# Review on "Visualizing probability distributions across bivariate cyclic temporal granularities"

This research was motivated by the desire to understand the Australia smart meters data, which was collected half-hourly for two years at the household level. The paper provides a thorough review on time deconstruction for linear time granularities and develops a formal characterization of cyclic time granularities with cyclic calendar algebra for single- and multiple-order-up time granularities. The paper also proposed an approach to check the feasibility of creating plots for any two cyclic granularities. An R package, `gravitas`, was developed for systematically exploring large quantities of temporal data under the proposed workflow.

There are a few questions and comments to the authors, especially on the characterization and algebra part.

- Although the first several definitions were inherited from Bettini & De Sibi (2000), some terms used in the paper are not clearly defined and therefore could lead to confusion. Below are a few examples.
  - What is the definition of "period"? Is the period defined on the grouping or on the granularity? Is the period denoted by $P$ or $(R, P)$ in Definition 6? From Pages 6 Line 55 to Page 7 Line 7, period was referred to an integer (7), and two time granularities (one year and 400 years). Are they consistent?
  - What is the definition of "grouping"? Though "*$G$ groups into $H$*" was defined in Definition 5, there is not a notation of grouping $(G, H)$ until we see "*grouping (day, month)*" at Page 7 Line 7. Please rephrase the sentence "*each month is a grouping of the same number of days over years*" on Page 7 Line 5.
  - What is the definition of "aperiodic linear granularities"? On Page 11, Definition 10, $M_i$ are aperiodic linear granularities. If $M_i$ are aperiodic, then is $M$ also an aperiodic linear granularity? What is the relationship between aperiodic linear granularity and aperiodic cyclic granularity?
  - $w$ in $T_w$is not well defined for circular granularities in Section 3.2 & 3.5. The definition of $T_w$ uses $T_w$ itself in the formula making it hard to understand. Relevantly, what is the interpretation of $k$ where $w = 0, 1, ..., k-1$? In my opinion, the number of cycles should be defined for cyclic granularities, for example, $w_{B,G}(z) = \lfloor z/P(B,G) \rfloor$, which represent the cycle that the granule is in.

- Places that need clarification.
  - Page 7, Definition 6. Is "*the number of granules of H*" finite or infinite? If each granule $G(i)$ has the index $i \in \mathbb{Z}_{\geq 0}$, then the phrase is similar as "the number of non-negative integers".
  - Page 7, Lines 19-20. It is claimed that quasi-periodic relationship has a **finite** number of periods where granularities behave in an anomalous way. What does the "finite number of periods" mean and why is it finite? Does it mean a finite number of granularity groupings? Does it mean a finite number of irregular cycles? By Definition 4 of Bettini & De Sibi (2000), $E_1, \ldots, E_z$ are the granularity exceptions, but the number of granularity exceptions does not have to be finite.
  - Page 8, Definition 8, what is the "*period*" $P(B, G)$? Is it same as $P$ in Definition 6? Is it a mapping or function?
  - Page 10, Line 14, Q4 week-of-month is probably not a good example for quasi-circular granularities. By Definition 4, week cannot be grouped into month, because there could exist $week(i) \nsubseteq month(j)$ when the $i$th week crosses two months. Similarly, week is not finer than month by Definition 5. Since periodicity is defined on top of grouping, and circular is defined on top of periodicity, the scenario of Q4 cannot be defined. On the other hand, the period length for week-of-month is not an integer, but it is an integer for Q1 - Q3.
  - Page 10, Definition 9, is $R'$ similar as the period $P$ of the grouping $(B, G')$? Is $R'$ a constant or variable with irregular mapping?
  - Page 11, Definition 10, what is the interpretation of $n$ where $i = 1, ..., n$?
  - Page 14, Lines 26-27. Is z the original index for the bottom granularity? If yes, why $C_{\text{katun, baktun}}(z) = \lfloor 18 \times 20 \times z/20 \rfloor \mod 20$? Shouldn't it be $z \mod 20$? If not, then what is $z$? By Definition 8, $z \in \mathbb{Z}$ is the original time index based on the bottom granularity, but here $z$ seems to be defined as the time index based on granularity katun.
  - Page 11, lines 22-33. The explanation of Figure 3 is confusing. If the *week-of-the-semester week type* is a quasi-circular granularity, then the *day-of-the-semester week* type can be also quasi-circular granularity, as all days are nested in weeks. Actually the notation should not be "*week-of-the-semester week type*", but "semester week-of-the-semester week type". The relationship between $G$ and $H$ should be clarified. It will be clearer if the authors can make a granularity "semester day" ($U$) from $B$. Therefore we have the following granularities.
    - $Q_{H,M}$: quasi-circular granularity, "semester week"-of-the-"semester week type".
    - $Q_{U,M}$: quasi-circular granularity, "semester day"-of-the-"semester week type".

* $A_{B,M}$: aperiodic cyclic granularity, day-of-the-"semester week type".
* $A_{G,M}$: aperiodic cyclic granularity, week-of-the-"semester week type".

And the quasi-circular granularity $Q_{U,M}$ looks like

$$Q_{U,M} = |0, 0, ..., 0|0, 1, ..., 6|0, 1, ..., 41|0, 1, ..., 6|0, 1, ..., 48|0, 1, ..., 6|0, 1, ..., 15|.$$

- Page 20, Line 47, "*for weekdays the interquartile range of consumption reduces over the year, whereas this pattern is not true for weekends*" is hard to be justified. By Figure 4(c), the IQR is given by the height of blue (LV=F) boxes. They are the largest in Quarter 2 and smallest in Quarter 4 for both weekdays and weekends.

- Other questions of interest.
  - Page 6, Definition 5, should $S$ be a contiguous set of integers or not necessary?
  - Page 6, what is the difference between "*finer than*" and "*group into*" in Definitions 4 and 5? On Page 7, Line 34, they are used as alternative choices. Can you give an example that $G \preceq H$ does not imply $G \trianglelefteq H$, or vice versa? "*finer than*" is not used a lot in this paper. If the two concepts are similar, why not pick one of them?
  - Page 16, Line 13, *multiple order-up quasi-circular granularities*. The equations for this operation are not trivial to be ignored. Should be added if possible.
  - Can the time granularity concept be extended to continuous time domain?

- Other minor issues.
  - Page 9, Line 20, $C_{(G,H)}$ or $C_{G,H}$? Couldn't find parenthesis in the subscript of $C$ in other places of the paper.
  - Page 14, Lines 41-51, the subscripts for $C$ are sometimes with $G$, sometime not with $G$. Please make them consistent.
  - Page 23, Line 31, should be "Plot a shows granularity 2 faceted by granularity 1".