Due Date: 04/23/2025

Instructions: Please follow the guideline in *Assignments* section of the syllabus. To get full credit, you must show all your work. While submitting your homework, you need to submit 1) a document summarizing your solutions (the math programming models, calculations and/or the outputs of the codes) and 2) all codes as separate files (including .dat, .mod, .run and .out files). Each problem is 25 points. Upload your homework to canvas as a soft copy with the codes or handwritten calculations. Please submit them separately and do not zip files.

1. To produce a high-protein nutritional mix, a food processing company is blending several ingredients to meet nutrient requirements at minimum cost. The company is considering 5 different ingredients, each with its own cost per unit and contribution to 3 essential nutrients (Protein, Fat, and Carbohydrate). The goal is to create 3 types of nutritional products (Basic, Standard, and Premium), each requiring specific levels of nutrients per unit. The table below provides the cost per unit of each ingredient, and the amount of each nutrient (in grams) that each unit of ingredient contributes. The table also shows the minimum requirement for each nutrient in each product type.

		Nutrient Contribution (g/unit)		
Ingredient	**Cost ($/unit)**	**Protein**	**Fat**	**Carbohydrate**
1	2.50	20	5	30
2	1.80	15	10	25
3	3.10	25	8	20
4	2.20	18	4	35
5	2.90	22	6	28

		Minimum Nutrient Requirements (g/unit of product)		
Product		**Protein**	**Fat**	**Carbohydrate**
1	Basic	11	3	26
2	Standard	12	4	28
3	Premium	10	6	30

Table 1: Ingredient Costs and Nutrient Contributions

We seek a minimum cost blending plan for each product type.

(a) Define the sets, parameters and decision variables. Formulate the model.

(b) Enter and solve the model in AMPL.

(c) Assume we now have an additional ingredient (Ingredient 6), which has the cost \$3.40/unit. It has 30g protein, 5g fat, and 10g carbohydrate. Modify the model in (a) and solve it in AMPL. Does addition of this ingredient change the solution and total cost?

2. A petroleum company imports crude oil from different countries. Each country supplies crude oil of a specific grade and at a certain cost. The company operates several refineries located in different regions. Each refinery blends the incoming crude oil to produce refined products that meet quality specifications. The company wants to determine how much crude to purchase from each country and ship to each refinery, such that:

- All refineries are adequately supplied to meet their demand.

- The blended crude at each refinery meets required quality specifications (e.g., sulfur content, API gravity).
- The total cost, including purchase and transportation, is minimized.

Data are given as follows:

Crude Purchase & Transportation Costs (USD per barrel):

Country	**Houston**	**Rotterdam**
Saudi	50	7
Nigeria	60	4
Venezuela	40	6

Maximum Supply (barrels):

Country	**Supply**
Saudi	50,000
Nigeria	40,000
Venezuela	30,000

Refinery Demand (barrels):

Refinery	**Demand**
Houston	60,000
Rotterdam	50,000

Sulfur Content (% by weight):

Country	**Sulfur (%)**
Saudi	1.8
Nigeria	0.5
Venezuela	2.5

API Gravity (degrees):

Country	**API Gravity**
Saudi	32
Nigeria	35
Venezuela	28

Blending Constraints per Refinery:

Refinery	**Max Sulfur (%)**	**Min API Gravity**
Houston	1.8	30
Rotterdam	2.0	31

(a) Define the sets, parameters and decision variables. Formulate the model.

(b) Enter and solve this model using AMPL.

(c) To meet new mandates, the company now faces a global carbon penalty associated with crude from Venezuela. This penalty is \$10 per barrel. Modify the model in (a) and solve it using AMPL.

(d) The company decides to strategically store at least 43,000 barrels of Saudi crude. Modify the model in (a) and solve it using AMPL.

3. To support a network of distribution centers and regional warehouses, a logistics company must design a flow plan for delivering three types of commodities—Widgets, Gadgets, and Doodads—from suppliers to customers. Shipments must go through a set of transit hubs due to regulation and logistical efficiency. Each arc in the network has a capacity and a unit cost for each commodity. The goal is to minimize the total shipping cost while satisfying the supply and demand requirements and not exceeding arc capacities. The table below shows the arc capacities and shipping costs (in dollars per unit) for each commodity on selected arcs of the network.

Arc ($i \to j$)	**Capacity**	**Widgets**	**Gadgets**	**Doodads**
Supplier1 → Hub1	600	2	3	1
Supplier2 → Hub1	700	4	2	3
Supplier2 → Hub2	800	3	5	2
Hub1 → Customer1	500	6	4	5
Hub1 → Customer2	700	5	3	4
Hub2 → Customer2	400	3	6	2

Table 2: Capacities and Unit Shipping Costs

Node	**Widgets**	**Gadgets**	**Doodads**
Supplier1	200	225	100
Supplier2	150	125	120
Customer1	-250	-100	-150
Customer2	-100	-250	-70

Table 3: Supply (+) and Demand (−) Values at Nodes

We seek a minimum total cost shipping plan that routes flow from suppliers to customers through transit hubs, subject to capacity limits on each arc and the requirement that flow be conserved for each commodity.

(a) Define the sets, parameters and decision variables. Formulate the model.

(b) Enter and solve this model using AMPL.

(c) Assume instead of a separate capacity constraint for Hub from Supplier 2, we only have a constraint on the total supply capacity from Supplier 2. Let this capacity be 1500. Modify the model in (a) and solve the model in AMPL.

(d) Assume instead of a separate capacity constraint for Customer from Hub 1, we only have a constraint on the total supply capacity from Hub 1. Let this capacity be 700. Modify the model in (a) and solve the model in AMPL.

4. Given a directed graph $G = (N, A)$, and a set of commodities K, each with a source and a sink node, the objective is to maximize the total flow of all commodities, while respecting arc capacities. Data are given as follows:

Sets:

- $N = \{1, 2, 3, 4, 5, 6, 7, 8\}$: Nodes
- A: Set of arcs listed below
- $K = \{A, B, C\}$: Commodities

Arc and Capacities:

Arc (i, j)	Capacity cap_{ij}
(1,2)	15
(1,3)	10
(2,4)	12
(2,5)	10
(3,5)	5
(3,6)	8
(4,5)	7
(4,7)	10
(5,6)	10
(5,7)	5
(6,7)	15
(6,8)	12
(7,8)	10
(2,3)	5
(1,4)	6
(3,4)	4
(5,8)	10
(1,5)	8
(4,6)	5
(2,6)	7

Commodities:

Commodity k	Source s_k	Sink t_k
A	1	8
B	2	7
C	3	8

(a) Define the parameters and decision variables. Formulate this problem as a maximum flow model.

(b) Enter and solve this model using AMPL.

(c) How will you solution change if the capacities on arc $(6, 7)$ and $(6, 8)$ to be 1? Modify the model from (a) and solve it in AMPL.

(d) Assume instead of a separate capacity constraint on the total outflow from node 6, how will your solution change if there is a limit on the total outflow from node 6 is restricted to 2? Modify the model from (a) and solve it in AMPL.