Due Date: 05/14/2025

Instructions: Please follow the guideline in *Assignments* section of the syllabus. To get full credit, you must show all your work. While submitting your homework, you need to submit 1) a document summarizing your solutions (the math programming models, calculations and/or the outputs of the codes) and 2) all codes as separate files (including .dat, .mod, .run and .out files). Each problem has equal points. Upload your homework to canvas as a soft copy with the codes or handwritten calculations. Please submit them separately and do not zip files.

1. (Modified from HW5 Problem 1) A telecommunications operator must decide how much capacity (in Gbps) to add at each of its regional base-station sites. For each site, the cost of adding capacity follows a three-tier pricing structure:

   - Tier 0 covers small upgrades (up to 5 Gbps) at a low per-Gbps rate.
   - Tier 1 covers medium upgrades (greater than 5 and up to 10 Gbps) at a moderate per-Gbps rate, with a fixed offset to maintain continuity.
   - Tier 2 covers large upgrades (greater than 10 Gbps up to 15 Gbps) at a high per-Gbps rate, again with an offset for continuity.

   We give the rate as per-Gbps costs $\mathbf{c}_{ij}$ (in dollars):

Site i	$\mathbf{c}_{i0}$	$\mathbf{c}_{i1}$	$\mathbf{c}_{i2}$
1	100	120	150
2	90	110	140
3	95	115	145
4	105	125	155
5	85	105	135

Table 1: Per-Gbps Costs

   And the fixed offsets cost $\mathbf{d}_{ij}$ (in dollars) as:

Site i	$\mathbf{d}_{i0}$	$\mathbf{d}_{i1}$	$\mathbf{d}_{i2}$
1	0	250	500
2	0	200	450
3	0	220	470
4	0	270	520
5	0	180	430

Table 2: fixed offsets cost

   In addition, we give the rate as per-Gbps quality score $\mathbf{q}_{ij}$ (in dollars):

Site i	$\mathbf{q}_{i0}$	$\mathbf{q}_{i1}$	$\mathbf{q}_{i2}$
1	60	80	120
2	45	90	110
3	75	100	125
4	70	120	150
5	80	105	110

Table 3: Per-Gbps Quality Scores

The total added capacity for all sites should be larger than the demand $D = 30$. The goal is to determine the capacity to add at each site.

(a) We have two goals: 1. Minimize the largest single-site upgrade cost with the weight w, and 2. Maximize the total upgrade quality score with the weight $(1 - w)$. Formulate the model.

(b) Let $w = 0.3$. Enter and solve the model in AMPL.

(c) Assume the aspiration level of both goals is 1000. For example, the largest single-site upgrade cost should be less than or equal to 1000, and the total upgrade quality score should be larger than or equal to 1000. We want to minimize the worst aspiration level violation. Formulate the model. (Hint: refer to Section 5.3 in the lecture notes)

(d) Enter and solve the model in (c) in AMPL.

2. For the following pure binary model:

$$\begin{aligned} \min \quad & 4\mathbf{x}_1 + 2\mathbf{x}_2 + 6\mathbf{x}_3 + 7\mathbf{x}_4 \\ s.t. \quad & \mathbf{x}_1 + 3\mathbf{x}_2 + 2\mathbf{x}_3 + 4\mathbf{x}_4 \geq 6 \\ & \mathbf{x}_1 + 3\mathbf{x}_2 + \mathbf{x}_3 + 7\mathbf{x}_4 \leq 17 \\ & \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4 \in \{0, 1\} \end{aligned} \tag{1}$$

Solve the model by the branch and bound method. Plot the branch and bound diagram. At each node, you need to give the added constraint, the current optimal solution and the current optimal objective function value.

3. For the following mixed-integer model:

$$\begin{aligned} \max \quad & 4\mathbf{x}_1 - \mathbf{x}_2 - 3\mathbf{x}_3 + 7\mathbf{x}_4 \\ s.t. \quad & \mathbf{x}_1 + 2\mathbf{x}_2 - 4\mathbf{x}_3 + 3\mathbf{x}_4 \geq 7 \\ & 3\mathbf{x}_1 - 4\mathbf{x}_2 + 2\mathbf{x}_3 + 7\mathbf{x}_4 \leq 17 \\ & 5\mathbf{x}_1 + 3\mathbf{x}_2 + 4\mathbf{x}_3 + 2\mathbf{x}_4 \leq 12 \\ & \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \in \mathbb{Z}_+, \mathbf{x}_4 \geq 0. \end{aligned} \tag{2}$$

Solve the model by the branch and bound method. Plot the branch and bound diagram. At each node, you need to give the added constraint, the current optimal solution and the current optimal objective function value.