1) What are Eigenvalues and Eigenvectors? How are they related to the Eigen-Decomposition approach?

Explain with an example.

Ans- In Mathematics, an eigenvector corresponds to the real non zero eigenvalues which point in the direction stretched by the transformation whereas eigenvalue is considered as a factor by which it is stretched. In case, if the eigenvalue is negative, the direction of the transformation is negative.

Eigendecomposition of matrix: eigenvalue and eigenvector  
  
The same methodology in math. An integer can be decomposed into prime factors, for example, 20 = 2 \* 2 \* 5, which means 20 is not divisible by 3, and any integer multiple of 20 will be divisible by 5.

2) What is eigen decomposition and what is its significance in linear algebra?

Ans- In linear algebra, eigendecomposition is the factorization of a matrix into a canonical form, whereby the matrix is represented in terms of its eigenvalues and eigenvectors. Only diagonalizable matrices can be factorized in this way.

Eigenvalues are associated with eigenvectors in Linear algebra. Both terms are used in the analysis of linear transformations. Eigenvalues are the special set of scalar values that is associated with the set of linear equations most probably in the matrix equations.

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Ans- The eigenvectors of A are associated to an eigenvalue. Hence, if λ1 is an eigenvalue of A and AX=λ1X, we can label this eigenvector as X1. Note again that in order to be an eigenvector, X must be nonzero. There is also a geometric significance to eigenvectors.

Eigenvalues and Eigenvectors have their importance in linear differential equations where we want to find a rate of change or when we want to maintain relationships between two variables. Additionally, eigenvectors and eigenvalues are used in facial recognition techniques such as EigenFaces.

4) What is the significance of the spectral theorem in the context of the Eigen-Decomposition approach? How is it related to the diagonalizability of a matrix? Explain with an example.

Ans- The spectral theorem provides a sufficient criterion for the existence of a particular canonical form. Specifically, the spectral theorem states that if M equals the transpose of M, then M is diagonalizable: there exists an invertible matrix C such that C − 1 M C C^{-1} MC C−1MC is a diagonal matrix.

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5) How do you find the eigenvalues of a matrix and what do they represent?

Ans- To find the eigenvalues of a square matrix A: Find its characteristic equation using |A - λI| = 0, where I is the identity matrix of same order A. Solve it for λ and the solutions would give the eigenvalues.

Eigenvalues represent magnitude, or importance. Bigger Eigenvalues correlate with more important directions. Finally, we make an assumption that more variability in a particular direction correlates with explaining the behavior of the dependent variable

6) What are eigenvectors and how are they related to eigenvalues?

Ans- In Mathematics, an eigenvector corresponds to the real non zero eigenvalues which point in the direction stretched by the transformation whereas eigenvalue is considered as a factor by which it is stretched. In case, if the eigenvalue is negative, the direction of the transformation is negative.

In simple terms, eigenvalues and eigenvectors are the building blocks of linear transformations. Eigenvalues represent the scaling factor by which a vector is transformed when a linear transformation is applied, while eigenvectors represent the directions in which the transformation occurs

7) Can you explain the geometric interpretation of eigenvectors and eigenvalues?

Ans- Geometrically, an eigenvector, corresponding to a real nonzero eigenvalue, points in a direction in which it is stretched by the transformation and the eigenvalue is the factor by which it is stretched. If the eigenvalue is negative, the direction is reversed.

8) What are some real-world applications of eigen decomposition?

Ans- Eigendecomposition provides us with a tool to decompose a matrix by discovering the eigenvalues and the eigenvectors. This operation can prove useful since it allows certain matrix operations to be easier to perform and it also tells us important facts about the matrix itself.

It is used in car design especially car stereo system and also in decoupling three phase system. Eigendecomposition is particularly useful for analysing the structure of the data matrix in terms of the eigenvalues and eigenvectors.

9) Can a matrix have more than one set of eigenvectors and eigenvalues?

Ans- Since a nonzero subspace is infinite, every eigenvalue has infinitely many eigenvectors. (For example, multiplying an eigenvector by a nonzero scalar gives another eigenvector.) On the other hand, there can be at most n linearly independent eigenvectors of an n × n matrix, since R n has dimension n .

10) In what ways is the Eigen-Decomposition approach useful in data analysis and machine learning?

Discuss at least three specific applications or techniques that rely on Eigen-Decomposition.

Ans- Eigendecomposition provides us with a tool to decompose a matrix by discovering the eigenvalues and the eigenvectors. This operation can prove useful since it allows certain matrix operations to be easier to perform and it also tells us important facts about the matrix itself

This scaler multiple is known as Eigenvalue. Eigenvectors and Eigenvalues are key concepts used in feature extraction techniques such as Principal Component Analysis which is an algorithm used to reduce dimensionality while training a machine learning model.

Eigenvalues and eigenvectors are used for: Computing prediction and confidence ellipses. Principal Components Analysis (later in the course) Factor Analysis (also later in this course)