LU 5 - Individual Assignment: Discrete and Particle Filter

Sayeeda Begam Mohamed Ikbal

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Question

In the course material, it was mentioned that the particle filter is more efficient than the discrete filter. In your own words, briefly justify this statement. Please also find one scenario in which the discrete filter would have an advantage over the basic particle filter.

Why the Particle Filter is More Efficient than the Discrete Filter

The particle filter is generally considered more efficient than the discrete filter, especially in high-dimensional state spaces, due to the following reasons:

1. **Computational Complexity**
   The particle filter uses stochastic sampling and resampling, which operates with linear time complexity $O(n)$, where n is the number of particles. In contrast, the discrete filter's motion update has a quadratic time complexity $O(n^2)$, making it slower and less scalable for larger state spaces.

2. **Scalability**
   Unlike the discrete filter, which requires the state space to be uniformly divided into grids, the particle filter uses a finite set of particles to approximate the belief. This makes the particle filter computationally feasible for continuous and high-dimensional problems where discretizing the state space would be impractical or memory-intensive.

3. **Efficient Sampling**
   The particle filter allocates computational resources intelligently by placing more particles in regions with higher probability. This avoids wasting memory and compute power on states with near-zero probability, as happens with the discrete filter.

4. **Flexibility**
   Particle filters are better suited for non-linear and non-Gaussian systems, where the discrete filter often struggles. This adaptability makes the particle filter a preferred choice in many real-world robotics and localization problems.

A Scenario Where the Discrete Filter Has an Advantage

Despite its efficiency, the particle filter has limitations. The discrete filter can outperform the particle filter in specific scenarios, such as:

1. **Low-Dimensional and Discrete State Spaces**
   In environments with naturally low-dimensional or discrete state spaces (e.g., robot localization in a simple grid-based map), the discrete filter has the following advantages:

   - It evaluates all possible states systematically, providing accurate and deterministic updates without relying on random sampling.
   - The particle filter, in contrast, may fail if there are too few particles to adequately represent the belief distribution, potentially missing important regions of the state space.

2. **The Hijacked Robot Problem**
   If a robot is unexpectedly teleported to a new location, the particle filter might fail if its particles do not cover the new pose. The discrete filter, however, evaluates all states regardless of their initial probabilities, ensuring the new pose is eventually considered and detected.

   Example:
   In a 1D or 2D grid world with limited states, the discrete filter computes exact probabilities for each state without requiring resampling. This deterministic approach ensures all possible states are accounted for, making the discrete filter more robust in unexpected situations or when resources are limited.

Summary

In summary, while the particle filter excels in high-dimensional, continuous, and computationally intensive problems, the discrete filter is better suited for low-dimensional, deterministic problems or scenarios where exhaustive state evaluation is necessary.