Parsing

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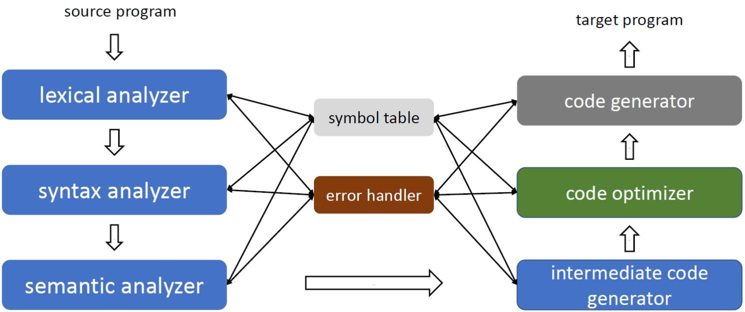
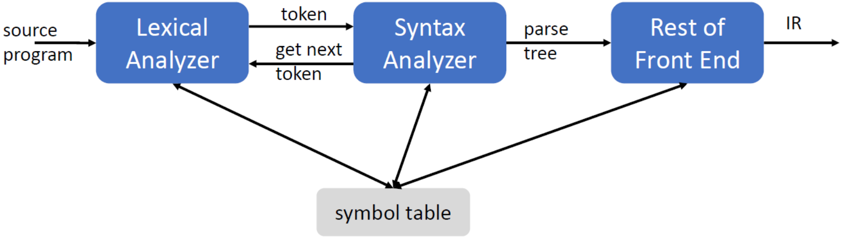
Overview of the compilation process



Parser link


source
program
Lexical
Analyzer
token
get next
token
Syntax
Analyzer
parse
tree
Rest of
Front End
IR
symbol table

Role of a parser

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- It verifies whether the string of tokens can be generated by the *grammar* of the source language .
- Outputs a parse tree that will be used by the rest of the phases.
- Reports syntax errors (Tries to recover in a intelligent fashion).

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- Universal Parsing -CYK algorithm.

Grammars

$$G = (V, T, R, S)$$

- V: Set of variables.
- T: Set of terminals.
- R: Set of Production rules.
- S: Start variable.

Example 1.1

$E \rightarrow E + T|T$
$T \rightarrow T * F|F$
$F \rightarrow (E)|id$

$V = \{E, T, F\}$, $T = \{+, *, (,), id\}$, $S = E$,
$R = \{E \rightarrow E + T, E \rightarrow T, \ldots\}$

Context-Free Grammars

$$G = (V, T, R, S)$$

- V: Set of variables. T: Set of terminals. S: Start variable.
- R: Set of Production rules of the form $A \rightarrow \alpha$, where $A \in V$, $\alpha \in \{V \cup T\}^*$.

Example 1.2 (CFG)

$E \rightarrow E + T|T$
$T \rightarrow T * F|F$
$F \rightarrow (E)|id$

Example 1.3 (NOT CFG)

$E \rightarrow E + T|T$
$TE \rightarrow T * F|F$
$F) \rightarrow (E)|id$

Derivations

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If the start symbol $S \overset{*}{\implies} \alpha$, α is said to be a *sentential form* of G.

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A sentence of G is a sentential form without any variables.

Derivations

Grammar

$E \rightarrow E + T | T$

$T \rightarrow T * F | F$

$F \rightarrow (E) | id$

String to be derived

$id + id$

$$
\begin{aligned}
E &\implies E + T \\
&\implies E + F \\
&\implies T + F \\
&\implies F + F \\
&\implies F + id \implies id + id
\end{aligned}
$$

At each step of the production, we have two choices to make:

- Which variable to choose
- Which production to choose for corresponding variable

Derivations

Leftmost Derivation: At each step, the leftmost variable is chosen for production, denoted as $\alpha \underset{lm}{\Longrightarrow} \beta$.

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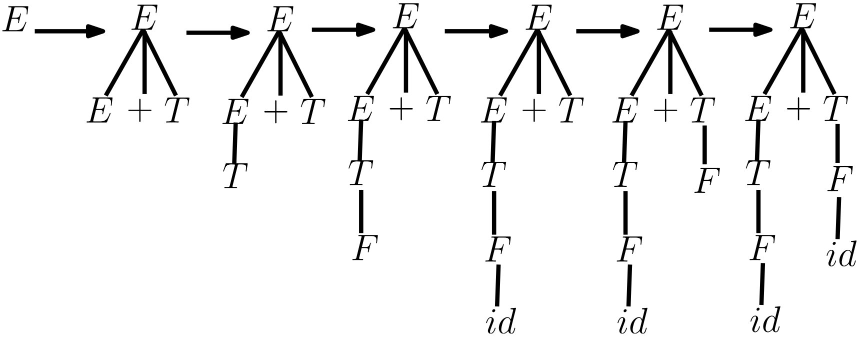
Parse Tree

A parse tree is a graphical representation of derivation.

- The start symbol S is the root of the tree.
- Each internal node is a variable - they represent application of productions.
- leaves are labelled with terminals (or may be with variables in case of a partial parse tree)
- leaves of a parse tree chosen in left to right order produces a sentential form - also known as Yield of the tree.

Parse Tree

$E \implies E + T \implies T + T \implies F + T \implies id + T \implies$
$id + F \implies id + id$



Parse Tree

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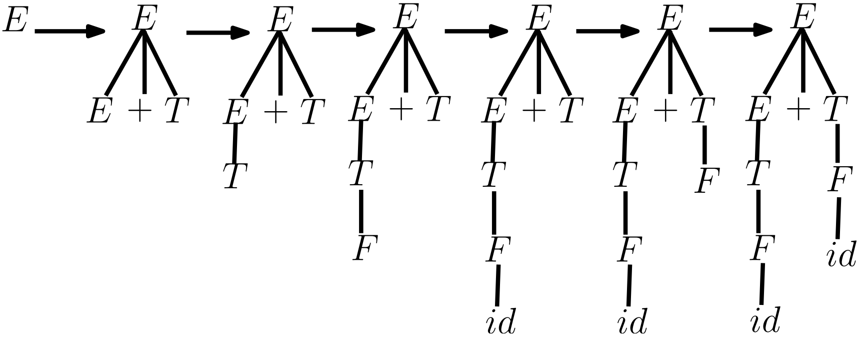


The role of parsing is: Given a grammar $G(V, T, R, S)$ and a string w of terminals, can w be generated from S using the set of rules given in R - if so, generate the corresponding parse tree.

Expressive power: CFG vs NFA

For every NFA, there exists a CFG that generates the language accpeted by the NFA.

- For each state i in the NFA, create a variable A_i in the grammar.
- If state i has a transition to state j on input a, add the rule $A_i \rightarrow aA_j$.
- If state i is accepting state, add the rule $A_i \rightarrow \epsilon$.
- If state i is start state, make A_i the start variable.

However, every language generated by a CFG may not have an equivalent NFA (see with $a^n b^n$).

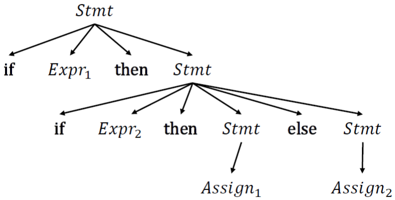
Can the problem of balanced parenthesis be solved with NFAs?

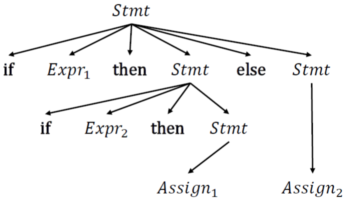
Ambiguous Grammar - Dangling else

A grammar $G(V, T, R, S)$ is said to be ambiguous if certain sentence can be derived from S using rules in R using two distinct left most (or right most) derivations. There are multiple parse trees yielding the same sentence.

$$
\begin{aligned}
Stmt \rightarrow\ & \textbf{if}\ Expr\ \textbf{then}\ Stmt \\
\mid\ & \textbf{if}\ Expr\ \textbf{then}\ Stmt\ \textbf{else}\ Stmt \\
\mid\ & Assign
\end{aligned}
$$

$$\textbf{if}\ Expr_1\ \textbf{then}\ \textbf{if}\ Expr_2\ \textbf{then}\ Assign_1\ \textbf{else}\ Assign_2$$





Ambiguous Grammar - Fixing Dangling else

In all programming languages, an else is matched with the closest unmatched if-then.

Modified grammar:

$$
\begin{aligned}
\mathit{Stmt} &\rightarrow \textbf{if } \mathit{Expr} \textbf{ then } \mathit{Stmt} \\
&\quad\mid \textbf{if } \mathit{Expr} \textbf{ then } \mathit{ThenStmt} \textbf{ else } \mathit{Stmt} \\
&\quad\mid \mathit{Assign} \\
\mathit{ThenStmt} &\rightarrow \textbf{if } \mathit{Expr} \textbf{ then } \mathit{ThenStmt} \textbf{ else } \mathit{ThenStmt} \\
&\quad\mid \mathit{Assign}
\end{aligned}
$$

$$\textbf{if } \mathit{Expr}_1 \textbf{ then if } \mathit{Expr}_2 \textbf{ then } \mathit{Assign}_1 \textbf{ else } \mathit{Assign}_2$$

$$\Downarrow$$

$$
\begin{aligned}
\mathit{Stmt} &\rightarrow \textbf{if } \mathit{Expr} \textbf{ then } \mathit{Stmt} \\
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\end{aligned}
$$

Ambiguous Grammar

Consider the grammar. $E \rightarrow E + E | E * E | id$. There are two leftmost derivations of the string $id + id * id$.

$$\begin{aligned} E &\Longrightarrow E + E \\ &\Longrightarrow id + E \\ &\Longrightarrow id + E * E \\ &\Longrightarrow id + id * E \\ &\Longrightarrow id + id * id \end{aligned}$$

$$\begin{aligned} E &\Longrightarrow E * E \\ &\Longrightarrow E + E * E \\ &\Longrightarrow id + E * E \\ &\Longrightarrow id + id * E \\ &\Longrightarrow id + id * id \end{aligned}$$

To remove ambiguity, encode precedence in the grammar.

$$\begin{gathered} E \rightarrow E + T | T \\ T \rightarrow T * F | F \\ F \rightarrow id \end{gathered}$$

Increasing order of Priority

$$\Downarrow$$

Ambiguous Grammar: Precedence encoding

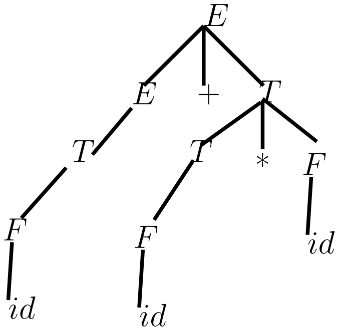
$$E \rightarrow E + T | T$$
$$T \rightarrow T * F | F$$
$$F \rightarrow id$$

Increasing order of Priority

$\Downarrow$

Corresponding Leftmost derivation:

$$\begin{aligned} E &\Longrightarrow E + T \\ &\Longrightarrow T + T \\ &\Longrightarrow F + T \\ &\Longrightarrow id + T \\ &\Longrightarrow id + T * F \\ &\Longrightarrow id + F * F \\ &\Longrightarrow id + id * F \\ &\Longrightarrow id + id * id \end{aligned}$$



Left Recursion

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- Direct left recursion: there is a production of the form $A \Longrightarrow A\alpha$.
- Indirect left recursion: there is a production of the form $A \Longrightarrow S\alpha$ and $S \stackrel{*}{\Longrightarrow} A\beta$.

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Left recursion introduces problems during Topdown parsing - it can be avoided by rewriting the rules.

Direct Left Recursion Elimination

$$A \rightarrow A\alpha_1 | A\alpha_2 | \ldots | A\alpha_m | \beta_1 | \ldots | \beta_n$$

$$A \rightarrow \beta_1 A' | \beta_2 A' | \ldots | \beta_n A'$$
$$A' \rightarrow \alpha_1 A' | \alpha_2 A' | \ldots | \alpha_m A' | \epsilon$$

Direct Left Recursion Elimination example

Example 1.4

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\begin{aligned}
E &\rightarrow E + T | T \\
T &\rightarrow T * F | F \\
F &\rightarrow id.
\end{aligned}
$$

Direct Left Recursion Elimination example

Example 1.4

$$E \to E + T | T$$
$$T \to T * F | F$$
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After removal of Left recursion,

$E \to TE'$
$E' \to +TE' | \epsilon$
$T \to FT'$
$T' \to *FT' | \epsilon$
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Indirect Left Recursion

$$S \rightarrow Aa|b$$
$$A \rightarrow Sc|\epsilon$$

There is indirect left recursion since $S \implies Aa \implies Sda$.

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$S \to Aa|b$
$A \to Sc|\epsilon$

When $i = 2$, $A \to Sc$ is replaced with $A \to Aac|bc|\epsilon$. Removing immediate LR, the grammar becomes
$S \to Aa|b$
$A \to bcA'|A'$
$A' \to acA'|\epsilon$

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$$E \rightarrow b \;.$$

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$$S' \rightarrow eS | \epsilon$$
$$E \rightarrow b \;.$$

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The Key problem is to choose the correct production from the set of rules.

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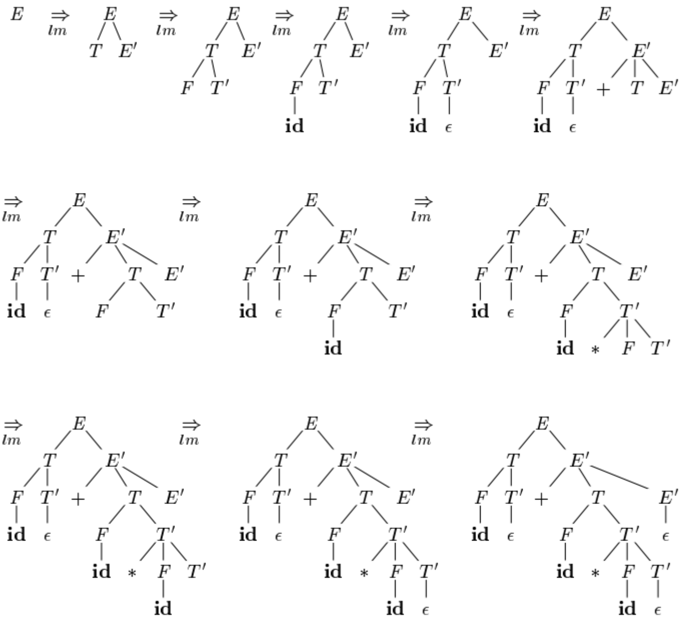
The derivation of the string $id + id * id$ according to Top-Down parse given in the following figure.

Top-Down Parse



Recursive Descent Parsing

A Recursive Descent parsing program consists of a set of procedures, one for each variable.
Execution begins at the start symbol procedure $S()$.
Success if it scans the entire input and halts.

```
void A() {
        Choose an A-production, A → X1 X2 ··· Xk;
        for ( i = 1 to k ) {
                if ( Xi is a nonterminal )
                        call procedure Xi();
                else if ( Xi equals the current input symbol a )
                        advance the input to the next symbol;
                else /* an error has occurred */;
        }
}
```

Process above is nondeterministic - which A-production to choose.

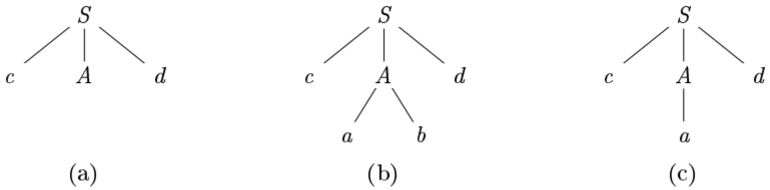
Recursive Descent - Backtracking

$$S \rightarrow cAd$$
$$A \rightarrow ab|a$$

Recursive Descent - Backtracking

$$S \to cAd$$
$$A \to ab|a$$

We want to construct the parse tree for the derivation of string $w = cad$.



Left recursion may cause infinite loop.

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If $A \stackrel{*}{\Longrightarrow} a\beta$, then $a \in FIRST(A)$.
Why this is helpful:
Let $A \rightarrow \alpha|\beta$ and $FIRST(\alpha) \cap FIRST(\beta) = \phi$. Then by looking at the next input symbol a, we can say for sure which A-production to choose next.

$FOLLOW(X)$

Definition 1.7 ($FOLLOW(X)$)

The set of terminals a such that there exists a production of the form $S \stackrel{*}{\Longrightarrow} \alpha X a \beta$. If X is the rightmost symbol in some sentential form, then \$ is added to $FOLLOW(X)$. (\$ is a special endmarker symbol)

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3. If $X \rightarrow \epsilon$, then add ϵ to $FIRST(X)$.

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3. If $A \rightarrow \alpha X$ or $A \rightarrow \alpha X \beta$ and $FIRST(\beta)$ contains ϵ, then everything in $FOLLOW(A)$ is added to $FOLLOW(X)$.

Example

$$E \to TE' \ E' \to +TE'|\epsilon$$
$$T \to FT' \ T' \to *FT'|\epsilon$$
$$F \to (E)|id.$$

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2. If $X \rightarrow X_1X_2 \ldots X_k$ is a production, $a \in FIRST(X)$ if $X_1X_2 \ldots X_{l-1} \stackrel{*}{\Longrightarrow} \epsilon$ and $a \in FIRST(X_l)$.
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$$FIRST(T) = FIRST(F)$$

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$LL(1)$ Grammars

Recursive Descent parsers without backtracking can be constructed for a class of grammars known as $LL(1)$ grammars.

Leftmost derivation

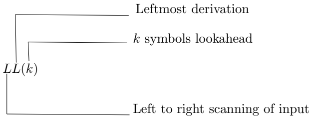
k symbols lookahead

$LL(k)$

Left to right scanning of input

$LL(1)$ Grammars

Recursive Descent parsers without backtracking can be constructed for a class of grammars known as $LL(1)$ grammars.



Definition 1.8 ($LL(1)$ Grammar)

Whenever $A \rightarrow \alpha|\beta$ are two distinct productions in the grammar,

1. For no terminal a do both α and β derive strings starting with a.
2. Atmost one of α or β derives ϵ.
3. If $\beta \stackrel{*}{\Longrightarrow} \epsilon$, then α does not derive any string starting with a terminal in $FOLLOW(A)$.

$LL(1)$ Grammars

Whenever $A \rightarrow \alpha|\beta$ are two distinct productions in the grammar,

1. For no terminal *a* do both α and β derive strings starting with *a*.
2. Atmost one of α or β derives ϵ.
3. If $\beta \stackrel{*}{\Longrightarrow} \epsilon$, then α does not derive any string starting with a terminal in $FOLLOW(A)$.

(1) and (2) ensure that $FIRST(\alpha) \cap FIRST(\beta) = \phi$. (3) ensures that if $\beta \stackrel{*}{\Longrightarrow} \epsilon$, then $FIRST(\alpha) \cap FOLLOW(A) = \phi$.

Algorithm for predictive parsing table

Input: CFG G
Output: Parsing table M Method: For each production $A \rightarrow \alpha$, do the following.

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Output: Parsing table M Method: For each production $A \rightarrow \alpha$, do the following.

1. For each terminal $a \in FIRST(\alpha)$, add $A \rightarrow \alpha$ to $M[A, a]$.
2. If $\epsilon \in FIRST(\alpha)$, then for each terminal $b \in FOLLOW(A)$, add $A \rightarrow \alpha$ to $M[A, b]$. If $\epsilon \in FIRST(\alpha)$ and $\$ \in FOLLOW(A)$, add $A \rightarrow \alpha$ to $M[A, \$]$.

Empty entries in the table are errors.

Example 1

$$E \to TE' \; E' \to +TE'|\epsilon$$
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$FOLLOW(F) = \{*, +,), \$\}$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \mathbf{id}$			$F \rightarrow (E)$		

Example 2

$$S \to iEtSS'|a$$
$$S' \to eS|\epsilon \; E \to b.$$

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$FIRST(E) = \{b\}$

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$$S \rightarrow iEtSS'|a$$
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$$FIRST(E) = \{b\}$$
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$$S \rightarrow iEtSS'|a$$
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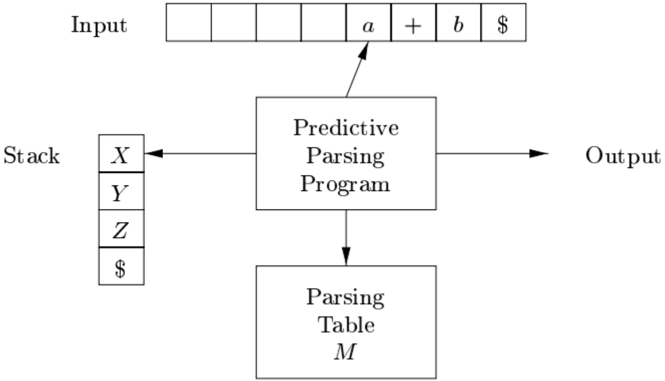
NON - TERMINAL	INPUT SYMBOL					
	a	b	e	i	t	$\$$
S	$S \to a$			$S \to iEtSS'$		
S'			$S' \to \epsilon$ $S' \to eS$			$S' \to \epsilon$
E		$E \to b$				

Nonrecursive (Table driven) Predictive Parser

Maintains a stack explicitly rather than implicitly via recursive calls.
If $S \xRightarrow[lm]{*} w\alpha$, and w is the matched input, then the stack contains α.



Algorithm for Nonrecursive Predictive parser

Input: A parsing table derived from the grammar G and input string w

Output: If $w \in L(G)$, a leftmost derivation of w; else raise an error

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Method: Initially, the stack contains $S\$$ with S at the top; the input pointer points to the leftmost symbol of w.

let a be the first symbol of w;
let X be the top stack symbol;
while ($X \neq \$$) { /* stack is not empty */
    if ($X = a$) pop the stack and **let** a be the next symbol of w;
    else if (X is a terminal) *error*();
    else if ($M[X, a]$ is an error entry) *error*();
    else if ($M[X, a] = X \rightarrow Y_1 Y_2 \cdots Y_k$) {
        output the production $X \rightarrow Y_1 Y_2 \cdots Y_k$;
        pop the stack;
        push $Y_k, Y_{k-1}, \ldots, Y_1$ onto the stack, with Y_1 on top;
    }
    let X be the top stack symbol;
}

Example 1

$E \rightarrow TE'$ $E' \rightarrow +TE'|\epsilon$
$T \rightarrow FT'$ $T' \rightarrow *FT'|\epsilon$

$F \rightarrow (E)|id.$

The sequence of moves made by the parser in derivation of $id + id * id$.

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MATCHED	STACK	INPUT	ACTIONS
	$E\$$	$id + id * id\$$	

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MATCHED	STACK	INPUT	ACTIONS
	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	Using $E \to TE'$

Example 1

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id	$T'E'\$$	$+id * id\$$	Match id

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id	$E'\$$	$+id * id\$$	Using $T' \to \epsilon$

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id	$T'E'\$$	$+id * id\$$	Match id
id	$E'\$$	$+id * id\$$	Using $T' \to \epsilon$
id	$+TE'\$$	$+id * id\$$	Using $E' \to +TE'$

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id	$E'\$$	$+id * id\$$	Using $T' \to \epsilon$
id	$+TE'\$$	$+id * id\$$	Using $E' \to +TE'$
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id	$T'E'\$$	$+id * id\$$	Match id
id	$E'\$$	$+id * id\$$	Using $T' \to \epsilon$
id	$+TE'\$$	$+id * id\$$	Using $E' \to +TE'$
id+	$TE'\$$	$id * id\$$	Match $+$
id+	$FT'E'\$$	$id * id\$$	Using $T \to FT'$

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	$TE'\$$	$id + id * id\$$	Using $E \rightarrow TE'$
	$FT'E'\$$	$id + id * id\$$	Using $T \rightarrow FT'$
	$idT'E'\$$	$id + id * id\$$	Using $F \rightarrow id$
id	$T'E'\$$	$+id * id\$$	Match id
id	$E'\$$	$+id * id\$$	Using $T' \rightarrow \epsilon$
id	$+TE'\$$	$+id * id\$$	Using $E' \rightarrow +TE'$
id+	$TE'\$$	$id * id\$$	Match $+$
id+	$FT'E'\$$	$id * id\$$	Using $T \rightarrow FT'$
id+	$idT'E'\$$	$id * id\$$	Using $F \rightarrow id$

Example 1

$E \to TE'$ $E' \to +TE'|\epsilon$
$T \to FT'$ $T' \to *FT'|\epsilon$

$F \to (E)|id.$

The sequence of moves made by the parser in derivation of $id + id * id$.

MATCHED	STACK	INPUT	ACTIONS
	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	Using $E \to TE'$
	$FT'E'\$$	$id + id * id\$$	Using $T \to FT'$
	$idT'E'\$$	$id + id * id\$$	Using $F \to id$
id	$T'E'\$$	$+id * id\$$	Match id
id	$E'\$$	$+id * id\$$	Using $T' \to \epsilon$
id	$+TE'\$$	$+id * id\$$	Using $E' \to +TE'$
id+	$TE'\$$	$id * id\$$	Match $+$
id+	$FT'E'\$$	$id * id\$$	Using $T \to FT'$
id+	$idT'E'\$$	$id * id\$$	Using $F \to id$
id+id	$T'E'\$$	$*id\$$	Match id

MATCHED	STACK	INPUT	ACTIONS
	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	Using $E \rightarrow TE'$
	$FT'E'\$$	$id + id * id\$$	Using $T \rightarrow FT'$
	$idT'E'\$$	$id + id * id\$$	Using $F \rightarrow id$
id	$T'E'\$$	$+id * id\$$	Match id
id	$E'\$$	$+id * id\$$	Using $T' \rightarrow \epsilon$
id	$+TE'\$$	$+id * id\$$	Using $E' \rightarrow +TE'$
id+	$TE'\$$	$id * id\$$	Match $+$
id+	$FT'E'\$$	$id * id\$$	Using $T \rightarrow FT'$
id+	$idT'E'\$$	$id * id\$$	Using $F \rightarrow id$
id+id	$T'E'\$$	$*id\$$	Match id

MATCHED	STACK	INPUT	ACTIONS
	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	Using $E \rightarrow TE'$
	$FT'E'\$$	$id + id * id\$$	Using $T \rightarrow FT'$
	$idT'E'\$$	$id + id * id\$$	Using $F \rightarrow id$
id	$T'E'\$$	$+id * id\$$	Match id
id	$E'\$$	$+id * id\$$	Using $T' \rightarrow \epsilon$
id	$+TE'\$$	$+id * id\$$	Using $E' \rightarrow +TE'$
id+	$TE'\$$	$id * id\$$	Match $+$
id+	$FT'E'\$$	$id * id\$$	Using $T \rightarrow FT'$
id+	$idT'E'\$$	$id * id\$$	Using $F \rightarrow id$
id+id	$T'E'\$$	$*id\$$	Match id
id+id	$*FT'E'\$$	$*id\$$	Using $T' \rightarrow *FT'$

MATCHED	STACK	INPUT	ACTIONS
	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	Using $E \rightarrow TE'$
	$FT'E'\$$	$id + id * id\$$	Using $T \rightarrow FT'$
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id+	$TE'\$$	$id * id\$$	Match $+$
id+	$FT'E'\$$	$id * id\$$	Using $T \rightarrow FT'$
id+	$idT'E'\$$	$id * id\$$	Using $F \rightarrow id$
id+id	$T'E'\$$	$*id\$$	Match id
id+id	$*FT'E'\$$	$*id\$$	Using $T' \rightarrow *FT'$
id+id*	$FT'E'\$$	$id\$$	Match $*$

MATCHED	STACK	INPUT	ACTIONS
	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	Using $E \to TE'$
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id+id	$T'E'\$$	$*id\$$	Match id
id+id	$*FT'E'\$$	$*id\$$	Using $T' \to *FT'$
id+id*	$FT'E'\$$	$id\$$	Match $*$
id+id*	$idT'E'\$$	$id\$$	Using $F \to id$

MATCHED	STACK	INPUT	ACTIONS
	$E\$$	$id + id * id\$$	
	$TE'\$$	$id + id * id\$$	Using $E \rightarrow TE'$
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id+	$FT'E'\$$	$id * id\$$	Using $T \rightarrow FT'$
id+	$idT'E'\$$	$id * id\$$	Using $F \rightarrow id$
id+id	$T'E'\$$	$*id\$$	Match id
id+id	$*FT'E'\$$	$*id\$$	Using $T' \rightarrow *FT'$
id+id*	$FT'E'\$$	$id\$$	Match $*$
id+id*	$idT'E'\$$	$id\$$	Using $F \rightarrow id$
id+id*id	$T'E'\$$	$\$$	Match id

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id+id*id	$T'E'\$$	$\$$	Match id
id+id*id	$E'\$$	$\$$	Using $T' \rightarrow \epsilon$
id+id*id	$\$$	$\$$	Using $E' \rightarrow \epsilon$

Bottom-Up Parsing

Construction of the parse tree for a input string beginning at the leafs working up towards the root.

$E \rightarrow E + T | T \;\; T \rightarrow T * F | F \;\; F \rightarrow id$.

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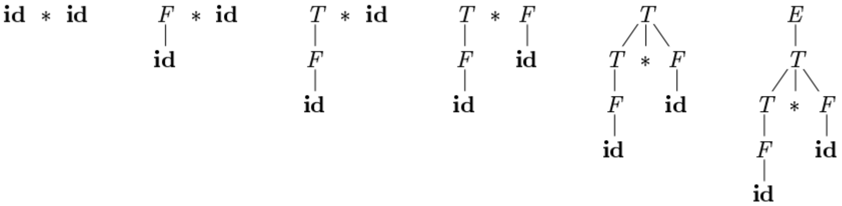

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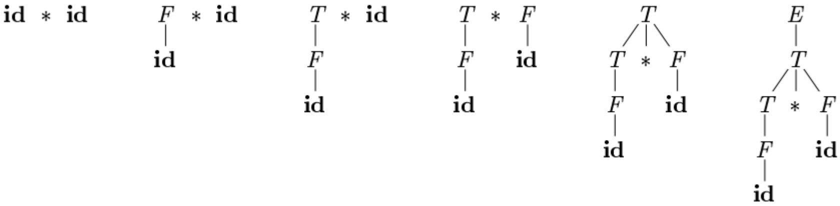

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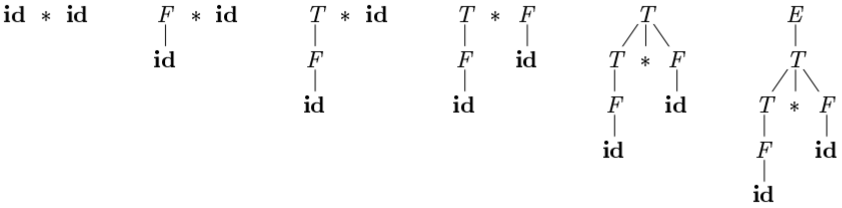

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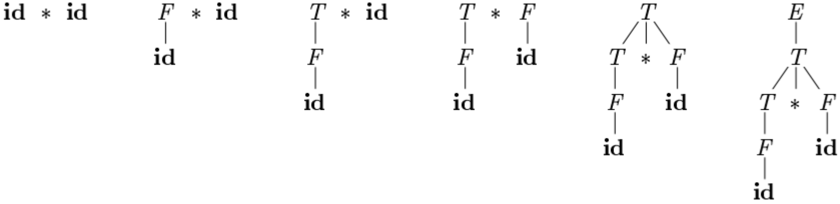

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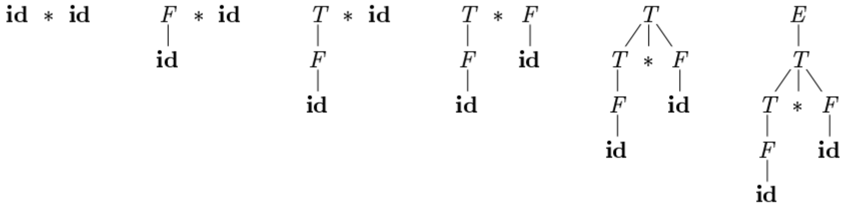

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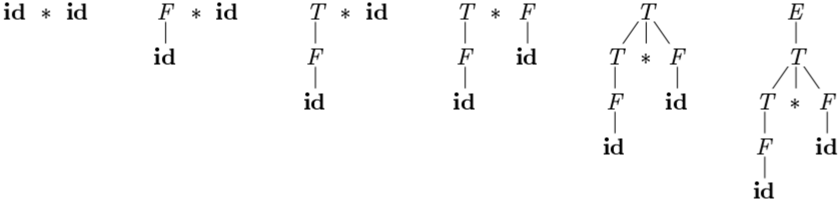

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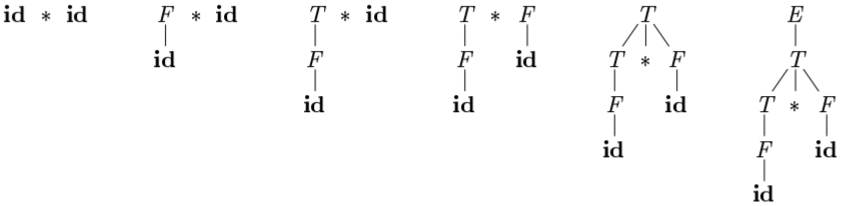

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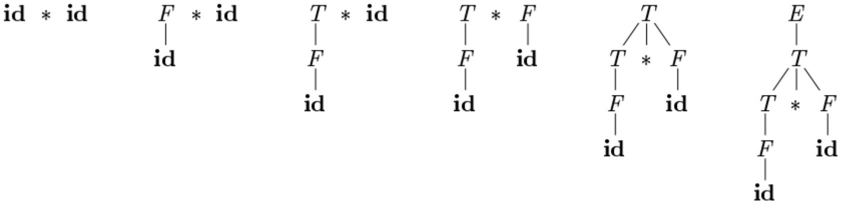

Consider the rightmost derivation of $id * id$ from E.
$E \rightarrow T \rightarrow T * F \rightarrow T * id \rightarrow F * id \rightarrow id * id.$
Can you see any connection between the parse tree construction and derivation?

Reductions

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- Key decisions are when to reduce and what production to apply.
- Goal of BU parsing is to create the derivation tree in reverse.

Handle

Definition 1.9

A handle is a substring that matches the body of a production, whose reduction represents one step along the reverse of the rightmost derivation.

RIGHT SENTENTIAL FORM	HANDLE	REDUCING PRODUCTION
$\mathbf{id}_1 * \mathbf{id}_2$	$\mathbf{id}_1$	$F \to \mathbf{id}$
$F * \mathbf{id}_2$	F	$T \to F$
$T * \mathbf{id}_2$	$\mathbf{id}_2$	$F \to \mathbf{id}$
$T * F$	$T * F$	$T \to T * F$
T	T	$E \to T$

Handle and Pruning

Definition 1.10

Formally, if $S \underset{rm}{\overset{*}{\rightarrow}} \alpha A \gamma \rightarrow \alpha\beta\gamma$, then $A \rightarrow \beta$ is the handle of $\alpha\beta\gamma$.

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1. Locate the handle β_n in r_n
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3. Repeat until S is reached.

Shift-Reduce Parsing

STACK	INPUT	ACTIONS

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$\$F$	$*id_2\$$	reduce by $T \rightarrow F$

Shift-Reduce Parsing

STACK	INPUT	ACTIONS
$\$$	$id_1 * id_2\$$	shift
$\$id_1$	$*id_2\$$	reduce by $F \to id$
$\$F$	$*id_2\$$	reduce by $T \to F$
$\$T$	$*id_2\$$	shift

Shift-Reduce Parsing

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$\$$	$id_1 * id_2\$$	shift
$\$id_1$	$*id_2\$$	reduce by $F \rightarrow id$
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$\$T$	$*id_2\$$	shift
$\$T*$	$id_2\$$	shift

Shift-Reduce Parsing

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$\$$	$id_1 * id_2\$$	shift
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$\$F$	$*id_2\$$	reduce by $T \to F$
$\$T$	$*id_2\$$	shift
$\$T*$	$id_2\$$	shift
$\$T * id_2$	$\$$	reduce by $F \to id$

Shift-Reduce Parsing

STACK	INPUT	ACTIONS
$\$$	$id_1 * id_2\$$	shift
$\$id_1$	$*id_2\$$	reduce by $F \to id$
$\$F$	$*id_2\$$	reduce by $T \to F$
$\$T$	$*id_2\$$	shift
$\$T*$	$id_2\$$	shift
$\$T * id_2$	$\$$	reduce by $F \to id$
$\$T * F$	$\$$	reduce by $T \to T * F$

Shift-Reduce Parsing

STACK	INPUT	ACTIONS
$\$$	$id_1 * id_2\$$	shift
$\$id_1$	$*id_2\$$	reduce by $F \rightarrow id$
$\$F$	$*id_2\$$	reduce by $T \rightarrow F$
$\$T$	$*id_2\$$	shift
$\$T*$	$id_2\$$	shift
$\$T * id_2$	$\$$	reduce by $F \rightarrow id$
$\$T * F$	$\$$	reduce by $T \rightarrow T * F$
$\$T$	$\$$	reduce by $E \rightarrow T$

Shift-Reduce Parsing

STACK	INPUT	ACTIONS
$\$$	$id_1 * id_2\$$	shift
$\$id_1$	$*id_2\$$	reduce by $F \rightarrow id$
$\$F$	$*id_2\$$	reduce by $T \rightarrow F$
$\$T$	$*id_2\$$	shift
$\$T*$	$id_2\$$	shift
$\$T * id_2$	$\$$	reduce by $F \rightarrow id$
$\$T * F$	$\$$	reduce by $T \rightarrow T * F$
$\$T$	$\$$	reduce by $E \rightarrow T$
$\$E$	$\$$	accept

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- cannot decide whether to shift or reduce - shift-reduce conflict.
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Example 1.11 ($S \rightarrow iEtS | iEtSeS | \ldots$)

Stack - $iEtS$ Remaining input - $e \ldots \$$

Here, we cannot decide whether to reduce $iEtS$ by $S \rightarrow iEtS$ or to shift e.

$LR(k)$ Parsers

L - Left to right scanning of input

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The class of grammars for LR methods are superset of LL methods.

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Example 1.12 ($A \rightarrow XYZ$ yields 4 items)

$A \rightarrow .XYZ$
$A \rightarrow X.YZ$
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$A \to X.YZ$
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$A \to \epsilon$ yields $A \to .$

$A \to .XYZ$ - We expect to see string derived from XYZ.

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$A \rightarrow XYZ.$ - We have seen string derived from XYZ; It is time to reduce XYZ to A.

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this ensures the start symbol never lying in the body of any production.
Acceptance occurs when the parser is about to reduce by $S' \rightarrow S$..

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Let I contains $[E' \rightarrow .E]$, then $CLOSURE(I)$ contains the following items.

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$E \rightarrow .E + T$
$E \rightarrow .T$
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- NonKernel items: All items with . at the leftmost position except $S' \rightarrow S$.

GOTO function

If $[A \to \alpha.X\beta] \in I$, $GOTO(I, X)$ contains $CLOSURE([A \to \alpha X.\beta])$.

Example 1.15 (I contains two items: $E' \to E$ and $E \to E. + T$)

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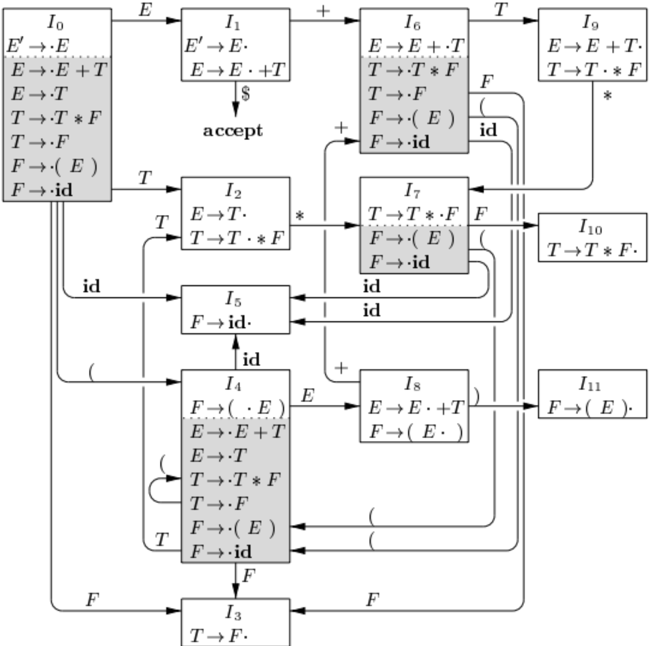
$F \rightarrow .id$

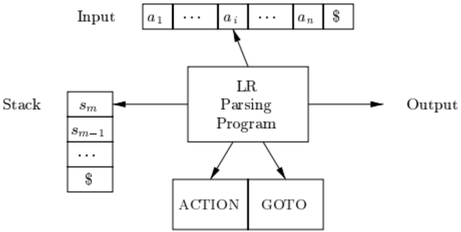
Algorithm for $LR(0)$ automaton construction

void *items*(G') {
    $C = \{\text{CLOSURE}(\{[S' \to \cdot S]\})\}$;
    repeat
        for (each set of items I in C)
            for (each grammar symbol X)
                if ($\text{GOTO}(I, X)$ is not empty and not in C)
                    add $\text{GOTO}(I, X)$ to C;
    until no new sets of items are added to C on a round;
}

The $LR(0)$ automaton



LR parsing algorithm



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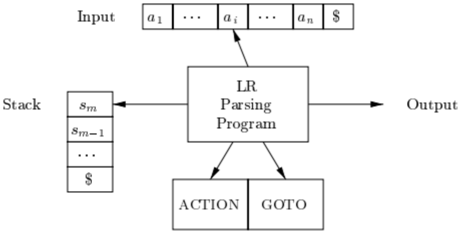

Shifts states rather than symbols on to the stack.

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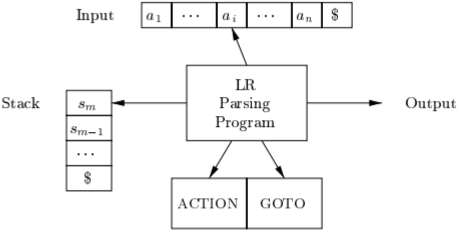

Shifts states rather than symbols on to the stack.
Each state except the start state has a grammar symbol associated with it. (HOW?)

LR parsing algorithm

Structure of the Parsing table

Definition 1.16

$ACTION[i, a]$

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3. Accept: S' at the top, \$ in input - Parser accepts and halts.
4. Error:

Definition 1.17

If $GOTO[I_i, A] = I_j$, GOTO maps a state i and variable A to state j.

LR parsing algorithm

Parser Configuration

It is a pair $(s_0 s_1 \ldots s_m, a_i a_{i+1} \ldots a_n)$ -
represents the right sentential form $X_1 X_2 \ldots X_m a_i a_{i+1} \ldots a_n$, where X_i is associated with s_i.

LR algorithm- Parser Behaviour

Let parser configuration be $(s_0 s_1 \ldots s_m, a_i a_{i+1} \ldots a_n)$

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Output - SLR parsing table with functions $ACTION$ and $GOTO$

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3. For variables A, if $GOTO(I_i, A) = I_j$, set $GOTO[i, A] = j$.
4. Entries not defined by above rules are errors.
5. The initial state of the parser is constructed from the set of items containing $[S' \to .S]$.

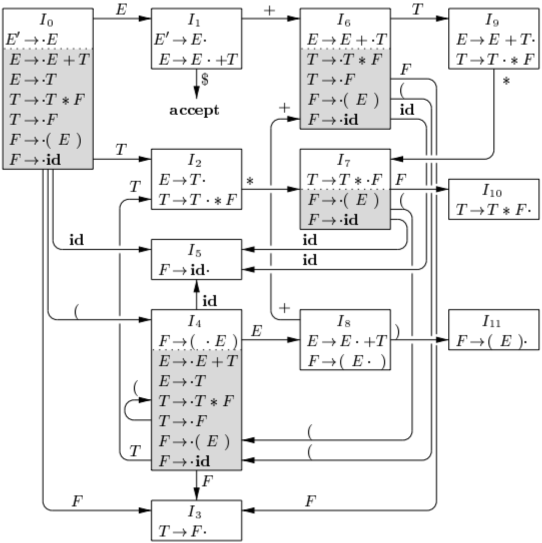
Example

(0) $E' \to E$ (4) $T \to F$
(1) $E \to E + T$ (5)$F \to (E)$
(2) $E \to T$ (6) $F \to id$

(3) $T \to T * F$

$FIRST(E') = FIRST(E) = FIRST(T) = FIRST(F) = \{(, id)\}$
$FOLLOW(E') = \{\$\}$, $FOLLOW(E) = \{\$,), +\}$

$FOLLOW(T) = \{\$,), +, *\}$



STATE	ACTIONS						GOTO		
	id	$+$	*	(	)	$	E	T	F

STATE	ACTIONS						GOTO		
	id	$+$	*	(	)	$	E	T	F
0	s5			s4			1	2	3

STATE	ACTIONS						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			

STATE	ACTIONS						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			

STATE	ACTIONS						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			

STATE	ACTIONS						GOTO		
	id	+	*	(	)	\$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3

STATE	ACTIONS						GOTO		
	id	$+$	$*$	(	)	\$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			

STATE	ACTIONS						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3

STATE	ACTIONS						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10

STATE	ACTIONS						GOTO		
	id	$+$	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				

STATE	ACTIONS						GOTO		
	id	$+$	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			

STATE	ACTIONS						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
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5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			

STATE	ACTIONS						GOTO		
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6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Moves of the SLR parser on $id * id + id$

	STACK	SYMBOLS	INPUT	ACTION
(1)	0		$\mathbf{id} * \mathbf{id} + \mathbf{id}\,\$$	shift (s5)
(2)	0 5	$\mathbf{id}$	$* \mathbf{id} + \mathbf{id}\,\$$	reduce by $F \to \mathbf{id}$ (r6)
(3)	0 3	F	$* \mathbf{id} + \mathbf{id}\,\$$	reduce by $T \to F$ (r4)
(4)	0 2	T	$* \mathbf{id} + \mathbf{id}\,\$$	shift (s7)
(5)	0 2 7	$T *$	$\mathbf{id} + \mathbf{id}\,\$$	shift (s5)
(6)	0 2 7 5	$T * \mathbf{id}$	$+ \mathbf{id}\,\$$	reduce by $F \to \mathbf{id}$ (r6)
(7)	0 2 7 10	$T * F$	$+ \mathbf{id}\,\$$	reduce by $T \to T * F$ (r3)
(8)	0 2	T	$+ \mathbf{id}\,\$$	reduce by $E \to T$ (r2)
(9)	0 1	E	$+ \mathbf{id}\,\$$	shift (s6)
(10)	0 1 6	$E +$	$\mathbf{id}\,\$$	shift (s5)
(11)	0 1 6 5	$E + \mathbf{id}$	$\$$	reduce by $F \to \mathbf{id}$ (r6)
(12)	0 1 6 3	$E + F$	$\$$	reduce by $T \to F$ (r4)
(13)	0 1 6 9	$E + T$	$\$$	reduce by $E \to E + T$ (r1)
(14)	0 1	E	$\$$	accept

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$I_2 \xrightarrow{=} I_6$

Show that the following grammar is $LL(1)$, but not $SLR(1)$

$S \to AaAb | BbBa$, $A \to \epsilon$, $B \to \epsilon$

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Neither $LL(1)$ is proper subset of $SLR(1)$ or viceversa.

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3. For variables A, if $GOTO(I_i, A) = I_j$, set $GOTO[i, A] = j$.
4. Entries not defined by above rules are errors.
5. The initial state of the parser is constructed from the set of items containing $[S' \rightarrow S]$.

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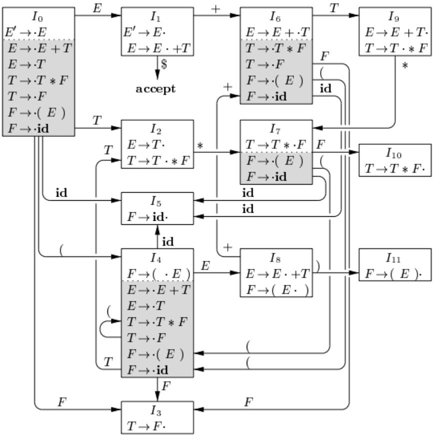
  2. if $\beta_2 = \epsilon$, then we have the handle at the top and its time to reduce.



In the grammar above, $E + T*$ is a viable prefix and the parser is at state 7 after reading $E + T*$. 7 contains items $T \to T * F$, $F \to (E)$, $F \to id$, which are valid items for $E + T*$.

$$
\begin{aligned}
E' &\underset{rm}{\rightarrow} E \\
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Problems with $SLR(1)$ parser

$$
\begin{array}{lll@{\qquad}lll}
I_0: & S' \to \cdot S & & I_5: & L \to \mathbf{id}\cdot \\
 & S \to \cdot L = R & & & \\
 & S \to \cdot R & & I_6: & S \to L = \cdot R \\
 & L \to \cdot * R & & & R \to \cdot L \\
 & L \to \cdot\mathbf{id} & & & L \to \cdot * R \\
 & R \to \cdot L & & & L \to \cdot\mathbf{id} \\
 & & & & \\
I_1: & S' \to S\cdot & & I_7: & L \to *R\cdot \\
 & & & & \\
I_2: & S \to L\cdot = R & & I_8: & R \to L\cdot \\
 & R \to L\cdot & & & \\
 & & & I_9: & S \to L = R\cdot \\
I_3: & S \to R\cdot & & & \\
 & & & & \\
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SLR parser at state 2 performs reduce "$R \to L$" on input $=$, as well as shift I_6 to $S \to L = .R$.

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However, no sentential form of the grammar begins with $R = \cdots$.

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I_0: $S' \to \cdot S$
$S \to \cdot L = R$
$S \to \cdot R$
$L \to \cdot * R$
$L \to \cdot \mathbf{id}$
$R \to \cdot L$

I_1: $S' \to S \cdot$

I_2: $S \to L \cdot = R$
$R \to L \cdot$

I_3: $S \to R \cdot$

I_4: $L \to * \cdot R$
$R \to \cdot L$
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I_5: $L \to \mathbf{id} \cdot$

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=

SLR parser at state 2 performs reduce "$R \to L$" on input $=$, as well as shift I_6 to $S \to L = .R$.
However, no sentential form of the grammar begins with $R = \cdots$.
Thus, state 2 corresponding to viable prefix L should not call for reduce "$R \to L$" on input $=$.

LR(1) items

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The look ahead has no effect on $[A \rightarrow \alpha.\beta, a]$ if $\beta \neq \epsilon$.
An item $[A \rightarrow \alpha., a]$ calls for reduction only if the next input symbol is a.
The set of such a's are a subset of $FOLLOW(A)$.

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$$S \to BB$$
$$B \to aB|b.$$

$S \underset{rm}{\overset{*}{\to}} aaBab \underset{rm}{\to} aaaBab$

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- $\delta = aa$
- $\alpha = a$
- $\beta = B$
- $A = aB$
- $w = ab$.

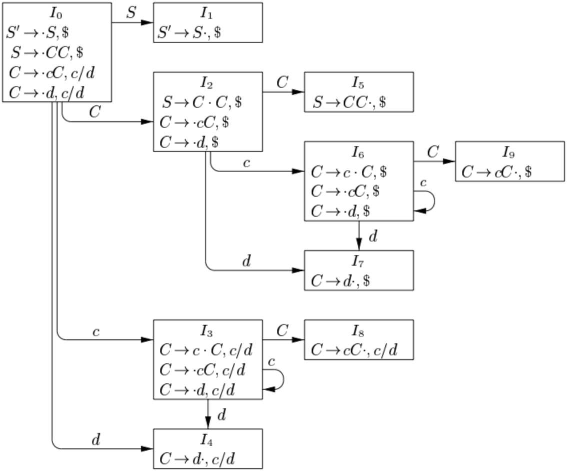
Constructing $LR(1)$ set of items

```
SetOfItems CLOSURE(I) {
    repeat
        for ( each item [A → α·Bβ, a] in I )
            for ( each production B → γ in G' )
                for ( each terminal b in FIRST(βa) )
                    add [B → ·γ, b] to set I;
    until no more items are added to I;
    return I;
}

SetOfItems GOTO(I, X) {
    initialize J to be the empty set;
    for ( each item [A → α·Xβ, a] in I )
        add item [A → αX·β, a] to set J;
    return CLOSURE(J);
}

void items(G') {
    initialize C to {CLOSURE({[S' → ·S, $]})};
    repeat
        for ( each set of items I in C )
            for ( each grammar symbol X )
                if ( GOTO(I, X) is not empty and not in C )
                    add GOTO(I, X) to C;
    until no new sets of items are added to C;
}
```

$$(0)\ S' \rightarrow S$$
$$(1)\ S \rightarrow CC$$
$$(2)\ S \rightarrow cC$$
$$(3)\ C \rightarrow d$$



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   - if $[S' \rightarrow S., \$]$ is in I_i, set $ACTION[i, \$] =$accept.
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   - if $[A \rightarrow \alpha.a\beta, b]$ is in I_i, and $GOTO(I_i, a) = I_j$, set $ACTION[i, a] = sj$, for terminal a.
   - if $[A \rightarrow \alpha., a]$ is in I_i, set $ACTION[i, a] = r`A \rightarrow \alpha'$.
   - if $[S' \rightarrow S., \$]$ is in I_i, set $ACTION[i, \$] =$accept.
     if any conflciting actions result from these rules, the grammar is not $SLR(1)$.
3. For variables A, if $GOTO(I_i, A) = I_j$, set $GOTO[i, A] = j$.

Construction of Cannonical $LR(1)$ parsing table

Input - Augmented grammar G'
Output - CLR(1) parsing table with functions $ACTION$ and $GOTO$

1. Construct $C = \{I_0, I_1, \ldots, I_n\}$, the collection of $LR(1)$ items. State i is constructed from I_i
2. $ACTION$ for state i
   - if $[A \rightarrow \alpha.a\beta, b]$ is in I_i, and $GOTO(I_i, a) = I_j$, set $ACTION[i, a] = sj$, for terminal a.
   - if $[A \rightarrow \alpha., a]$ is in I_i, set $ACTION[i, a] = r`A \rightarrow \alpha'$.
   - if $[S' \rightarrow S., \$]$ is in I_i, set $ACTION[i, \$] =$accept.
     if any conflciting actions result from these rules, the grammar is not $SLR(1)$.
3. For variables A, if $GOTO(I_i, A) = I_j$, set $GOTO[i, A] = j$.
4. Entries not defined by above rules are errors.

Construction of Cannonical $LR(1)$ parsing table

Input - Augmented grammar G'
Output - CLR(1) parsing table with functions $ACTION$ and $GOTO$

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   - if $[A \rightarrow \alpha.a\beta, b]$ is in I_i, and $GOTO(I_i, a) = I_j$, set $ACTION[i, a] = sj$, for terminal a.
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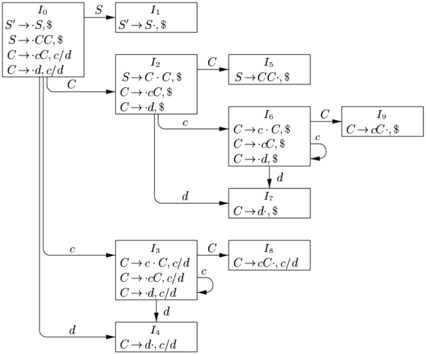
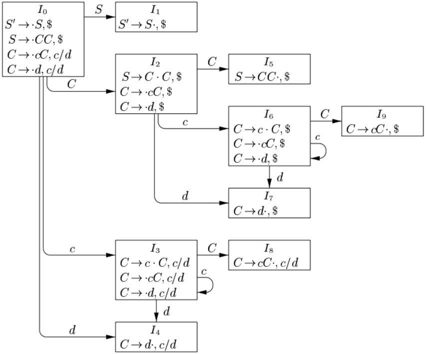
   - if $[S' \rightarrow S., \$]$ is in I_i, set $ACTION[i, \$] =$accept.
     if any conflciting actions result from these rules, the grammar is not $SLR(1)$.
3. For variables A, if $GOTO(I_i, A) = I_j$, set $GOTO[i, A] = j$.
4. Entries not defined by above rules are errors.
5. The initial state of the parser is constructed from the set of items containing $[S' \rightarrow S]$.



STATE	ACTION			GOTO	
	c	d	$\$$	S	C
0	s3	s4		1	2
1			acc		
2	s6	s7			5
3	s3	s4			8
4	r3	r3			
5			r1		
6	s6	s7			9
7			r3		
8	r2	r2			
9			r2		

LALR parser

The problem with $CLR(1)$ parsers is that the number of states is huge.
$LALR(1)$ parsers have the same number of states as $SLR(1)$



Difference between I_4 and I_7: Consider the string $c * dc * d$.

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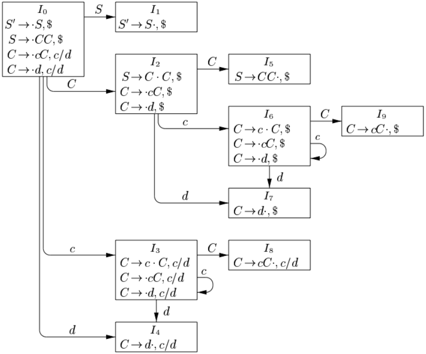

Difference between I_4 and I_7: Consider the string $c * dc * d$.

After seeing $c * d$, the parser enters the configuration $033\ldots34$.

LALR parser

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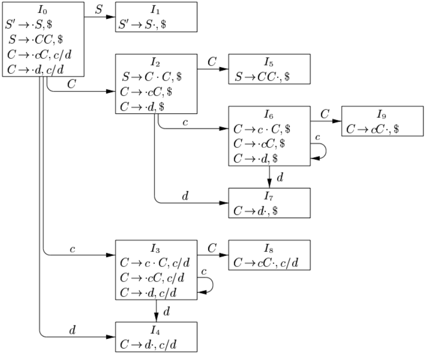
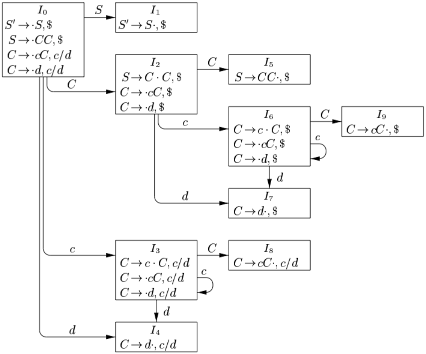
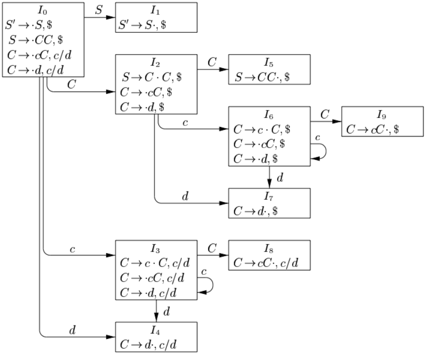


Difference between I_4 and I_7: Consider the string $c * dc * d$.
After seeing $c * d$, the parser enters the configuration $033 \ldots 34$.
On seeing a \$ next, the parser declares error

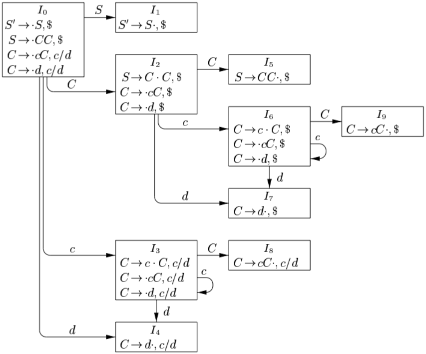


On seeing the next c, the parser reduces $[C \to d., c]$, pops 4 and pushes 8 to enter $033\ldots38$



On seeing the next c, the parser reduces $[C \to d., c]$, pops 4 and pushes 8 to enter $033\ldots38$

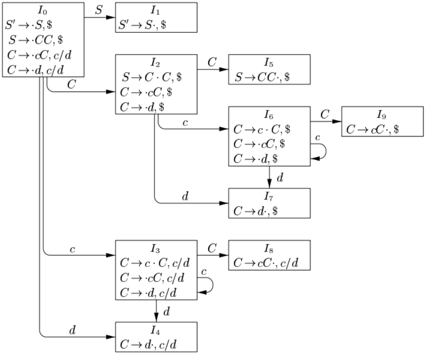
At 8, it reduces $[C \to cC., c]$ until all 3's are popped out: then it enters 02.



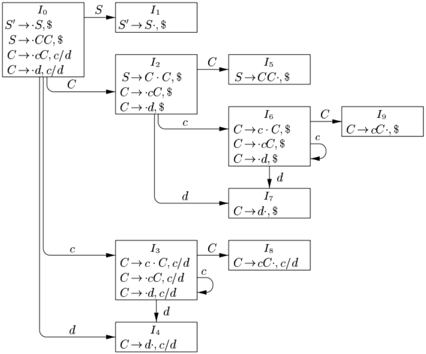
On seeing the next c, the parser reduces $[C \to d., c]$, pops 4 and pushes 8 to enter $033\ldots38$

At 8, it reduces $[C \to cC., c]$ until all 3's are popped out: then it enters 02.

It enters $0266\ldots67$ on seeing the next $c * d$.



On seeing the next c, the parser reduces $[C \to d., c]$, pops 4 and pushes 8 to enter $033\ldots38$
At 8, it reduces $[C \to cC., c]$ until all 3's are popped out: then it enters 02.
It enters $0266\ldots67$ on seeing the next $c * d$.
On seeing \$, it reduces $[C \to d., \$]$, enters $0266\ldots69$.



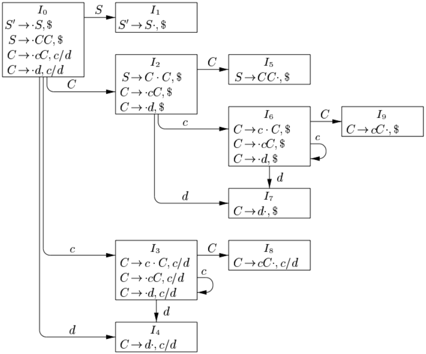
On seeing the next c, the parser reduces $[C \to d., c]$, pops 4 and pushes 8 to enter $033\ldots38$

At 8, it reduces $[C \to cC., c]$ until all 3's are popped out: then it enters 02.

It enters $0266\ldots67$ on seeing the next $c * d$.

On seeing \$, it reduces $[C \to d., \$]$, enters $0266\ldots69$.

At 9, it reduces $[C \to cC., \$]$ until all 6's are popped out: then it enters 025.



On seeing the next c, the parser reduces $[C \to d., c]$, pops 4 and pushes 8 to enter $033\ldots38$

At 8, it reduces $[C \to cC., c]$ until all 3's are popped out: then it enters 02.

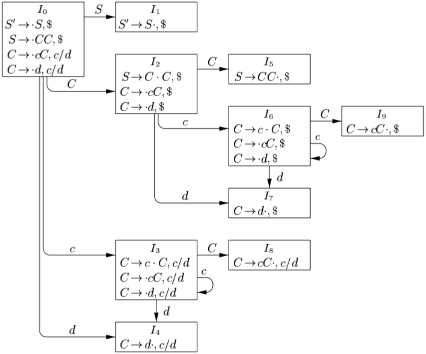
It enters $0266\ldots67$ on seeing the next $c * d$.

On seeing \$, it reduces $[C \to d., \$]$, enters $0266\ldots69$.

At 9, it reduces $[C \to cC., \$]$ until all 6's are popped out: then it enters 025.

It then reduces $[S \to CC., \$]$, pops 2 and 5, enters 01 where it accepts on \$

LALR set of items



Replace I_4 and I_7 by I_{47} - $[C \to d., c|d|\$]$
GOTO's to I_4 and I_7 non enters I_{47}.
ACTION of I_{47} is to reduce on $c, d, \$$.
On $ccd\$$ as input, the parser raises error if I_4 and I_7 are different; the parser reduces at I_{47}.

LALR set of items

Merging of states cannot introduce new shift-reduce conflict

Let $\{[A \rightarrow \alpha., a], [B \rightarrow \gamma.a\eta, b]\} \in I_{ij}$ for the new shift-reduce conflict.
Then, I_i must contain $[A \rightarrow \alpha., a]$ and $[B \rightarrow \gamma.a\eta, c]$: the shift-reduce conflict exists in I_i as well.

Core - the set of items in a state, not considering the lookaheads.

LALR parsing table construction

Input - Augmented grammar G'
Output - CLR(1) parsing table with functions $ACTION$ and $GOTO$

LALR parsing table construction

Input - Augmented grammar G'
Output - CLR(1) parsing table with functions *ACTION* and *GOTO*

1. Construct $C = \{I_0, I_1, \ldots, I_n\}$, the collection of $LR(1)$ items.

LALR parsing table construction

Input - Augmented grammar G'
Output - CLR(1) parsing table with functions *ACTION* and *GOTO*

1. Construct $C = \{I_0, I_1, \ldots, I_n\}$, the collection of $LR(1)$ items.
2. For each core, find the items with same core and merge. Let $C' = \{J_0, J_1, \ldots, J_n\}$ be the resulting set of items.

LALR parsing table construction

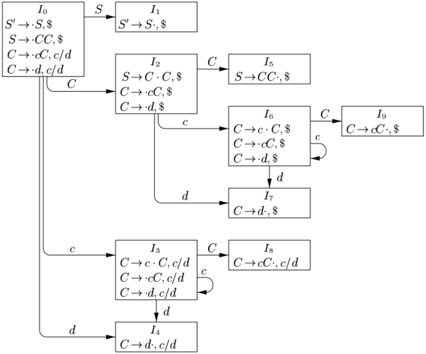
Input - Augmented grammar G'
Output - CLR(1) parsing table with functions $ACTION$ and $GOTO$

1. Construct $C = \{I_0, I_1, \ldots, I_n\}$, the collection of $LR(1)$ items.
2. For each core, find the items with same core and merge. Let $C' = \{J_0, J_1, \ldots, J_n\}$ be the resulting set of items.
3. $ACTION$ for state i: same as $CLR(1)$ parser.

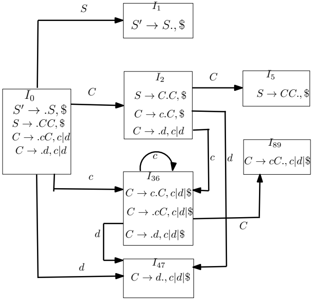
LALR parsing table construction

Input - Augmented grammar G'
Output - CLR(1) parsing table with functions $ACTION$ and $GOTO$

1. Construct $C = \{I_0, I_1, \ldots, I_n\}$, the collection of $LR(1)$ items.
2. For each core, find the items with same core and merge. Let $C' = \{J_0, J_1, \ldots, J_n\}$ be the resulting set of items.
3. $ACTION$ for state i: same as $CLR(1)$ parser.
4. If $J = \{I_1, \ldots, I_l\}$. Then $GOTO(J, X) = K$, where K has the same core as $GOTO(I_1, X)$.



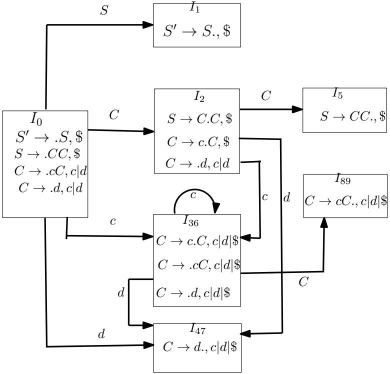
I_0
$S' \to \cdot S, \$$
$S \to \cdot CC, \$$
$C \to \cdot cC, c/d$
$C \to \cdot d, c/d$
S
I_1
$S' \to S\cdot, \$$
C
I_2
$S \to C \cdot C, \$$
$C \to \cdot cC, \$$
$C \to \cdot d, \$$
C
I_5
$S \to CC\cdot, \$$
c
I_6
$C \to c \cdot C, \$$
$C \to \cdot cC, \$$
$C \to \cdot d, \$$
C
I_9
$C \to cC\cdot, \$$
c
d
d
I_7
$C \to d\cdot, \$$
c
I_3
$C \to c \cdot C, c/d$
$C \to \cdot cC, c/d$
$C \to \cdot d, c/d$
C
I_8
$C \to cC\cdot, c/d$
c
d
d
I_4
$C \to d\cdot, c/d$



S
I_1
$S' \to S., \$$
I_0
$S' \to .S, \$$
$S \to .CC, \$$
$C \to .cC, c|d$
$C \to .d, c|d$
C
I_2
$S \to C.C, \$$
$C \to c.C, \$$
$C \to .d, c|d$
C
I_5
$S \to CC., \$$
c
d
I_{89}
$C \to cC., c|d|\$$
c
c
I_{36}
$C \to c.C, c|d|\$$
$C \to .cC, c|d|\$$
$C \to .d, c|d|\$$
C
d
d
I_{47}
$C \to d., c|d|\$$

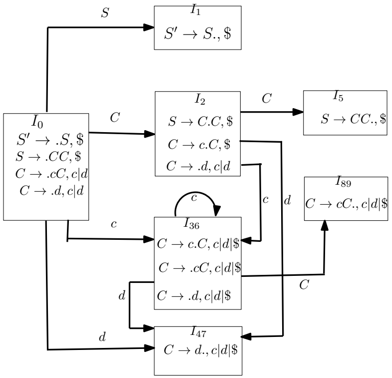
LALR parsing table

STATE	ACTION			GOTO	
	c	d	$\$$	S	C
0	s36	s47		1	2
1			acc		
2	s36	s47			5
36	s36	s47			89
47	r3	r3	r3		
5			r1		
89	r2	r2	r2		



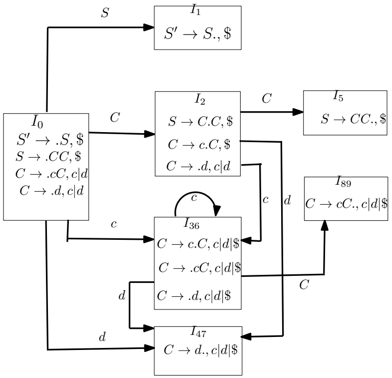
Sample input: *ccdcd*\$

0



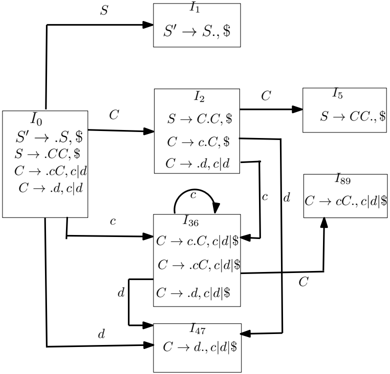
Sample input: $ccdcd\$$

$0 \xrightarrow{c} 036$



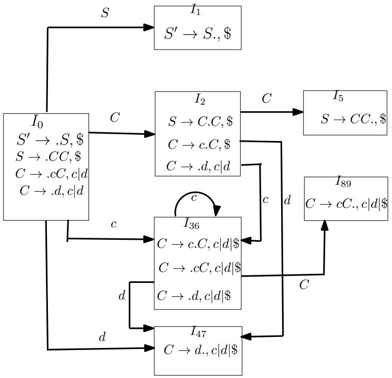
Sample input: *ccdcd*\$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636$



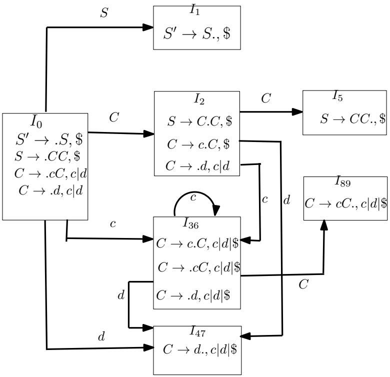
Sample input: $ccdcd\$$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636 \xrightarrow{d} 0363647$



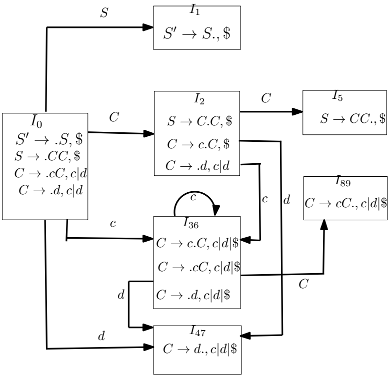
Sample input: *ccdcd*\$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636 \xrightarrow{d} 0363647 \xrightarrow{reduce C \to d} 0363689$



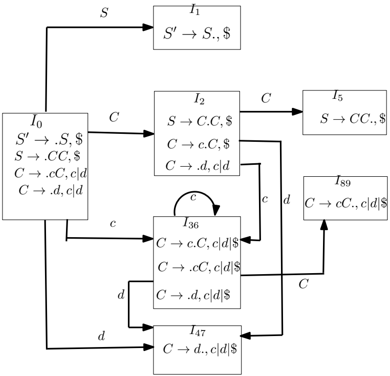
Sample input: *ccdcd*\$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636 \xrightarrow{d} 0363647 \xrightarrow{reduce C\to d} 0363689 \xrightarrow{reduce C\to cC}$
03689



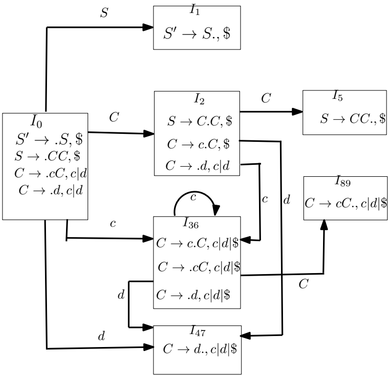
Sample input: *ccdcd*\$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636 \xrightarrow{d} 0363647 \xrightarrow{reduce C \to d} 0363689 \xrightarrow{reduce C \to cC}$
$03689 \xrightarrow{reduce C \to cC} 02$



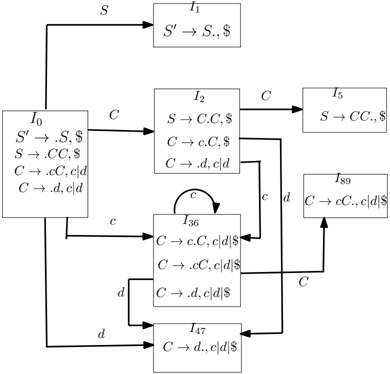
Sample input: *ccdcd*\$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636 \xrightarrow{d} 0363647 \xrightarrow{reduce C \to d} 0363689 \xrightarrow{reduce C \to cC}$
$03689 \xrightarrow{reduce C \to cC} 02 \xrightarrow{c} 0236$



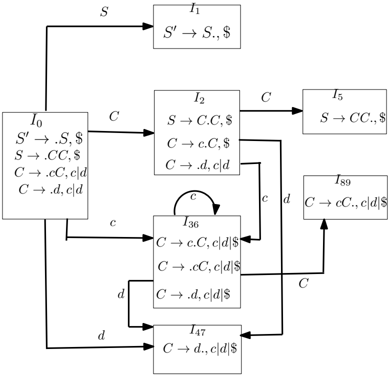
Sample input: *ccdcd*\$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636 \xrightarrow{d} 0363647 \xrightarrow{reduce C\to d} 0363689 \xrightarrow{reduce C\to cC}$
$03689 \xrightarrow{reduce C\to cC} 02 \xrightarrow{c} 0236 \xrightarrow{d} 023647$



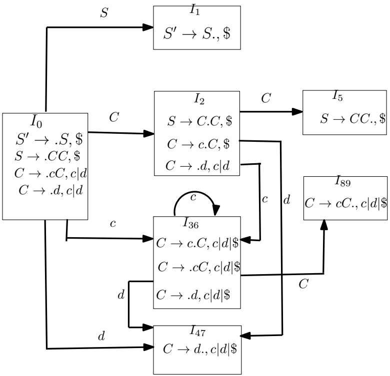
Sample input: *ccdcd*\$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636 \xrightarrow{d} 0363647 \xrightarrow{reduce C \to d} 0363689 \xrightarrow{reduce C \to cC}$
$03689 \xrightarrow{reduce C \to cC} 02 \xrightarrow{c} 0236 \xrightarrow{d} 023647 \xrightarrow{reduce C \to d}$
023689



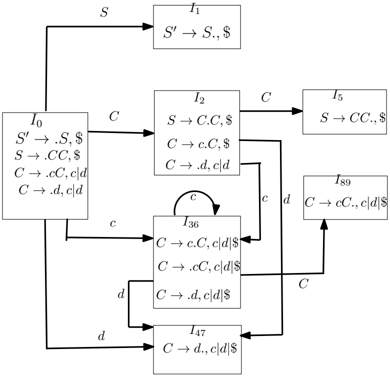
Sample input: *ccdcd*\$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636 \xrightarrow{d} 0363647 \xrightarrow{reduce C \to d} 0363689 \xrightarrow{reduce C \to cC}$
$03689 \xrightarrow{reduce C \to cC} 02 \xrightarrow{c} 0236 \xrightarrow{d} 023647 \xrightarrow{reduce C \to d}$
$023689 \xrightarrow{reduce C \to cC} 025$



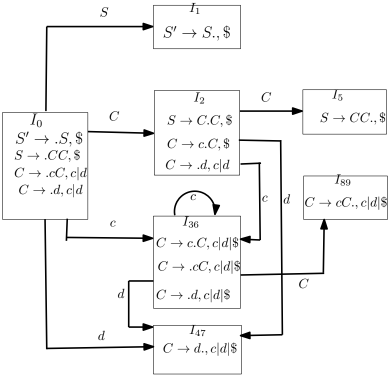
Sample input: *ccdcd*\$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636 \xrightarrow{d} 0363647 \xrightarrow{reduce C \to d} 0363689 \xrightarrow{reduce C \to cC}$
$03689 \xrightarrow{reduce C \to cC} 02 \xrightarrow{c} 0236 \xrightarrow{d} 023647 \xrightarrow{reduce C \to d}$
$023689 \xrightarrow{reduce C \to cC} 025 \xrightarrow{\$} accept.$



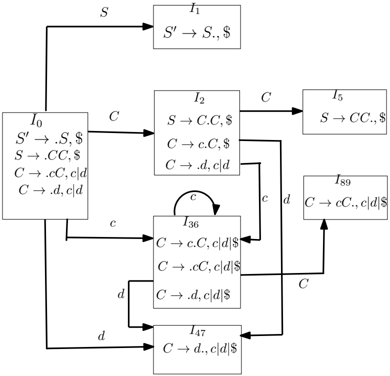
Sample input: *ccd*\$

0



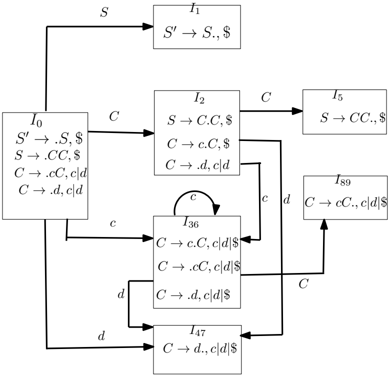
Sample input: *ccd*\$

$0 \xrightarrow{c} 036$



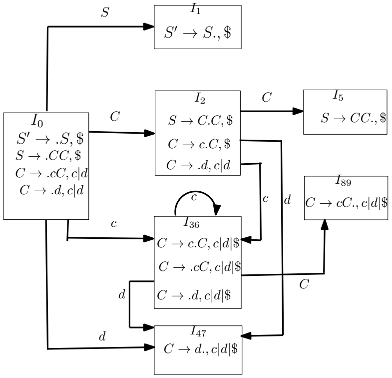
Sample input: *ccd*\$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636$



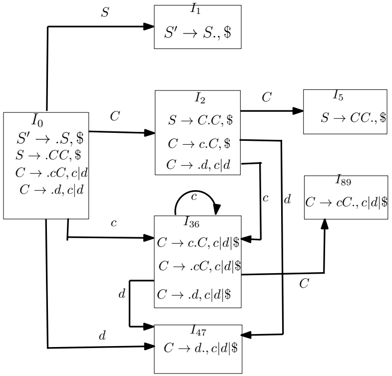
Sample input: *ccd*\$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636 \xrightarrow{d} 0363647$



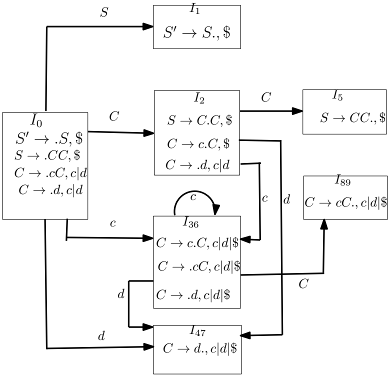
Sample input: *ccd*\$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636 \xrightarrow{d} 0363647 \xrightarrow{reduce C \to d} 0363689$



Sample input: *ccd*\$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636 \xrightarrow{d} 0363647 \xrightarrow{reduce C \to d} 0363689 \xrightarrow{reduce C \to cC}$ 03689



Sample input: *ccd*\$

$0 \xrightarrow{c} 036 \xrightarrow{c} 03636 \xrightarrow{d} 0363647 \xrightarrow{reduce C \to d} 0363689 \xrightarrow{reduce C \to cC}$
$03689 \xrightarrow{reduce C \to cC} 02 \xrightarrow{\$} error.$

Parser generator: Bison

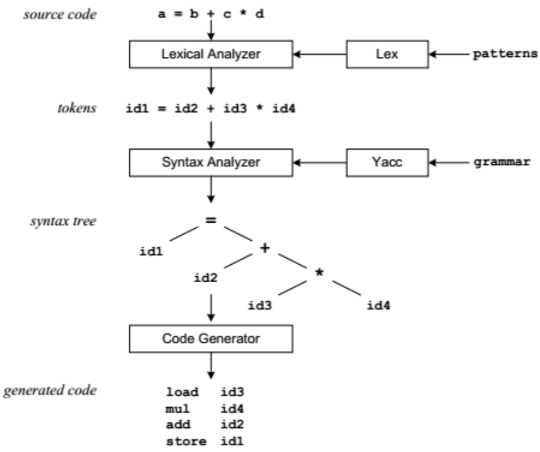


Figure 1: Compilation Sequence

Parser generator: Bison

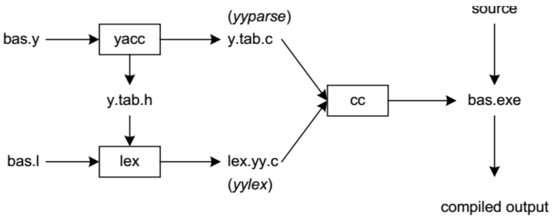


Figure 2: Building a Compiler with Lex/Yacc

Figure 2 illustrates the file naming conventions used by lex and yacc. We'll assume our goal is to write a BASIC compiler. First, we need to specify all pattern matching rules for lex (**bas.l**) and grammar rules for yacc (**bas.y**). Commands to create our compiler, **bas.exe**, are listed below:

bison -d bas.y #creates bas.tab.c and bas.tab.h
flex bas.l #creates lex.yy.c
gcc bas.tab.c lex.yy.c -o bas.exe #creates the executable file bas.exe

Example

(1) $E \to E + E$
(2) $E \to E * E$

(3) $E \to id$

(r1) $E \to E + E$
(r2) $\to E + E * E$
(r3) $\to E + E * id$
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(r3) $\to id + id * id$

1	. x + y * z	shift	
2	x . + y * z	reduce(r3)	
3	E . + y * z	shift	
4	E + . y * z	shift	
5	E + y . * z	reduce(r3)	
6	E + E . * z	shift	
7	E + E * . z	shift	
8	E + E * z .	reduce(r3)	
9	E + E * E .	reduce(r2)	emit multiply
10	E + E .	reduce(r1)	emit add
11	E .	accept	

The grammar is ambiguous and has shift-reduce conflict.

Shift-reduce conflict: Yacc chooses shift over reduce.
Reduce-reduce conflict: Yacc chooses the first production in the listing.

```
... definitions ...
%%
... rules ...
%%
... subroutines ...
```

The definitions section consists of token declarations and C code bracketed by "%{" and "%}".
The BNF grammar is placed in the rules section and user subroutines are added in the subroutines section.

Bison Stacks

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- For example, when lex returns an INTEGER token, yacc shifts this token to the parse stack.
- At the same time the corresponding yylval is shifted to the value stack.
- The parse and value stacks are always synchronized so finding a value related to a token on the stack is easily accomplished.

Bison program for calculator

```
%{
    #include <stdio.h>
    int yylex(void);
    void yyerror(char *);
%}

%token INTEGER

%%

program:
        program expr '\n'         { printf("%d\n", $2); }
        |
        ;

expr:
        INTEGER                   { $$ = $1; }
        | expr '+' expr           { $$ = $1 + $3; }
        | expr '-' expr           { $$ = $1 - $3; }
        ;

%%

void yyerror(char *s) {
    fprintf(stderr, "%s\n", s);
}

int main(void) {
    yyparse();
    return 0;
```

With left-recursion, we have specified that a program consists of zero or more expressions. Each expression terminates with a newline. When a newline is detected we print the value of the expression.

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%%

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    fprintf(stderr, "%s\n", s);
}

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```

When we apply the rule expr: expr '+' expr $\{$ \$\$ $=$ \$1 $+$ \$3; $\}$ we replace the right-hand side of the production in the parse stack with the left-hand side of the same production. In this case we pop expr '+' expr and push expr. We have reduced the stack by popping three terms off the stack and pushing back one term.

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%%

void yyerror(char *s) {
    fprintf(stderr, "%s\n", s);
}

int main(void) {
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```

“\$1” denotes the first term on the right-hand side of the production, “\$2” for the second, and so on. “\$\$” designates the top of the stack after reduction has taken place. The above action adds the value associated with two expressions, pops three terms off the value stack, and pushes back a single sum.

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    int yylex(void);
    void yyerror(char *);
%}

%token INTEGER

%%

program:
        program expr '\n'         { printf("%d\n", $2); }
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        INTEGER                   { $$ = $1; }
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%%

void yyerror(char *s) {
    fprintf(stderr, "%s\n", s);
}

int main(void) {
    yyparse();
    return 0;
```

%token INTEGER
This definition declares an INTEGER token.
Yacc generates a parser in file y.tab.c and an include file, y.tab.h:
#ifndef YYSTYPE
#define YYSTYPE int
#endif
#define INTEGER 258
extern YYSTYPE yylval;

Lex program for calculator

```
%{
#include <stdlib.h>
void yyerror(char *);
#include "y.tab.h"
%}

%%

[0-9]+      {
                yylval = atoi(yytext);
                return INTEGER;
            }

[-+\n]      return *yytext;

[ \t]       ; /* skip whitespace */

.           yyerror("invalid character");

%%

int yywrap(void) {
    return 1;
}
```

Lex includes this file and utilizes the definitions for token values.
To obtain tokens yacc calls yylex.
Function yylex has a return type of int that returns a token.

Values associated with the token are returned

by lex in variable yylval.