Problem set - 2

Physics of binary evolution

Sayantani Datta

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$\boxed{\text{Mass transfer}}$: Assume that the more massive component of the binary, m_1, undergoes mass transfer ($dm_1/dt < 0$), where some fraction of the mass (β) may be lost due to various mechanisms like stellar winds.

① <u>Conservative case</u> : For $\beta = 0$, calculate whether mass transfer in a binary shrinks the orbit or expands it as a function of mass ratio.

$\Rightarrow$ Let us assume that the more massive star m_1, loses matter at a rate of $\dot{m}_1 < 0$.

In the conservative case, all mass lost by m_1 is captured by m_2.

$$\text{so,} \quad \dot{m}_2 = -\dot{m}_1 \qquad \text{--- (1)}$$

Differentiating angular momentum of the system, we get,

$$J = \mu \sqrt{GMa} \qquad \text{--- (2)}$$

$$\text{where,} \quad \mu = \frac{m_1 m_2}{M}, \quad M = m_1 + m_2.$$

$$J^2 = \mu^2 \times GMa$$

$$\Rightarrow \quad a = \frac{J^2}{\mu^2 GM} \qquad \text{--- (3)}$$

$$\frac{da}{dt} = \frac{2J\dot{J}}{\mu^2 G M} + \frac{J^2}{GM}\frac{(-2)}{\mu^3}\dot{\mu} + \frac{J^2}{\mu^2 G}\frac{(-1)}{M^2}\dot{M} \quad \text{—— } (4)$$

For a conservative system, $\dot{J}=0$
& $\dot{M}=0$

so, $$\dot{a} = -\frac{2J^2}{GM\mu^3}\dot{\mu} = -2a\frac{\dot{\mu}}{\mu}$$

$$\Rightarrow \frac{\dot{a}}{a} = -2\frac{\dot{\mu}}{\mu} \quad \text{—— } (5)$$

Reduced mass is given by,

$$\mu = \frac{m_1 m_2}{M}$$

$$\Rightarrow \dot{\mu} = \frac{1}{M}(\dot{m}_1 m_2 + m_1 \dot{m}_2) \quad [\dot{M}=0]$$

$$\Rightarrow \frac{\dot{u}}{u} = \frac{1}{m_1 m_2}\left(\dot{m}_1 m_2 + m_1 \dot{m}_2\right)$$

$$\Rightarrow \frac{\dot{u}}{u} = \left(\frac{\dot{m}_1}{m_1} + \frac{\dot{m}_2}{m_2}\right)$$

$$= \frac{\dot{m}_1}{m_1}\left(1 + \frac{\dot{m}_2}{m_2} \times \frac{m_1}{\dot{m}_1}\right)$$

Using Eq^n – 1 we get,

$$\frac{\dot{u}}{u} = \frac{\dot{m}_1}{m_1}\left(1 - \frac{m_1}{m_2}\right) \quad \text{———} \quad 6$$

Substituting 6 in 5 we get,

$$\boxed{\frac{\dot{a}}{a} = -2\left(1 - \frac{m_1}{m_2}\right)\frac{\dot{m}_1}{m_1}} \quad \text{———} \quad 7$$

case 1 :- $m_1 > m_2$ & $\dot{m}_1 < 0$

$$\therefore \frac{\dot{a}}{a} = -\alpha_q \frac{\dot{m}_1}{m_1} \qquad \text{———} \quad (8)$$

where, α_q depends on the mass ratio.

For a given mass transfer rate, Eq^n (6) tells that final seperation will be lesser than initial seperation.

$\boxed{\text{Hence, for } m_1 > m_2 \text{, the orbits shrink during mass transfer}}$

case 2 :- $m_1 < m_2$ & $\dot{m}_1 < 0$

Equation (5) gives,

$$\frac{\dot{a}}{a} = \alpha_q \frac{\dot{m}_1}{m_1} \qquad \text{———} \quad (9)$$

Eq^n (7) tells that for a given rate of $\dot{m}_1$, final seperation will be larger than initial seperation.

$$\boxed{\text{For, } m_1 < m_2 \text{, the orbits expands during mass transfer}}$$

(2) Non-Conservative Case :

Evaluate the change in the orbital seperation by considering some non-zero values of β. Calculate the change as a function of different values.

$\Rightarrow$ Let $\beta \in [0, 1]$ be the fraction of ejected matter from m_1.

so, $\dot{m}_2 = -(1-\beta)\dot{m}_1 \geq 0$ —— (10)

Using Eq^n (4) :—

$$\dot{a} = \frac{2J\dot{J}}{J^2}a - 2\frac{\dot{\mu}}{\mu}a - Ma\frac{\dot{M}}{M^2}$$

$$\Rightarrow \frac{\dot{a}}{a} = 2\frac{\dot{J}}{J} - 2\frac{\dot{\mu}}{\mu} - \frac{\dot{M}}{M} \quad \text{—— (11)}$$

from (10), $\dot{m}_2 + \dot{m}_1 \equiv \dot{M} = \beta\dot{m}_1$ —— (12)

and $\mu = \frac{m_1 m_2}{M}$

$$\Rightarrow \dot{\mu} = \frac{1}{M}(\dot{m}_1 m_2 + m_1\dot{m}_2) - \frac{m_1 m_2}{M^2}\dot{M}$$

$$\Rightarrow \frac{\dot{\mu}}{\mu} = \left(\frac{\dot{m}_1}{m_1} + \frac{\dot{m}_2}{m_2}\right) - \frac{\beta\dot{m}_1}{M} \quad \text{—— (13)}$$

Using (12) & (13) in (11) :-

$$\frac{\dot{a}}{a} = \frac{2\dot{J}}{J} - 2\left(\frac{\dot{m}_1}{m_1} + \frac{\dot{m}_2}{m_2}\right) + \frac{2\beta\dot{m}_1}{M} - \frac{\beta\dot{m}_1}{M}$$

$$= \frac{2\dot{J}}{J} - 2\left(\frac{\dot{m}_1}{m_1} + \frac{\dot{m}_2}{m_2}\right) + \frac{\beta\dot{m}_1}{M}$$

$$= \frac{2\dot{J}}{J} - 2\left(\frac{\dot{m}_1}{m_1} + \frac{\beta\dot{m}_1 - \dot{m}_1}{m_2}\right) + \frac{\beta\dot{m}_1}{M}$$

$$= \frac{2\dot{J}}{J} - 2\left(\frac{\dot{m}_1}{m_1} + \frac{\beta\dot{m}_1}{m_2} - \frac{\dot{m}_1}{m_2}\right) + \frac{\beta\dot{m}_1}{M}$$

$$= \frac{2\dot{J}}{J} - 2\left((\beta-1)\frac{\dot{m}_1}{m_2} + \frac{\dot{m}_1}{m_1}\right) + \frac{\beta\dot{m}_1}{M}$$

$$\Rightarrow \boxed{\frac{\dot{a}}{a} = \frac{2\dot{J}}{J} - 2\left[(\beta-1)\frac{m_1}{m_2} + 1\right]\frac{\dot{m}_1}{m_1} + \frac{\beta\dot{m}_1}{M}}$$

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For, $\beta = 0.5$

Let us consider that some angular momentum is also lost along with m_1 :- $\dot{J}<0$ & $\dot{m}_1<0$ changing the signs appropriately in Eq 14

$$\frac{\dot{a}}{a} = -\frac{2\dot{J}}{J} + 2\left[-\frac{1}{2}\frac{m_1}{m_2}+1\right]\frac{\dot{m}_1}{m_1} - \frac{1}{2}\frac{\dot{m}_1}{M}$$

$$= -\frac{2\dot{J}}{J} - \left[\frac{m_1}{m_2} - 2\right]\frac{\dot{m}_1}{m_1} - \frac{1}{2}\frac{\dot{m}_1}{M}$$

If $q = \frac{m_1}{m_2} > 2$

$$\frac{\dot{a}}{a} = -\frac{2\dot{J}}{J} - \alpha_q\frac{\dot{m}_1}{m_1} - \frac{1}{2}\frac{\dot{m}_1}{M}$$

Orbits will shrink

If $q = \frac{m_1}{m_2} = 2$

$$\frac{\dot{a}}{a} = -\frac{2\dot{J}}{J} - \frac{1}{2}\frac{\dot{m}_1}{M}$$

Orbits will shrink

If $q = \frac{m_1}{m_2} < 2$

$$\frac{\dot{a}}{a} = -\frac{2\dot{J}}{J} + \alpha_q \frac{\dot{m}_1}{m_1} - \frac{1}{2}\frac{\dot{m}_1}{M}$$

Competing effects

For $\beta = 0$

This reduces to conservative case where both mass and angular momentum is conserved, $\dot{M} = \dot{J} = 0$

For $\beta = 1$

Equation (10) gives,

$$\dot{m}_2 = 0$$

which means the mass of the acceptor is not increasing as all the matter during transfer is lost.

Putting $\beta = 1$ in eqn (14) gives us,

$$\frac{\dot{a}}{a} = \frac{2\dot{J}}{J} - 2\frac{\dot{m}_1}{m_1} + \frac{\dot{m}_1}{M}$$

$$= \frac{2\dot{J}}{J} - \frac{2\dot{m}_1}{m_1}\left(1 - \frac{1}{2}\frac{m_1}{M}\right)$$

Considering that both angular momentum & mass is lost, $\dot{J} < 0$ & $\dot{m}_1 < 0$

and also folding in $\frac{m_1}{M} < 1$

we get,

$$\frac{\dot{a}}{a} = -\frac{2\dot{J}}{J} + \frac{2\dot{m}_1}{m_1}\left(1 - \frac{1}{2}\frac{m_1}{M}\right)$$

Two competing effects.

1st term may tend to shrink the binary, second term may tend to widen it.