ACTIVE SEGMENTATION MOMENTS AND TEXTURE FEATURES

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1. Image Moments

Spatial and central moments are important statistical properties of an image. Mathematically, the image moment is generally defined as the inner product of the image intensity function f(x,y) and a certain basis function $P_{m,n}$. In the continuous approximation, the moments of a function are computed by the integral

$$M_{m,n} = \iint\limits_{I} P_{m,n}(x, y) f(x, y) dx dy$$

where $P_{m,n}(x, y)$ is polynomial, parameterized by the integers m and n. depending on whether the basis functions satisfy orthogonality, the image moments can be classified into orthogonal moments and non-orthogonal moments.

2. Raw and Central Moments

For example, the raw image moments are given by the homogeneous form $P_{m,n}(x, y) = x^m y^n$. The moments, can be referred to the center of the image frame or to the center of mass of the image (x_c, y_c), in which case, $P_{m,n}(x, y) = (x - x_c)^m (y - y_c)^n$. The two main problems with such a choice is that the moments contain redundant information because the homogeneous polynomials are not orthogonal; also the

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computation loses numerical precision due to cancellation of large terms. Mathematically, a better choice of polynomials is a polynomial from an orthogonal family. Such polynomials enjoy an expansion property, that is

$$f(x, y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} M_{m,n} P_{m,n}(x, y)$$

Useful examples of such orthogonal families are the Legendre and Zernike polynomials.

3. Legendre moments

The Legendre polynomials form an orthogonal set on the interval $[-1, 1]$ The Legendre polynomials enjoy a two term recurrence relation

$$(n + 1)L_{n+1}(x) = (2n + 1)xL_n(x) - nL_{n-1}(x), \quad L_0(x) = 1, \quad L_1(x) = x \tag{1}$$

that was used for their computation in the present paper. The advantage of the recursion relation is that all Legendre moments up to a user-defined order can be computed simultaneously.

4. Zernike moments

The Zernike polynomials are normalized on the unit disk in the complex plane. The radial Zernike polynomials can be defined for $n - m$ even as:

$$R_n^m(r) = \sum_{l=0}^{(n-m)/2} \frac{(-)^l (n-l)!}{l!((n+m)/2 - l)!((n-m)/2 - l)!} r^{n-2l} \tag{2}$$

and 0 otherwise. The present paper implemented a recursive computation method given by the formula [2]

$$R_n^m(r) = r\left(R_{n-1}^{|m-1|}(r) + R_{n-1}^{m+1}(r)\right) - R_{n-2}^m(r), \quad R_0^0 = 1 \tag{3}$$

The orthogonal Zernike polynomials then are

$$V_{mn}(r, \theta) := R_n^m(r)e^{-im\theta} \tag{4}$$

The normalization of the polynomials for grayscale images is not an issue because they have a fixed dynamic range, so an image can be always normalized to unit range prior to computation of an image moment.

5. Haralick features

Haralick features [1] are coded as following 0 – Angular 2$^{\text{nd}}$ Moment; 1 – Contrast; 2 – Correlation; 3 – Dissimilarity; 4 – Energy; 5 – Entropy; 6 – Homogeneity.

6. ImageJ statistics

The following ImageJ statistics are computed: Area, mean, stdev, min, max, centroid, center of mass, perimeter, ellipse, shape descriptors, Ferret's diameter, integrated density, median, skewness, kurtosis, area fraction.

References

[1] R. M. Haralick, K. Shanmugam, and I. Dinstein. Textural features for image classification. *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-3(6):610–621, nov 1973.

[2] Barmak Honarvar Shakibaei and Raveendran Paramesran. Recursive formula to compute zernike radial polynomials. *Optics Letters*, 38(14):2487, jul 2013.