

Supplementary materials for the main submission entitled - A new metric for automatic discovery of periodic patterns in time series

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Appendix A recalls the necessary notations and

1 Recalling notations

Consider two cyclic granularities A and B , such that $A = \{a_j : j = 1, 2, \dots, J\}$ and $B = \{b_k : k = 1, 2, \dots, K\}$ with A placed across x-axis and B across facets. Let $v = \{v_t : t = 0, 1, 2, \dots, T-1\}$ be a continuous measured variable observed across T time points. Let the four elementary designs be D_{null} where there is no difference in distribution of v for A or B , D_{var_f} denotes the set of designs where there is difference in distribution of v for B and not for A . Similarly, D_{var_x} denotes the set of designs where difference is observed only across A . Finally, $D_{var_{all}}$ denotes those designs for which difference is observed across both A and B .

2 Behavior of raw weighted distance measure

2.1 Tuning parameter

How does the tuning parameter affects the value of wpd under different designs? The tuning parameter is used to put relative weight-age to the difference in distributions within and between facets. So it is interesting to see how the value of wpd changes for these two designs.

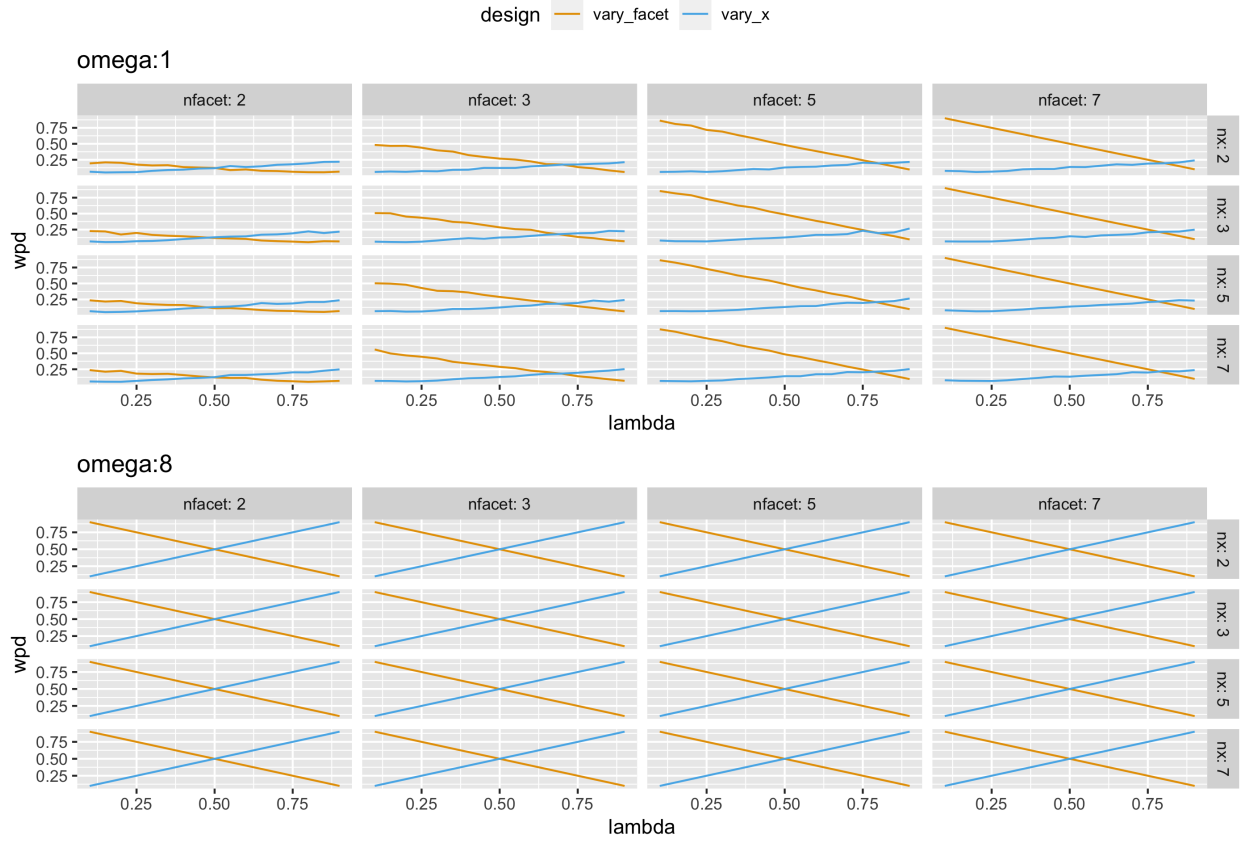
2.1.1 Simulation design

Observations are generated from Normal(0,1) distribution for each combination of nx and $nfacet$ from the following sets: $nx = nfacet = \{2, 3, 5, 7, 14, 20\}$. Let $\omega = \{1, 2, \dots, 10\}$ denotes the variable denoting the

increment in mean of the distribution. The values of lambda ranges from 0.1, 0.2, ..., 0.9. Two designs are considered D_{var_x} and D_{var_f} and the values of wpd is being computed for all these different values of the considered variable.

2.1.2 Results

Figure ?? shows how the value of wpd changes for $\lambda = 0.1, 0.2, \dots, 0.9$ for the two different designs D_{var_x} and D_{var_f} for two values of increment in mean $\omega = 1, 8$. For a lower value of ω , the two designs intersect at $\lambda > 0.7$ and for a higher ω , the two designs intersect at $\lambda = 0.5$. The value of wpd increases with λ for D_{var_x} and decreases with increasing λ for D_{var_f} . Figure 1 shows the value of λ for which the two designs intersect across different values of ω . It can be observed that as the value of ω ($\omega > 4$) increase, the value of λ at which the two designs intersect converge is $\lambda = 0.5$.



3 Normalization

3.1 Simulation design

Observations are generated from Gamma(2,1), G(0.5, 1), N(0,1), N(0, 5) and N(5, 1) distribution for each combination of nx and $nfacet$ from the following sets: $nx = nfacet = \{2, 3, 5, 7, 14, 20, 31, 50\}$ to cover a wide range of levels from very low to moderately high. Each combination is being referred to as a *panel*. That is, data is being generated for each of the panels $\{nx = 2, nfacet = 2\}, \{nx = 2, nfacet = 3\}, \{nx = 2, nfacet = 5\}, \dots, \{nx = 50, nfacet = 31\}, \{nx = 50, nfacet = 50\}$. For each of the 64 panels, $ntimes = 500$ observations are drawn for each combination of the categories. That is, if we consider the panel $\{nx = 2, nfacet = 2\}$, 500 observations are generated for each of the combination of categories from the panel, namely, $\{(1, 1), (1, 2), (2, 1), (2, 2)\}$. The values of wpd is obtained for each of the panels. The measurement variable for each combination of categories in a panel are drawn from the same distribution

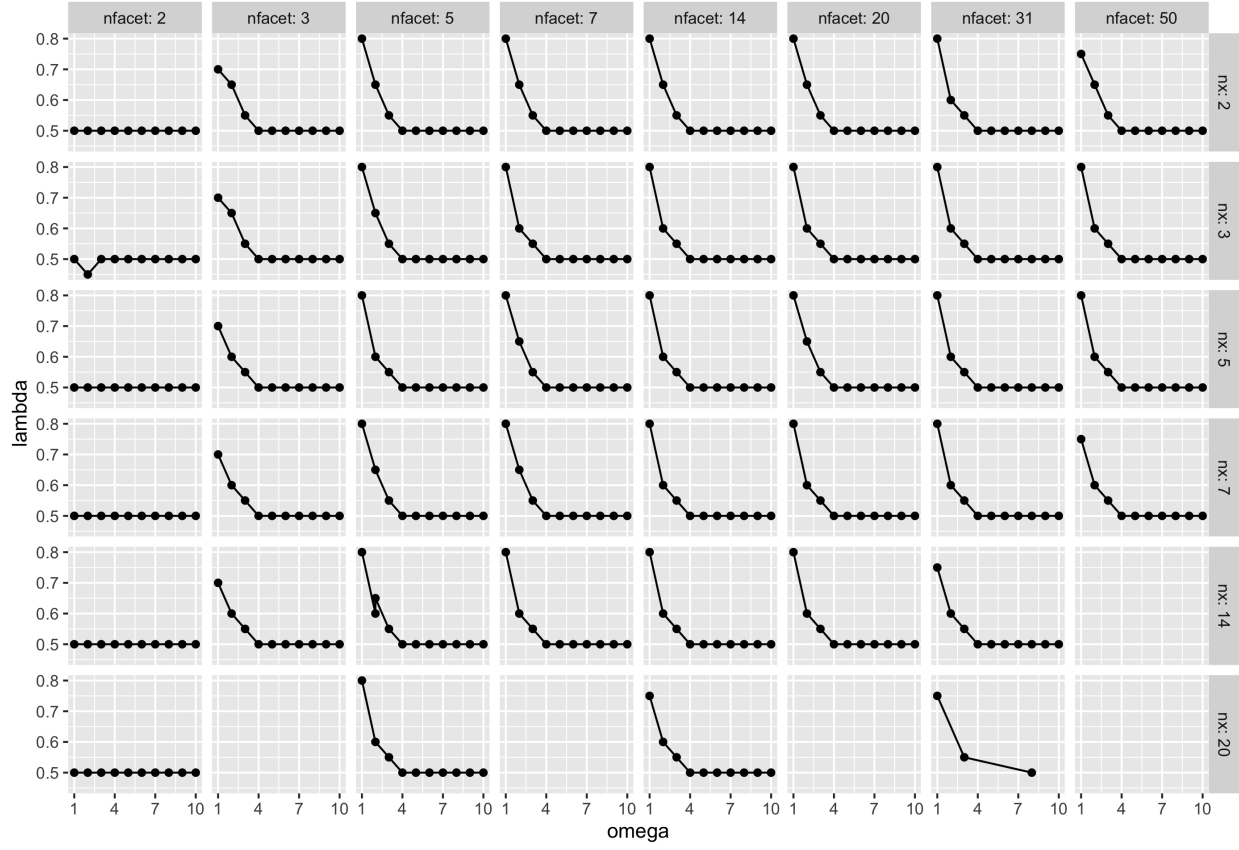


Figure 1: For most panels it is observed that the most common value of the tuning parameter for which the designs interact is 0.5, which implies any value greater than 0.5 could be chosen to up-weight the within-facet distances and down-weight the between-facet distances for most situations.

and hence the design corresponds to D_{null} . Furthermore, this entire method is repeated for each panels $nsim = 200$ times, so that the distribution of wpd under D_{null} could be observed.

3.2 Different underlying distributions

Since the measure wpd is essentially set up to detect “differences” in distributions irrespective of underlying distribution, it would be ideal if it has minimal dependency on the type, location and scale of the initial distribution. To that end, some data pre-processing through the Normal Score Transform (NQT) has been applied in order to make most asymmetrical distributed measured variables more normal-like. Figure 2 shows ridge plots of raw wpd for a Gamma(0.5,1), Gamma(2,1) before NQT. It is observed that for the underlying distribution Gamma(2,1), location and scale of the distribution of wpd changes from top-left panel to bottom-right panel. Moreover, the location and scale of the distribution of wpd for different underlying distribution Gamma(0.5,1), Gamma(2,1). Figure 3 shows the the distributions of wpd under same underlying distributions but after performing NQT. It is observed that within each panel, the distributions of the wpd looks same, however, the distributions change from extreme top-left panel to bottom-right panels. Similar observations could be made in Figure ?? for different underlying normal distributions $N(0,1)$, $N(5,1)$ and $N(0,5)$. This implies, NQT has atleast been able to bridge the gap in distribution of wpd for different non-normal underlying distributions.

3.3 Different number of comparisons

4 Location and scale

Next, we study the distribution of wpd for different nx and $nfacet$ in various ways.

4.1 Increment

5 Behavior of normalised distance measure

5.1 sample size

5.2 number of permutations

5.3 designs

6 Ranking and selecting harmonies

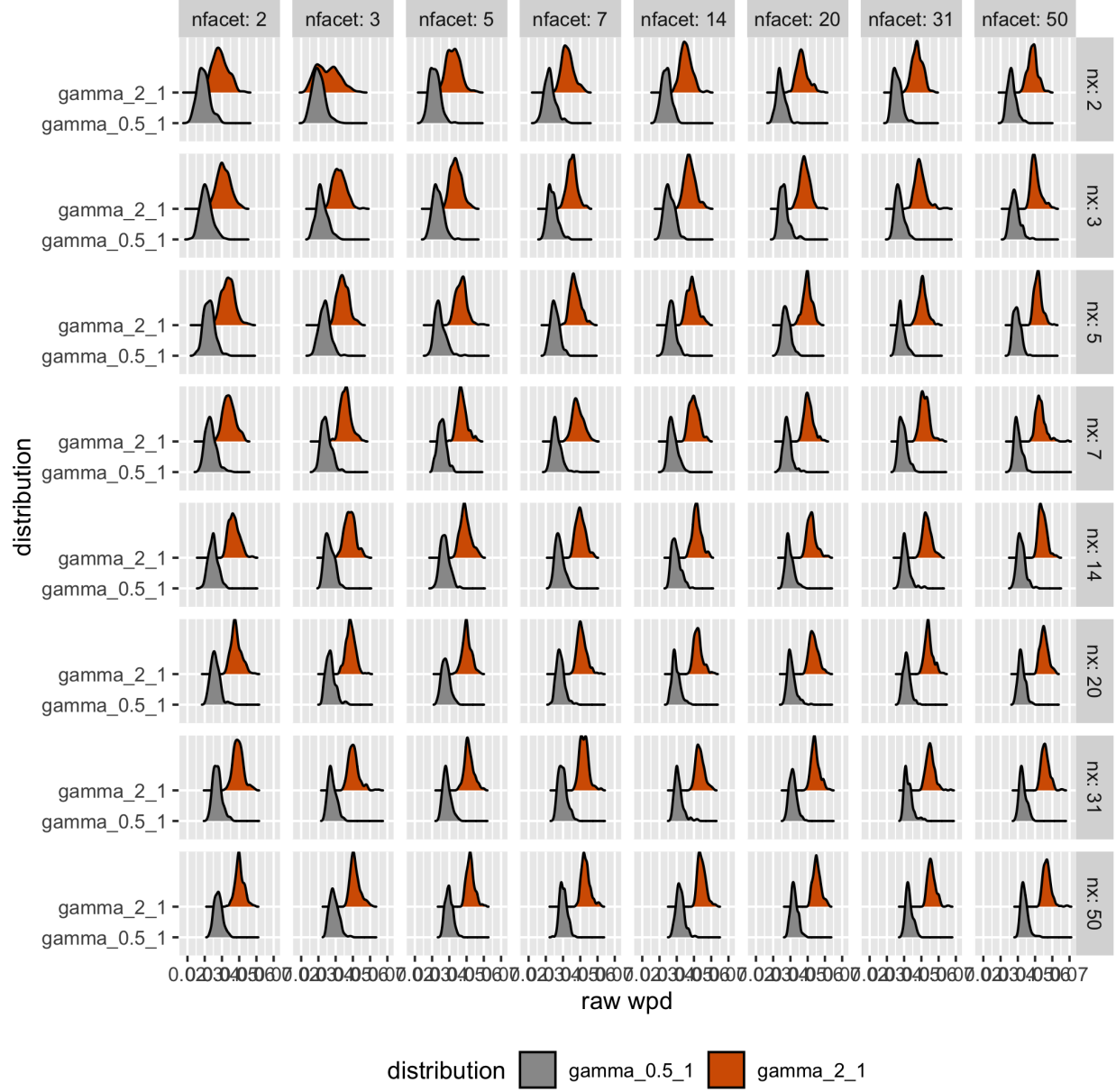


Figure 2: Ridge plots of raw wpd is shown for Gamma(0.5,1), Gamma(2,1) distribution without NQT. The densities change across different facet and x levels and also looks different for the two distributions, which implies wpd value is affected by the change in the shape paramter of the gamma distribution.

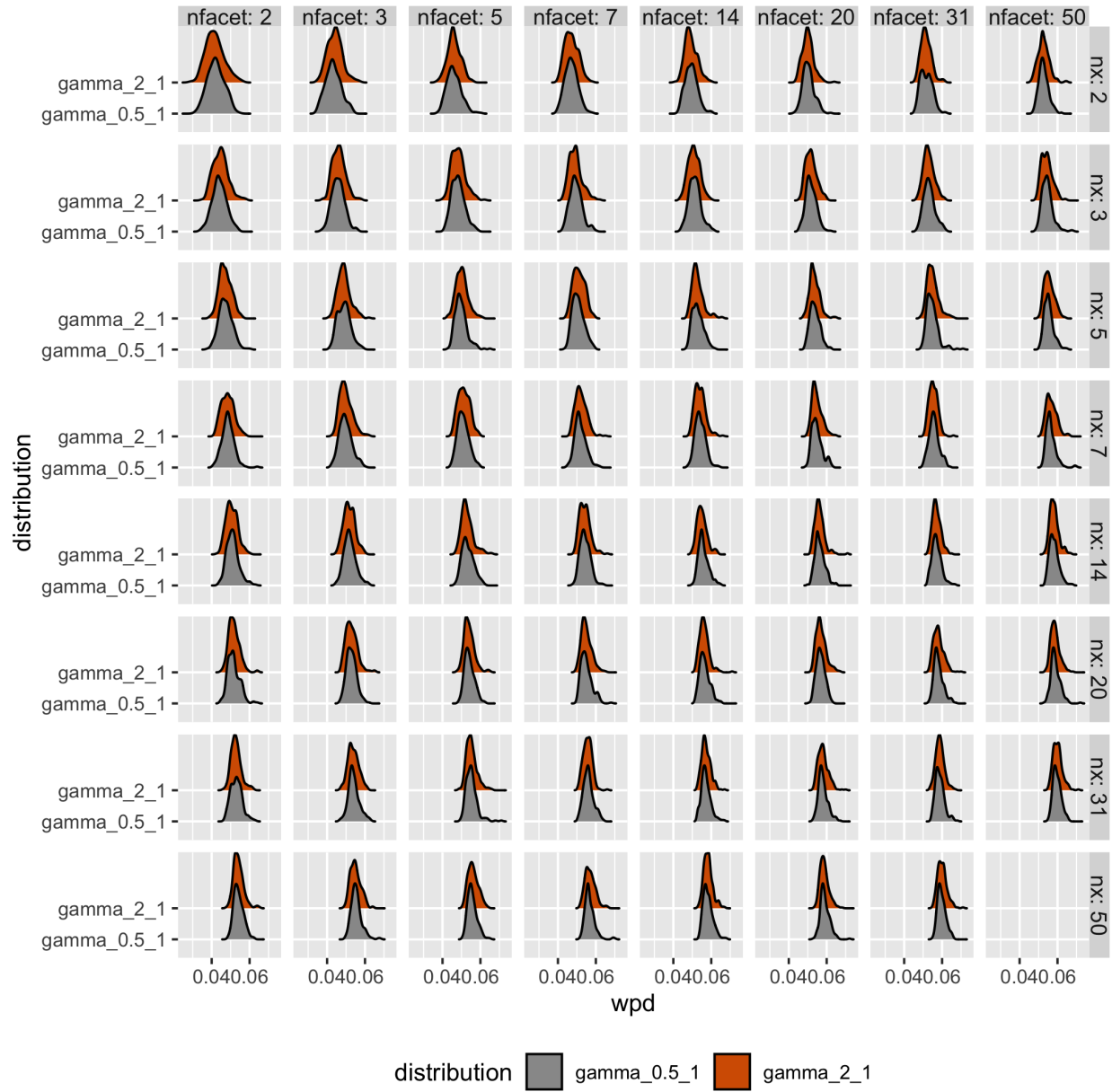


Figure 3: Ridge plots of raw wpd is shown for Gamma(0.5,1), Gamma(2,1) distribution. The densities change across different facet and x levels but look same for the two distributions, which implies wpd value is unaffected by the change in the shape parameter of the gamma distribution

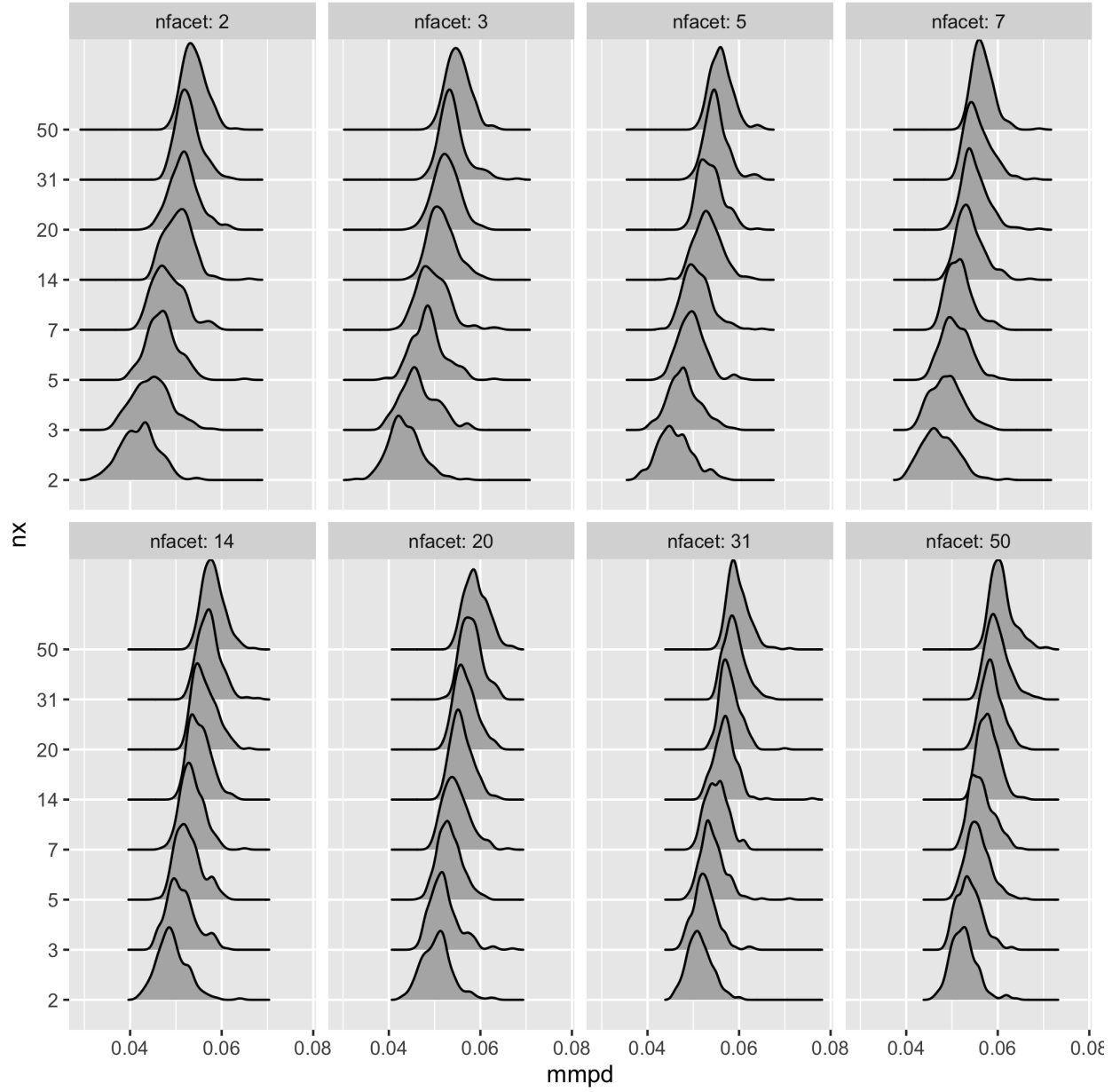


Figure 4: Ridge plots of raw wpd is shown for $N(0,5)$ distribution. For each panel, it could be seen that the location shifts to the right for increasing x levels. Across each panel, the scale of the distribution seems to change for low/moderately lower values and higher values of n facets and left tails are longer for lower facet levels.