LOGISTIC MAP

$$x_{n+1} \;=\; F(x_n) \;=\; r\; x_n(1-x_n)$$

Parameter : $0 < r \leq 4$ $\qquad x \geq 0$

Simple idealised model of seasonal variations of a population

Relevant to seasonally breeding population in which generations do not overlap

Temperate zone insects, including many economically important crop and orchard pests, are of this kind

Simple mathematical models with very complicated dynamics, Robert May, *Nature* 1976

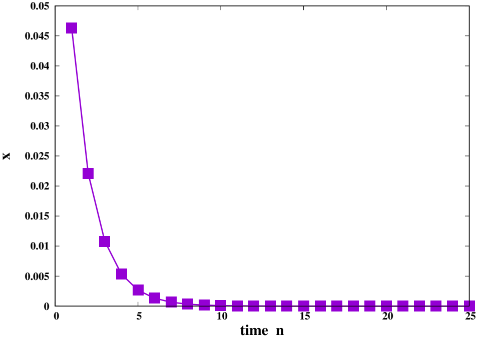
First, check out the Time Series of the Linear Map:

$$x_{n+1} = r\, x_n$$

- If $r < 1$: Extinction as $x_n \to 0$ as $n \to \infty$
- If $r > 1$: Exponential Growth as $x_n \to \infty$ as $n \to \infty$

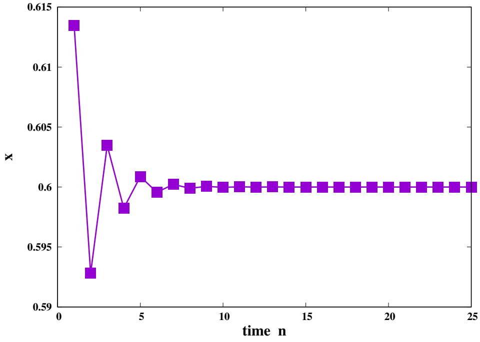
Next, check out the timeseries of the Logistic Map for different r

I. Time series when $r < 1$



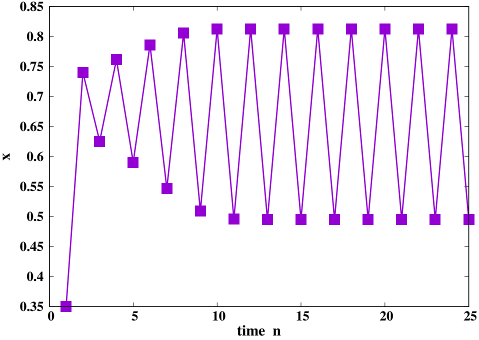
$r = 0.5$ After transience $x^* = 0$

II. Time series when $1 < r < 3$



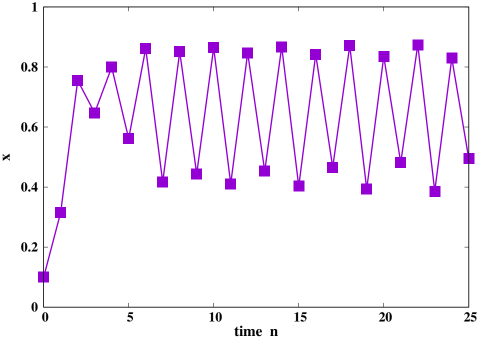
$r \ = \ 2.5$ After transience $x^* = 1 - \frac{1}{r} = \frac{3}{5}$

III. Time series when $r > 3$



$r \;=\; 3.25$ After transience Period 2-cycle: $x_0 \rightarrow x_1 \rightarrow x_0 \ldots$

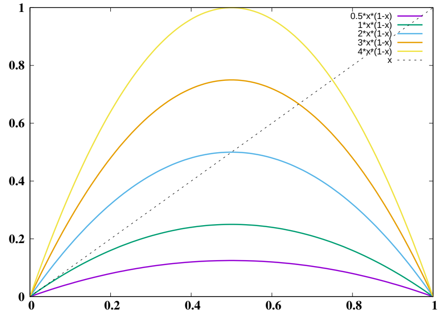
IV. Time series when $3.449 \cdots < r < 3.544 \ldots$



$r = 3.5$

After transcience Period 4-cycle: $x_0 \to x_1 \to x_2 \to x_3 \to x_0 \ldots$

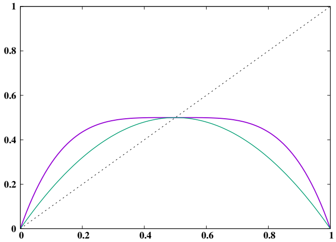
Graphical Solution of the Fixed Point



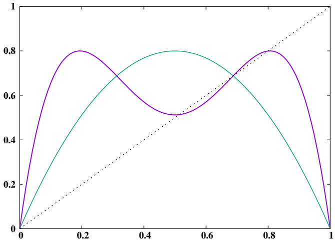
Solid lines: x_{n+1} for parameter $r = 0.5, 1, 2, 3$ and 4
Dashed: 45^0 line

Intersections: Fixed Point solutions
Slope at intersections: Stability of the Fixed Points

Now consider the two times iterated logistic map F^2



$r = 2$



$r = 3.2$

- The fixed point of F is also fixed point of F^2
- Two new solutions (intersections) for $r > 3$