Gaussian Beam

(Lab Report)

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Objective

The objective of this lab was to obtain the “Beam Waist” and “Divergence” of a He-Ne laser by fitting gaussian surfaces on different intensity profiles of the laser.

Contents

1 Definition

In optics, a Gaussian beam is a highly monochromatic electromagnetic beam whose amplitude envelope in the transverse plane is given by a Gaussian function. it also has a Gaussian intensity (irradiance) profile. When such a beam is refocused by a lens, the transverse phase dependence is altered; this results in a different Gaussian beam. The electric and magnetic field amplitude profiles along any such circular Gaussian beam (for a given wavelength and polarization) are determined by a single parameter: the so-called waist w_o.

2 Procedure

2.1 Parameters

1. Distance between successive planes is 25 cm,
2. The focal length of the lens is 3 cm,
3. Diameter of collimated beam is approximately 2 mm.
4. The laser is a red He-Ne laser of wavelength 632 nm.
5. For images at z_1 and z_2, 1 cm = 160 pixels.
6. For images at z_3 and z_4, 1 cm = 236 pixels.

2.2 Setup

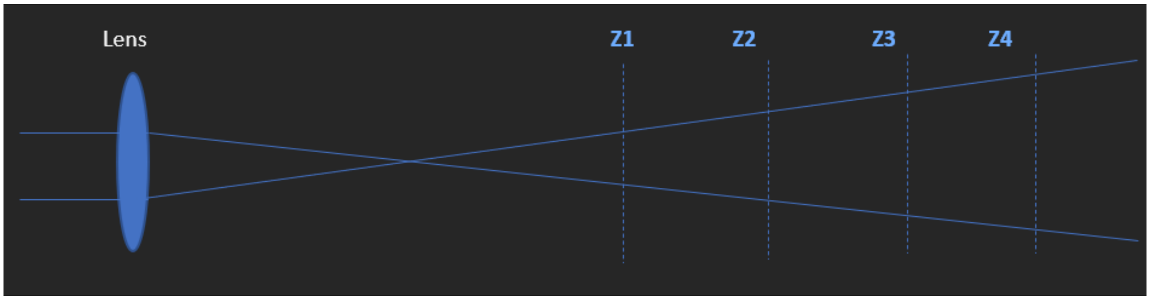
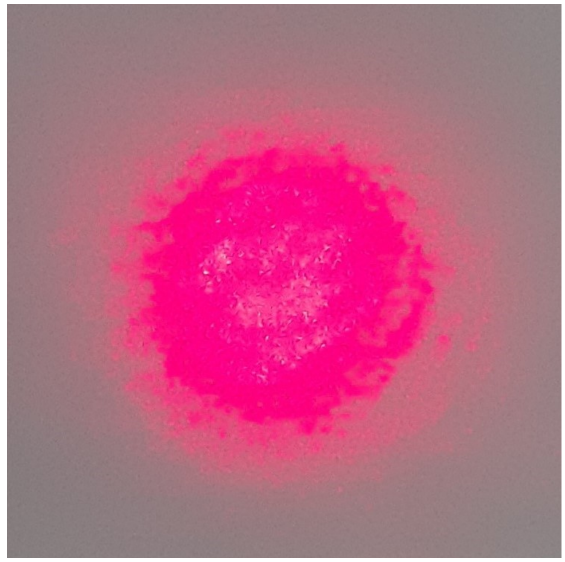


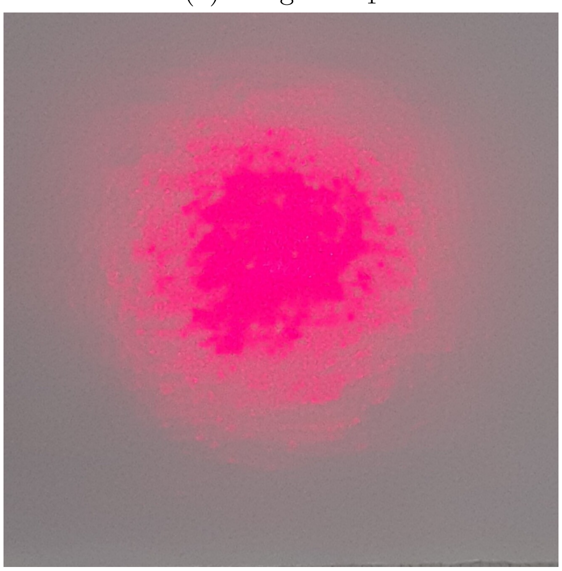
Figure 1: Ray Diagram of Setup

2.3 Observations

(a) Image at z_1



(b) Image at z_2

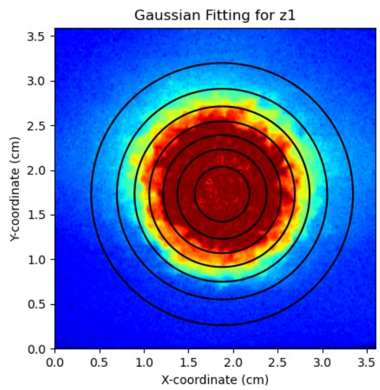


(c) Image at z_3

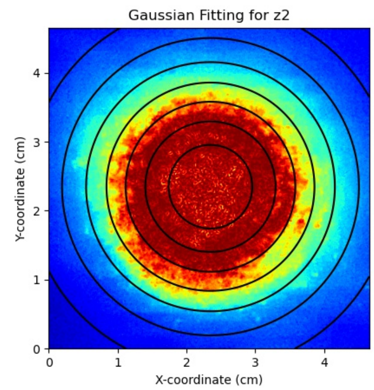
(d) Image at z_4

Figure 2: Images of Laser Beam at intervals of 25 cm.

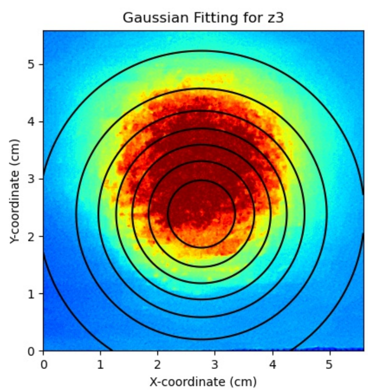
2.4 Gaussian Fit



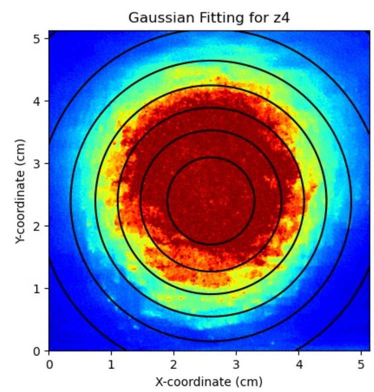
(a) Gaussian Fit for image at z_1



(b) Gaussian Fit for image at z_2



(c) Gaussian Fit for image at z_3



(d) Gaussian Fit for image at z_4

Figure 3: Gaussian Fit for all images.

Equation used for Gaussian surface:

$$f(x, y) = D + A \exp\left\{-\frac{(x - x_o)^2 + (y - y_o)^2}{2\sigma^2}\right\} \tag{1}$$

where D is offset, A is amplitude, $(x_o,\ y_o)$ is center, and σ is the standard deviation.

	A	x_o	y_o	σ	D
z_1	188.78 ± 0.0950	1.87 ± 0.0003	1.72 ± 0.0003	0.70 ± 0.0004	103.35 ± 0.0513
z_2	155.99 ± 0.0742	2.35 ± 0.0003	2.34 ± 0.0003	1.30 ± 0.0008	119.92 ± 0.0856
z_3	145.76 ± 0.0230	2.76 ± 0.0001	2.38 ± 0.0001	1.23 ± 0.0002	129.90 ± 0.0163
z_4	141.70 ± 0.0435	2.59 ± 0.0002	2.39 ± 0.0002	1.51 ± 0.0006	132.50 ± 0.0510

Table 1: Fitting parameters and there errors

Using the fitting parameters the following were determined:

$w(z_1)$	1.4097891328612377 cm
$w(z_2)$	2.614863530550012 cm
$w(z_3)$	2.4631081312127363 cm
$w(z_4)$	3.0368550386562196 cm

Table 2: Beam radius at different z values

3 Divergence (θ)

Let the distance between focus and z_1 be x cm. Then using similarity of triangles we can say,

$$\begin{aligned} \frac{x}{x+25} &= \frac{w(z_1)}{w(z_2)} \\ \implies \frac{x}{x+25} &= \frac{1.41}{2.61} \\ \implies \frac{x}{x+25} &= 0.540 \\ \implies (1-0.54)x &= 13.50 \\ \implies x &= 29.36 \text{ cm} \end{aligned} \tag{2}$$

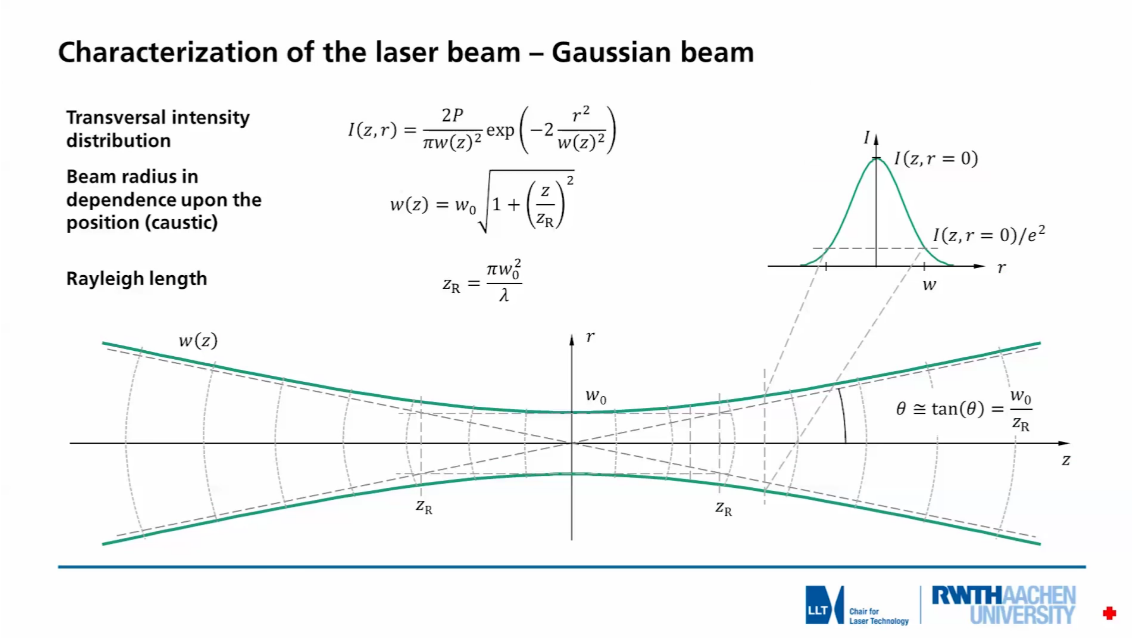


Figure 4: Geometric definitions of different Gaussian beam characteristics

We know,

$$\tan\theta = \frac{w(z_o)}{z_R} \tag{3}$$

Using similarity of triangles,

$$\begin{aligned}
\tan\theta &= \frac{w(z_1)}{x} \\
\implies \tan\theta &= \frac{1.41}{29.36} \\
\implies \tan\theta &= 0.048 \\
\implies \theta &= \arctan(0.048) \\
\implies \theta &= 2.7^\circ
\end{aligned} \tag{4}$$

Hence divergence is 2.7°.

4 Beam Waist (w_o)

We know,

$$\tan\theta = \frac{w_o}{z_R} \text{ and } z_R = \frac{\pi w_o^2}{\lambda} \implies \tan\theta = \frac{\lambda}{\pi w_o} \tag{5}$$

$$\begin{aligned} &\implies 0.047 = \frac{632\times 10^{-9}}{\pi w_o} \\ &\implies w_o = 4.265\times 10^{-6}\ \text{m} \\ &\implies w_o = 4.265\times 10^{-3}\ \text{mm} \end{aligned} \tag{6}$$

5 Error Analysis

5.1 Root mean squared error

RMS(z_1)	13.202312326637943
RMS(z_2)	11.286342871310694
RMS(z_3)	7.887848068630636
RMS(z_4)	9.502495868079983

Table 3: RMS error at different z values

5.2 Coefficient of determination

$r^2(z_1)$	0.9298868479311381
$r^2(z_2)$	0.9224667491681414
$r^2(z_3)$	0.9589927045109969
$r^2(z_4)$	0.9301858936509008

Table 4: RMS error at different z values

6 Conclusion

Divergence (θ) = 2.7°
Beam Waist = 4.265×10^{-3} mm