Supplementary materials for the main submission entitled - Detecting distributional differences between temporal granularities for exploratory time series analysis

Contents

1 Recalling notations

Consider two cyclic granularities A and B, such that $A = \{a_j : j = 1, 2, \dots, J\}$ and $B = \{b_k : k = 1, 2, \dots, K\}$ with A placed across x-axis and B across facets. Let $v = \{v_t : t = 0, 1, 2, \dots, T-1\}$ be a continuous measured variable observed across T time points. The pairwise distances between pairs $(a_j b_k, a'_j b'_k)$ could be within-facets or within-facets a seen in Figure 4 of the main paper. Let the four elementary designs be D_{null} where there is no pairwise difference in distribution of v across A or B, D_{var_f} denotes the set of designs where there is difference in distribution of v for B and not for A. Similarly, D_{var_x} denotes the set of designs where difference is observed only across A. Finally, $D_{var_{all}}$ denotes those designs for which difference is observed across both A and B. The following method is deployed for generating different distributions across different combinations for non-null designs - suppose the distribution of the combination of first levels of x and facet category is $N(\mu, \sigma)$ and μ_{jk} denotes the mean of the combination $(a_j b_k)$, then $\mu_{j.} = \mu + j\omega$ (for design D_{var_x}) and $\mu_{.k} = \mu + k\omega$ (for design D_{var_f}). Table 1 shows an example of how initial distributions are assigned in a panel with $nfacet = 3$ and $nx = 2$ for different designs using $\omega = 1$.

2 Behavior of raw weighted distance measure

2.1 Tuning parameter

How does the tuning parameter affects the value of *wpd* under different designs? The tuning parameter is used to put relative weight-age to the difference in distributions within and between facets. So it is interesting to see how the value of *wpd* changes for these two designs. But the tuning parameter might have a different impact depending on the value of ω and different levels of x and facets. Hence, a simulation study is conducted to see the impact of ω, nx, $nfacet$ and λ together on the values of nx and $nfacet$.

Table 1: Simulation setup for a panel with 3 facet levels and 2 x-axis levels for different designs starting from an initial distribution N(0, 1) for the combination (a_1, b_1).

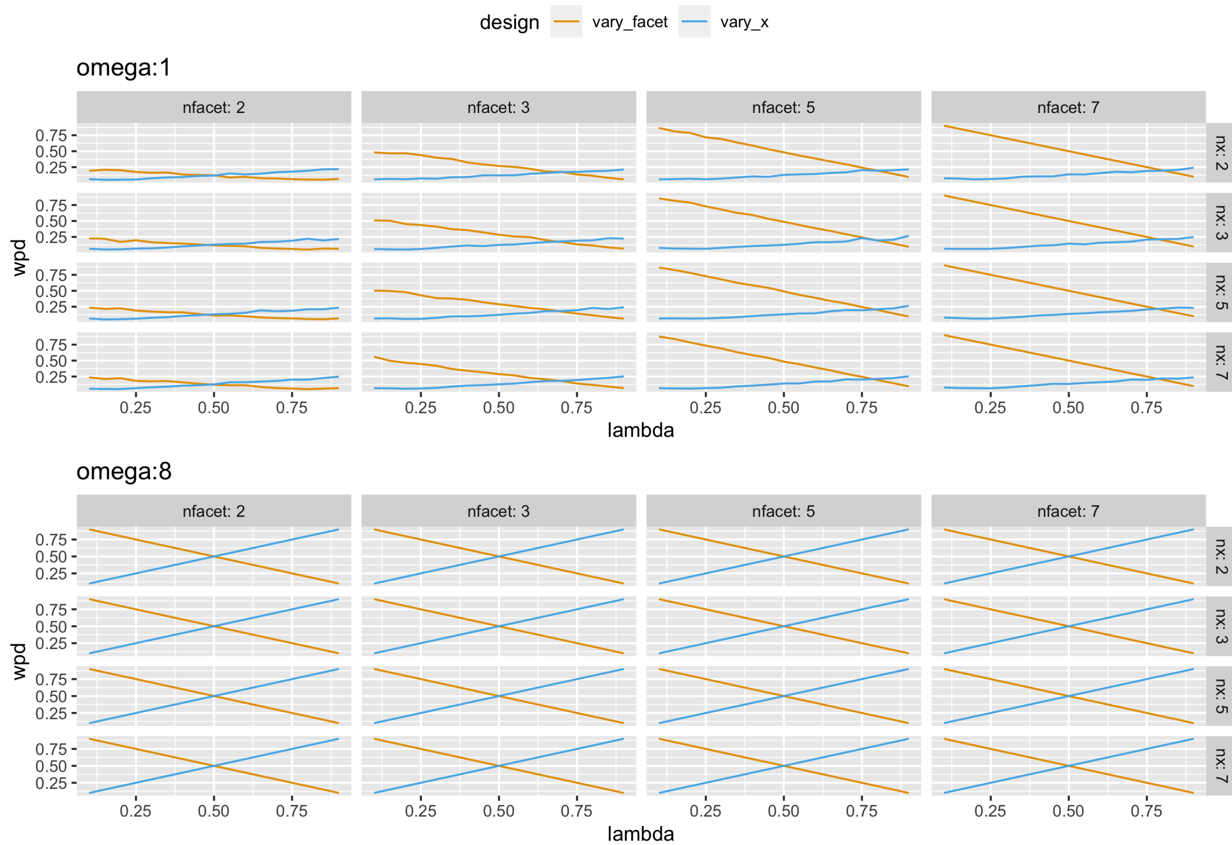
x	facet	D_{var_f}	D_{var_x}	$D_{var_{all}}$
a_1	b_1	N(0, 1)	N(0, 1)	N(0, 1)
a_2	b_1	N(0, 1)	N(1, 1)	N(1, 1)
a_1	b_2	N(1, 1)	N(0, 1)	N(2, 1)
a_2	b_2	N(1, 1)	N(1, 1)	N(3, 1)
a_1	b_3	N(2, 1)	N(0, 1)	N(4, 1)
a_2	b_3	N(2, 1)	N(1, 1)	N(5, 1)

2.1.1 Simulation design

Observations are generated from Normal(0,1) distribution for each combination of nx and $nfacet$ from the following sets: $nx = nfacet = \{2, 3, 5, 7, 14, 20\}$. Let $\omega = \{1, 2, \ldots, 10\}$ denotes the variable denoting the increment in mean of the distribution. The values of lambda ranges from $0.1, 0.2, \ldots, 0.9$. Two designs are considered D_{var_x} and D_{var_f} and the values of *wpd* is being computed for all these different values of the considered variable.

2.1.2 Results

Figure **??** shows how the value of *wpd* changes for $\lambda = 0.1, 0.2, \ldots, 0.9$ for the two different designs D_{var_x} and D_{var_f} for two values of increment in mean $\omega = 1, 8$. For a lower value of ω, the two designs intersect at $\lambda > 0.7$ and for a higher ω, the two designs intersect at $\lambda = 0.5$. The value of *wpd* increases with λ for D_{var_x} and decreases with increasing λ for D_{var_f}. Figure 1 shows the value of λ for which the two designs intersect across different values of ω. It can be observed that as the value of ω ($\omega > 4$) increase, the value of λ at which the two designs intersect converge is $\lambda = 0.5$.



2.2 Underlying distributions

Since the measure *wpd* is essentially set up to detect "differences" in distributions irrespective of underlying distribution, it would be ideal if it has minimal dependency on the type, location and scale of the initial distribution. To that end, some data pre-processing through the Normal Score Transform (NQT) has been applied in order to make most asymmetrical distributed measured variables more normal-like. Figure 2 shows ridge plots of raw *wpd* for a Gamma(0.5,1), Gamma(2,1) before NQT. It is observed that for the underlying distribution Gamma(2,1), location and scale of the distribution of wpd changes from top-left panel to bottom-right panel. Moreover, the location and scale of the distribution of *wpd* for different underlying distribution Gamma(0.5,1), Gamma(2,1). Figure 3 shows the the distributions of *wpd* under same underlying distributions but after performing NQT. It is observed that within each panel, the distributions of the *wpd* looks same, however, the distributions change from extreme top-left panel to bottom-right panels. Similar observations could be made in Figure **??** for different underlying normal distributions N(0,1), N(5,1) and N(0,5). This implies, NQT has atleast been able to bridge the gap in distribution of *wpd* for different non-normal underlying distributions.

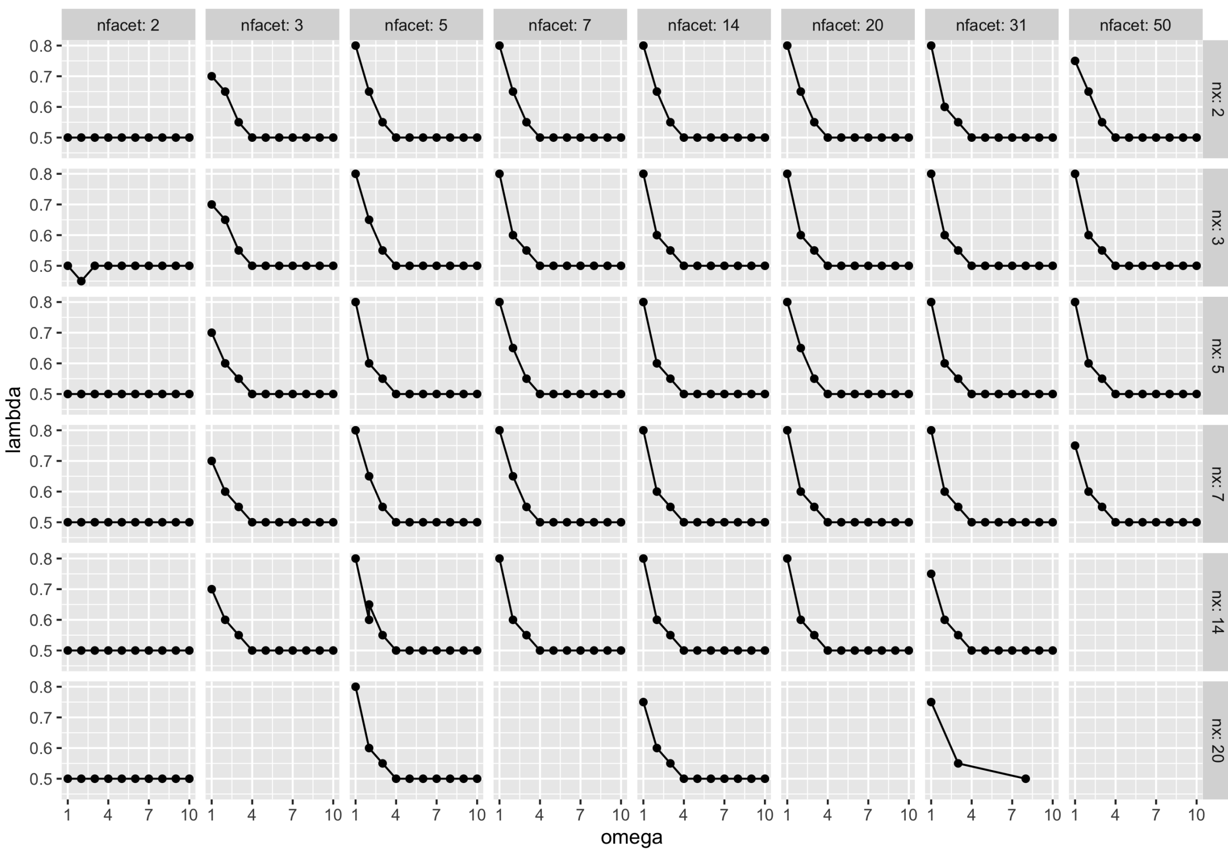


Figure 1: For most panels it is observed that the most common value of the tuning paramter for which the designs interact is 0.5, which implies any value greater than 0.5 could be chosen to up-weigh the within-facet distances and down-weigh the between-facet distances for most sitatuions.

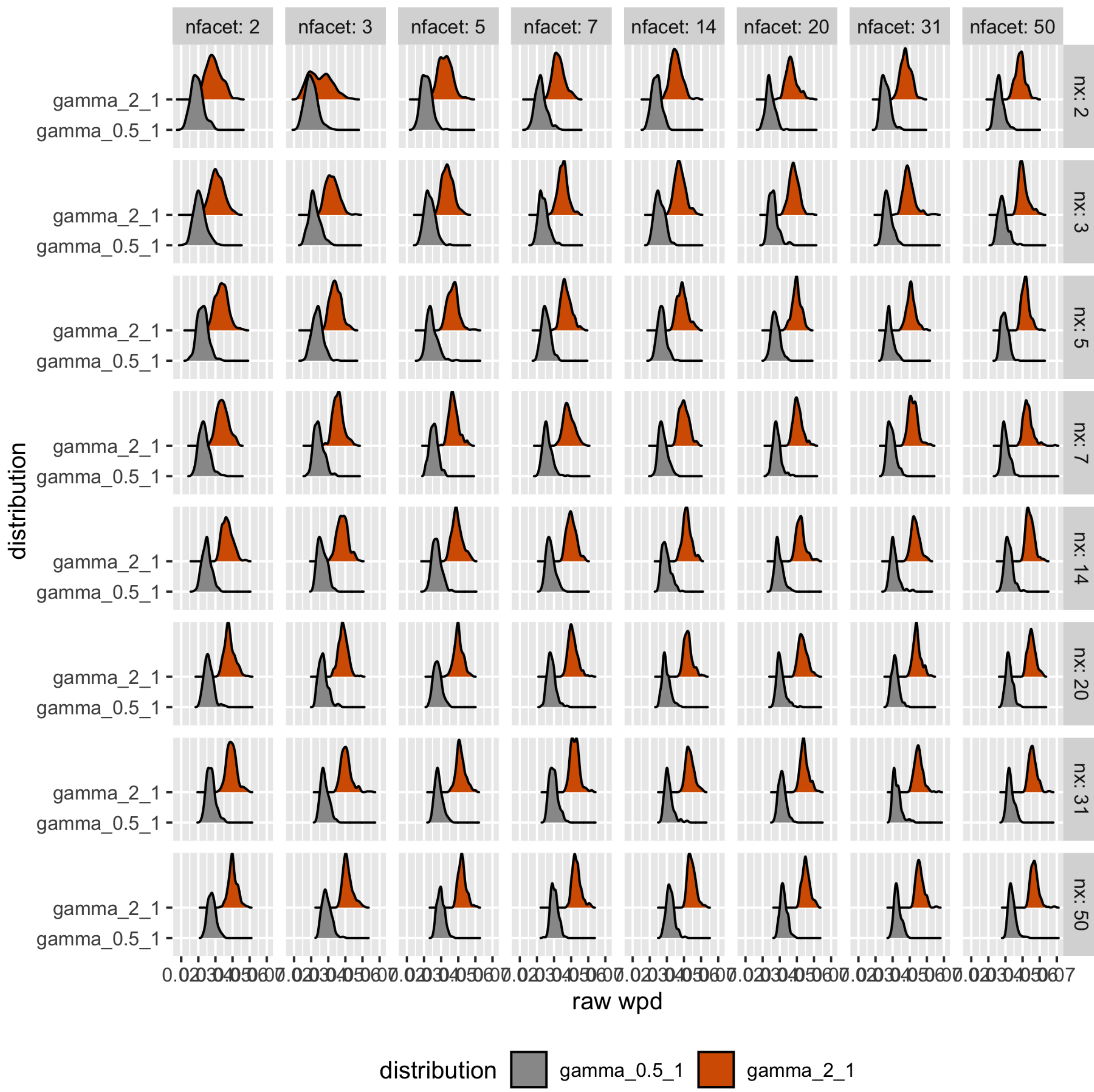


Figure 2: Ridge plots of raw wpd is shown for Gamma(0.5,1), Gamma(2,1) distribution without NQT. The densities change across different facet and x levels and also looks different for the two distributions, which implies wpd value is affected by the change in the shape paramter of the gamma distribution.

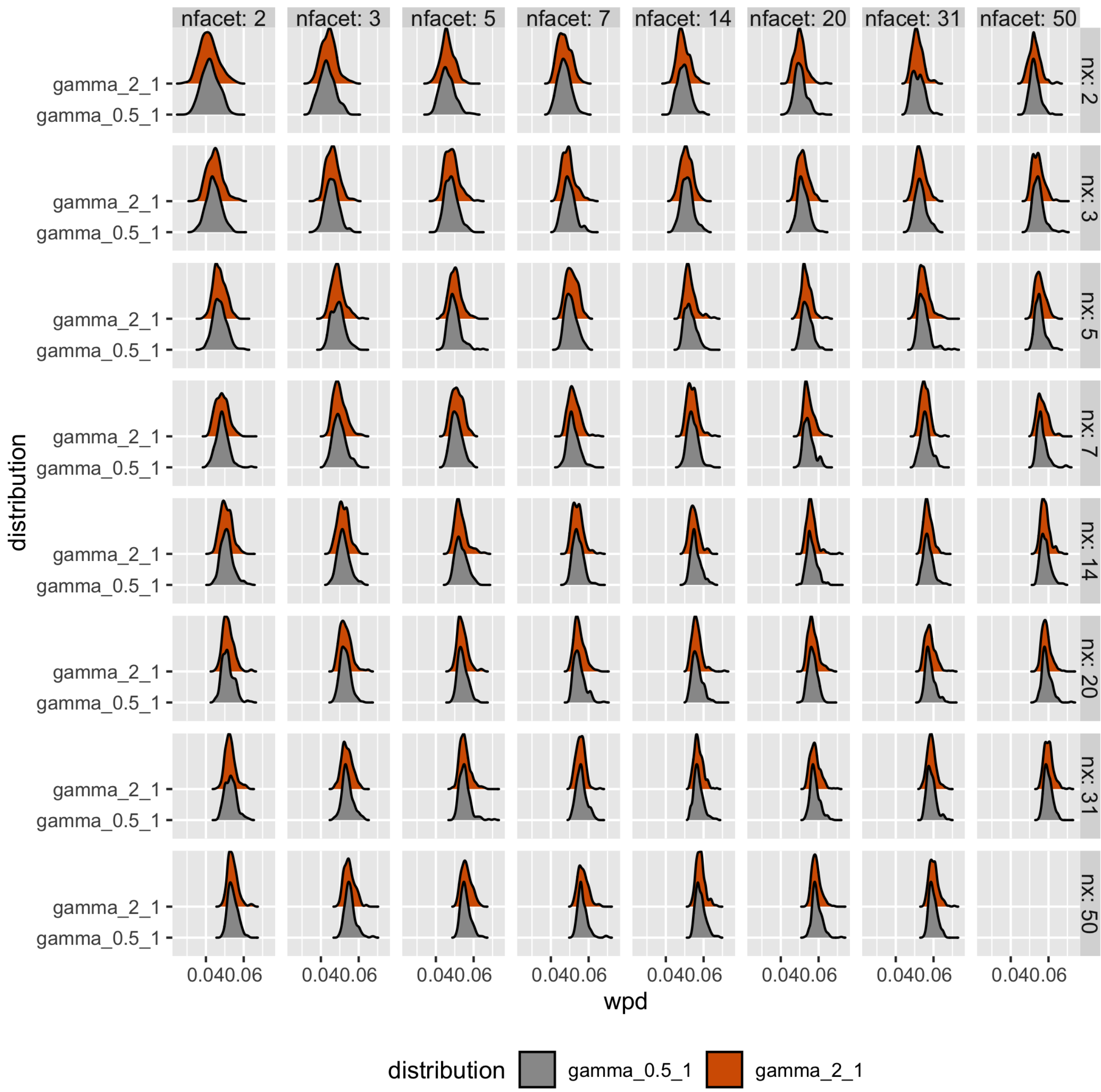


Figure 3: Ridge plots of raw wpd is shown for Gamma(0.5,1), Gamma(2,1) distribution. The densities change across different facet and x levels but look same for the two distributions, which implies wpd value is unaffected by the change in the shape paramter of the gamma distribution

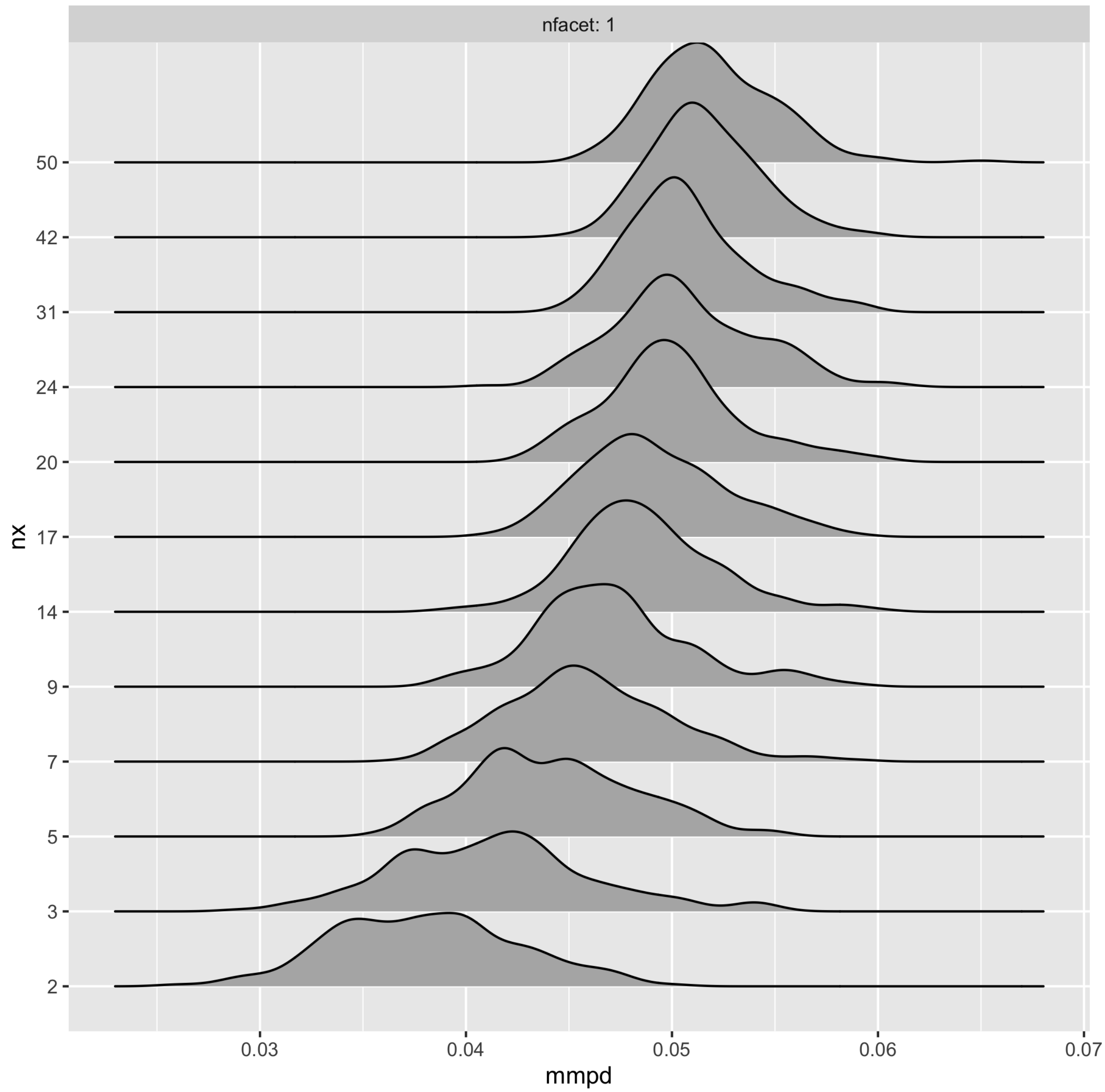
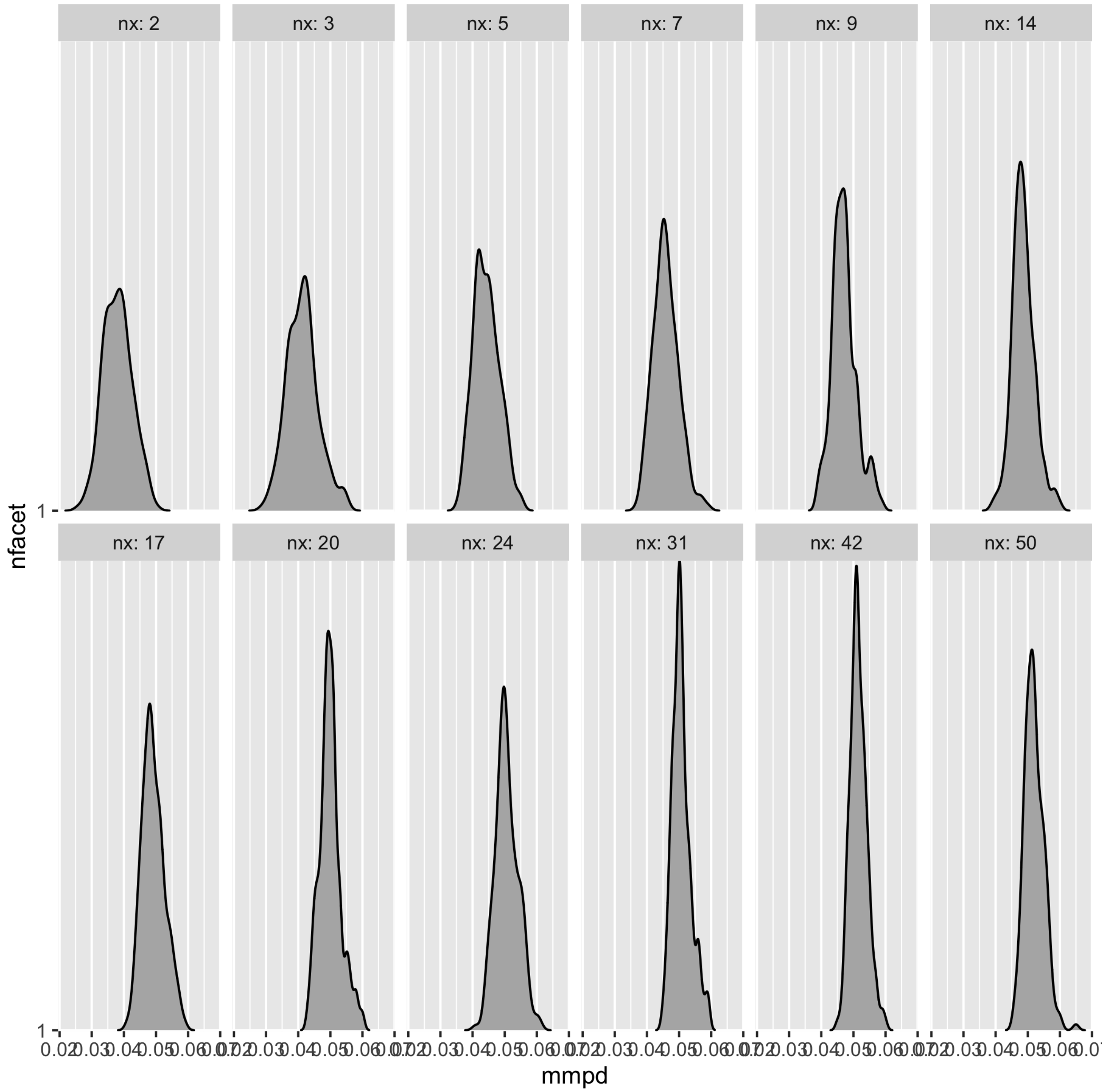
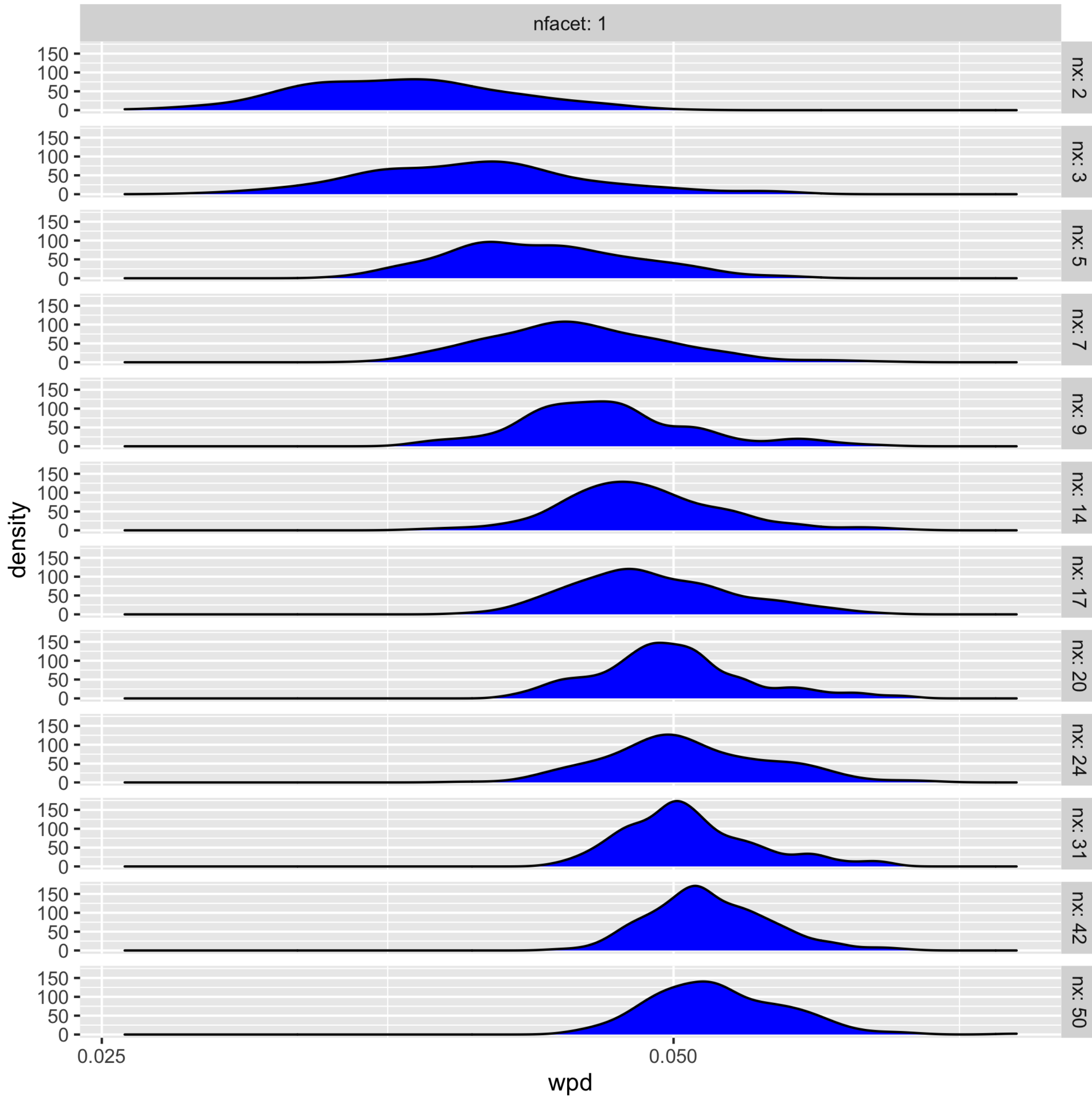


Figure 4: Ridge plots of raw wpd is shown for N(0,5) distribution. For each panel, it could be seen that the location shifts to the right for increasing x levels. Across each panel, the scale of the distribution seems to change for low/moderately lower values and higher values of nfacets and left tails are longer for lower facet levels.

2.3 Case: $\mathbf{m = 1}$




nfacet: 1
density
nx: 2
nx: 3
nx: 5
nx: 7
nx: 9
nx: 14
nx: 17
nx: 20
nx: 24
nx: 31
nx: 42
nx: 50
0.025
0.050
wpd

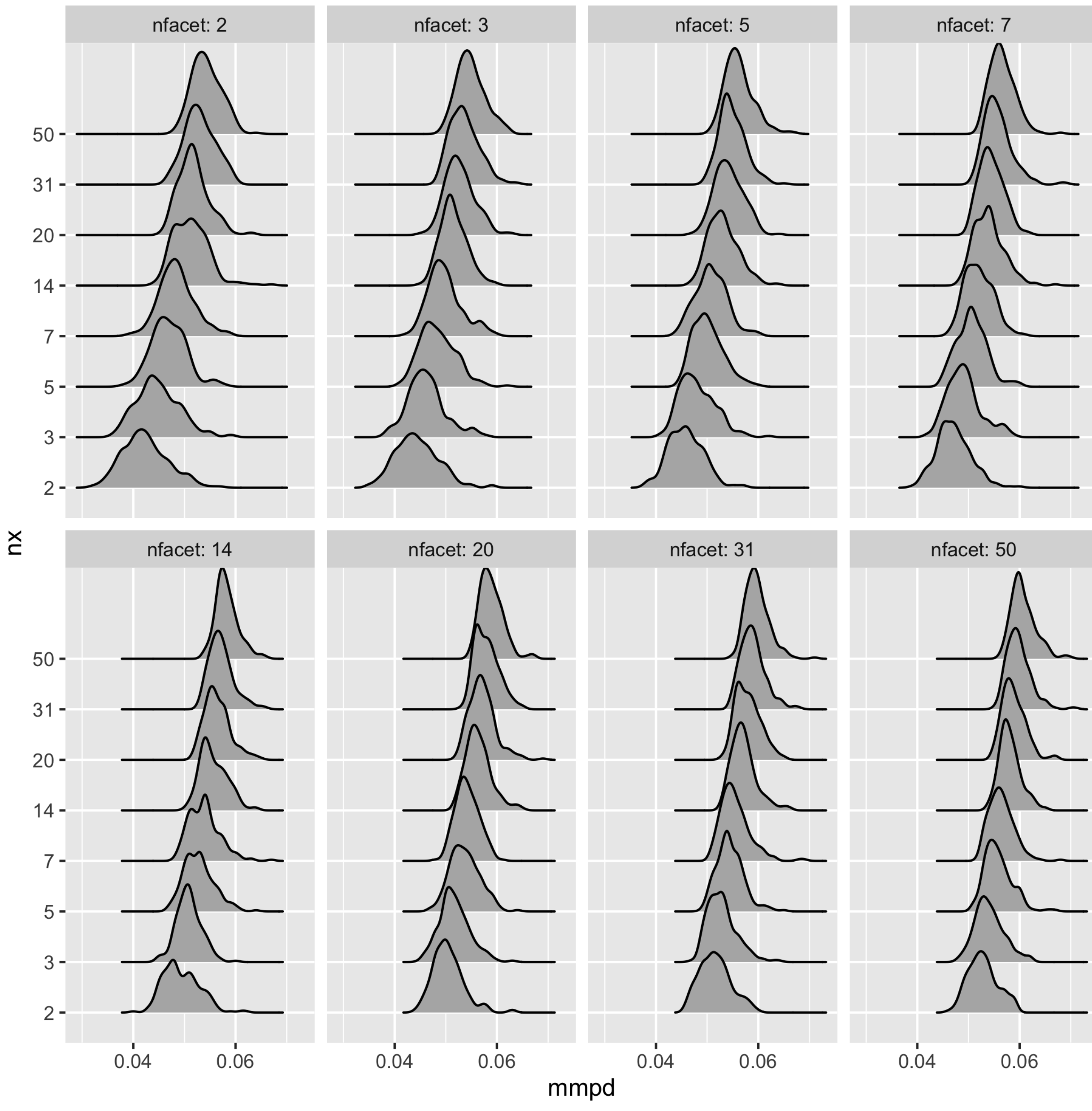
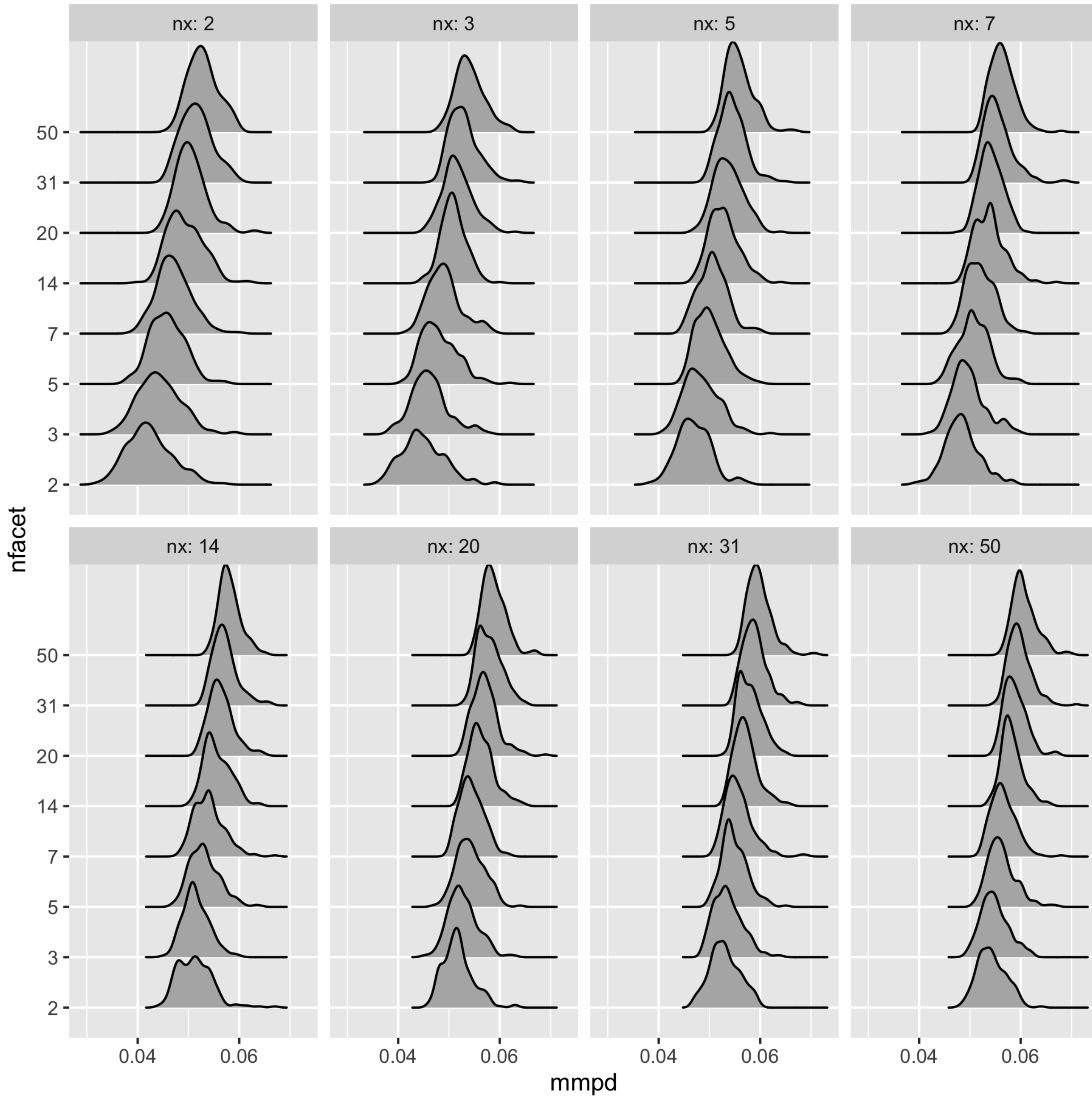
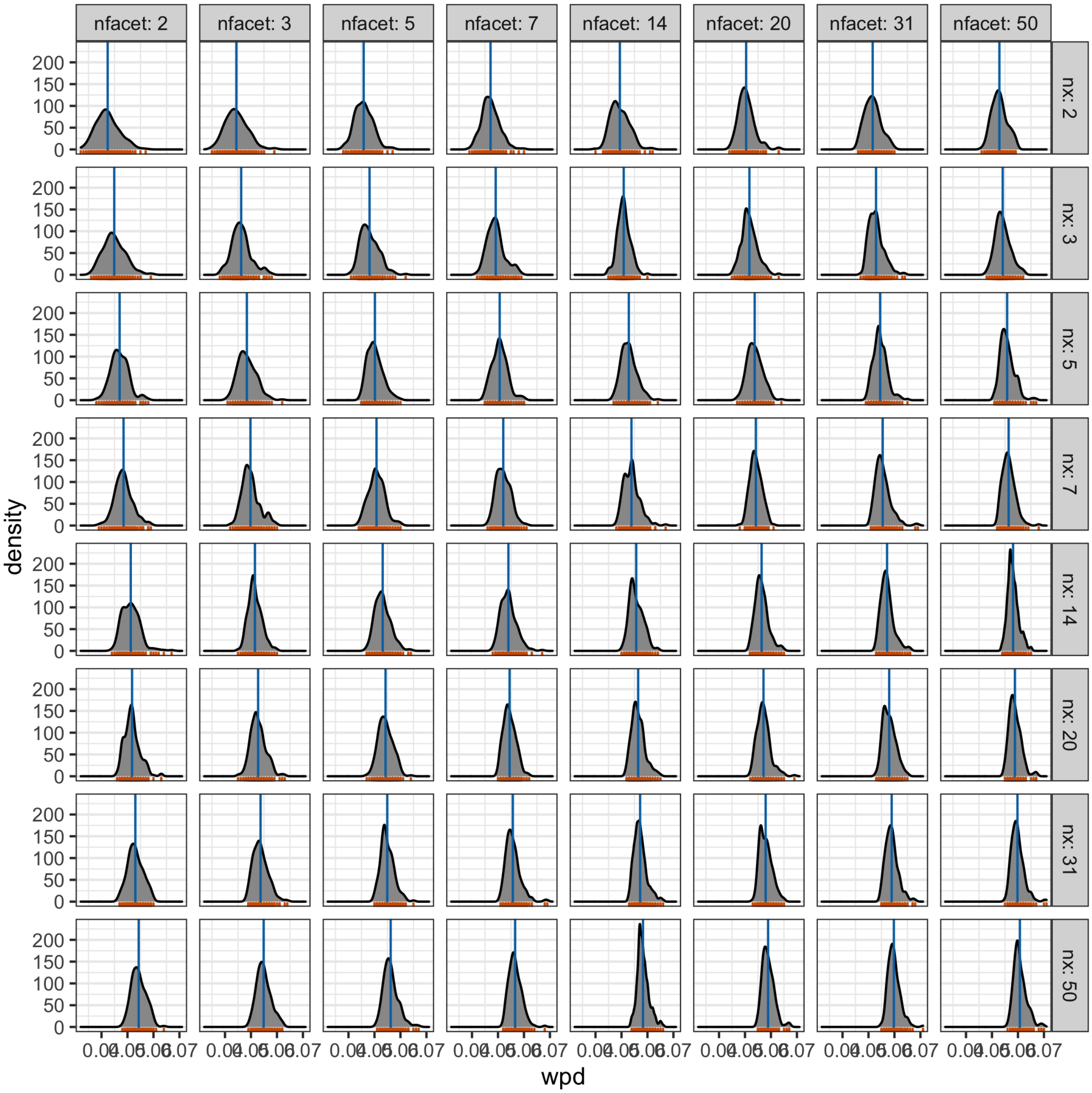


Figure 5: Ridge plots of raw wpd is shown for N(0,5) distribution. For each panel, it could be seen that the location shifts to the right for increasing x levels. Across each panel, the scale of the distribution seems to change for low/moderately lower values and higher values of nfacets and left tails are longer for lower facet levels.

2.4 Case: $\mathbf{m = 2}$





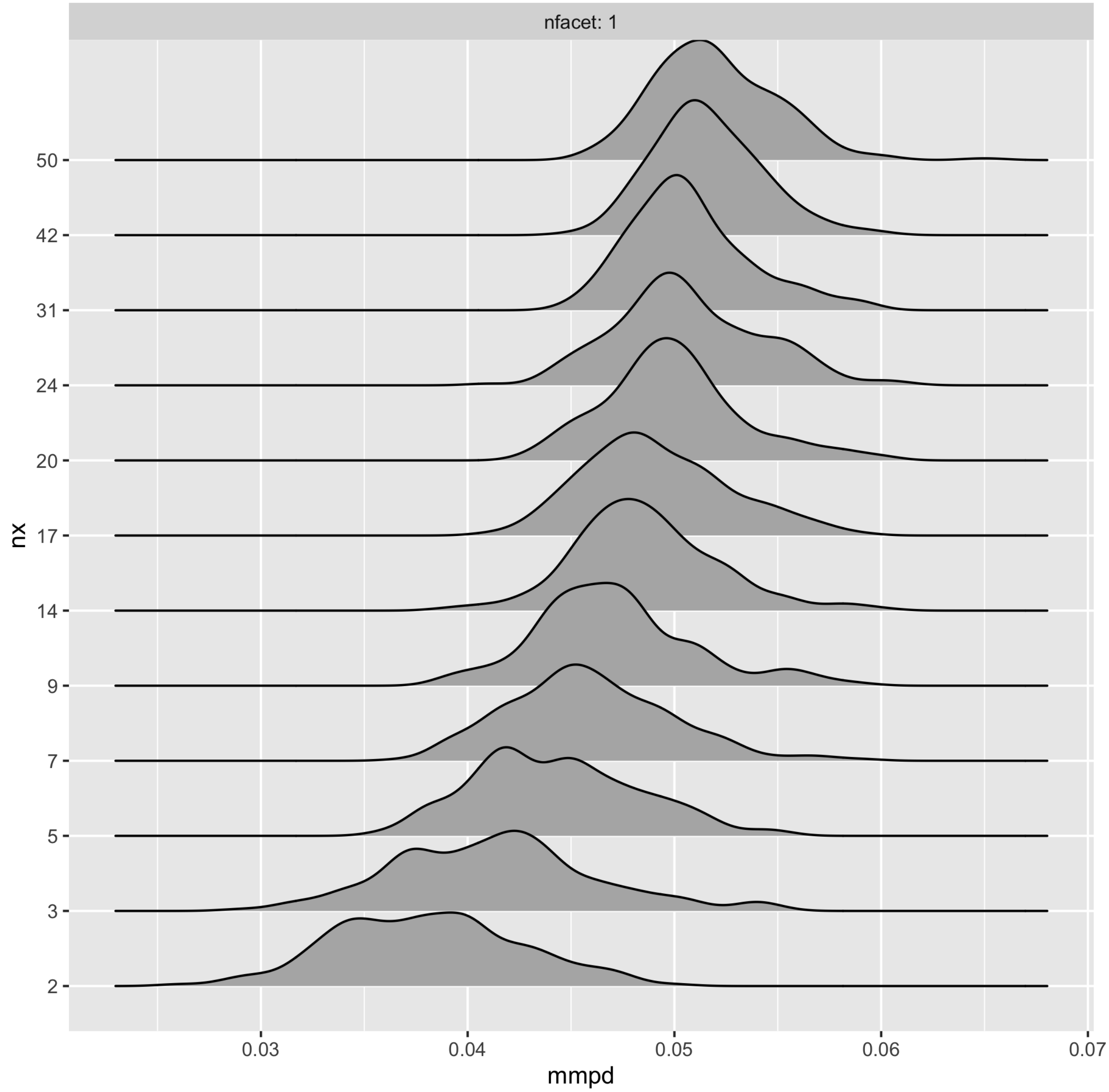
3 Adjusted wpd

3.1 Case: m = 1

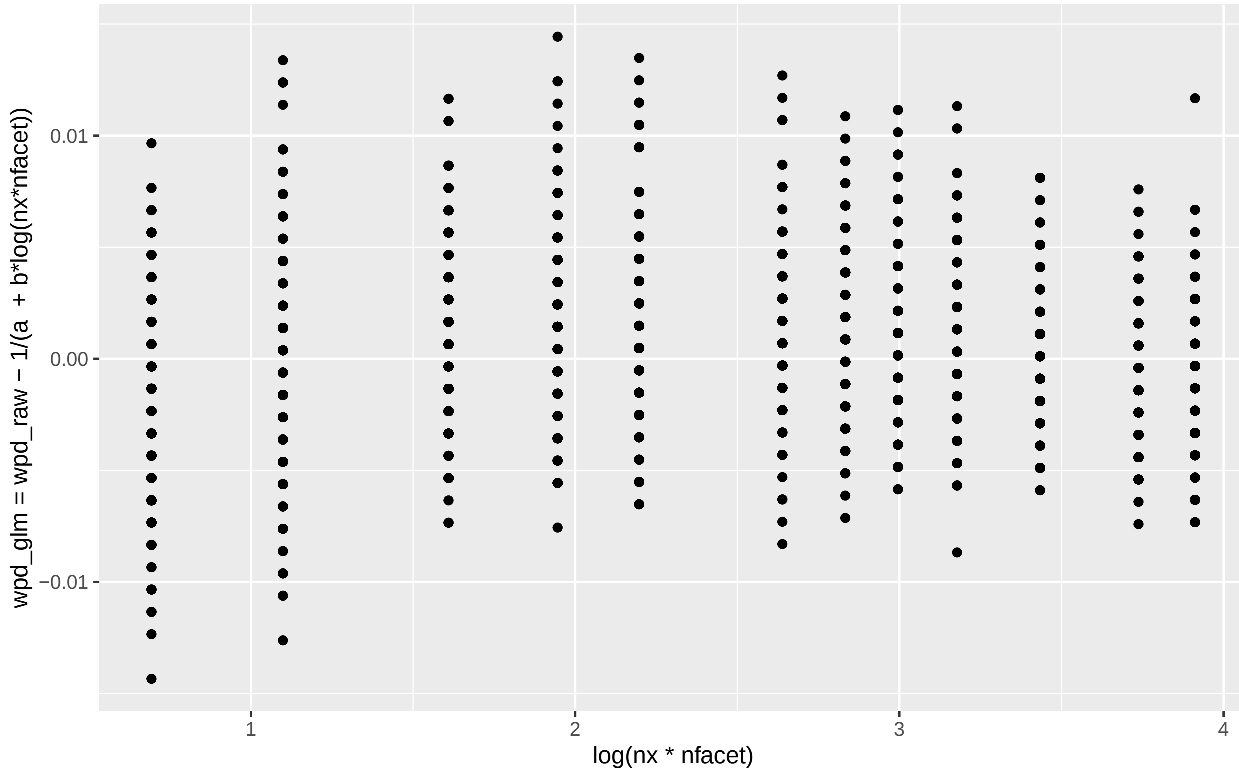
3.1.1 Simulation design

Observations are generated from a N(0,1) distribution for each $nx = \{2, 3, 5, 7, 9, 14, 17, 20, 24, 31, 42, 50\}$ to cover a wide range of levels from very low to moderately high. $ntimes = 500$ observations are drawn for each combination of the categories, that is, for a panel with $nx = 3$, 500 observations are simulated for each of the categories. This design corresponds to D_{null} as each combination of categories in a panel are drawn from the same distribution. Furthermore, the data is simulated for each of the categories $nsim = 200$ times, so that the distribution of wpd under D_{null} could be observed. The values of wpd is obtained for each of the panels. $wpd_{l,s}$ denotes the value of wpd obtained for the l^{th} panel and s^{th} simulation.

3.1.2 Permutation approach



3.1.3 Modeling approach



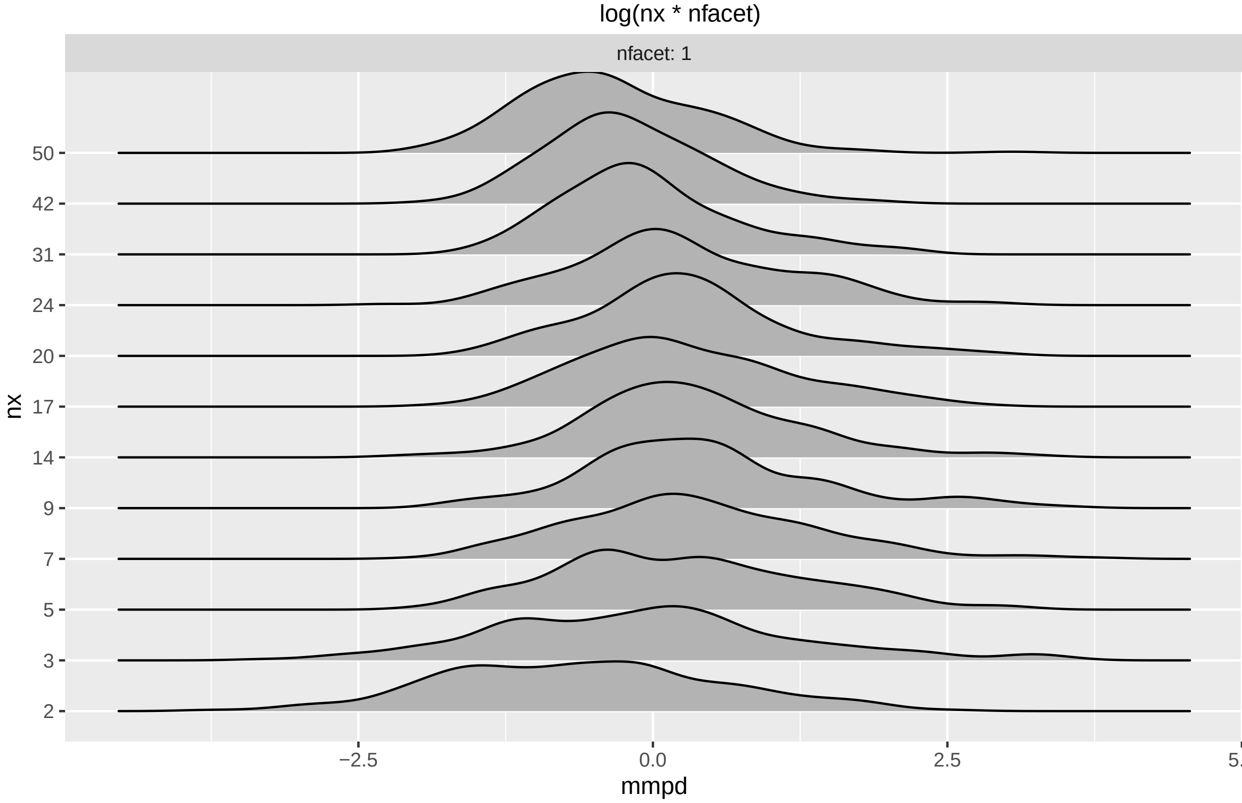
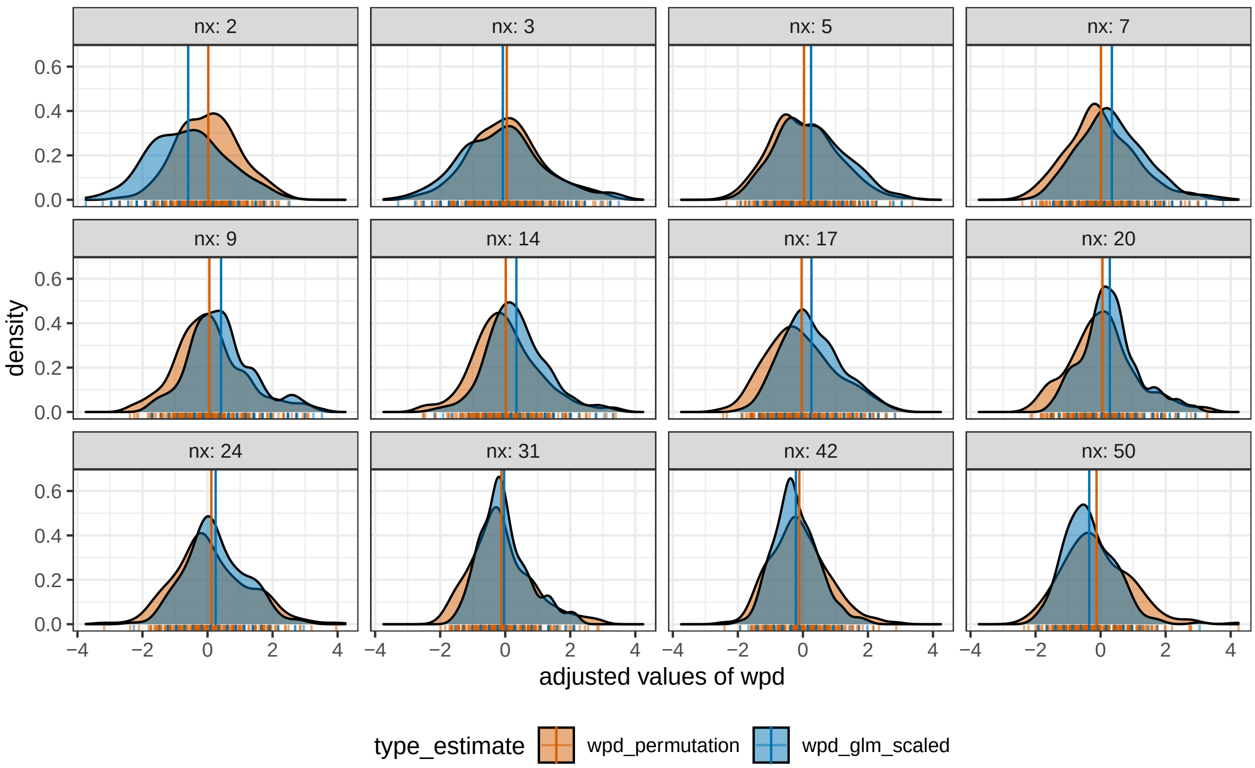


Table 2: Results of generalised linear model to capture the relationship between wpd_{raw} and number of comparisons.

term	estimate	std.error	statistic	p.value
(Intercept)	23.3996082	0.2247005	104.13688	0
log('nx * nfacet')	-0.9571158	0.0439971	-21.75408	0

3.1.4 Combination approach



3.2 Case: m = 2

3.2.1 Simulation design

Observations are generated from Gamma(2,1), G(0.5, 1), N(0,1), N(0, 5) and N(5, 1) distribution for each combination of nx and $nfacet$ from the following sets: $nx = nfacet = \{2, 3, 5, 7, 14, 20, 31, 50\}$ to cover a wide range of levels from very low to moderately high. Each combination is being referred to as a *panel*. That is, data is being generated for each of the panels $\{nx = 2, nfacet = 2\}, \{nx = 2, nfacet = 3\}, \{nx = 2, nfacet = 5\}, \dots, \{nx = 50, nfacet = 31\}, \{nx = 50, nfacet = 50\}$. For each of the 64 panels, $ntimes = 500$ observations are drawn for each combination of the categories. That is, if we consider the panel $\{nx = 2, nfacet = 2\}$, 500 observations are generated for each of the combination of categories from the panel, namely, $\{(1, 1), (1, 2), (2, 1), (2, 2)\}$. The values of *wpd* is obtained for each of the panels. The measurement variable for each combination of categories in a panel are drawn from the same distribution and hence the design corresponds to D_{null} . Furthermore, this entire method is repeated for each panels $nsim = 200$ times, so that the distribution of *wpd* under D_{null} could be observed.

3.2.2 Permutation approach

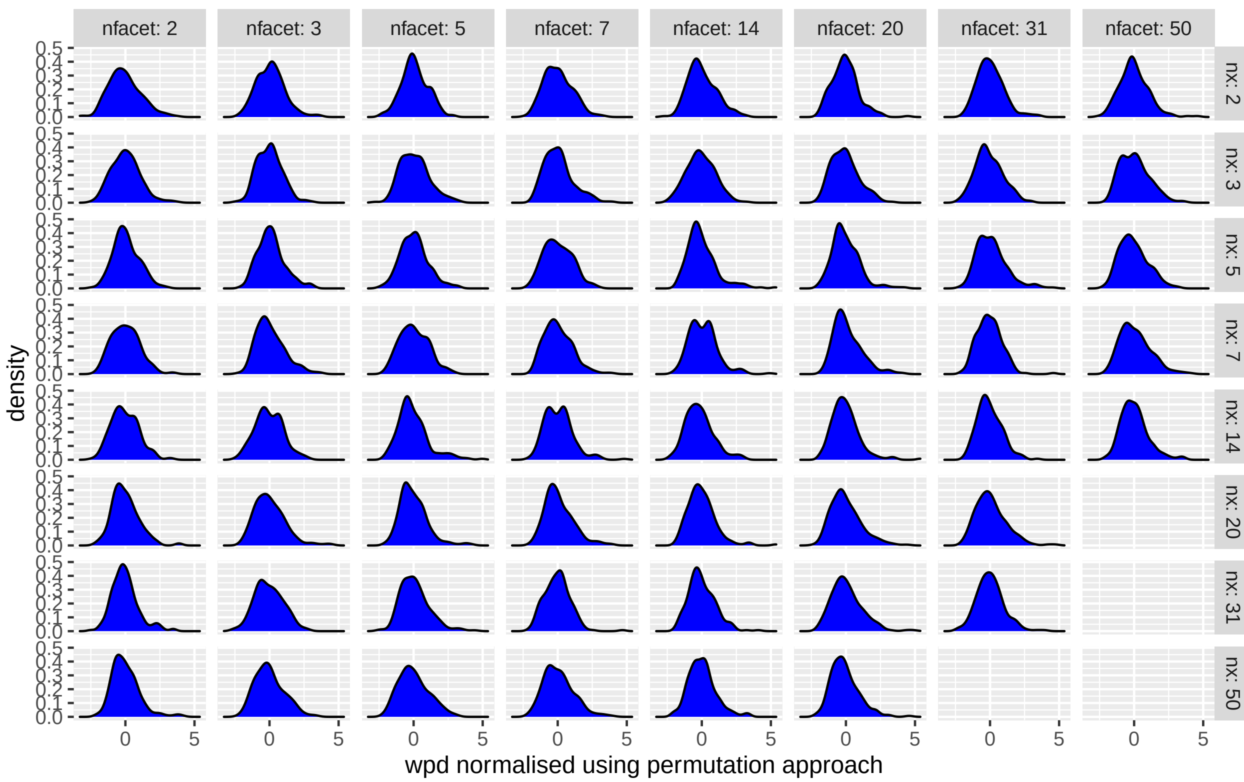


Figure 6: Distribution of wpd_{perm} is plotted across different nx and $nfacet$ categories. Both shape and scale of the distributions are now similar for different panels under the null design.

3.2.3 Modeling approach

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#> [1] 0.9886192
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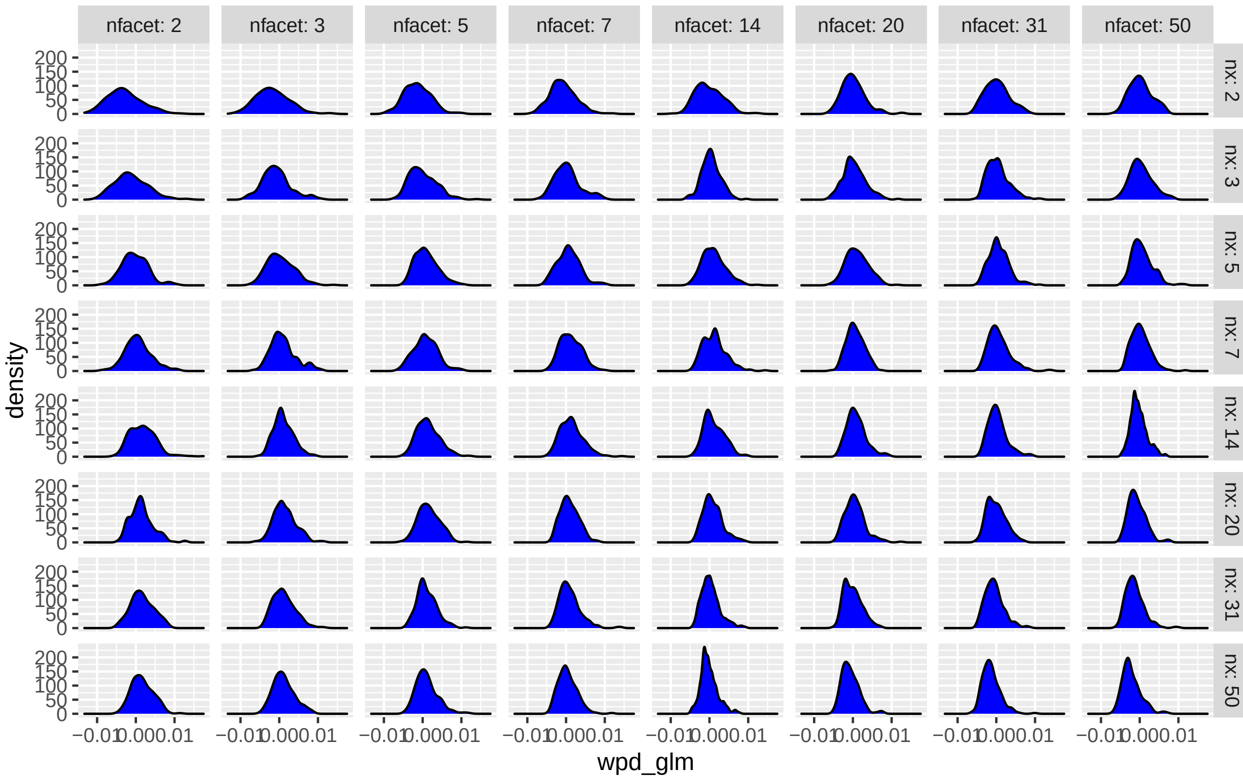


Figure 7: The distribution of wpd_{glm} is plotted. The distributions are more similar across higher nx and nfacet and dissimilar for fewer nc and nfacets.

3.2.4 Combination approach

4 Ranking and selecting harmonies

Simulation design

Observations are generated from a N(0,1) distribution for each combination of nx and $nfacet$ from the following sets: $nx = \{3, 7, 14\}$ and $nfacet = \{2, 9, 10\}$. This would result in 9 panels, viz, $(3, 2), (3, 9), (3, 10), \dots, (14, 9), (14, 10)$. Few experiments were conducted. In the first scenario, data for all panels are simulated using the null design D_{null}. In other scenarios, data simulated from the panel $(14, 2)$ and $(3, 10)$ are under $D_{vary_{all}}$. Moreover, $\omega = \{0.5, 2, 5\}$ are considered to examine if the proposed test is able to capture subtle differences and non-subtle differences when we shift from the null design. In the last scenario, we consider the panel $(3, 2), (7, 9), (14, 10)$ to be under D_{null}, the panels $(7, 2), (14, 9)$ to be under D_{var_f}. $(14, 2), (3, 10)$ under D_{var_x} and the rest under $D_{var_{null}}$. This is done to check if the consequent ranking procedure leads to designs like $D_{vary_{all}}$ to be chosen first followed by $D_{vary_{all}}$. We generate only one data set each for which these scenarios were simulated and consider this as the original data set. We generate 1000 repetitions of this experiment with different seeds.

Results

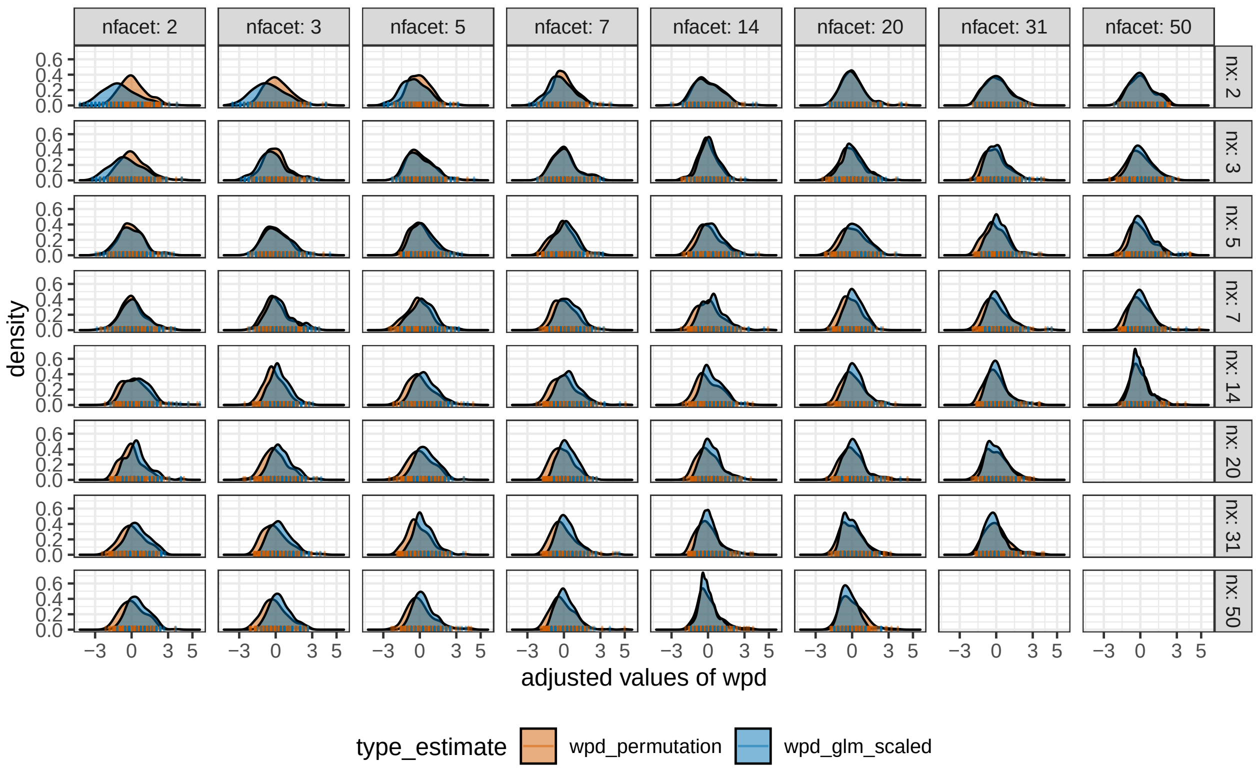
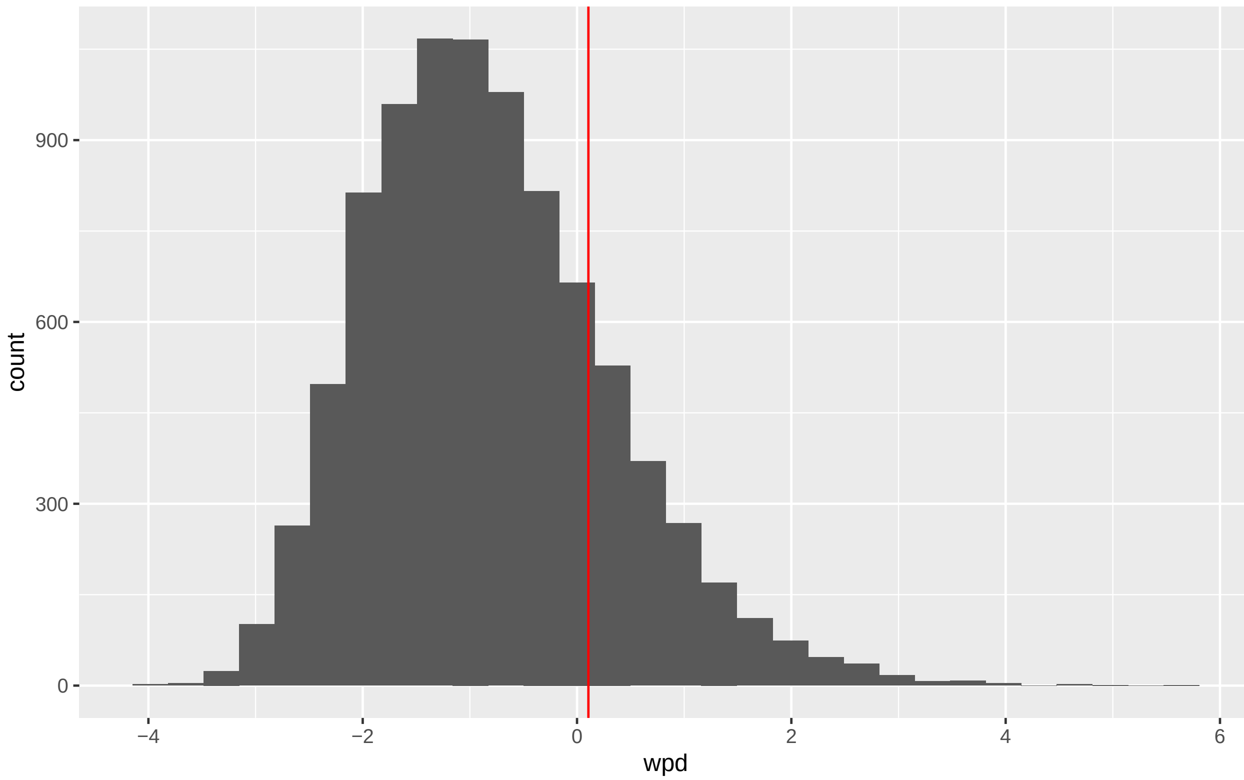


Figure 8: The distribution of wpd_{perm} and $wpd_{glm-scaled}$ are overlaid to compare the location and scale across different nx and $nfacet$. wpd_{norm} takes the value of wpd_{perm} for lower levels, and $wpd_{glm-scaled}$ for higher levels to to alleviate the problem of computational time in permutation approaches. This is possible as the distribution of the adjusted measure looks similar for both appraches for higher levels.



900
600
300
0
count
−4
−2
0
2
4
6
wpd