Due Date: 04/16/2025

Instructions: Please follow the guideline in *Assignments* section of the syllabus. To get full credit, you must show all your work. While submitting your homework, you need to submit 1) a document summarizing your solutions (the math programming models, calculations and/or the outputs of the codes) and 2) all codes as separate files (including .dat, .mod, .run and .out files). Each problem is 25 points. Upload your homework to canvas as a soft copy with the codes or handwritten calculations. Please submit them separately and do not zip files.

1. (a) Use AMPL to solve the following problem and compare the results with those for Example 1.4.1 (see HW#1 solution).

$$\begin{aligned} \min_{(x,y,z)} \quad & -25\,x - 18\,y - 20\,z \\ \text{s.t.} \quad & -20\,x - 10\,y - 15\,z \geq -1000, \\ & -10\,x - 10\,y - 20\,z \geq -800, \\ & x \geq 0, y \geq 0, z \geq 0. \end{aligned} \tag{2.6.9}$$

   (b) Use AMPL to solve the following problem and compare the results with those for Example 1.5.1 (see HW#1 solution).

$$\begin{aligned} \min_{(\mathbf{x_1},\mathbf{x_2},\mathbf{x_3},\mathbf{x_4},\mathbf{x_5})} \quad & -45\mathbf{x}_1 - 90\mathbf{x}_2 - 58\mathbf{x}_3 - 50\mathbf{x}_4 - 95\mathbf{x}_5 & \\ \text{s.t.} \quad & -18\mathbf{x}_1 - 47\mathbf{x}_2 - 25\mathbf{x}_3 - 3\mathbf{x}_4 - 23\mathbf{x}_5 & \geq -25, \\ & -5\mathbf{x}_1 - 8\mathbf{x}_2 - 5\mathbf{x}_3 - 15\mathbf{x}_4 - 22\mathbf{x}_5 & \geq -10, \\ & 3\mathbf{x}_1 + 6\mathbf{x}_2 + 10\mathbf{x}_3 + 10\mathbf{x}_4 + 12\mathbf{x}_5 & \leq 5, \\ & 0 \leq \mathbf{x}_1 \leq 1, & \\ & 0 \leq \mathbf{x}_2 \leq 1, & \\ & 0 \leq \mathbf{x}_3 \leq 1, & \\ & 0 \leq \mathbf{x}_4 \leq 1, & \\ & 0 \leq \mathbf{x}_5 \leq 1. & \end{aligned} \tag{2.6.10}$$

   (c) Use AMPL to solve the following problem and compare the results with those for

Example 1.5.1 (see HW#1 solution).

$$\begin{array}{rll}
\max_{(\mathbf{x_1},\mathbf{x_2},\mathbf{x_3},\mathbf{x_4},\mathbf{x_5},\mathbf{s_1},\mathbf{s_2},\mathbf{s_3})} & 45\mathbf{x}_1+90\mathbf{x}_2+58\mathbf{x}_3+50\mathbf{x}_4+95\mathbf{x}_5+0\mathbf{s}_1+0\mathbf{s}_2+0\mathbf{s}_3 & \\
\text{s.t.} & 18\mathbf{x}_1+47\mathbf{x}_2+25\mathbf{x}_3+3\mathbf{x}_4+23\mathbf{x}_5+\mathbf{s}_1 & =25, \\
& 5\mathbf{x}_1+8\mathbf{x}_2+5\mathbf{x}_3+15\mathbf{x}_4+22\mathbf{x}_5+\mathbf{s}_2 & =10, \\
& 3\mathbf{x}_1+6\mathbf{x}_2+10\mathbf{x}_3+10\mathbf{x}_4+12\mathbf{x}_5+\mathbf{s}_3 & =5, \\
& 0\le \mathbf{x}_1 \le 1, & \\
& 0\le \mathbf{x}_2 \le 1, & \\
& 0\le \mathbf{x}_3 \le 1, & \\
& 0\le \mathbf{x}_4 \le 1, & \\
& 0\le \mathbf{x}_5 \le 1, & \\
& \mathbf{s}_1 \ge 0, & \\
& \mathbf{s}_2 \ge 0, & \\
& \mathbf{s}_3 \ge 0. &
\end{array} \tag{2.6.11}$$

(d) (Problem 2.11, Chapter 2 of Rardin) Write each of the following as compactly as possible using summation and ”for all“ indexed notation.

i. $\min 3\mathbf{y}_{3,1}+3\mathbf{y}_{3,2}+4\mathbf{y}_{4,1}+4\mathbf{y}_{4,2}$

ii. $\max 1\mathbf{y}_{1,3}+2\mathbf{y}_{2,3}+3\mathbf{y}_{3,3}+4\mathbf{y}_{4,3}$

iii. $\max \alpha_1\mathbf{y}_{1,4}+\alpha_2\mathbf{y}_{2,4}+\ldots+\alpha_p\mathbf{y}_{p,4}$

iv. $\min \delta_1\mathbf{y}_1+\delta_2\mathbf{y}_2+\ldots+\delta_t\mathbf{y}_t$

v.

$$\mathbf{y}_{1,1}+\mathbf{y}_{1,2}+\mathbf{y}_{1,3}+\mathbf{y}_{1,4}=\mathbf{s}_1$$
$$\mathbf{y}_{2,1}+\mathbf{y}_{2,2}+\mathbf{y}_{2,3}+\mathbf{y}_{2,4}=\mathbf{s}_2$$
$$\mathbf{y}_{3,1}+\mathbf{y}_{3,2}+\mathbf{y}_{3,3}+\mathbf{y}_{3,4}=\mathbf{s}_3$$

vi.

$$\mathbf{a}_{1,1}\mathbf{y}_1+\mathbf{a}_{2,1}\mathbf{y}_2+\mathbf{a}_{3,1}\mathbf{y}_3+\mathbf{a}_{4,1}\mathbf{y}_4=\mathbf{c}_1$$
$$\mathbf{a}_{1,2}\mathbf{y}_1+\mathbf{a}_{2,2}\mathbf{y}_2+\mathbf{a}_{3,2}\mathbf{y}_3+\mathbf{a}_{4,2}\mathbf{y}_4=\mathbf{c}_2$$
$$\mathbf{a}_{1,3}\mathbf{y}_1+\mathbf{a}_{2,3}\mathbf{y}_2+\mathbf{a}_{3,3}\mathbf{y}_3+\mathbf{a}_{4,3}\mathbf{y}_4=\mathbf{c}_3$$

2. (Problem 2.29, Chapter 2 of Rardin) A small engineering consulting firm is establishing its plan for the next year. The director and the three partners are to meet to decide which projects to pursue.

   Preliminary research has been done on eight projects. The expected profit for each project is given in Table 2.18 together with the number of person-days of background preparation each will require and the computer processing unit (CPU) time (in hours) each will use. Excluding downtime, it is estimated that 1000 CPU hours will be available through the year. Presently there are 10 engineers (including the director and the partners); each works 240 days per year. At most three engineers could be let go, and management does not want to hire any new engineers for next year, due to market uncertainties. A minimum of 3 projects need to be selected, so each partner will be in charge of at least one project for the year. The director has four favorite projects (3, 4, 5, and 8), and the company needs to select at least one of these.

Project	Profit	Person-Days	CPU
1	2.1	550	200
2	0.5	400	150
3	3.0	300	400
4	2.0	350	450
5	1.0	450	300
6	1.5	500	150
7	0.6	350	200
8	1.8	200	600

Table 2.18: Problem 2.29

The firm wishes to formulate an optimization model to determine which projects to undertake, assuming that projects must be selected on an all-or-nothing basis.

(a) Let the decision variables for the model be $(j = 1, \ldots, 8)$,

$$\mathbf{x}_i = \begin{cases} 1 & \text{if project } j \text{ is selected} \\ 0 & \text{otherwise} \end{cases} .$$

Formulate the objective function to maximize total profit from the projects selected.

(b) Formulate constraints to enforce the minimum and maximum engineer person-days available with different numbers laid off.

(c) Formulate constraints to enforce the limit on computer time, meet the requirement to select at least three projects, and include at least one of the director's favorites.

(d) Enter and solve your model with AMPL. Provide the optimal solutions.

3. (Problem 2.30, Chapter 2 of Rardin) A major expansion of the Brisbane airport will require moving substantial quantities of earth from 4 sites where it is surplus to 7 locations where it is needed. Table 2.19 shows the haul distances (hundreds of meters) between points, as well as the quantity available $\left(m^3\right)$ at each surplus site. Quantities needed are 247 cubic meters at the extension, 394 at the dry pond, 265 along roads, 105 in the parking area, 90 at the fire station, 85 in the industrial park, and 145 along the perimeter road. The site engineer wishes to compute a minimum total distance times volume plan for accomplishing the required earth moving.

(a) Let the decision variables for the model be $\mathbf{x}_{ij}$ = cubic meters moved from surplus i to need j, where $i = 1, \ldots, 4$, $j = 1, \ldots, 7$. Formulate an objective function to minimize total distance times volume movement.

(b) Formulate a system of 4 constraints, assuring that the full available amount is moved from each surplus site.

(c) Formulate a system of 7 constraints, assuring that the required amount is moved to each needed location.

Need Site	Surplus Site			
	Apron	Term.	Cargo	Access
Extension	26	28	20	26
Dry pond	12	14	26	10
Roads	10	12	20	4
Parking	18	20	2	16
Fire station	11	13	6	24
Industrial park	8	10	22	14
Perimeter road	20	22	18	21
Quantity available	660	301	271	99

Table 2.19: Problem 2.30

(d) Enter and solve your model with AMPL, while ensuring that all decision variables are non-negative. Provide the optimal solutions.

4. (Problem 10.34, Chapter 10 of Rardin) Senior design students are negotiating which of the four members of the team will take primary responsibility for each of the four project tasks the team must complete. Table 2.20 shows the composite ratings (0 to 100) they have prepared to estimate the ability of each member to manage each task.

Member	Task Rating			
	1	2	3	4
1	90	78	45	69
2	11	71	50	89
3	88	90	85	93
4	40	80	65	39

Table 2.20: Problem 10.34

(a) The team wants to find a maximum total score plan that allocates exactly one task to each team member. Formulate a linear assignment problem (LP) to choose an optimal plan.

(b) Solve your model in (a) with AMPL. Provide the optimal solutions.

(c) Change the model in (a) to have two students work on the same task and each student can take up to two tasks. The total task rating will be the sum of individual member ratings.

(d) Solve your model in (c) with AMPL. Provide the optimal solutions.