Due Date: 04/30/2025

Instructions: Please follow the guideline in *Assignments* section of the syllabus. To get full credit, you must show all your work. While submitting your homework, you need to submit 1) a document summarizing your solutions (the math programming models, calculations and/or the outputs of the codes) and 2) all codes as separate files (including .dat, .mod, .run and .out files). Each problem is 25 points. Upload your homework to canvas as a soft copy with the codes or handwritten calculations. Please submit them separately and do not zip files.

1. We are given a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ with eight nodes, and arcs listed in Table 1. The capacity and the outflow gains from each arc is also given in this table.

Arc (i,j)	**Capacity $\mathbf{u}_{ij}$**	**Gain $\mathbf{g}^1_{ij}$**	**Gain $\mathbf{g}^2_{ij}$**
(1,2)	15	1.0	1.0
(1,3)	10	1.2	0.2
(2,4)	12	1.1	0.1
(2,5)	10	1.3	0.3
(3,4)	10	0.95	0.5
(3,6)	8	1.4	1.2
(4,7)	10	1.2	0.2
(5,7)	5	1.0	1.0
(6,5)	6	1.1	1.2
(6,8)	9	1.3	1.3
(7,6)	7	0.9	2.0
(7,8)	12	1.5	1.9

Table 1: Arc capacities and gain factors

Our goal is to determine the flow on each arc that maximizes the total flow accumulated at the sink node 8, subject to flow balance and capacity constraints.

   (a) Assume there is only one commodity with the gain $\mathbf{g}^1_{ij}$. The source is node 1. Formulate the model as a network gain problem.

   (b) Enter and solve your model with AMPL.

   (c) Now assume that we have supplies of an additional commodity available at node 2 with the gain $\mathbf{g}^2_{ij}$. Now change your model to ensure that the total flow reaching at node 8 is maximized.

   (d) Enter and solve your model with AMPL.

2. (Modified Version of Problem 2.39, Chapter 2 of Rardin) To improve tax compliance, the Texas Comptroller's staff regularly audits at corporate home offices the records of out-of-state corporations doing business in Texas. Texas is considering the opening of a series of small offices near these corporate locations to reduce the travel costs now associated with such out-of-state audits. The following table shows the fixed cost (in thousands of dollars) of operating such offices at 5 sites i, the number of audits required in each of 5 states j, and the travel cost (in thousands of dollars) per audit performed in each state from a base at any of the proposed office sites. We seek a minimum total cost auditing plan.

   (a) Formulate the model as a facility location problem.

   (b) Enter and solve your model with AMPL.

Tax Site	Fixed Cost	Cost to Audit Tax Corporate Location: **1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	160	0	0.4	0.8	0.4	0.8	0.3	0.6	0.5
2	49	0.7	0	0.8	0.4	0.4	0.5	0.3	0.6
3	246	0.6	0.4	0	0.5	0.4	0.6	0.5	0.6
4	86	0.6	0.4	0.9	0	0.4	0.4	0.2	0.4
5	100	0.9	0.4	0.7	0.4	0	0.7	0.3	0.5
Audits		400	100	300	100	200	300	200	400

Table 2: Tax Sites

(c) Assume that each site can serve exactly two states and each state can be served by only one site. Modify your model in (a).

(d) Enter and solve your model with AMPL.

3. A company must assign 8 employees to cover 3 shifts per day (Morning, Afternoon, Night) over 7 days. Each shift requires exactly one employee, and each employee can work at most one shift per day. Employees have stated preferences for certain shifts. Table 3 provides employee preferences for different shift types. Our goal is to maximize their preferences.

Employee	**Morning**	**Afternoon**	**Night**
1	4	2	5
2	1	5	5
3	3	5	3
4	4	3	4
5	3	3	5
6	4	4	3
7	5	1	5
8	2	5	4

Table 3: Shift Preference

(a) Formulate the model.

(b) Enter and solve the model in AMPL.

(c) Assume each employee can work at most three shifts per week and no employee should work the Night shift on day d and Morning shift on day $d+1$. Modify the model in (a).

(d) Enter and solve the model in AMPL.

4. A project consists of a set of tasks $\mathcal{T} = \{1, 2, \ldots, 8\}$, each of which requires a certain duration to complete. Tasks must be scheduled such that:

- Each task starts after all its prerequisite tasks are finished.
- The objective is to minimize the time at which the last task finishes and there is an additional penalty for finishing each task late, where the penalty increase linearly with the number of days a task is delayed and the delay of each task has different weights.

The tasks and precedence relations are given as follows:

- Task 1 must precede tasks 2 and 3

Task	**Task duration in days d_i**	**Penalty weight α_i**
1	4	1.0
2	3	0.8
3	2	1.2
4	5	1.5
5	3	1.0
6	4	0.7
7	2	1.3
8	3	0.9

Table 4: Task duration

- Task 2 must precede task 4
- Task 3 must precede task 5
- Task 4 and task 5 must precede task 6
- Task 6 must precede task 7
- Task 5 must precede task 8

(a) Formulate the model.

(b) Enter and solve your model in AMPL.

(c) Due to limited manpower, you want to avoid overlap of tasks. Modify your model in (a). (Hint: Add constraints with additional binary decision variables to ensure that for any two tasks that have no precedence relations, either task i finishes before task j starts, or task j finishes before task i starts.)

(d) Enter and solve your model in AMPL.