Homework 2: International Real Business Cycle

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In this assignment, we introduce an additional country to the model and study its implications. In this economy, the world consists of two countries: Home and Foreign. There is a representative agent in both countries producing the same goods that are traded across the border. There is a one-period bond that is traded at price P_t^B and pays one unit of final goods in the next period. The home country with a population share of π solves the following problem:

$$\max_{C_t, K_{t+1}, B_{t+1}, H_t} \quad \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \kappa \frac{H_t^{1+1/\psi}}{1+1/\psi} \right),$$

s.t.

$$C_t + K_{t+1} + P_t^B B_{t+1} = Z_t K_t^{\alpha} H_t^{1-\alpha} + (1-\delta)K_t + B_t - \frac{\phi_k}{2}(K_{t+1} - K_t)^2 - \frac{\phi_b}{2} P_t^B (B_{t+1} - \bar{B})^2.$$

The foreign country with a population share of $(1-\pi)$ solves an analogous problem:

$$\max_{C_t^*, K_{t+1}^*, B_{t+1}^*, H_t^*} \quad \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{*1-\gamma}}{1-\gamma} - \kappa^* \frac{H_t^{*1+1/\psi}}{1+1/\psi} \right),$$

s.t.

$$C_t^* + K_{t+1}^* + P_t^B B_{t+1}^* = Z_t^* K_t^{*\alpha} H_t^{*1-\alpha} + (1-\delta)K_t^* + B_t^* - \frac{\phi_k}{2}(K_{t+1}^* - K_t^*)^2 - \frac{\phi_b}{2} P_t^B (B_{t+1} - \bar{B}^*)^2.$$

The bond market clears every single period:

$$\pi B_t + (1-\pi) B_t^* = 0,$$

and Z_t and Z_t^* follow VAR(1) process in logs:

$$\ln \mathbf{Z}_t = \begin{bmatrix} \ln Z_{t+1} \\ \ln Z_{t+1}^* \end{bmatrix} = \begin{bmatrix} \rho & \tilde{\rho} \\ \tilde{\rho} & \rho^* \end{bmatrix} \begin{bmatrix} \ln Z_t \\ \ln Z_t^* \end{bmatrix} + \boldsymbol{\varepsilon}_{t+1}, \quad \boldsymbol{\varepsilon} \sim N(\mathbf{0}, V),$$

where $\tilde{\rho}$ is a parameter that captures the spillover between Home and Foreign countries and V is the covariance matrix of innovations between the home and foreign countries.

1. What are the state variables? What are the control variables?
2. What does the bond market clearing condition imply about the goods market clearing condition?
3. Set the parameter values as follows: $\beta = 0.98$, $\rho = \rho^* = 0.999$, $\tilde{\rho} = 0.0$, $\phi_k = 0.1$, $\alpha = 0.33$, $\delta = 0.025$, $\gamma = 2$, $\psi = 1$, $\pi = 0.7$. Furthermore, set $\bar{B} = \bar{B}^* = 0$ and $\phi_b = 0.0001$. Now solve

for the steady state. However, this time, set the steady-state labor $H = H^* = 1/3$ and solve for κ and κ^* that are consistent with the steady-state labor hours of 1/3.

4. Solve the model and report linear decision rules for the state variables. You may either use Matlab or Julia to solve the model.

5. Plot impulse responses of output, consumption, investment, and hours in both countries to a positive productivity shock in the home country. Create a 2x2 panel of figures, with each figure containing two lines, one for each country. How does the home country's productivity shock affect variables in the foreign country?

6. Simulate the model for 1500 periods and drop the first 500 periods. HP filter all variables using `hp_filter` function from the QuantEcon package. Report second moments of output, consumption, and investment, including cross-country correlations.

7. Now set $\rho = \rho^* = 0.9$ and re-solve the model. Re-simulate and re-calculate second moments, including cross-country correlations. What are the similarities? Differences? Why?