NORTH SOUTH UNIVERSITY

DEPARTMENT OF MATHEMATICS & PHYSICS
ASSIGNMENT # 01, FALL 2021
(MAT 125) INTRODUCTION TO LINEAR ALGEBRA
DUE DATE: NOVEMBER 22, 2021

Submitted by:

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Number of solved problems:

N.B.:

1. Please use **A4** size papers and add this sheet as a cover page
2. Assignment will not be **accepted** after the due date
3. Your score will be **zero** for any copy or plagiarism

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SHOW ALL THE NECESSARY CALCULATION TO GET FULL CREDIT.

Solve the following linear system of equations using Gaussian Elimination method (1-2):

1. $x + 2y + 3z = 3$
   $2x + 3y + 8z = 4$
   $5x + 8y + 19z = 11$

2. $x_1 - x_2 + x_3 - x_4 + x_5 = 1$
   $2x_1 - x_2 + 3x_3 + 4x_5 = 2$
   $3x_1 - 2x_2 + 2x_3 + x_4 + x_5 = 1$
   $x_1 + x_3 + 2x_4 + x_5 = 0$

Solve the following linear system of equations using Gauss-Jordan Elimination method (3-4):

3. $x + 2y - 3z = 4$
   $x + 3y + z = 11$
   $2x + 5y - 4z = 13$
   $2x + 6y + 2z = 22$

4. $10x_2 - 4x_3 + x_4 = 1$
   $x_1 + 4x_2 - x_3 + x_4 = 2$
   $3x_1 + 2x_2 + x_3 + 2x_4 = 5$
   $-3x_1 - 8x_2 + 2x_3 - 2x_4 = -4$
   $x_1 - 6x_2 + 3x_3 = 1$

5. Forward elimination changes $A\boldsymbol{x} = \boldsymbol{b}$ to a row reduced $R\boldsymbol{x} = \boldsymbol{d}$; the general solution is
   $$\boldsymbol{x} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$$
   What is the 3×3 reduced row echelon matrix R and what is $\boldsymbol{d}$?

Find the conditions on a, b, and c so that the following linear system of equations have a solution (7-8):

6. $-2x + y + z = a$
   $x - 2y + z =$
   $x + y - 3z = c$

7. $x + 2y - 3z = a$
   $2x + 6y - 11z = b$
   $2x - 4y + 14z = 2c$

8. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ and $A = P + Q$, where P is a symmetric matrix and Q is a skew- symmetric matrix. Compute the matrix Q. (Hint: A is a symmetric matrix if $A^T = A$ and A is a skew-symmetric matrix if $A^T = -A$).

9. Find the inverse of $A = \begin{bmatrix} 2 & 5 & 3 \\ 1 & 0 & 8 \\ 1 & 2 & 3 \end{bmatrix}$ using row operations and verify your answer.

10. Find the inverse using $B = \begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 1 & -4 \end{bmatrix}$ using adjoint matrix.

11. Suppose $A(t) = \begin{bmatrix} e^t & 0 & 0 \\ 0 & \cos t & \sin t \\ 0 & -\sin t & \cos t \end{bmatrix}$, show that $A(t)^{-1}$ exists and then find it.

Solve the following linear system of equations using Inverse matrix method (12-13):

12. $x + y + z = 6$
$2x + 3y + 4z = 20$
$3x - 2y + z = 2$

13. $x + y + z - 2t = -4$
$x - 2y + 3z + 4t = 10$
$2x + 3y - z + 2t = 9$
$4x - y + 2z - t = -7$

14. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1} and hence solve the system of equations

$$3x + 4y + 7z = 14$$
$$2x - y + 3z = 4$$
$$x + 2y - 3z = 0$$

15. If $A = \begin{bmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{bmatrix}$, then compute the determinant of A.

16. By using the properties of determinant prove that

$$\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$$

17. Find the determinant using the row operations $\begin{vmatrix} 1 & 2 & 1 & 2 \\ 3 & 1 & -2 & 3 \\ -1 & 0 & 3 & 1 \\ 2 & 3 & 2 & -1 \end{vmatrix}$

18. Solve the following linear system of equations using Cramer's Rule
$$x + y + z - 2t = -4$$
$$x - 2y + 3z + 4t = 10$$
$$2x + 3y - z + 2t = 9$$
$$4x - y + 2z - t = -7$$

Problem related to Euclidean n-space:

19. Show that there are infinitely many vectors in $\mathbb{R}^3$ with Euclidean norm 1 whose Euclidean inner product with $(1, -2,4)$ is zero.

20. Find two vectors of norm 1 that are orthogonal to the vectors $u = (2,1, -4,0)$, $v = (-1, -1,2,2)$, and $w = (3,2,5,4)$.