Q2:- Pure binary model.

Step 1: relax binary constraints ⇒ $x_1, x_2, x_3, x_4 \in [0,1]$

obj:

$$\begin{aligned} \min\ & 4x_1 + 2x_2 + 6x_3 + 7x_4 \\ \text{st}\ & x_1 + 3x_2 + 2x_3 + 4x_4 \geq 6 \\ & x_1 + 3x_2 + x_3 + 7x_4 \leq 17 \\ & 0 \leq x_i \leq 1 \quad \forall\, i \in 1, 2, 3, 4 \end{aligned}$$

Step 2 → Optimal solution is $x_1 = 0$, $x_2 = 1$, $x_3 = 0$ &
$x_4 = 0.75$

obj value: 7.25

But, x_4 fractional.

Step 3 → Branching on x_4.

$x_1 = 1$, $x_2 = 1$, $x_3 = 1$, $\boxed{x_4 = 0}$

obj value = 12

Step 4 → Other Node.

$x_1 = \{0,1\}$ $x_2 = \{0,1\}$ $x_3 = \{0,1\}$, $\boxed{x_4 = 1}$

Solution (optimal): $x_1 = 0$, $x_2 = 1$, $x_3 = 0$, $x_4 = 1$

obj value = 9

↓

New best solution

Branch & Bound Dgm:

Node 0

$x = (0, 1, 0, 0.75)$

Obj $= 7.25$

$x_4 = 0$

Node 1

$x = (1, 1, 1, 0)$

Obj $= 12$

$x_4 = 1$

Node 2

$x = (0, 1, 0, 1)$

Obj $= 9$

Ans →

$x_1 = 0,\ x_2 = 1,\ x_3 = 0,\ x_4 = 1.$

Q3. Step 1: relaxing integer constraints x_1, x_2, x_3.

Obj

$$\begin{aligned} \max\ & 4x_1 - x_2 - 3x_3 + 7x_4 \\ \text{s.t}\ & x_1 + 2x_2 - 4x_3 + 3x_4 \geq 7 \\ & 3x_1 - 4x_2 + 2x_3 + 7x_4 \leq 17 \\ & 5x_1 + 3x_2 + 4x_3 + 2x_4 \leq 12 \\ & x_1, x_2, x_3 \geq 0,\ x_4 \geq 0 \end{aligned}$$

Solving above →
(in AMPL)

$x_1 = 0,\ x_2 = 1.724,\ x_3 = 0,$
$x_4 = 3.41$

Obj Val $= 22.417.$

Step 2 → Branch on x_2 [round down]
($x_2 \leq 2$) → added constraint

~~Optimal sol. $x_i \equiv 0.7241, 1, 0, 2.689$~~
~~Val = 20.72~~

~~Node~~

Optimal solⁿ ⇒ $x_1 = 0$, $x_2 = 1.72$, $x_3 = 0$, $x_4 = 3.4$
Val $= 22.1724$

Node 2 → $x \geq 2$

Solⁿ ⇒ $x_i \equiv 0, 2, 0, 3$
Val $= 19$

All integer values.

\# Node 1 can't be branched further.

Root Node
$x = (0, 1.724, 0, 3.41)$
obj $= 22.17$

Node 1
$x = (0, 1.70, 2.84)$
obj $= 22.17$

Node 2
$x = (0, 2, 0, 3)$
obj $= 19$

Ans → $x_1 = 0$, $x_2 = 2$, $x_3 = 0$, $x_4 = 3$
Val = 19.

Q1. (a) x_{ij} : amt added to site i of Tier j

y_{ij} $\begin{cases} 1 \\ 0 \end{cases}$ if plan of Tier j is chosen for site i

$x_{ij} \geq 0 \quad \forall i, \forall j$

$x_{i1} \leq 5 y_{i1} \quad \forall i$

$5y_{i2} \leq x_{i2} \leq 10 y_{i2} \quad \forall i$

$10y_{i3} \leq x_{i3} \leq 15 y_{i3} \quad \forall i$

obj → min $w \left(\sum_{j=J} \sum_{i=I} x_{ij} c_{ij} + d_{ij} y_{ij} \right)$

$- (1-w) \left(\sum_{j=J} \sum_{i=I} x_{ij} q_{ij} \right)$

(c) Penalty1 $= 1000 - \left(\sum_{j=J} \sum_{i=I} x_{ij} c_{ij} + d_{ij} y_{ij} \right)$

Penalty2 $= 1000 - \left(\sum_{j=J} \sum_{i=I} x_{ij} q_{ij} \right)$

worst violation $= \max (P1, P2)$

obj → ~~w Penalty1~~ min worst violation.