LU 5 - Individual Assignment: CSP Formalization

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Introduction

The 6-Queen Problem is a variation of the N-Queens Problem where the objective is to place 6 queens on a 6x6 chessboard in such a way that no two queens can attack each other. This can be formulated as a Constraint Satisfaction Problem (CSP), where the task is to assign values to variables (representing the positions of the queens) subject to constraints that prevent conflicts between queens.

1. Variables

Each queen must be placed in a separate column, and its position in each column is represented by a variable. Thus, the variables correspond to the columns of the chessboard, and each variable indicates the row in which a queen is placed within that column. Let the variables be:

$$Q_1, Q_2, Q_3, Q_4, Q_5, Q_6$$

Here, Q_i represents the row number of the queen in column i, where i ranges from 1 to 6, corresponding to the 6 columns of the chessboard.

2. Domain

The domain of each variable is the set of possible row positions on the 6x6 chessboard. Since there are 6 rows, each variable Q_i can take any value from 1 to 6, indicating that the queen can be placed in any of the 6 rows within its respective column. Thus, the initial domain for each variable is:

$$\text{dom}(Q_1) = \text{dom}(Q_2) = \text{dom}(Q_3) = \text{dom}(Q_4) = \text{dom}(Q_5) = \text{dom}(Q_6) = \{1, 2, 3, 4, 5, 6\}$$

As the problem progresses and constraints are applied, the domains will reduce to specific values based on the placement of the queens.

3. Constraints

The constraints ensure that no two queens attack each other. Specifically:

- **No two queens are in the same row:** For all $i \neq j$, $Q_i \neq Q_j$. This ensures that no two queens can share the same row.
- **No two queens are in the same column:** This constraint is inherently satisfied since each variable represents a different column on the chessboard, and each queen must be placed in a separate column.
- **No two queens are on the same diagonal:** For all $i \neq j$, the absolute difference between the row positions of two queens must not equal the absolute difference between their column positions. This is mathematically expressed as:

  $$|Q_i - Q_j| \neq |i - j|$$

  This ensures that no two queens share the same diagonal, whether ascending or descending.

Solution 1

Solution Matrix:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Variables:

- $Q_1 = 2$ (Queen in column 1 is placed at row 2)
- $Q_2 = 4$ (Queen in column 2 is placed at row 4)
- $Q_3 = 6$ (Queen in column 3 is placed at row 6)
- $Q_4 = 1$ (Queen in column 4 is placed at row 1)
- $Q_5 = 3$ (Queen in column 5 is placed at row 3)
- $Q_6 = 5$ (Queen in column 6 is placed at row 5)

Initial Domain:

Initially, each variable Q_i can be placed in any of the 6 rows, so the domain for each variable is:

$$\{1, 2, 3, 4, 5, 6\}$$

Constraints:

1. No two queens can be placed in the same row.
2. No two queens can be placed in the same column.
3. No two queens can be placed on the same diagonal.

Table:

Variable	**Initial Domain**	**Domain After Update**
Q_1	$\{1, 2, 3, 4, 5, 6\}$	$\{2\}$
Q_2	$\{1, 2, 3, 4, 5, 6\}$	$\{4\}$
Q_3	$\{1, 2, 3, 4, 5, 6\}$	$\{6\}$
Q_4	$\{1, 2, 3, 4, 5, 6\}$	$\{1\}$
Q_5	$\{1, 2, 3, 4, 5, 6\}$	$\{3\}$
Q_6	$\{1, 2, 3, 4, 5, 6\}$	$\{5\}$

Solution 2

Solution Matrix:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Variables:

- $Q_1 = 3$ (Queen in column 1 is placed at row 3)
- $Q_2 = 6$ (Queen in column 2 is placed at row 6)
- $Q_3 = 2$ (Queen in column 3 is placed at row 2)
- $Q_4 = 4$ (Queen in column 4 is placed at row 4)
- $Q_5 = 5$ (Queen in column 5 is placed at row 5)
- $Q_6 = 1$ (Queen in column 6 is placed at row 1)

Initial Domain:

Initially, each variable Q_i can be placed in any of the 6 rows, so the domain for each variable is:

$$\{1, 2, 3, 4, 5, 6\}$$

Constraints:

1. No two queens can be placed in the same row.
2. No two queens can be placed in the same column.
3. No two queens can be placed on the same diagonal.

Table:

Variable	Initial Domain	Domain After Update
Q_1	$\{1,2,3,4,5,6\}$	$\{3\}$
Q_2	$\{1,2,3,4,5,6\}$	$\{6\}$
Q_3	$\{1,2,3,4,5,6\}$	$\{2\}$
Q_4	$\{1,2,3,4,5,6\}$	$\{4\}$
Q_5	$\{1,2,3,4,5,6\}$	$\{5\}$
Q_6	$\{1,2,3,4,5,6\}$	$\{1\}$

Solution 3

Solution Matrix:

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Variables:

- $Q_1 = 5$ (Queen in column 1 is placed at row 5)
- $Q_2 = 3$ (Queen in column 2 is placed at row 3)
- $Q_3 = 1$ (Queen in column 3 is placed at row 1)
- $Q_4 = 4$ (Queen in column 4 is placed at row 4)
- $Q_5 = 6$ (Queen in column 5 is placed at row 6)
- $Q_6 = 2$ (Queen in column 6 is placed at row 2)

Initial Domain:

Initially, each variable Q_i can be placed in any of the 6 rows, so the domain for each variable is:

$$\{1,2,3,4,5,6\}$$

Constraints:

1. No two queens can be placed in the same row.
2. No two queens can be placed in the same column.
3. No two queens can be placed on the same diagonal.

Table:

Variable	Initial Domain	Domain After Update
Q_1	$\{1, 2, 3, 4, 5, 6\}$	$\{5\}$
Q_2	$\{1, 2, 3, 4, 5, 6\}$	$\{3\}$
Q_3	$\{1, 2, 3, 4, 5, 6\}$	$\{1\}$
Q_4	$\{1, 2, 3, 4, 5, 6\}$	$\{4\}$
Q_5	$\{1, 2, 3, 4, 5, 6\}$	$\{6\}$
Q_6	$\{1, 2, 3, 4, 5, 6\}$	$\{2\}$