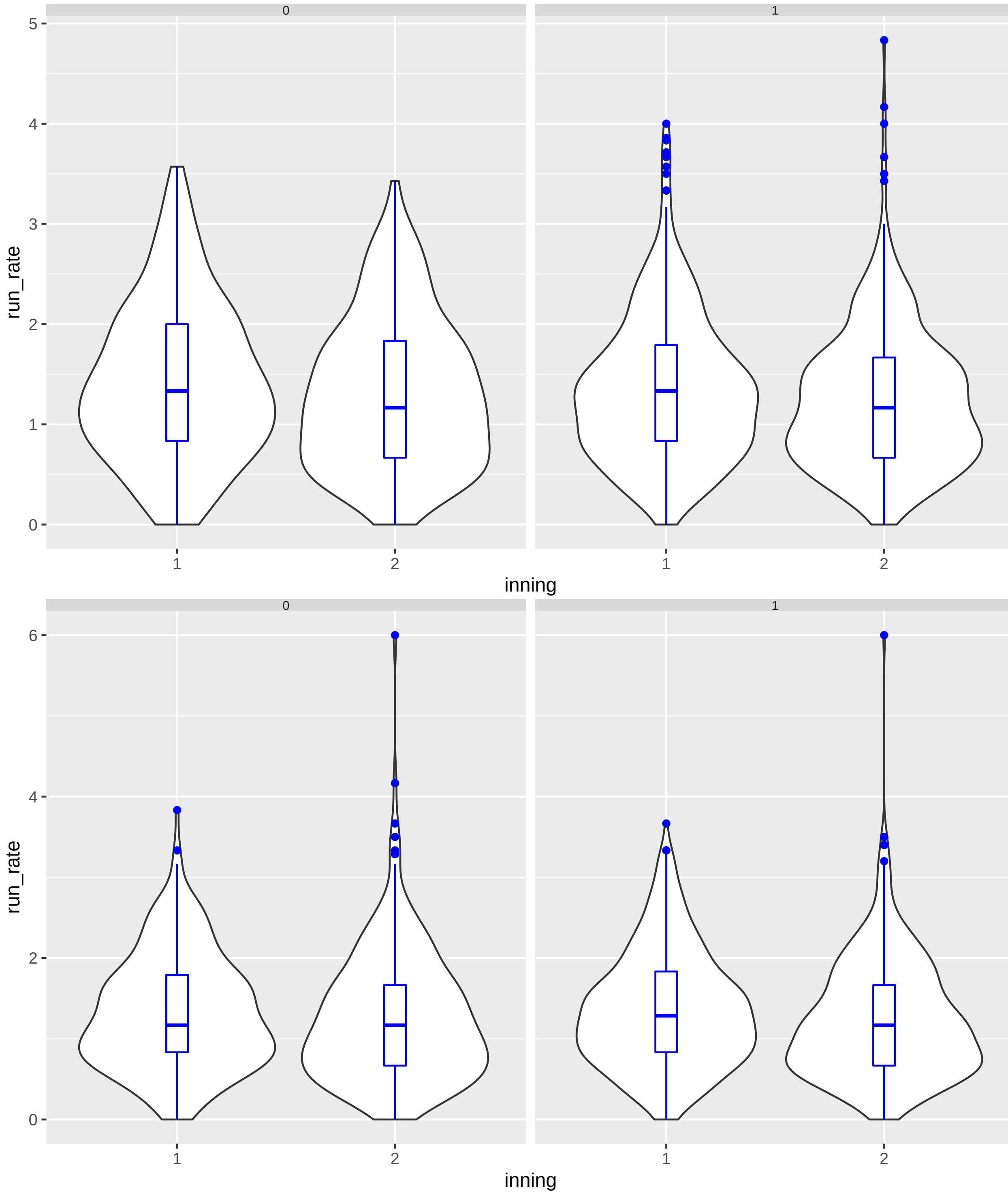


Figure 4: Violin and box plots of run rate across innings of the match and matches won or lost for Mumbai Indians(top) and Chennai Super Kings(bottom).The violins for CSK has long right tails in 2nd innings for cases when they lost or won. MI has long right tail only when they won the match.

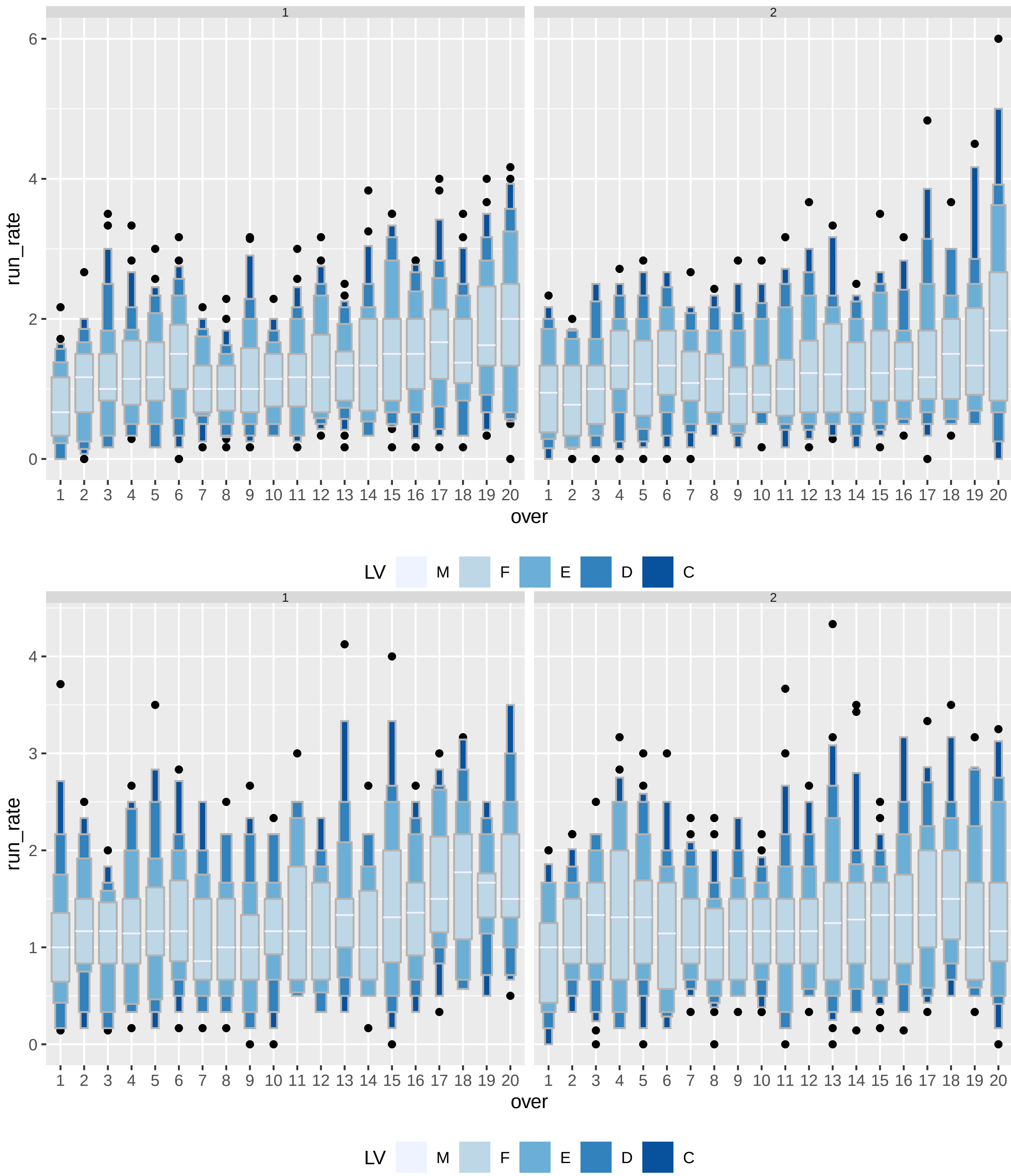


Figure 5: Letter value plot of run rate across overs of an inning and innings of the match for (top) Mumbai Indians and (bottom) Chennai Super Kings.

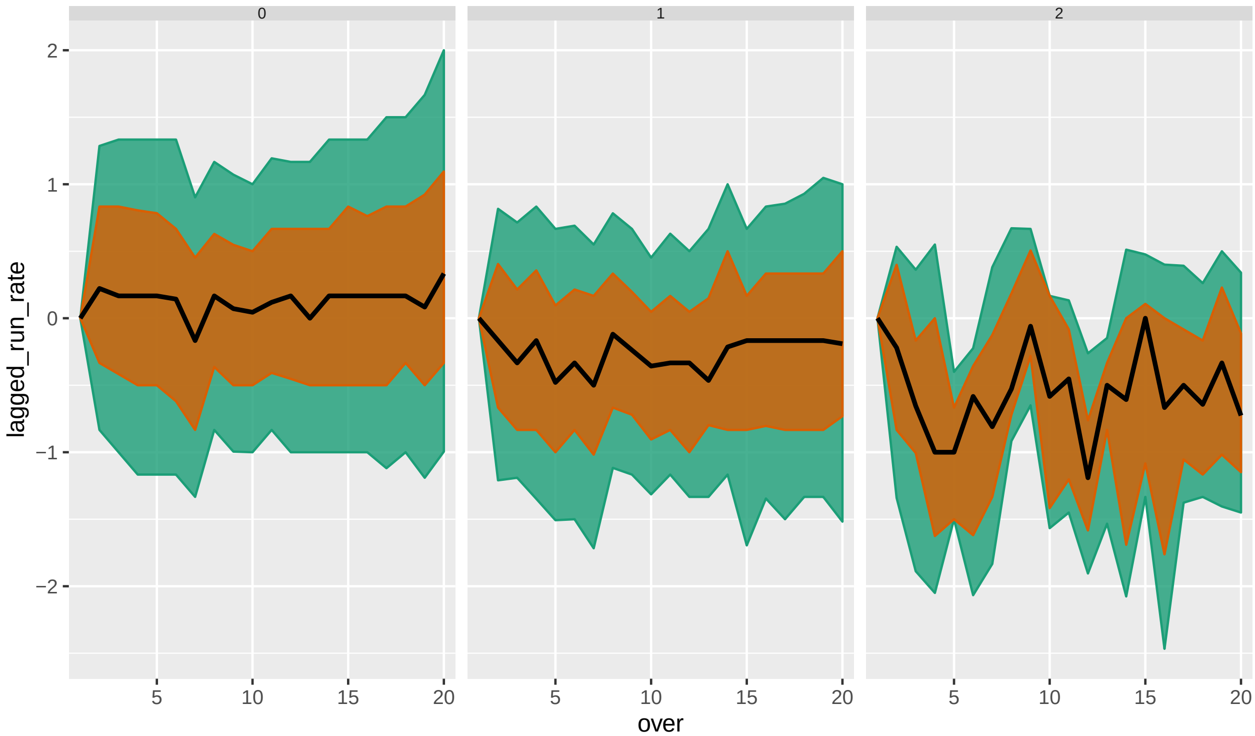


Figure 6: Distribution of lagged run rate across overs of the innings and dismissals by catch and catch and bowled.

```
#>   <fct>          <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1 0               1039   997   972   978   962   957  1002  1012   993
#> 2 1                120   145   171   165   180   181   138   124   140
#> 3 2                  5    10     8     8     9    10     6    10     8
#> 4 3                  0     0     0     0     0     0     0     0     0
#> # ... with 11 more variables: `10` <dbl>, `11` <dbl>, `12` <dbl>,
#> #   `13` <dbl>, `14` <dbl>, `15` <dbl>, `16` <dbl>, `17` <dbl>,
#> #   `18` <dbl>, `19` <dbl>, `20` <dbl>
```

Q4: Is run rate set to reduce in overs where number of dot balls are more/number of wickets are more?

A dot ball is a delivery bowled without any runs scored off it. The number of dot balls is reflective of the quality of bowling in the game. Run rate of an over should ideally decrease if the number of dot balls increase or the batsman can likely to go for big shots and get dismissed. Figure 7 shows that for any over, increase in dot balls lead to decrease in run rate.

“‘

7 Discussion

Acknowledgements

8 Bibliography

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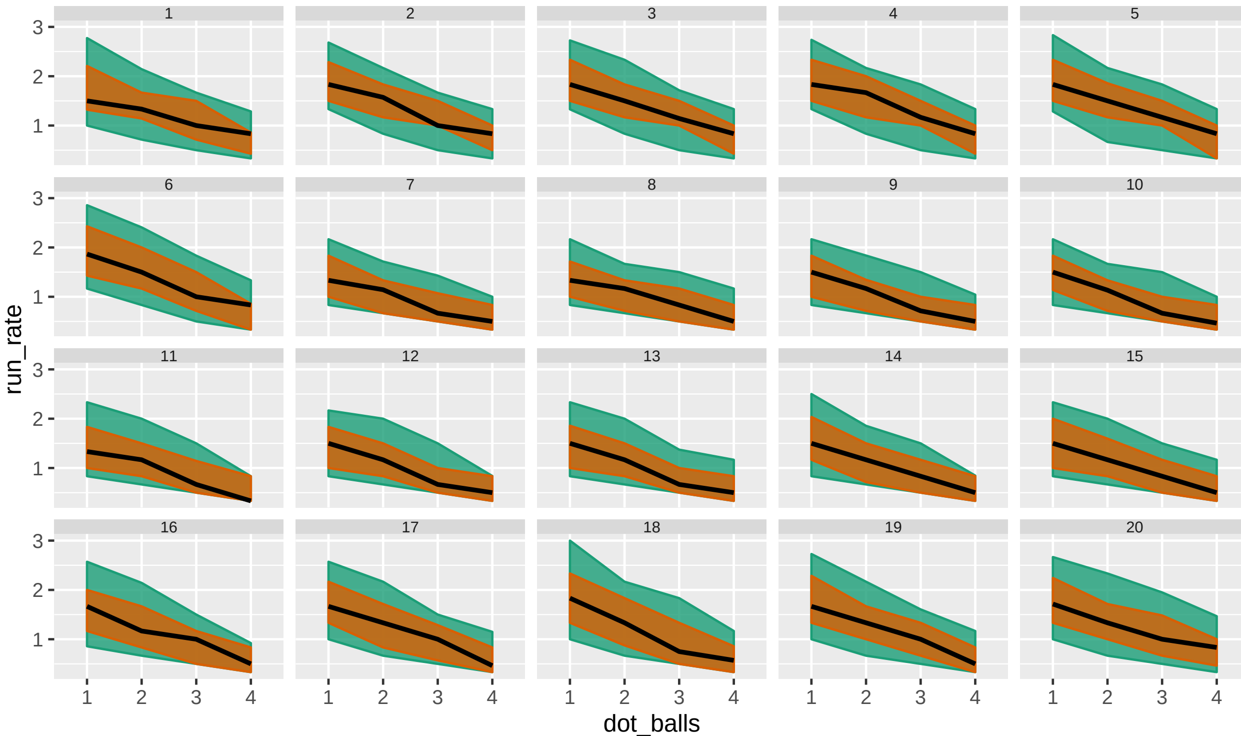

Figure 7: Distribution of run rate across overs of the innings and number of dot balls in an over.

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