IEMS 313 Lab 2

April 7, 2025

1 Data-Independent Model

1.1 Problem Statement

Consider a company producing 3 products, each requiring certain amounts of resources. The company aims to maximize profit, but the available resources and the profit per product are not yet determined. We will define a data-independent model using indices for the products and resources.

1.2 Formulation of the Model

Let:

- $i \in I = \{1, 2, 3\}$ be the index set of the 3 products.
- $j \in J = \{1, 2, \ldots, m\}$ be the index set of m resources.
- x_i be the quantity of product i produced.
- p_i be the profit per unit of product i.
- r_{ij} be the amount of resource j required to produce one unit of product i.
- R_j be the total available amount of resource j.

The objective is to maximize the total profit:

$$Z = \sum_{i \in I} p_i x_i$$

Subject to the resource constraints:

$$\sum_{i \in I} r_{ij} x_i \leq R_j \quad \text{for all } j \in J$$

Non-negativity constraints:

$$x_i \geq 0 \quad \text{for all } i \in I$$

1.3 Explanation

When formulating optimization problems, it is crucial to use general notation like indexed sets or summations rather than explicitly listing every variable and constraint, especially when dealing with large numbers of products, resources, or constraints. This approach allows the model to scale easily, adapting to changes in the number of products or resources without requiring significant rewrites. It also makes the model more concise and easier to read.

1.4 Data

Product	**Profit** (p_i)	**Resource 1** (r_{i1})	**Resource 2** (r_{i2})
1	5	2	1
2	4	1	2
3	6	3	1

Resource	**Availability** (R_j)
1	100
2	80

1.5 Solve the Model in AMPL

Similarly as Lab1, we want to create .mod file, .dat file and .run file to save the commands and outputs to .out file.

.mod file

- Define sets
- Define parameters (pay attention to their dimensions and units)
- Define decision variables (pay attention to their dimensions and units)
- Formulate the objective function
- Define constraints

.dat file Give values to sets and parameters you define in .mod file.

.run file Run the model.

.out file Show the results.

2 Practice Problem

You are managing a logistics company that needs to how to size distribution centers (DCs) to serve several retail stores across different cities. The goal is to minimize the total costs, which include both the cost of sizing a distribution center and the transportation costs of delivering goods from the distribution centers to the stores. Each distribution center has a capacity that you select with a unit cost, and each store has a specific demand that must be met. Additionally, not every distribution center can serve every store due to geographical limitations, so only certain DC-store pairs are feasible.

Your task is to decide:

1. How to size the distribution centers, and

2. How much each distribution center should ship to each store in order to meet demand while minimizing costs.

2.1 Key Considerations

- **Sizing Costs**: Each distribution center has a capacity that you select and a unit cost per unit of capacity.
- **Transportation Costs**: The transportation cost between a distribution center and a store depends on the amount of goods shipped.
- **Capacity**: Given the sizing decision, the capacity of each distribution center limits how much it can ship.
- **Demand**: Each store has a known demand that must be met.

2.2 Example Data

- Distribution centers: $I = \{1, 2\}$
- Stores: $J = \{A, B, C\}$
- Feasible pairs: $A = \{(1, A), (1, B), (2, B), (2, C)\}$

DC i	c_i (Unit Sizing Cost)
1	5
2	4

Store j	d_j (Demand)
A	30
B	40
C	20

DC i	Store j	t_{ij} (Transportation Cost)
1	A	2
1	B	3
2	B	1
2	C	4

Sets and Indices

- $i \in I$: Index set of distribution centers (DCs).
- $j \in J$: Index set of retail stores.
- $(i, j) \in A \subseteq I \times J$: Set of feasible DC-store pairs.

Parameters

- c_i: Unit sizing cost for DC i.
- t_{ij}: Transportation cost per unit from DC i to store j, for $(i, j) \in A$.
- d_j: Demand at store j.

Decision Variables

- $s_i \geq 0$: Capacity chosen for DC i.
- $x_{ij} \geq 0$: Amount shipped from DC i to store j, for $(i, j) \in A$.

Objective Function

$$\min \sum_{i \in I} c_i s_i + \sum_{(i,j) \in A} t_{ij} x_{ij}$$

Constraints

(1) Demand satisfaction: $$\sum_{i:(i,j) \in A} x_{ij} \geq d_j, \forall j \in J$$

(2) Capacity limit: $$\sum_{j:(i,j) \in A} x_{ij} \leq s_i, \forall i \in I$$

(4) Non-negativity: $$x_{ij} \geq 0, \quad s_i \geq 0, \forall i \in I, \forall (i, j) \in A$$