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Ans to the Q no - 1

Here, $W = \left\{ \begin{bmatrix} x & y \\ z & w \end{bmatrix} \middle| \; x - 2w = z + y \right\}$

All the matrix in W also belong to M_{22}

$\therefore W \subset M_{22}$.

let, $\vec{u} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$ $\vec{v} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in W$

Then,

$$\vec{u} + \vec{v} = \begin{bmatrix} x & y \\ z & w \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} x+a & y+b \\ z+c & w+d \end{bmatrix}$$

Now here, $(x+a) - 2(w+d) \cancel{= x+z+c+y+b}$.

$$= x + a - 2w - 2d$$

$$= (x - 2w) + (a - 2d)$$

$$= z + y + c + b$$

$= (z+c) + (y+b)$

$\therefore \vec{u} + \vec{v} \in W$

$\therefore$ W is closed under addition

let. k be any scalar. ~~and~~

$$\vec{u} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

Then $k\vec{u} = k \begin{bmatrix} x & y \\ z & w \end{bmatrix}$

$$= \begin{bmatrix} kx & ky \\ kz & kw \end{bmatrix}$$

Now, $kx - 2kw$

$\Rightarrow = k(x-2w)$

$= k(z+y)$

$= kz + ky.$

$\therefore k\vec{u} \in W$

$\therefore$ W is closed under scalar

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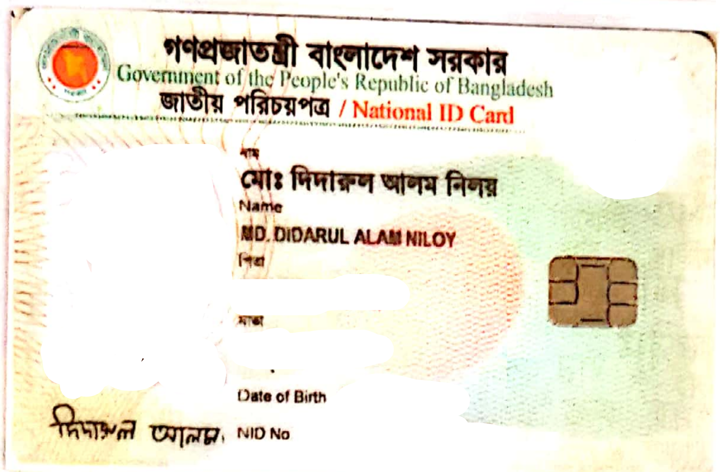
multiplication

$\therefore$ W is a subspace of M_{22}

Thus w is ~~sy~~ sytisfy by colse under addition and close under multiflication then w is a subspace of M_{22}

(Shown)



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Ans to the Q no - 2

To ~~to prove~~ show S is a basis for P_2

we need to show.

S is spans of P_2

S is linearly independent.

$\therefore$ $P_1 k_1 + P_2 k_2 + P_3 k_3 = 0$ for linearly independent

~~again $P_1 k_1 + P_2 k_2 + P_3 k_3 = \vec{b}$ for spans of P_2.~~

Now, $k_1(-1+2x+3x^2) + k_2(2+4x-6x^2) + k_3(-3+6x) = (0,0,0)$

Here, ~~augmented matrix~~.

$-k_1 + 2k_2 - 3k_3 = 0$

$-2k_1 + 4k_2 + 6k_3 = 0$

$3k_1 - 6k_2 + 0 = 0$

now augmented matrix.

$$\left[\begin{array}{ccc|c} -1 & 2 & -3 & 0 \\ -2 & 4 & 6 & 0 \\ 3 & -6 & 0 & 0 \end{array}\right]$$

again,

$P_1 K_1 + P_2 K_2 + P_3 K_3 = \vec{b}$ for ~~pa~~ show span.

$\therefore$ augmented matrix will be

$$\begin{bmatrix} -1 & 2 & -3 & b_1 \\ -2 & 4 & 6 & b_2 \\ 3 & -6 & 0 & b_3 \end{bmatrix}$$

we know that

if $\det(A) \neq 0$ then it will be linearly independent and spans of P_2.

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now $\det(A) = \begin{bmatrix} -1 & 2 & -3 \\ -2 & 4 & 6 \\ 3 & -6 & 0 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 2 & -3 \\ 0 & 0 & 12 \\ 0 & 0 & -9 \end{bmatrix}$$

$\det(A) = -1 \times 0 \times (-9) = 0.$

Thus det is 0 then

now, here,

$V = 1 - 3x + 2x^2$

Now, $-k_1 + 2k_2 - 3k_3 = 1$

$-2k_1 + 4k_2 + 6k_3 = -3$

$3k_1 - 6k_2 = 2$

Now $\left[\begin{array}{ccc|c} -1 & 2 & -3 & 1 \\ -2 & 4 & 6 & -3 \\ 3 & -6 & 0 & 2 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ -2 & 4 & 6 & -3 \\ 3 & -6 & 0 & 2 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & 0 & 12 & -5 \\ 0 & 0 & -9 & 5 \end{array}\right]$

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$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & 0 & 1 & -5/12 \\ 0 & 0 & -9 & 5 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & 0 & 1/4 \\ 0 & 0 & 1 & -5/12 \\ 0 & 0 & 0 & 5/4 \end{array}\right]$$

Here $0 = 5/4$ is inconsisten.

$\therefore$ The system is inconsisten.

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Ans to the Q no-3

Given matrix,

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ -2 & 2 & 1 \end{bmatrix}$$

now, here,

$$\lambda I - A = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ -2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda-1 & 0 & 1 \\ -1 & \lambda & 0 \\ 2 & -2 & \lambda-1 \end{bmatrix}$$

not $\det(\lambda I - A) = \lambda^3 - 2\lambda^2 - \lambda + 2$.

The eigen vale of A using cubic charatenistie.

$\therefore \lambda^3 - 2\lambda^2 - \lambda + 2 = 0$

$\Rightarrow \lambda^3 - 2\lambda^3 - \lambda + 2 = 0$

$\Rightarrow \lambda^2(\lambda - 2) - 1(\lambda - 2) = 0$

$\Rightarrow (\lambda - 2)(\lambda^2 - 1) = 0.$

$\therefore \lambda = 2, \pm 1.$

The eiganvalue are $\lambda = 2, \lambda = -1, \lambda = 1$.

Now, $\lambda = 2$

$\therefore \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 0 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 1 \\ 0 & -2 & 1 \end{bmatrix}$

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$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1/2 \\ 0 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

Here, $x_1 + x_3 = 0$

$\Rightarrow x_1 = -x_3$

$x_2 + \frac{x_3}{2} = 0$

$\therefore x_3 = -\frac{x_3}{2}$

$x_3 = s$

$x_1 = -s$

$x_2 = \frac{-s}{2}$

$$\therefore x \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -s \\ -s/2 \\ s \end{bmatrix} = s \begin{bmatrix} -1 \\ -1/2 \\ 1 \end{bmatrix}$$

$\therefore$ eigen space $\begin{bmatrix} -1 \\ -1/2 \\ 1 \end{bmatrix}$

for $\lambda = 2$.

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now, for $\lambda = 1$

$$\begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

hence, $x_1 = x_2$

$x_3 = 0$

now let $x_2 = s$.

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} s \\ s \\ 0 \end{bmatrix} = s \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore$ eigen space $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

for $\lambda = 1$.

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Ans to the Q no-4

Given, $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5$ column vectors

$$\therefore A = \begin{bmatrix} 1 & 3 & 1 & -2 & 1 \\ -3 & 9 & 3 & 3 & 0 \\ -1 & -3 & -1 & 2 & -1 \\ 2 & 6 & 2 & 1 & 4 \end{bmatrix}$$

Now, let RREF

$$R = \begin{bmatrix} 1 & 3 & 1 & -2 & 1 \\ 0 & 18 & 6 & -3 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 2 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{5} & \cancel{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 1 & -2 & 1 \\ 0 & 1 & 1/3 & -1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 2 \end{bmatrix}$$

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$$= \begin{bmatrix} 1 & 3 & 1 & -2 & 1 \\ 0 & 1 & 1/3 & -1/6 & 1/6 \\ 0 & 0 & 0 & 5 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 1 & -2 & 1 \\ 0 & 1 & 1/3 & -1/6 & 1/6 \\ 0 & 0 & 0 & 1 & 2/5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 1 & 0 & 9/5 \\ 0 & 1 & 1/3 & 0 & 7/30 \\ 0 & 0 & 0 & 1 & 2/5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{11}{10} \\ 0 & 1 & 1/3 & 0 & 7/30 \\ 0 & 0 & 0 & 1 & 2/5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{matrix} \vec{\omega}_1 & \vec{\omega}_2 & \vec{\omega}_3 & \vec{\omega}_4 & \vec{\omega}_5 \end{matrix} \end{array}$$

Here leading 1's in

~~RREF~~

col (R) = { w_1, w_2, w_4).

$\therefore$ colum space col(A) = { v_1, v_2, v_4 }

~~$\therefore$ w_3 and w_5~~

~~Now, let~~ ~~w_3~~ =

Here w_3 and w_5 are free ver

$$\therefore w_3 = \frac{1}{3} w_2$$

$$w_5 = \frac{11}{10} w_1 + \frac{7}{10} w_2 + \frac{2}{5} w_4.$$

$\therefore$

$$v_3 = \frac{v_3}{2}$$

$$v_5 = \frac{11}{10} v_1 + \frac{7}{30} v_2 + \frac{2}{5} v_4.$$