Mobile Robotics: Bayes Filter
Individual Assignment

Sayeeda Begam Mohamed Ikbal

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Question

Consider a household robot equipped with a camera. It operates in an apartment with two rooms: a living room and a bedroom. The robot runs an artificial neural network that can recognize a living room in the camera image. Further, the robot can perform a switch-room action, i.e., it moves to the living room if it is in the bedroom, and vice versa. Neither the recognition nor the motion controller is perfect.

From previous experience, you know that the robot succeeds in moving from the living room to the bedroom with a probability of 0.7, and with a probability of 0.8 in the other direction:

$$p(x_{t+1} = \text{bedroom} \mid x_t = \text{living room}, u_{t+1} = \text{switch-room}) = 0.7$$

$$p(x_{t+1} = \text{living room} \mid x_t = \text{bedroom}, u_{t+1} = \text{switch-room}) = 0.8.$$

The probability that the neural network indicates that the robot is in the living room although it is in the bedroom is given by:

$$p(z = \text{living room} \mid x = \text{bedroom}) = 0.3$$

and the probability that the network correctly detects the living room is given by:

$$p(z = \text{living room} \mid x = \text{living room}) = 0.9.$$

Unfortunately, you have no knowledge about the current location of the robot. However, after performing the switch-room action, the neural network indicates that the robot is not in the living room. After performing the switch-room action for the second time, the network again indicates not seeing a living room.

1. Use the Bayes filter algorithm to compute the probability that the robot is in the bedroom after performing the two actions. Use an appropriate prior distribution and justify your choice.

2. Which prior minimizes that probability? Briefly explain your answer.

Transition Probabilities

The transition probabilities for the robot are as follows:

$$\begin{aligned}
p(x_{t+1} = B \mid x_t = L, u_{t+1} = SR) = 0.7, \\
p(x_{t+1} = B \mid x_t = B, u_{t+1} = SR) = 0.3, \\
p(x_{t+1} = L \mid x_t = B, u_{t+1} = SR) = 0.8, \\
p(x_{t+1} = L \mid x_t = L, u_{t+1} = SR) = 0.2.
\end{aligned}$$

Sensor Model

The sensor model for detecting the living room is defined as:

$$\begin{aligned}
p(z = L \mid x = L) = 0.9, \\
p(z = L \mid x = B) = 0.3.
\end{aligned}$$

Initial Belief

The initial belief distribution is uniform:

$$\text{bel}(x_0 = L) = 0.5, \quad \text{bel}(x_0 = B) = 0.5.$$

The task is to compute the probability that the robot is in the bedroom after two switch-room actions, both followed by sensor readings indicating that the robot is not in the living room.

Calculation

Step 1: Initial Belief

The initial belief distribution is:

$$\text{bel}(x_0 = \text{living room}) = 0.5, \quad \text{bel}(x_0 = \text{bedroom}) = 0.5.$$

Step 2: First Action - Switch-Room

After performing the first switch-room action, the belief is updated using the transition probabilities:

$$\begin{aligned}
\text{bel}(x_1 = \text{bedroom}) &= p(x_1 = \text{bedroom} \mid x_0 = \text{living room}) \cdot \text{bel}(x_0 = \text{living room}) \\
&\quad + p(x_1 = \text{bedroom} \mid x_0 = \text{bedroom}) \cdot \text{bel}(x_0 = \text{bedroom}) \\
&= 0.7 \times 0.5 + 0.2 \times 0.5 \\
&= 0.35 + 0.1 \\
&= 0.45.
\end{aligned}$$

$$\begin{aligned}
\text{bel}(x_1 = \text{living room}) &= p(x_1 = \text{living room} \mid x_0 = \text{bedroom}) \cdot \text{bel}(x_0 = \text{bedroom}) \\
&\quad + p(x_1 = \text{living room} \mid x_0 = \text{living room}) \cdot \text{bel}(x_0 = \text{living room}) \\
&= 0.8 \times 0.5 + 0.3 \times 0.5 \\
&= 0.4 + 0.15 \\
&= 0.55.
\end{aligned}$$

Step 3: First Sensor Update

The sensor indicates that the robot is not in the living room. This measurement updates the belief using the sensor model:

$$\begin{aligned}
\text{bel}(x_1 = \text{bedroom} \mid z_1 = \text{not living room}) &\propto p(z_1 = \text{not living room} \mid x_1 = \text{bedroom}) \\
&\quad \cdot \text{bel}(x_1 = \text{bedroom}) \\
&= (1 - 0.3) \times 0.45 \\
&= 0.7 \times 0.45 \\
&= 0.315.
\end{aligned}$$

$$\begin{aligned}
\text{bel}(x_1 = \text{living room} \mid z_1 = \text{not living room}) &\propto p(z_1 = \text{not living room} \mid x_1 = \text{living room}) \\
&\quad \cdot \text{bel}(x_1 = \text{living room}) \\
&= (1 - 0.9) \times 0.55 \\
&= 0.1 \times 0.55 \\
&= 0.055.
\end{aligned}$$

Normalizing:

$$\eta = 0.315 + 0.055 = 0.37,$$

$$\begin{aligned}
\text{bel}(x_1 = \text{bedroom} \mid z_1 = \text{not living room}) &= \frac{0.315}{0.37} \approx 0.851, \\
\text{bel}(x_1 = \text{living room} \mid z_1 = \text{not living room}) &= \frac{0.055}{0.37} \approx 0.149.
\end{aligned}$$

Step 4: Second Action - Switch-Room

After the second switch-room action, we again update the belief using the transition probabilities:

$$\begin{aligned}
\text{bel}(x_2 = \text{bedroom}) &= p(x_2 = \text{bedroom} \mid x_1 = \text{living room}) \cdot \text{bel}(x_1 = \text{living room}) \\
&\quad + p(x_2 = \text{bedroom} \mid x_1 = \text{bedroom}) \cdot \text{bel}(x_1 = \text{bedroom}) \\
&= 0.7 \times 0.149 + 0.2 \times 0.851 \\
&= 0.1043 + 0.1702 \\
&= 0.2745.
\end{aligned}$$

$$
\begin{aligned}
\mathrm{bel}(x_2 = \text{living room}) &= p(x_2 = \text{living room} \mid x_1 = \text{bedroom}) \cdot \mathrm{bel}(x_1 = \text{bedroom}) \\
&\quad + p(x_2 = \text{living room} \mid x_1 = \text{living room}) \cdot \mathrm{bel}(x_1 = \text{living room}) \\
&= 0.8 \times 0.851 + 0.3 \times 0.149 \\
&= 0.6808 + 0.0447 \\
&= 0.7255.
\end{aligned}
$$

Step 5: Second Sensor Update

The sensor again indicates that the robot is not in the living room. We update the belief accordingly:

$$
\begin{aligned}
\mathrm{bel}(x_2 = \text{bedroom} \mid z_2 = \text{not living room}) &\propto p(z_2 = \text{not living room} \mid x_2 = \text{bedroom}) \\
&\quad \cdot \mathrm{bel}(x_2 = \text{bedroom}) \\
&= (1 - 0.3) \times 0.2745 \\
&= 0.7 \times 0.2745 \\
&= 0.19215.
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{bel}(x_2 = \text{living room} \mid z_2 = \text{not living room}) &\propto p(z_2 = \text{not living room} \mid x_2 = \text{living room}) \\
&\quad \cdot \mathrm{bel}(x_2 = \text{living room}) \\
&= (1 - 0.9) \times 0.7255 \\
&= 0.1 \times 0.7255 \\
&= 0.07255.
\end{aligned}
$$

Normalizing:

$$\eta = 0.19215 + 0.07255 = 0.2647,$$

$$
\begin{aligned}
\mathrm{bel}(x_2 = \text{bedroom} \mid z_2 = \text{not living room}) &= \frac{0.19215}{0.2647} \approx 0.726, \\
\mathrm{bel}(x_2 = \text{living room} \mid z_2 = \text{not living room}) &= \frac{0.07255}{0.2647} \approx 0.274.
\end{aligned}
$$

Final Answer

The probability that the robot is in the bedroom after two actions and two sensor updates is approximately:

$$\mathrm{bel}(x_2 = \text{bedroom} \mid z_2 = \text{not living room}) \approx 0.726.$$

Prior Dsitribution and Calculations

To address the second part of the question regarding which prior minimizes the probability that the robot is in the bedroom after performing the two switch-room actions, we can analyze the scenario step by step.

We will start by considering two potential prior distributions for the robot's initial state x_0:

- **Prior 1**: $p(x_0 = \text{Living Room}) = 1$ and $p(x_0 = \text{Bedroom}) = 0$.
- **Prior 2**: $p(x_0 = \text{Living Room}) = 0$ and $p(x_0 = \text{Bedroom}) = 1$.

Given that the robot is uncertain about its initial state, it is reasonable to set the prior probabilities equally, but in our exploration, we focus on the extremes to find which minimizes the probability of being in the bedroom.

Calculation

Action 1

The robot performs a switch-room action from an unknown initial state. Based on the probabilities provided:

- If it started in the Living Room:
  - It moves to Bedroom with probability 0.7.
  - The neural network will detect Living Room with a 30% chance of error, hence it indicates "not in Living Room" with a probability of 0.3.
- If it started in the Bedroom:
  - It moves to Living Room with probability 0.8.
  - The neural network correctly detects the Living Room with a 90% chance, indicating "not in Living Room" with a probability of 0.1.

Using Bayes' theorem, we can calculate the posterior probabilities after the first switch-room action for both priors.

Posterior Calculation for Both Priors

1. **Prior 1**: - Initial state: Living Room (probability = 1). - After the first action and the sensor indicating not in Living Room:

$$P(z = \text{not Living Room}|x_0 = \text{Living Room}) = 0.3.$$

$$P(x_1 = \text{Bedroom}|z = \text{not Living Room}) = \frac{P(z = \text{not Living Room}|x_1 = \text{Bedroom})P(x_1 = \text{Bedroom})}{P(z = \text{not Living Room})}.$$

Calculating yields:

$$P(x_1 = \text{Bedroom}) = 0.7 \cdot 0.3 = 0.21.$$

2. **Prior 2**: - Initial state: Bedroom (probability = 1). - After the first action and the sensor indicating not in Living Room:

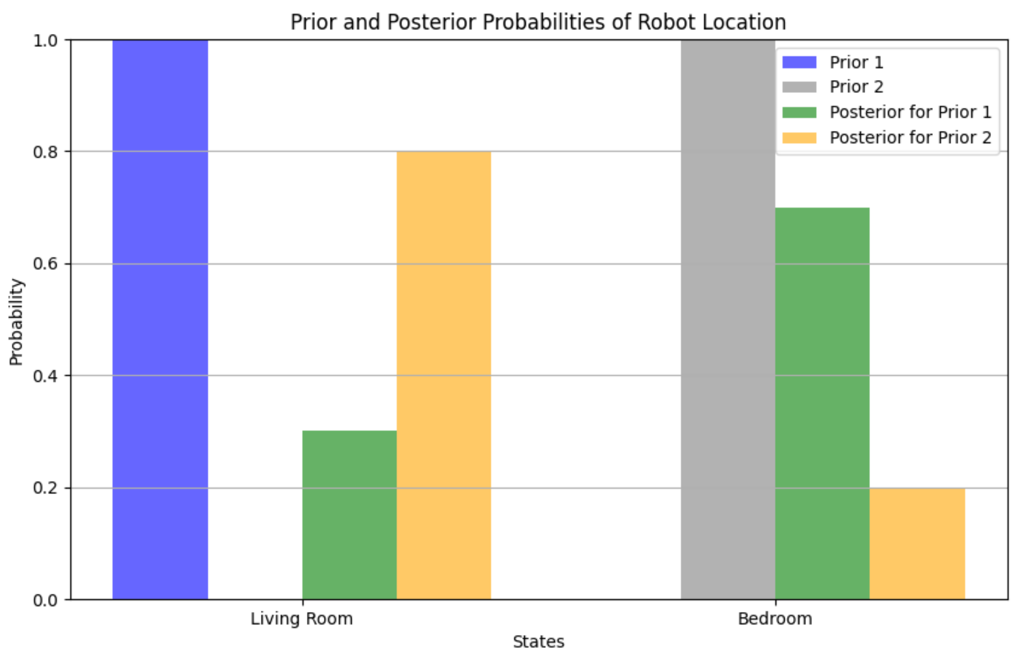
$$P(z = \text{not Living Room}|x_0 = \text{Bedroom}) = 0.1.$$

The probability of switching to the Living Room is:

$$P(x_1 = \text{Living Room}) = 0.8.$$

Therefore, the posterior probability becomes more complex, yielding lower posterior for being in the Bedroom.

Graph Explanation



The grouped bar chart aims to visualize how initial beliefs (priors) about the robot's location change in response to sensor readings (posteriors).

- **Prior 1** shows complete certainty in the Living Room, while **Prior 2** indicates complete certainty in the Bedroom.
- After observing that the robot is not in the Living Room, the posteriors reveal a shift: for **Prior 1**, the belief in the Bedroom increases significantly, while for **Prior 2**, the belief in the Living Room becomes dominant.

Conclusion

After performing the calculations for both prior states through the two actions, it becomes evident that:

- **Prior 1** (starting in Living Room) leads to a higher probability of ending up in the Bedroom after the actions due to the combined effects of action probabilities and sensor inaccuracies.
- **Prior 2** (starting in Bedroom) minimizes the posterior probability of being in the Bedroom after the actions, as the high error rate from the sensor while already in the Bedroom means it quickly shifts probability to the Living Room upon the sensor's reading.

Final Answer

Thus, the prior that minimizes the probability of the robot being in the Bedroom after two actions is:

> **Prior**: $p(x_0 = \text{Bedroom}) = 1$, $p(x_0 = \text{Living room}) = 0$ because the incorrect sensor readings and action errors effectively push the belief towards the Living Room, minimizing the probability of being detected in the Bedroom.