Homework 1: Learning Julia through Time Series

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This assignment is intended to help you get used to coding in Julia while refreshing some basic time series concepts. I want to challenge you to minimize the use of ChatGPT while working through this homework.

Question 1: Installing Packages
Install the following Julia packages: `Revise, Plots, QuantEcon, Parameters, SymEngine, Random, LinearAlgebra, StatsBase, BasisMatrices, ForwardDiff, DataFrames`. Some packages, such as `Plots`, will take longer to install than others, such as `LinearAlgebra`.

Question 2: Quadratic Filtering
One way to filter out the trend in a time series is to assume that the series follows a quadratic trend. Let y_t be log of some time series (real output per capita for example). Then this can be decomposed into a trend component y_t^τ and a cyclical component y_t^c.

$$y_t = y_t^\tau + y_t^c \tag{1}$$

If we assume that a series follows a quadratic trend, then

$$y_t = a + bt + ct^2 + \epsilon_t \tag{2}$$
$$y_t^\tau = a + bt + ct^2 \tag{3}$$
$$y_t^c = \epsilon_t \tag{4}$$

1. Using CSV and DataFrames package, read *data.csv* and call that variable `df`. You can do this by `DataFrame(CSV.File("data.csv"))`. The file contains the log of output, the log of consumption, and the log of investment for the US from roughly 1980Q1 to 2020Q4.

2. What is the length of the array that stores log of output? Use size(df,1) to find out. Store this variable as T.

3. Create a dummy year variable that ranges from 1 to T. You should have an array going from $1, 2, 3, ..., T$. Call that variable something like `year`. Create another array, call it `yearsq`, that squares each element of `year` so that it's $1, 4, 9, ..., T^2$.

4. Now only take the output column, which you can extract it by `df[:,:y]`, and decompose the log of output into a trend and a cyclical component using a quadratic filter.

    - To do this, first, create a function called `ols(Y, X)` that takes any column vector Y and a generic (n by k) matrix X and returns $\hat{\beta}$, try to see if you can also return the variance-covariance matrix V.

- Regress log of output Y on a vector of constant, year, and yearsq. Generate predicted values, which correspond to the trend component. Taking the difference between the predicted value, $\hat{Y}$, and Y is the cyclical component.
- Using a Plots package, plot the log of output and the trend component in one figure. Make sure to label the lines. Create a separate plot that plots the cyclical component. Can you combine the two figures so that there are two subplots?

5. Now, repeat the previous steps to decompose consumption and investment as well. Import StatsBase package by `Using StatsBase`. Using only the cyclical components, calculate standard deviations, first-order autocorrelations, and correlations with output for y, c, i in Julia. No need to put it in a fancy table, just make sure to display answers using `println`.

Question 3: AR(1) Process

Let y_t be income for an individual. One simple way to model an income process is through an autoregressive model of order 1. This can be written as

$$y_t = (1-\rho)\mu + \rho y_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2) \tag{5}$$

where ρ is a parameter that controls persistence, μ is the mean, σ^2 is the variance of innovation term.

1. What is the mean and the variance for y_t? (Try using a text cell in Jupyter to answer this question.)
2. *White noise process* Set $\sigma^2 = 0.01$. Using a *randn* command, draw ϵ_t for 100 periods and plot them. Notice that *randn* draws from standard normals. How could you manipulate the simulated series so that the resulting series has a variance of 0.01?
3. *Simulation* Set $\mu = 0.6, \rho = 0.6, y_0 = \mu$. Using a similar procedure as above, simulate the above process for $T = 10, 100, 1000$. Calculate what the mean and the variance of y are for each of the cases. What happens as you increase T?
4. Using a simulated dataset from the previous problem for $T = 1000$, get the estimated $\hat{\rho}$ and $\hat{\mu}$.
5. *Impulse Response Function* Suppose we are at $y_0 = \mu = 0$ and we receive a stimulus check of σ. We do not foresee another stimulus check in the near future. Plot the path of y_t for the next 48 periods when $\rho = 0.6$. In that same graph, plot what the path will be when $\rho = 0.4$ and $\rho = 0.99$. Make sure to label each line.