Due Date: 05/09/2025

Instructions: Please follow the guideline in *Assignments* section of the syllabus. To get full credit, you must show all your work. While submitting your homework, you need to submit 1) a document summarizing your solutions (the math programming models, calculations and/or the outputs of the codes) and 2) all codes as separate files (including .dat, .mod, .run and .out files). Each problem is 25 points. Upload your homework to canvas as a soft copy with the codes or handwritten calculations. Please submit them separately and do not zip files.

1. A telecommunications operator must decide how much capacity (in Gbps) to add at each of its regional base-station sites. For each site, the cost of adding capacity follows a three-tier pricing structure:

   - Tier 0 covers small upgrades (up to 5 Gbps) at a low per-Gbps rate.
   - Tier 1 covers medium upgrades (greater than 5 and up to 10 Gbps) at a moderate per-Gbps rate, with a fixed offset to maintain continuity.
   - Tier 2 covers large upgrades (greater than 10 Gbps up to 15 Gbps) at a high per-Gbps rate, again with an offset for continuity.

   We give the rate as per-Gbps costs $\mathbf{c}_{ij}$ (in dollars):

Site i	$\mathbf{c}_{i0}$	$\mathbf{c}_{i1}$	$\mathbf{c}_{i2}$
1	100	120	150
2	90	110	140
3	95	115	145
4	105	125	155
5	85	105	135

   Table 1: Per-Gbps Costs

   And the fixed offsets cost $\mathbf{d}_{ij}$ (in dollars) as:

Site i	$\mathbf{d}_{i0}$	$\mathbf{d}_{i1}$	$\mathbf{d}_{i2}$
1	0	250	500
2	0	200	450
3	0	220	470
4	0	270	520
5	0	180	430

   Table 2: fixed offsets cost

   The total added capacity for all sites should be larger than the demand $D = 30$. The goal is to determine the capacity to add at each site.

   (a) Minimize the total upgrade cost among all sites. Formulate the model.

   (b) Enter and solve the model in AMPL.

   (c) Minimize the largest single-site upgrade cost. Modify the model in (a).

   (d) Enter and solve the model in AMPL.

2. For the cybersecurity network design, each node in the graph below represents a server, and each edge represents a direct secure communication link between two servers.

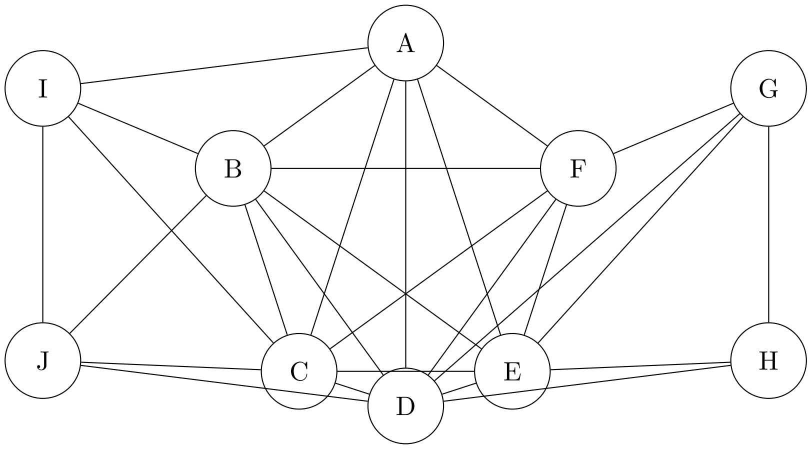

Figure 1: Network of servers with secure communication links.

(a) Companies often need to identify a subset of computers or servers that are fully connected with secure communication links. Such a subset should form a *clique* in the network graph. The objective is to maximize the number of servers included. Formulate the model as a maximum clique problem.

(b) Enter and solve the model in AMPL.

(c) Now, assume that each server needs monitoring. The company has a set of sensor configurations (predefined coverage zones), each of which covers a subset of servers within communication range. Each sensor configuration incurs a cost. The goal is to minimize the total cost, subject to every node is covered by exactly one selected sensor configuration. The sensor configurations and costs of each zone are given in Table 3. Formulate the model.

Zone	**Covered Nodes**	**Cost** ($\mathbf{c}_z$)
1	{A, B, D}	40
2	{C, E}	50
3	{B, C, D, I}	35
4	{E, F, G}	45
5	{F, H}	38
6	{G, I, J}	42

Table 3: Sensor configurations and costs

(d) Enter and solve your model in AMPL.

3. A delivery company must ship a quantity $\mathbf{D}_i$ of items i to a customer using different transportation modes. Each shipping mode $j \in \mathcal{J}$ has a cost per unit $\mathbf{c}_{ij}$ and a delivery time per unit $\mathbf{s}_{ij}$ for each type of item.

   If the weighted average delivery time exceeds a deadline t, a fixed penalty p is incurred. We give the data as follows:

   (a) The company aims to minimize the total shipping cost and the total penalty cost. Formulate the model.

   (b) Enter and solve the model in AMPL.

   (c) Each item should be shipped by exactly 2 modes and you can assume at least one unit of items is shipped if the mode is chosen. Modify the model in (a).

Item i	Demand $\mathbf{D}_i$	Deadline $\mathbf{t}_i$ (hours)	Penalty $\mathbf{p}_i$
1	50	100	1000
2	70	120	800
3	40	90	1200

Table 4: Item demands, deadlines, and penalties

Shipping cost $\mathbf{c}_{ij}$	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Item 1	5	8	4	7	6
Item 2	6	7	5	8	7
Item 3	5	9	4	6	5
Shipping time $\mathbf{s}_{ij}$					
Item 1	20	15	25	18	22
Item 2	19	14	24	17	20
Item 3	21	13	23	16	19

Table 5: Cost per unit

(d) Enter and solve your model in AMPL.

4. (Modified Version of Problem 2.39, Chapter 2 of Rardin) To improve tax compliance, the Texas Comptroller's staff regularly audits at corporate home offices the records of out-of-state corporations doing business in Texas. Texas is considering the opening of a series of small offices near these corporate locations to reduce the travel costs now associated with such out-of-state audits. The following table shows the fixed cost (in thousands of dollars) of operating such offices at 5 sites i, the number of audits required in each of 5 states j, and the travel cost (in thousands of dollars) per audit performed in each state from a base at any of the proposed office sites. We seek a minimum total cost auditing plan.

		Cost to Audit Tax Corporate Location:							
Tax Site	Fixed Cost	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	160	0	0.4	0.8	0.4	0.8	0.3	0.6	0.5
2	49	0.7	0	0.8	0.4	0.4	0.5	0.3	0.6
3	246	0.6	0.4	0	0.5	0.4	0.6	0.5	0.6
4	86	0.6	0.4	0.9	0	0.4	0.4	0.2	0.4
5	100	0.9	0.4	0.7	0.4	0	0.7	0.3	0.5
Audits		400	100	300	100	200	300	200	400

Table 6: Tax Sites

Now we have an additional objective: Maximize the overall service quality. Assume the objective 1 (minimize the cost) has the weight w and the objective 2 (maximize the serive quality) has the weight $(1-w)$. The table for the service quality is given as:

(a) Assume the weight $w = 0.5$. Formulate the model as a multiobjective problem.

(b) Enter and solve your model with AMPL.

(c) Let $w = 0.1, 0.2, \ldots, 0.9$. Plot the optimal objective function values with different w.

(d) We assume that we have the target goal for the cost to be $T_c = 300$ and the service quality to be $T_q = 1700$. Now, instead of minimizing cost or maximizing quality directly, we minimize

Tax site	Quality Score to Location							
	1	2	3	4	5	6	7	8
1	0.80	0.70	0.60	0.75	0.24	0.84	0.23	0.45
2	0.65	0.85	0.55	0.70	0.32	0.28	0.40	0.55
3	0.70	0.80	0.90	0.65	0.35	0.34	0.56	0.40
4	0.60	0.75	0.50	0.85	0.44	0.80	0.20	0.35
5	0.20	0.85	0.25	0.45	0.60	0.23	0.34	0.70

Table 7: Service quality scores

total "deficiency": cost exceeding T_c and quality falling below T_q. Assume they have the same weight, $w = 0.5$. Modify the model in (a).

(e) Enter and solve your model with AMPL.