NORTH SOUTH UNIVERSITY

DEPARTMENT OF MATHEMATICS & PHYSICS
ASSIGNMENT # 01, FALL 2021
(MAT 125) INTRODUCTION TO LINEAR ALGEBRA
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Submitted by:

Name: Md. Abu Sayed Fahim
ID#: 1812534642
Serial #:
Email: abu.fahim@northsouth.edu
Name of the Department: ECE

Date of Submission: 22nd November, 2021

Submitted to:

Mohammad Mahmud Hasan
Senior Lecturer
Office: SAC 1156
Department of Mathematics & Physics
North South University
E-Mail: mohammad.hasan02@northsouth.edu

Number of solved problems: 20

N.B.:

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Answer No - 01

$$x + 2y + 3z = 3$$
$$2x + 3y + 8z = 4$$
$$5x + 8y + 19z = 11$$

$$[A|b] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 2 & 3 & 8 & 4 \\ 5 & 8 & 19 & 11 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -1 & 2 & -2 \\ 0 & -2 & 4 & -4 \end{array}\right] \qquad \begin{array}{l} R_2' = R_2 + R_1(-2) \\ R_3' = R_3 + R_1(-5) \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & -2 & 2 \\ 0 & -2 & 4 & -4 \end{array}\right] \qquad R_2' = R_2(-1)$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad R_3' = R_3 + R_2(2)$$

$$x + 2y + 3z = 3$$
$$y - 2z = 2$$

if, $y = 2z + 2$

then, $x = 3 - 2y - 3z$

$= 3 - 2(2z + 2) - 3z$

$= 3 - 4z - 4 - 3z$

$= -1 - 7z$

So, $x = -1 - 7z$

$y = 2z + 2$

let, $z = p$

$x = -1 - 7p$; $y = 2p + 2$

set, $p = 0$ then, $x = -1, y = 2, z = 0$

set, $p = 1$, then, $x = -8, y = 4, z = 0$

So, the system is consistent and it has infinite many solution.

Answer No - 02

$$x_1 - x_2 + x_3 - x_4 + x_5 = 1$$

$$2x_1 - x_2 + 3x_3 + 4x_5 = 2$$

$$3x_1 - 2x_2 + 2x_3 + x_4 + x_5 = 1$$

$$x_1 + x_3 + 2x_4 + x_5 = 0$$

$$(A|b) = \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 2 & -1 & 3 & 0 & 4 & 2 \\ 3 & -2 & 2 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 2 & 0 \\ 0 & 1 & -1 & 2 & -2 & -2 \\ 0 & 1 & 0 & 3 & 0 & -1 \end{array}\right] \quad \begin{array}{l} R_2' = R_2 + (-2)R_1 \\ R_3' = R_3 + (-3)R_1 \\ R_4' = R_4 + (-1)R_1 \end{array}$$

$$= \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 2 & 0 \\ 0 & 0 & -2 & 0 & -4 & -2 \\ 0 & 0 & -1 & 1 & -2 & -1 \end{array}\right] \quad \begin{array}{l} R_3' = R_3 + (-1)R_2 \\ R_4' = R_4 + (-1)R_2 \end{array}$$

$$= \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & -1 & 1 & -2 & -1 \end{array}\right] \quad R_3' = R_3 \times \left(-\frac{1}{2}\right)$$

$$= \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right] \quad R_4' = R_3 + R_4$$

Here, x_5 is free variable.

Let, $\cancel{x_5}$ =

$$x_1 - x_2 + x_3 - x_4 + x_5 = 1$$

$$x_2 + x_3 + 2x_4 + 2x_5 = 0$$

$$x_3 + 2x_5 = 1$$

$$x_4 = 0$$

So, x_1, x_2, x_3, x_4 are leading variable and x_5 is free variable.

Let, $x_5 = P$

So, $x_4 = 0$

$\therefore x_3 + 2x_5 = 1$

$\Rightarrow x_3 + 2.P = 1$

$x_3 = 1 - 2P$

$\therefore x_2 + x_3 + 2x_4 + 2x_5 = 0$

$\Rightarrow x_2 = -x_3 - 2x_4 - 2x_5$

$= -1 - 2P - 2.0 - 2P$

$= -1 - 2P - 2P = 1 - 4P$

$\therefore x_1 - x_2 + x_3 - x_4 + x_5 = 1$

$\Rightarrow x_1 = 1 + 1 - 4P - 1 + 2P - P$

$= 1 - 3P$

Therefore, $x_1 = 1 - 3P$

$x_2 = 1 - 4P$

$x_3 = 1 - 2P$, $x_4 = 0$, $x_5 = P$

the system is consistent and has many solution.

Answer No - 03

$$x + 2y - 3z = 4$$

$$x + 3y + z = 11$$

$$2x + 5y - 4z = 13$$

$$2x + 6y + 2z = 22$$

$$(A|b) = \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 1 & 3 & 1 & 11 \\ 2 & 5 & -4 & 13 \\ 2 & 6 & 2 & 22 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 1 & 4 & 7 \\ 0 & 1 & 2 & 5 \\ 0 & 2 & 8 & 14 \end{array}\right] \quad \begin{array}{l} R_2' = R_2 + (-1)R_1 \\ R_3' = R_3 + (-2)R_1 \\ R_4' = R_4 + (-2)R_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 1 & 4 & 7 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R_3' = R_3 + (-1)R_2 \\ R_4' = R_4 + (-1)R_2 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 1 & 4 & 7 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad R_3' = \left(-\frac{1}{2}\right) R_3$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 0 & 7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{array}{l} R_1' = R_1 + 3R_3 \\ R_2' = R_2 + (-4) R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad R_1' = R_1 + (-2) R_2$$

Therefore, $x = 1$

$y = 3$

$z = 1$

Answer No - 04

$$10x_2 - 4x_3 + x_4 = 1$$

$$x_1 + 4x_2 - x_3 + x_4 = 2$$

$$3x_1 + 2x_2 + x_3 + 2x_4 = 5$$

$$-3x_1 - 8x_2 + 2x_3 - 2x_4 = -4$$

$$x_1 - 6x_2 + 3x_3 = 1$$

$$(A|b) = \left[\begin{array}{cccc|c} 0 & 10 & -4 & 1 & 1 \\ 1 & 4 & -1 & 1 & 2 \\ 3 & 2 & 1 & 2 & 5 \\ -3 & -8 & 2 & -2 & -4 \\ 1 & -6 & 3 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 4 & -1 & 1 & 2 \\ 0 & 10 & -4 & 1 & 1 \\ 3 & 2 & 1 & 2 & 5 \\ -3 & -8 & 2 & -2 & -4 \\ 1 & -6 & 3 & 0 & 1 \end{array}\right] \quad R_1 \rightleftharpoons R_2$$

$$= \left[\begin{array}{cccc|c} 1 & 4 & -1 & 1 & 2 \\ 0 & 10 & -4 & 1 & 1 \\ 0 & -10 & 4 & -1 & -1 \\ 0 & 4 & -1 & 1 & 2 \\ 0 & -10 & 4 & -1 & -1 \end{array}\right] \qquad \begin{array}{l} R_3' = R_3 + (-3) R_1 \\ R_4' = R_4 + (3) R_1 \\ R_5' = R_5 + (-1) R_1 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 4 & -1 & 1 & 2 \\ 0 & 1 & -\frac{2}{5} & \frac{1}{10} & \frac{1}{10} \\ 0 & -10 & 4 & -1 & -1 \\ 0 & 4 & -1 & 1 & 2 \\ 0 & -10 & 4 & -1 & -1 \end{array}\right] \qquad R_2' = \left(\frac{1}{10}\right) R_2$$

$$= \left[\begin{array}{cccc|c} 1 & 4 & -1 & 1 & 2 \\ 0 & 1 & -\frac{2}{5} & \frac{1}{10} & \frac{1}{10} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{5} & \frac{3}{5} & \frac{8}{5} \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{array}{l} R_3' = R_3 + (10) R_1 \\ R_4' = R_4 + (-4) R_1 \\ R_5' = R_5 + (10) R_1 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 4 & -1 & 1 & 2 \\ 0 & 1 & -\frac{2}{5} & \frac{1}{10} & \frac{1}{10} \\ 0 & 0 & \frac{3}{5} & \frac{3}{5} & \frac{8}{5} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad R_3 \rightleftharpoons R_4$$

$$= \left[\begin{array}{cccc|c} 1 & 4 & -1 & 1 & 2 \\ 0 & 1 & -\frac{2}{5} & \frac{1}{10} & \frac{1}{10} \\ 0 & 0 & 1 & 1 & \frac{8}{3} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad R_3' = \left(\frac{5}{3}\right) R_3$$

$$= \left[\begin{array}{cccc|c} 1 & 4 & -1 & 1 & 2 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{7}{5} \\ 0 & 0 & 1 & 1 & \frac{8}{3} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad R_2' = R_2 + \left(\frac{2}{5}\right) R_3'$$

$$= \left[\begin{array}{cccc|c} 1 & 4 & 0 & 2 & \frac{14}{3} \\ 0 & 1 & 0 & \frac{1}{2} & \frac{7}{5} \\ 0 & 0 & 1 & 1 & \frac{8}{3} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad R_1' = R_1 + (1) R_3$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{14}{15} \\ 0 & 1 & 0 & \frac{1}{2} & \frac{7}{5} \\ 0 & 0 & 1 & 1 & \frac{8}{3} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad R_1' = R_1 + (-4) R_2$$

Here, x_1, x_2, x_3 are leading variable and x_4 free variable.

Let, $x_4 = p$

$$x_1 = -\frac{14}{15}$$

$$x_2 + \frac{1}{2}p = -\frac{14}{15}$$

$$\Rightarrow x_2 = -\frac{14}{15} - \frac{p}{2}$$

$$= \frac{-28 - 15P}{30}$$

$$x_3 + p = \frac{8}{3}$$

$$\Rightarrow x_3 = \frac{8}{3} - p$$

Therefore, $x_1 = -\frac{14}{15}$.

$$x_2 = \frac{-28 - 15P}{30}$$

$$x_3 = \frac{8}{3} - p$$

The system is consistant and has infinite many solution.

Answer No - 05

Here,

$$x = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$$

R is row echelon form it must have,

$$d = \begin{bmatrix} 4 & 0 & 0 \end{bmatrix}^t$$

So,

$$x = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 2s \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} 5t \\ 0 \\ t \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 2s + 5t \\ s \\ t \end{bmatrix}$$

if, $y = s$, $z = t$

$$x = 4 + 2s + 5t$$
$$y = s$$
$$z = t \quad \therefore R = \begin{bmatrix} 1 & -2 & -5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Ans:

Answer No - 06

$$-2x + y + z = a$$

$$x - 2y + z = 0$$

$$x + y - 3z = c$$

$$(A|b) = \left[\begin{array}{ccc|c} -2 & 1 & 1 & a \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -3 & c \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ -2 & 1 & 1 & a \\ 1 & 1 & -3 & c \end{array}\right] \quad R_1 \rightleftharpoons R_2$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & -3 & 3 & a \\ 0 & 3 & -4 & c \end{array}\right] \quad \begin{array}{l} R_2' = R_2 + (2) R_1 \\ R_3' = R_3 + (-1) R_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & -\frac{a}{3} \\ 0 & 3 & -4 & c \end{array}\right] \quad R_2' = \left(-\frac{1}{3}\right) R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & \frac{-a}{3} \\ 0 & 0 & -1 & a+c \end{array}\right] \quad R_3' = R_3 + (-3)R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & \frac{-a}{3} \\ 0 & 0 & 1 & -a-c \end{array}\right] \quad R_3' = (-1)R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & a+c \\ 0 & 1 & 0 & \frac{-4a-3c}{3} \\ 0 & 0 & 1 & -a-c \end{array}\right] \quad \begin{array}{l} R_1' = R_1 + (-1)R_3 \\ R_2' = R_2 + (1)R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-5a-3c}{3} \\ 0 & 1 & 0 & \frac{-4a-3c}{3} \\ 0 & 0 & 1 & -a-c \end{array}\right] \quad R_1' = R_1 + (2)R_2$$

$$x = \frac{-5a-3c}{3}$$

$$y = \frac{-4a-3c}{3}$$

$$z = -a-c$$

Answer No: 7

$$x + 2y - 3z = a$$

$$2x + 6y - 11z = b$$

$$2x - 4y + 14z = 2c$$

$$(A|b) = \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 2 & 6 & -11 & b \\ 2 & -4 & 14 & 2c \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & 2 & -5 & b-2a \\ 0 & -8 & 20 & 2c-2a \end{array}\right] \quad \begin{array}{l} R_2' = R_2 + (-2)R_1 \\ R_3' = R_3 + (-2)R_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & 1 & \frac{-5}{2} & \frac{b-2a}{2} \\ 0 & -8 & 20 & 2c-2a \end{array}\right] \quad R_2' = R_2\left(\frac{1}{2}\right)$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & 1 & \frac{-5}{2} & \frac{b-2a}{2} \\ 0 & 0 & 0 & 4b+2c-10a \end{array}\right] \quad R_3' = R_3 + (8)R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3a-b \\ 0 & 1 & \frac{-5}{2} & \frac{b-2a}{2} \\ 0 & 0 & 0 & 4b+2c-10a \end{array}\right] \quad R_1' = R_1 + (-2)R_2$$

Here, x and y leading variable and z free variable.

If, $z = p$

$$x = 3a - b - 2p$$

$$y = \frac{b-2a}{2} - \frac{-5p}{2}$$

Answer.

Answer NO - 08

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$

and $A = P + Q$

where, $P^T = P$

$Q^T = -Q$

So, $P + Q = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ ——— (i)

$\Rightarrow (P+Q)^T = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}^T$ [Transpose both side]

$\Rightarrow P^T + Q^T = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$

$\Rightarrow P - Q = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$ ——— (ii)

(i) - (ii), (substracting (ii) from (i))

$$P + Q - P + Q = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$$

$$\Rightarrow 2Q = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\Rightarrow \left(\frac{1}{2}\right) 2Q = \frac{1}{2} \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\therefore Q = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Answer:

Answer No - 09

$$[A|I] = \left[\begin{array}{ccc|ccc} 2 & 5 & 3 & 1 & 0 & 0 \\ 1 & 0 & 8 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 8 & 0 & 1 & 0 \\ 2 & 5 & 3 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \quad R_1 \rightleftarrows R_2$$

$$\left[\begin{array}{cccccc} 1 & 0 & 8 & 0 & 1 & 0 \\ 0 & 5 & -13 & 1 & -2 & 0 \\ 0 & 2 & -5 & 0 & -1 & 1 \end{array}\right] \quad \begin{array}{l} R_2' = R_2 + (-2)R_1 \\ R_3' = R_3 + (-1)R_1 \end{array}$$

$$\left[\begin{array}{cccccc} 1 & 0 & 8 & 0 & 1 & 0 \\ 0 & 1 & -3 & 1 & 0 & -2 \\ 0 & 2 & -5 & 0 & -1 & 1 \end{array}\right] \quad R_2' = R_2 + (-2)R_3$$

$$\left[\begin{array}{cccccc} 1 & 0 & 8 & 0 & 1 & 0 \\ 0 & 1 & -3 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 & -1 & 5 \end{array}\right] \quad R_3' = R_3 + (-2)R_2$$

$$= \begin{bmatrix} 1 & 0 & 8 & 0 & 1 & 0 \\ 0 & 1 & 0 & -5 & -3 & 13 \\ 0 & 0 & 1 & -2 & -1 & 5 \end{bmatrix} \quad R_2' = R_2 + (3)R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 16 & 9 & -40 \\ 0 & 1 & 0 & -5 & -3 & 13 \\ 0 & 0 & 1 & -2 & -1 & 5 \end{array}\right] \quad R_1' = R_1 + (-8)R_3$$

$$= [\,I \mid A^{-1}\,]$$

Thus,

$$A^{-1} = \begin{bmatrix} 16 & 9 & -40 \\ -5 & -3 & 13 \\ -2 & -1 & 5 \end{bmatrix}.$$

Answer:

Answer No - 10

$$B = \begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 1 & -4 \end{bmatrix}; \quad B^{-1} = \frac{1}{|B|} Adj(B)$$

$$|B| \neq 0$$

$$|B| = \begin{vmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 1 & -4 \end{vmatrix}$$

$$= 3(0-3) - 4(-4-6) - 1(1-0)$$

$$= -9 + 40 - 1$$

$$= 30$$

$|B| \neq 0$; B^{-1} exist.

Co factor of $[B]$;

$$C_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 3 \\ 1 & -4 \end{vmatrix} = (0-3) = -3$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} = (-1)(-4-6) = 10$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = (1-0) = 1$$

$$C_{21} = (-1)^{2+1} \begin{vmatrix} 4 & -1 \\ 1 & -4 \end{vmatrix} = (-1)(-16+1) = 15$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 3 & -1 \\ 2 & -4 \end{vmatrix} = (-12+2) = -10$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix} = (-1)(3-8) = 5$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 4 & -1 \\ 0 & 3 \end{vmatrix} = (12-0) = 12$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} = (-1)(9+1) = -10$$

$$C_{33} = (-1)^{33} \begin{vmatrix} 3 & 4 \\ 1 & 0 \end{vmatrix} = (0-4) = -4$$

$$Adj(B) = \begin{bmatrix} -3 & 10 & 1 \\ 15 & -10 & 5 \\ 12 & -10 & -4 \end{bmatrix}^T = \begin{bmatrix} -3 & 15 & 12 \\ 10 & -10 & -10 \\ 1 & 5 & -4 \end{bmatrix}$$

Therefore,

$$B^{-1} = \frac{1}{30} \begin{bmatrix} -3 & 15 & 12 \\ 10 & -10 & -10 \\ 1 & 5 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{10} & \frac{1}{2} & \frac{2}{5} \\ \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ \frac{1}{30} & \frac{1}{6} & -\frac{2}{15} \end{bmatrix}$$

Answer:

Answer No - 11

$$A(t) = \begin{bmatrix} e^t & 0 & 0 \\ 0 & \cos t & \sin t \\ 0 & -\sin t & \cos t \end{bmatrix}$$

$$|A(t)| = e^t(\cos^2 t + \sin^2 t)$$

$$= e^t$$

$\therefore$ $|A(t)| \neq 0$; A^{-1} exist

Co factor of $A(t)$

$$C_{11} = \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} = (\cos^2 t + \sin^2 t) = 1$$

$$C_{12} = (-1)\begin{vmatrix} 0 & \sin t \\ 0 & \cos t \end{vmatrix} = (-1) \times 0 = 0$$

$$C_{13} = \begin{vmatrix} 0 & \cos t \\ 0 & -\sin t \end{vmatrix} = 0$$

$$C_{21} = (-1)\begin{vmatrix} 0 & 0 \\ -\sin t & \cos t \end{vmatrix} = 0$$

$$c_{22} = \begin{vmatrix} e^t & 0 \\ 0 & \cos t \end{vmatrix} = e^t \cos t.$$

$$c_{23} = (-1) \begin{vmatrix} e^t & 0 \\ 0 & -\sin t \end{vmatrix} = -e^t \sin t$$

$$c_{31} = \begin{vmatrix} 0 & 0 \\ \cos t & \sin t \end{vmatrix} = 0$$

$$c_{32} = (-1) \begin{vmatrix} e^t & 0 \\ 0 & \sin t \end{vmatrix} = -e^t \sin t$$

$$c_{33} = \begin{vmatrix} e^t & 0 \\ 0 & \cos t \end{vmatrix} = e^t \cos t$$

$$Adj(A(t)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^t \cos t & -e^t \sin t \\ 0 & -e^t \sin t & e^t \cos t \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^t \cos t & -e^t \sin t \\ 0 & -e^t \sin t & e^t \cos t \end{bmatrix}$$

Therefore,

$(A(t))^{-1}$

$$= \frac{1}{e^t}\begin{bmatrix} 1 & 0 & 0 \\ 0 & e^t\cos t & -e^t\sin t \\ 0 & e^t\sin t & e^t\cos t \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{e^t} & 0 & 0 \\ 0 & \cos t & -\sin t \\ 0 & \sin t & \cos t \end{bmatrix}$$

Answer.

Answer No: 12

$$x + y + z = 6$$

$$2x + 3y + 4z = 20$$

$$3x - 2y + z = 2$$

$$(A|b) = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 3 & 4 & 20 \\ 3 & -2 & 1 & 2 \end{array}\right]$$

$$|A| = 1(3+8) - 1(2-12) + 1(-4-9)$$

$$= 11 + 10 - 13 = 21$$

So, A^{-1} exist.

Co factors of $[A]$,

$$C_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 4 \\ -2 & 1 \end{vmatrix} = (3+8) = 11$$

$$C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} = (-1)(2-12) = 10$$

$$C_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = (-4-9) = -13$$

$$C_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = (-1)(1+2) = -3$$

$$C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = (1-3) = -2$$

$$C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = (-1)(-2-3) = 5$$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = (4-3) = 1$$

$$C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} = (-1)(4-2) = -2$$

$$C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = (3-2) = 1$$

$$\mathrm{Adj}(A) = \begin{bmatrix} 11 & 10 & -13 \\ -3 & -2 & 5 \\ 1 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} 11 & -3 & 1 \\ 10 & -2 & -2 \\ -13 & 5 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{8}\begin{bmatrix} 11 & -3 & 1 \\ 10 & -2 & -2 \\ -13 & 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{11}{8} & -\frac{3}{8} & \frac{1}{8} \\ -\frac{5}{4} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{13}{8} & \frac{5}{8} & \frac{1}{8} \end{bmatrix}$$

Now, multiplying the inverse matrix with the solution vector.

$$= \begin{bmatrix} \frac{11}{8} & -\frac{3}{8} & \frac{1}{8} \\ \frac{5}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{13}{8} & \frac{5}{8} & \frac{1}{8} \end{bmatrix} \times \begin{bmatrix} 6 \\ 20 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{11}{8} \times 6 - \frac{3}{8} \times 20 + \frac{1}{8} \times 2 \\ \frac{5}{4} \times 6 - \frac{1}{4} \times 20 - \frac{1}{4} \times 2 \\ -\frac{13}{8} \times 6 + \frac{5}{8} \times 20 + \frac{1}{8} \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Therefore, $x = 1$

$y = 2$

$z = 3$

Answer:

Answer No - 13

$$x + y + z - 2t = -4$$

$$x - 2y + 3z + 4t = 10$$

$$2x + 3y - z + 2t = 9$$

$$4x - y + 2z - t = -7$$

$$(A|b) = \left[\begin{array}{cccc|c} 1 & 1 & 1 & -2 & -4 \\ 1 & -2 & 3 & -4 & 10 \\ 2 & 3 & -1 & 2 & 9 \\ 4 & -1 & 2 & -1 & -7 \end{array}\right]$$

$$[A|I] = \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 3 & -4 & 0 & 1 & 0 & 0 \\ 2 & 3 & -1 & 2 & 0 & 0 & 1 & 0 \\ 4 & -1 & 2 & -1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & -3 & 2 & -2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -3 & 6 & -2 & 0 & 1 & 0 \\ 0 & -5 & -2 & 7 & -4 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R_2' = R_2 + (-1)R_1 \\ R_3' = R_3 + (-2)R_1 \\ R_4' = R_4 + (-4)R_1 \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -10 & 22 & -9 & 1 & 0 & 0 \\ 0 & 1 & -3 & 6 & -2 & 0 & 1 & 0 \\ 0 & -5 & -2 & 7 & -4 & 0 & 0 & 0 \end{array}\right] \quad R_2' = R_2 + (-4)R_3$$

$$= \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -10 & 22 & -9 & \cancel{1} & 0 & 0 \\ 0 & 0 & 7 & -16 & 7 & -\cancel{1} & \cancel{1} & 0 \\ 0 & 0 & -52 & 117 & -49 & 5 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R_3' = R_3 + (-1)R_2 \\ R_4' = R_4 + (5)R_2 \end{array}$$

$$= \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -10 & 22 & -9 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{16}{7} & 1 & -\frac{1}{7} & \frac{\cancel{1}}{7} & 0 \\ 0 & 0 & -52 & 117 & -49 & 5 & 0 & 0 \end{array}\right] \quad R_3' = R_3\left(\frac{1}{7}\right)$$

$$= \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -10 & 22 & -9 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{16}{7} & 1 & -\frac{1}{7} & 0 & 0 \\ 0 & 0 & 0 & -\frac{13}{7} & -49 & 5 & 0 & 0 \end{array}\right] \quad R_4' = R_4 + (52)R_3$$

$$= \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -10 & 22 & -9 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{16}{7} & 1 & -\frac{1}{7} & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{343}{13} & \frac{35}{13} & 0 & 0 \end{array}\right] \quad R_4' = R_4\left(-\frac{7}{13}\right)$$

Answer No - 14

here,

$$3x + 4y + 7z = 14$$
$$2x - y + 3z = 4$$
$$x + 2y - 3z = 0$$

and $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$

$$|A| = 3(3-6) - 2(-12-14) + 1(12+7)$$
$$= 9 + 52 + 19$$
$$= 80$$

$\therefore A^{-1}$ exist

$C_{11} = (3-6) = -3$; $C_{12} = -(-12-14) = 26$

$C_{13} = (12+7) = 19$; $C_{21} = -(-6-3) = 9$

$C_{22} = (-9-7) = -16$; $C_{23} = -(9-14) = 5$

$C_{31} = (4+1) = 5$; $C_{32} = -(6-4) = -2$

$C_{33} = (-3-4) = -7$

$$Adj(A) = \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & 7 \end{bmatrix}^t = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{80} \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & 7 \end{bmatrix}$$

$$3x + 4y + 7z = 14$$
$$2x - y + 3z = 4$$
$$x + 2y - 3z = 0$$

$$[A \mid b] = \begin{bmatrix} 3 & 4 & 7 & 14 \\ 2 & -1 & 3 & 4 \\ 1 & 2 & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & -1 & 3 & 4 \\ 3 & 4 & 7 & 14 \end{bmatrix} \quad R_1 \rightleftharpoons R_3$$

$$= \begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & -5 & -3 & 4 \\ 0 & -2 & -2 & 14 \end{bmatrix} \quad \begin{matrix} R_2' = R_2 + (-2)R_1 \\ R_3' = R_3 + (-3)R_1 \end{matrix}$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 1 & 3 & -38 \\ 0 & -2 & -2 & 14 \end{array}\right] \quad R_2' = R_2 + (-3) R_3$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 1 & 3 & -38 \\ 0 & 0 & 4 & -62 \end{array}\right] \quad R_3' = R_3' + (2) R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 1 & 3 & -38 \\ 0 & 0 & 1 & -\frac{31}{2} \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 0 & -\frac{93}{2} \\ 0 & 1 & 0 & \frac{17}{2} \\ 0 & 0 & 1 & -\frac{31}{2} \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{127}{2} \\ 0 & 1 & 0 & \frac{17}{2} \\ 0 & 0 & 1 & -\frac{31}{2} \end{array}\right]$$

$x = -\frac{127}{2}$

$y = \frac{17}{2} \quad z = -\frac{31}{2}$ Answer:

Answer No : 15

$$A = \begin{bmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (x+y)(x-y) - (y+z)(z-y) + (z+x)(z-x)$$

$$\cancel{= (x+y)(x-y) -}$$

$$= x^2 - y^2 - z^2 - y^2 + z^2 - x^2$$

$$= 0$$

Answer No: 16

L.H.S,
$$\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= (b^2+c^2)\{(c^2+a^2)(a^2+b^2)-b^2c^2\}$$
$$-a^2\{b^2(a^2+b^2)-b^2c^2\}$$
$$+a^2\{b^2c^2-c^2(c^2+a^2)\}$$

$$= (b^2+c^2)(a^2c^2+b^2c^2+a^4+a^2b^2-b^2c^2)$$
$$-a^2(a^2b^2+b^4-b^2c^2)+a^2(b^2c^2-c^4-a^2c^2)$$

$$= a^2b^2c^2+b^4c^2+b^2a^4+a^2b^4-b^4c^2+a^2c^4+b^2c^4$$
$$+a^4c^2+a^2b^2c^2-b^2c^4-a^4b^2-a^2b^4+a^2b^2c^2$$
$$+a^2b^2c^2-a^2c^4-a^4c^2$$

$$= 4a^2b^2c^2$$

$= R.H.S$

$L.H.S = R.H.S$

(Proved)

Answer NO - 17

$$\text{Det} = \begin{vmatrix} 1 & 2 & 1 & 2 \\ 3 & 1 & -2 & 3 \\ -1 & 0 & 3 & 1 \\ 2 & 3 & 2 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 1 & 2 \\ 0 & -5 & -5 & -3 \\ 0 & 2 & 4 & 3 \\ 0 & -1 & 0 & -5 \end{vmatrix} \qquad \begin{array}{l} R_2' = R_2 + (-3)R_1 \\ R_3' = R_3 + (1)R_1 \\ R_4' = R_4 + (-2)R_1 \end{array}$$

$$= (-5)\begin{vmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & \frac{3}{5} \\ 0 & 2 & 4 & 3 \\ 0 & -1 & 0 & -5 \end{vmatrix} \qquad R_2' = \left(-\frac{1}{5}\right)R_2$$

$$= (-5)\begin{vmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & \frac{3}{5} \\ 0 & 0 & 2 & \frac{9}{5} \\ 0 & 0 & 1 & -\frac{22}{5} \end{vmatrix} \qquad \begin{array}{l} R_3' = R_3 + (-2)R_2 \\ R_4' = R_4 + (1)R_2 \end{array}$$

$$(-5)(2)\begin{vmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & -\frac{3}{5} \\ 0 & 0 & 1 & \frac{3}{10} \\ 0 & 0 & 1 & -\frac{28}{5} \end{vmatrix} \qquad R_3' = \left(\frac{1}{2}\right)R_3$$

$$= (-5)(2)\begin{vmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & -\frac{3}{5} \\ 0 & 0 & 1 & \frac{3}{10} \\ 0 & 0 & 0 & -\frac{59}{10} \end{vmatrix} \qquad R_4' = R_4 + (-1)R_3.$$

$$\therefore \det = (-10)1.\ 1.\ 1.\ -\frac{59}{10}$$

$$= -\frac{59}{10} \times (-10)$$

$$= 59$$

Answer No - 18

$$x + y + z - 2t = -4$$

$$x - 2y + 3z + 4t = 10$$

$$2x + 3y - z + 2t = 9$$

$$4x - y + 2z - t = -7$$

$$\begin{bmatrix} 1 & 1 & 1 & -2 \\ 1 & -2 & 3 & 4 \\ 2 & 3 & -1 & 2 \\ 4 & -1 & 2 & -1 \end{bmatrix}$$

$$\Delta = 1\{2(1-4) - 3(-3+2) + 4(6-1)\}$$
$$-1\{1(1-4) - 3(-2-8) + 4(4+4)\}$$
$$+1\{1(-3+2) + 2(-2-8) + 4(-2-12)\}$$
$$+2\{(6-1) + 2(4+4) + 3(-2-12)\}$$

$$= (6+3+20) - (-3+30+32) + (-1-20-56)$$
$$+ 2(5+16-42)$$

$$= 29 - 59 - 77 - 42 = -149$$

Replacing first column with solution vector.

$$\begin{bmatrix} -4 & 1 & 1 & -2 \\ 10 & -2 & 3 & 4 \\ 9 & 3 & -1 & 2 \\ -7 & -1 & 2 & -1 \end{bmatrix}$$

$$\Delta x = -4\{-2(1-4)-3(-3+2)+4(6-1)\}$$
$$-1\{10(1-4)-3(9+14)+4(18-7)\}$$
$$+1\{10(-3+2)+2(-9+14)+4(9+21)\}$$
$$+2\{10(6-1)+12(18-7)+3(-9+21)\}$$
$$= -4(6+3+20)-(-30-15+44)+(-10+10+48)$$
$$+2(50+22+36)$$
$$= -116+1+48+216 = 149$$

$$\therefore \Delta x = 149$$

Replacing second column with solution vector.

$$\begin{bmatrix} 1 & -4 & 1 & -2 \\ 1 & 10 & 3 & 4 \\ 2 & 9 & -1 & 2 \\ 4 & -7 & 2 & -1 \end{bmatrix}$$

$$\Delta Y = 1\{10(1-4)-3(-9+14)+4(18-7)\}$$
$$-4\{1(1-4)-3(-2-8)+4(4+4)\}$$
$$+1\{1(-9+14)-10(-2-8)+4(-14-36)\}$$
$$+2\{1(18-7)-10(4+4)+3(-14-36)\}$$

$$= (-30-15+44)+4(-3+30+32)$$
$$+(5+100+200)+2(11-80-150)$$

$$= -1+236-95-438 = -298$$

$$\Delta Y = -298.$$

Replacing third column with solution vector.

$$\begin{bmatrix} 1 & 1 & -4 & -2 \\ 1 & -2 & 10 & 4 \\ 2 & 3 & 9 & 2 \\ 4 & -1 & -7 & -1 \end{bmatrix}$$

$$\Delta z = 1\{-2(-9+14)-10(-3+2)+4(-21+7)\}$$
$$-1\{1(-9+14)-10(-2-8)+4(-14-36)\}$$
$$-4\{1(-3+2)+2(-2-8)+4(-2-12)\}$$
$$+2\{1(-21+7)+2(-14-36)+10(-2-12)\}$$

$$= (-10+10-48)-(5+100-200)$$
$$-4(-1-20-56)+2(-12-100-140)$$

$$= -48+95+308-504$$

$$= -149$$

Replacing fourth column with solution vector

$$\begin{bmatrix} 1 & 1 & 1 & -4 \\ 1 & -2 & 3 & 10 \\ 2 & 3 & -1 & 9 \\ 4 & -1 & 2 & -7 \end{bmatrix}$$

$$\Delta t = 1\{-2(-7-18) - 3(-21+9) + 10(6-1)\}$$
$$-1\{1(7-18) - 3(-14-36) + 10(4+4)\}$$
$$+1\{1(-21+9) + 2(-14-36) + 10(-2-12)\}$$
$$+4\{1(6-1) + 2(4+4) + 3(-2-12)\}$$

$$= (22+36+50) - (-11+150+80) + (12-100-140)$$
$$+4(5+16-42)$$

$$= 108 - 219 - 252 - 84 = -447$$

$$\Delta t = -447$$

Therefore,

$$x = \frac{\Delta x}{\Delta} = \frac{149}{-149} = -1$$

$$y = \frac{\Delta y}{\Delta} = \frac{-298}{-149} = 2$$

$$z = \frac{\Delta z}{\Delta} = \frac{-149}{-149} = 1$$

$$t = \frac{\Delta t}{\Delta} = \frac{-447}{-149} = 3$$

$$(x, y, z, t) = (1, 2, 1, 3)$$

Answer:

Answer No: 19

Let, $p = (x, y, z)$ be vector,

such that,

$$||p|| \Rightarrow \sqrt{x^2 + y^2 + z^2} = 1$$

$$\Rightarrow x^2 + y^2 + z^2 = 1 \quad \text{—— (1)}$$

$$(x, y, z), (1, -2, 4) = 0$$

$$\Rightarrow x - 2y + 4z = 0$$

$$\therefore x = 2y - 4z \quad \text{—— (11)}$$

(1) + (11)

$$(2y - 4z)^2 + y^2 + z^2 = 1$$

$$\Rightarrow 4y^2 - 16yz + 16z^2 + y^2 + z^2 = 1$$

$$\Rightarrow 5y^2 + 17z^2 - 16yz = 1$$

There are many solution of the system.

Therefore, There will be many (x, y, z) in $\mathbb{R}^3$.

Answer No: 20

Let, $x = (x_1, x_2, x_3, x_4)$ be a vector, which is orthogonal to u, v and w

Then, we have $(x, u) = 2x_1 + x_2 + x_3 = 0$

$(x, v) = -x_1 - x_2 + 2x_3 + 2x_4 = 0$ and

$(x, w) = 3x_1 + 2x_2 + 5x_3 + 4x_4 = 0$

Since,

$2x_1 + x_2 - 4x_3 = 0, \; -x_1 - x_2 + 2x_3 + 2x_4 = 0.$

we have,

$2x_1 + x_2 - 4x_3 + 2(-x_1 - x_2 + 2x_3 + 2x_4)$

$= 0 + 2.0$ and $3(-x_1 - x_2 + 2x_3 + 2x_4)$

$+ 3x_1 + 2x_2 + 5x_3 + 4x_4$

$= 3.0 + 0$ which gives,

$-x_2 + 4x_4 = 0$ and $-x_2 + 11x_3 + 10x_4 = 0$

Thus,

$$x_2 = 4x_4 \text{ and } x_3 = -\frac{6x_4}{11}$$

and $2x_1 + x_2 - 4x_3 = 0$

So, $$x_1 = -\frac{34x_4}{11}$$

$$\therefore x = \left(-\frac{34x_4}{11}, 4x_4, -\frac{6x_4}{11}, x_4\right)$$

Now, in order for x to be unitary we must have $||x|| = 1$

then, we must have

$$\sqrt{\left(-\frac{34x_4}{11}\right)^2 + (4x_4)^2 + \left(-\frac{6x_4}{11}\right)^2 + x_4^2} = 1$$

Since, $$\sqrt{\left(-\frac{34x_4}{11}\right)^2 + (4x_4)^2 + \left(-\frac{6x_4}{11}\right)^2 + (x_4)^2}$$

$$= \sqrt{\frac{1156}{121}x_4^2 + 16x_4^2 + \frac{36}{121}x_4^2 + x_4^2}$$

$$= \sqrt{\left(\frac{1156}{121} + 16 + \frac{36}{121} + 1\right) x_4^2}$$

$$= \sqrt{\frac{3249}{121}x_4^2}$$

$$= \frac{57}{11} x_4$$

$$= 1$$

So, $|x_4| = \frac{11}{57}$

$\therefore x_4 = \pm \frac{11}{57}$

Thus, $x = \left(\frac{34}{57}, -\frac{44}{57}, \frac{6}{57}, -\frac{11}{57} \right)$

and $x' = \left(-\frac{34}{57}, \frac{44}{57}, -\frac{6}{57}, \frac{11}{57} \right)$

are two unit vectors which are orthogonal to u, v and w.

Answer: