IEMS 313 Lab 3

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1 3-Dimension Index Problem

1.1 Problem Statement

(AMPL Book Chapter 4.1) A steel company operates three production plants—Gary, Cleveland, and Pittsburgh—from which it must ship three types of products (bands, coils, and plate) to seven regional distribution centers: Franklin (FRA), Detroit (DET), Lansing (LAN), Windsor (WIN), St. Louis (STL), Fresno (FRE), and Lafayette (LAF).

Each plant has a limited supply of each product, and each distribution center has a fixed demand for each product. The company also faces limits on how much total material (across all products) can be shipped between any origin-destination pair due to transportation capacity. The goal is to minimize total shipping cost, considering per-unit shipment costs that differ by origin, destination, and product type.

1.2 Formulation of the Model

Sets:

- $i \in \mathrm{O} = \{\mathrm{GARY, CLEV, PITT}\}$: set of origin plants.
- $j \in \mathrm{D} = \{\mathrm{FRA, DET, LAN, WIN, STL, FRE, LAF}\}$: set of destination centers.
- $p \in \mathrm{P} = \{\mathrm{bands, coils, plate}\}$: set of product types.

Parameters:

- $\mathrm{s}_{i,p}$: amount of product p available at origin i.

- $\mathrm{d}_{j,p}$: amount of product p required at destination j.
- $\mathrm{l}_{i,j}$: maximum shipment capacity (across all products) from i to j.
- $\mathrm{c}_{i,j,p}$: cost per unit of shipping product p from i to j.

Decision Variables:

$$x_{i,j,p} \geq 0 \quad \text{amount of product } p \text{ shipped from } i \text{ to } j$$

Objective:

$$\min \sum_{i \in \mathrm{O}} \sum_{j \in \mathrm{D}} \sum_{p \in \mathrm{P}} \mathrm{c}_{i,j,p} \cdot x_{i,j,p}$$

Constraints:

$$\begin{aligned}
&\text{(Supply)} && \sum_{j \in \mathrm{D}} x_{i,j,p} = \mathrm{s}_{i,p} && \forall i \in \mathrm{O}, p \in \mathrm{P} \\
&\text{(Demand)} && \sum_{i \in \mathrm{O}} x_{i,j,p} = \mathrm{d}_{j,p} && \forall j \in \mathrm{D}, p \in \mathrm{P} \\
&\text{(Capacity)} && \sum_{p \in \mathrm{P}} x_{i,j,p} \leq \mathrm{l}_{i,j} && \forall i \in \mathrm{O}, j \in \mathrm{D}
\end{aligned}$$

Table 1: Supply at Origins (in units)

Product	GARY	CLEV	PITT
Bands	400	700	800
Coils	800	1600	1800
Plate	200	300	300

Table 2: Demand at Destinations (in units)

Product	FRA	DET	LAN	WIN	STL	FRE	LAF
Bands	300	300	100	75	650	225	250
Coils	500	750	400	250	950	850	500
Plate	100	100	0	50	200	100	250

Table 3: Shipment Cost per Unit

Product	Origin	Destination	FRA	DET	LAN	WIN	STL	FRE	LAF
Bands	GARY		30	10	8	10	11	71	6
	CLEV		22	7	10	7	21	82	13
	PITT		19	11	12	10	25	83	15
Coils	GARY		39	14	11	14	16	82	8
	CLEV		27	9	12	9	26	95	17
	PITT		24	14	17	13	28	99	20
Plate	GARY		41	15	12	16	17	86	8
	CLEV		29	9	13	9	28	99	18
	PITT		26	14	17	13	31	104	20

2 Practice Problem

A logistics company is planning to establish delivery contracts between a set of suppliers, a set of warehouses, and a set of customers. Each supplier can send goods to any warehouse, which then redistributes the goods to customers. However, due to budget constraints and partnership agreements, each shipment from a supplier to a warehouse to a customer must be either fully accepted or not at all.

The cost of shipping from supplier i to warehouse j and then to customer k is known for each combination. The company wants to select a subset of these supplier–warehouse–customer routes to minimize total cost while ensuring:

1. Each customer is served exactly once.

2. No warehouse is overloaded beyond its capacity.

3. Each supplier can ship to at most a given number of customers.

2.1 Example Data

Table 4: Cost c_{ijk} of shipping from Supplier i to Customer k via Warehouse j

Supplier	Warehouse	Customer	Cost
S1	W1	C1	8
S1	W1	C2	7
S1	W1	C3	9
S1	W2	C1	10
S1	W2	C2	12
S1	W2	C3	6
S2	W1	C1	11
S2	W1	C2	6
S2	W1	C3	10
S2	W2	C1	9
S2	W2	C2	8
S2	W2	C3	7

Table 5: Warehouse Capacity and Supplier Customer Limits

	Capacity / Limit	Value
Warehouse W1	Capacity	2
Warehouse W2	Capacity	2
Supplier S1	Max customers	2
Supplier S2	Max customers	2