5-10-2023
promy Mrs

Why we need information and coding:

~~Transmitted message = Received signal (+Noise)~~
error

Received signal = Transmitted signal + Noise.

physical solution.

Source coding: Message কে bit এ convert করাকে
(~~data~~ যত কম bit দ্বারা সম্ভব সংক্ষিপ্ত represent করা) (Data compression)

Channel coding: Data channel এর সর্বাধিক transform করার সময় error detect করার capability (channel error detect করে)

lower probability deals with information
explain
higher probability ~~deals~~ implies with message.

Self information (math)

let, no of event = 'x'
'x'-event

(I_x – information)

$$I_x = \log_2 \frac{1}{P_x}$$

$$= -\log_2 P_x \quad \text{bit.}$$

$$I_x = -\log_{10} P_x \rightarrow \text{Hartler}$$

- Discrete source (text, message ...)

$x_1 = a, b, c, d.$

$x_2 = 00, 01, 11, 10.$

- Discrete memoryless source. (পরবর্তী অংশের কোনো কিছু নির্ভরশীল না. dice of ludo. 1, 3, 5)

- Discrete source with memory

(predictive text)
suggested match.

~~Data source~~

Image, Audio, Text.

X = Audio signal

x_1 (probability)

$$P_{x_1} = 0.25$$

$$P_{x_2} = 0.5$$

$$I_{x_1} = -\log_2(0.25)$$

$$= 2 \text{ bit.}$$

$$I_{x_2} = -\log_2(0.5)$$

$$= 1.$$

probability কম হলে represent করতে bit বেশি লাগে.

probability বেশি হলে represent করতে bit কম লাগে.

properties of self information:

$P_i = 1 \rightarrow$ message signal

1) $I(x_1) = 0$, if $P_i = 1$

2) $I(x_1) \geq 0$, $0 \leq P(x_i) \leq 1$

3) $I(x_1) > I(x_2)$ if $P_1 < P_2$

4) $I(x_1, x_2) = I(x_1) + I(x_2)$

$$I(x_1) = \log_2 \frac{1}{P_{x_1}}$$

$$I(x_2) = \log_2 \frac{1}{P_{x_2}}$$

$$I(x_1) + I(x_2) = \log_2 \left(\frac{1}{P_{x_1}} * \frac{1}{P_{x_2}} \right)$$

$$= I(x_1, x_2).$$

(*) $X = \{x_1, x_2, x_3\}$

$P(x_1) = 0.5$ $\quad I_{x_1} = -\log_2(0.5) = 1$ bit.

$P(x_2) = 0.25$ $\quad I_{x_2} = -\log_2(0.25) = 2$ bit

$P(x_3) = 0.125$ $\quad I_{x_3} = -\log_2(0.125) = 3$ bit

* $H(x_1) = -P_i \log_2 P_i$ where $x_1 \in X$

$$\therefore H(X) = \sum_{x_i \in X} H(x_1)$$

$$= -\sum_{1}^{n} P_i \log_2 P_i$$

Entropy : Avg. information per symbol.

$$\frac{\text{symbol probability}}{\text{self information}}$$

$$H(x) = \sum H(x_i)$$

$$= P_1 \log_2 \frac{1}{P_1} + P_2 \log_2 \frac{1}{P_2} + P_3 \log_2 \frac{1}{P_3}$$

$$= 0.5 \times (-1) + 0.25(-2) + 0.125(-3)$$

$$= -0.5 - 0.5 - 0.375$$

$$= 1.375$$

প্রত্যেক symbol কে minimum কতগুলো bit দ্বারা represent করা যায়

A, B, C, D, E, F

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}$

$$H(x) = -\frac{1}{2}\log_2 \frac{1}{2} - \frac{1}{4}\log_2 \frac{1}{4} - \frac{1}{8}\log_2 \frac{1}{8}$$

$$- \frac{1}{16}\log_2 \frac{1}{16} - \frac{1}{32}\log_2 \frac{1}{32}$$

$$- \frac{1}{64}\log_2 \frac{1}{64}$$

$$= 1.765 \text{ bit}$$

* Source coding এর জন্য entropy একটি limiting Factor. explain —

Bit length must be greater than or equal to ~~limiting~~ entropy.

* $n H(x)$

Information rate: $\xrightarrow[n/r]{nH(x)}$ $r\,H(x)$.

$\frac{1}{r}$ sec

$\frac{n}{r}$ sec

* information rate = entropy × symbol rate.

$$R = rH(x)$$

* Joint entropy: একাধিক ....

Weather	Picnic yes	Picnic NO
Sunny	✓	
Rainy		✗
Cloudy	✓	

weather → picnic

$$H(x,y) = \sum_{x \in X} \sum_{y \in Y} -p(x,y) \log_2 p(x,y)$$

$$P(x=0, y=0) = \frac{1}{4}$$

$$P(x=0, Y=1) = \frac{1}{4}$$

$$P(x=1, Y=0) = \frac{1}{4}$$

$$P(x=1, Y=1) = \frac{1}{4}$$

$$H(x,y) = \frac{1}{4}\log_2 4 + \frac{1}{4}\log_2 4 + \frac{1}{4}\log_2 4 + \frac{1}{4}\log_2 4$$

$$= \frac{1}{4} \times 2 + \frac{1}{4} \times 2 + \frac{1}{4} \times 2 + \frac{1}{4} \times 2$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$= 2$$

$x \backslash y$	0	1
0	$\frac{1}{4}$	$\frac{1}{8}$
1		

BPSK - 0.1

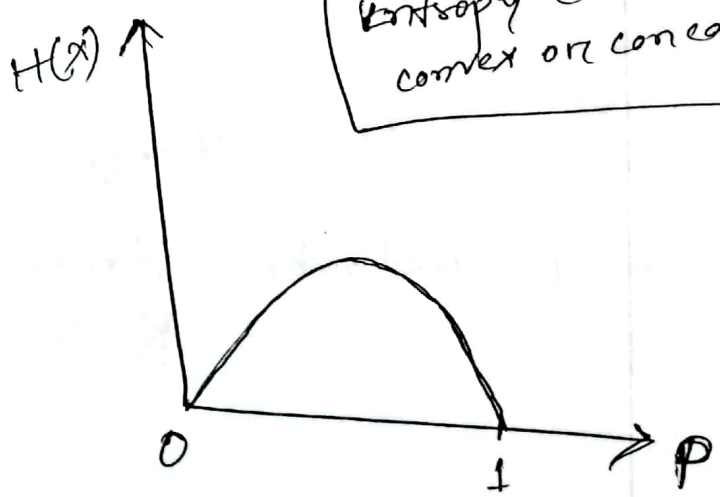
$0 \rightarrow p$

$1 \rightarrow 1-p$

$$\therefore \text{Entropy } H(x) = p\log_2 \frac{1}{p} + (1-p)\log_2 \frac{1}{(1-p)}$$

$$= 0 + (1-0)\log \frac{1}{(1-0)}$$

Entropy curve convex or concave?



Assignment: Why we use logarithmic for information cap.

* Relation between

Nat → Hartley → Bit

Nat – Decit → Bit

* Slide বানাতে লাগবে ফোকেরের ক্লাসের (P.P)

Khadija Mks
12-10-2023

$H(Y|X)$

X Y

$$H(Y|X) = -\sum_{x \in X} P(x) \sum_{y \in Y} P(y|x) \log P(y|x)$$

$$= -\sum_{x \in X} \sum_{y \in Y} P(x,y) \log P(y|x)$$

If 'x' and 'y' are independent then joint entropy is.

$$H(x,y) = H(x) + H(y)$$

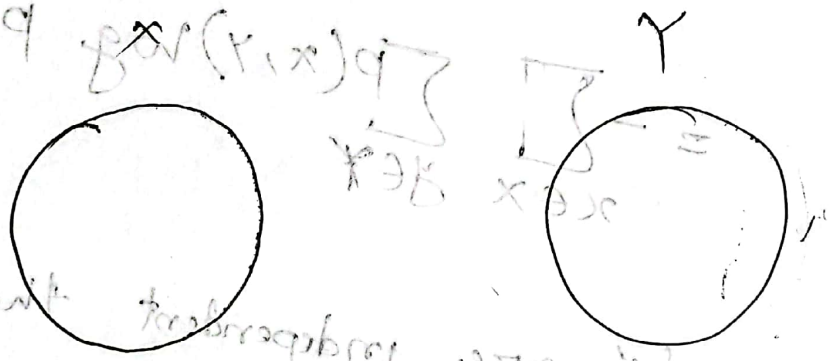
$$I(x;Y) = H(x) + H(y) - H(x,y)$$

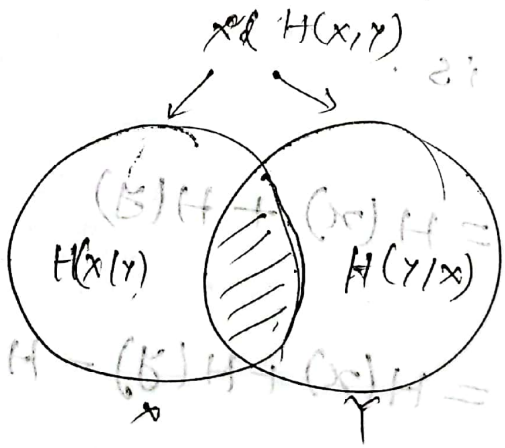
Measures the uncertainty reduction in x if we know y.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$H(X|Y) = -\sum_{y \in Y} P(y) \sum_{x \in X} P(x|y) \log P(x|y)$$

$$= \sum_{y \in Y} \sum_{x \in X} P(x,y) \log P(x|y)$$





Theorem 2.2.1: (Chain rule).

$$H(X,Y) = H(X) + H(Y/X)$$

Proof:

$$H(X,Y) = -\sum_{x\in X}\sum_{y\in Y} p(x,y)\log p(x,y)$$

$$= -\sum_{x\in X}\sum_{y\in Y} p(x,y)\log p(x)p(y|x)$$

$$= -\sum_{x\in X}\sum_{y\in Y} p(x,y)\log p(x)$$

$$-\sum_{x\in X}\sum_{y\in Y} p(x,y)\log(p/x)$$

$$= -\sum_{x\in X} p(x)\log p(x) - \sum_{x\in X}\sum_{y\in Y} p(x,y)\log p(y/x)$$

$$= H(X) + H(Y/X).$$

Mutule information can be represented as.

$$I(x,y) = \sum_{x \in X} \sum_{y \in Y} P(x,y) \log \frac{P(x,y)}{P(x)P(y)}$$

$$= \sum_{x,y} P(x,y) \log \frac{P(x|y)}{P(x)}$$

$$= -\sum_{x,y} P(x,y) \log P(x) + \sum_{x,y} P(x,y) \log P(x|y)$$

$$= -\sum_{x} P(x) \log P(x) - \left(-\sum_{x,y} P(x,y) \log P(x|y)\right)$$

$$= H(x) - H(x|Y)$$

00 01 10 10 11 11 01 01

10 00 10 10 11

01 10

$P(00) = \frac{2}{15}$

$P(01) = \frac{4}{15}$

$P(10) = \frac{6}{15}$

$P_{11} = \frac{3}{15}$

$= P_{00} \log_2(P_{00}) + P_{01} \log_2(P_{01})$

$+ P_{10} \log_2(P_{10}) + P_{11} \log_2(P_{11})$

$= 1.898$

19-10-2023
promy Mrs

Q: The output of an information source consists of 150 symbols. 32 of which occurs with a probability of $\frac{1}{64}$ and the remaining with a probability of $\frac{1}{236}$. The source emits 2000 symbols/sec.

Assuming that the symbols are chosen independently, find the $\underbrace{\text{average information}}_{\text{entropy}}$ rate of this source. $\boxed{R = n \times H}$

$$H = P_1 \log_2 \frac{1}{P_1} + P_2 \log_2 \frac{1}{P_2}$$

$$= 32\left(\frac{1}{64} \log_2 64\right) + \left(\frac{1}{236} \log_2 236\right) \times 118$$

$$= 6.9 \text{ bits/sym}$$

$$\therefore R = n \times H$$

$$= 2000 \times 6.9 = 13800 \text{ b.}$$

two type symbol → dot, dash

Q: The international morse code uses a sequence of dots and dashes to transmit letters of English alphabet. The dash is represented by a current pulse that has a duration of 3 units. and the dot has a duration of 1 unit. The probability of occurance of a dash is $\frac{1}{3}$ of the probability of a dot.

a) Calculate the information content of a dot and dash.

b) Calculate the average information in the dot-dash code.

Ans)

$$P_{dot} + P_{dash} = 1$$

$$P_{dash} = \frac{1}{3} P_{dot}$$

$$\therefore P_{dot} + \frac{1}{3} P_{dot} = 1$$

$$= \frac{\left(\frac{1}{3}+1\right) P_{dot}}{P_{dot}} = 1$$

$$\Rightarrow P_{dot} = \frac{3}{4}$$

$\therefore P_{dash} = 1 - \frac{3}{4} = \frac{1}{4}$

self information: $\boxed{I_d = \log_2 \frac{1}{P}}$

$P_{dot} = I_{P_{dot}} = \log \frac{1}{\frac{3}{4}} = 0.41$

$I_{P_{dash}} = \log \frac{1}{\frac{1}{4}}$

$= 2$

(b) $\cancel{I_{dash}}\ H = H_{dot} + H_{dash}$

$= P_{dot} \log_2 \frac{1}{P_{dot}} + P_{dash} \log_2 \frac{1}{P_{dash}}$

$= \frac{3}{4} \log_2 \frac{4}{3} + \frac{1}{4} \log_2 4$

$= 0.811.$

Convex function:

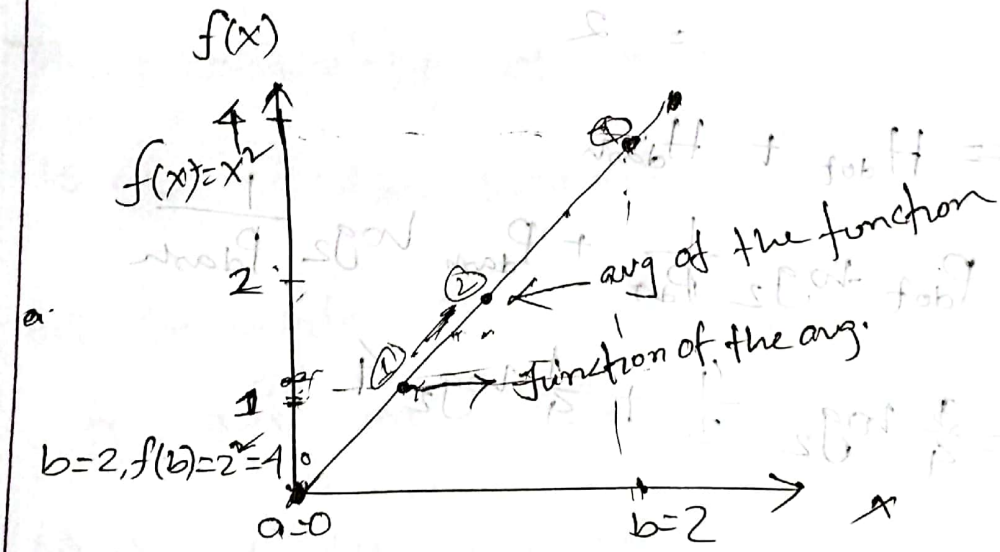
→ second derivative non negative.

→ function transmit করা হয় -
Either: data avg করে transmit
or: Transmit করে avg.

Jensen's Theorem:

For convex func

avg of funct > funct. of the avg.



f. of the avg $= \left(\frac{0+2}{2}\right)^2 = 1^2 = 1$

for concave avg of funct < funct of the avg.

* If f is a convex function and x is a random variable then

$$\longrightarrow \quad E\{f(x)\} \geq f(E\{x\})$$

expectation
mean probability

~~proof~~

proof :- $\sum E\{f(x)\} \geq f(E\{x\})$

$$-D(P\|q) = -\sum_{x \in X} P(x) \log \frac{(P(x))}{q(x)}$$

$$= \sum P(x) \log \frac{q(x)}{P(x)}$$

$E\{f(x)\} \leq f(E\{x\})$

$\frac{P(x)}{q(x)} = \text{Constant}$

$$\leq \log \sum_{x \in X} P(x) \frac{q(x)}{P(x)}$$

$$= \log \sum_{x \in x} q(x) \leq \log 1$$

$$-D(P\|q) = \log \sum_{x \in x} q(x) \leq 0$$

Gibbs information inequality.

30-10-2023
Promy Mres

$X = \{0, 1\}$

$p\{0\} = r \qquad p\{1\} = s$

$q\{0\} = 1-r \qquad q\{1\} = 1-s$

$$-D(p\|q) = -\sum_{x \in X} p(x) \log \frac{p(x)}{q(x)}$$

$$= \sum p(x) \log \frac{q(x)}{p(x)}$$

$$\leq \log \sum_{x \in X} p(x) \frac{q(x)}{p(x)}$$

$$\leq \log \sum_{x \in X} q(x)$$

$$\leq 0$$

Q: when, $D(P\|q) = D(q\|P) = 0$?

$x = \{0, 1\}$

	Actual	Estimation
$P\{0\} = \pi$	$P\{1\} = s$	$q\{1\} = s$
$q\{0\} = 1-\pi$	$q\{1\} = 1-s$	$q\{0\} = 1-s$

$T_x \rightarrow$ Actual

R_x – predict / Estim

$$D(P\|q) = (1-\pi)\log\frac{1-\pi}{1-s} + \pi\log\frac{\pi}{s}$$

$D(q\|P) = ?$

Ans

$$D(q\|P) = (1-s)\log\frac{1-s}{1-\pi} + s\log\frac{s}{\pi}$$

when, $\pi = s$.

$$D(P\|q) = D(q\|P) = 0 + 0 = 0$$

Q: when $D(P\|q) \neq D(q\|P)$

let, $\pi = \frac{1}{2}$, $s = \frac{1}{4}$

now $D(P\|q) = 1-$

$D(q\|P) =$

$\therefore D(P\|q) \neq D(q\|P)$

~~(Relative entropy)~~

Thomas A. Cover → Derivation (self)
Zensen's inequality proof

Source

probability $\propto \frac{1}{\text{self info}}$ bit

Image
text
Audio
Video
→ Source coding

	PDF (prob. Density Func)	FLC	VLC (variable length coding)
a	0.5	2	1
b	0.25	2	2
c	0.125	2	3
d	0.125	2	3
		= 8	= 9

Minimum ~~Entropy~~ Avg length : $(1 \times 0.5) + (2 \times 0.25) + (3 \times 0.125 \times 2)$

$L = 1.75$

Entropy : $0.5 \log_2 (0.5) + 0.25 \log_2 (0.25) + 0.125 \times \log_2 0.125 + 0.125 \times \log_2 0.125$

$H(s) = \cancel{0.6462} = 1.75$

$L \geq H(S)$ - Source coding boundary.

The source coding theorem states that for a DMS, x, with entropy $H(x)$ the average code word length L per symbol is bounded as:

$$L \geq H(x)$$

Coding efficiency $\eta = \frac{H(x)}{L}$

$\eta = 1$, efficient coding.

$\eta > 1$, signal loss high.

$\eta \leq 1$, ~~length L per symbol~~ for error-free condition communication.

Redundancy $r = 1 - \eta$

$$\eta_{VLC} = \frac{1.75}{9} = 0.194$$

VLC calculation করার সময় 9 use করা হয়েছে, Example দেখেন হয়, Example দেখেন

$$\eta_{FLC} = \frac{1.75}{8} = 0.218$$

2-10-2023

Promy MCS

Let,

$a = 0$

$b = 01$

$c = 001$

c b a

001 01 0 ← prefix free coding / prefix coding / Instantenous coding.

Case=1 (prefix free)

$a = 0$

$b = 10$

$c = 110$

$d = 1111$

কোনোর code word নয়

Case-II (

$a = 0$

$b = 01$

$c = 110$

$d = 0111$

b, d এর prefix

Not prefix free coding

a কোনোর codeword হয়। কোনোর codeword এর prefix হতে পারে না.

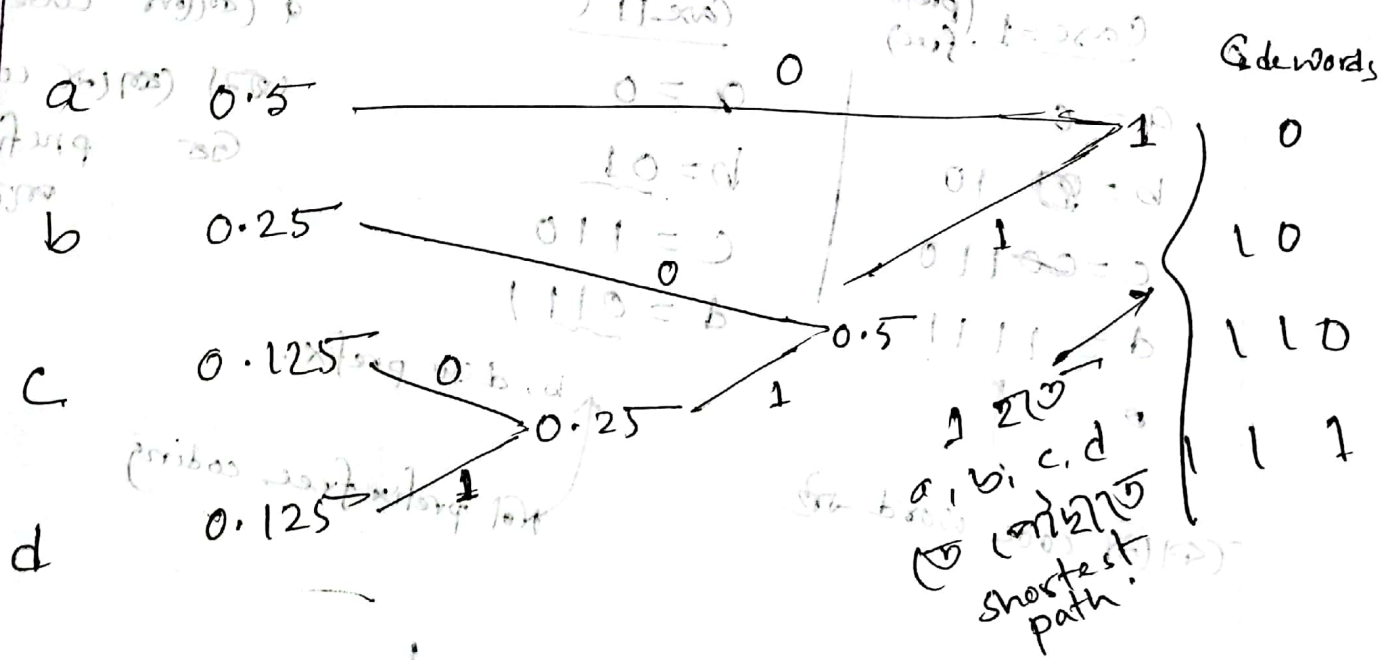
Kraft inequality : $\sum_{K=1}^{L} 2^{-n_K} \leq 1$,

for case 1 : $2^{-n_1} + 2^{-n_2} + 2^{-n_3} + 2^{-n_4}$

$= 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} = 0.937$

Prefix free coding maintains craft inequality. Doesn't mean craft inequality maintanence ~~is the~~ means its always ~~craft ineq~~ prefix free coding.

a, b, c, d → $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}$ Find the Huffman code.



probability কম হলে bit বেশি

Same probability তে bit সমান

$$H = -0.5 \log_2 0.5 + -0.25 \log_2 0.25 + -0.125 \log_2 0.125 + -0.125 \log_2 0.125$$

$$= 1.75$$

Avg length =

$$= 1.75$$

$$\eta = \frac{H}{L_{avg}} \cancel{= 100\%}$$

$$= \frac{1.75}{1.75} = 100\%$$

* Binary encoding:

a = 00

b = 01

c = 10

d = 11

$L_{avg} = 2$

$$\eta = \frac{1.75}{2} = 87.5\%$$

∴ Huffman coding is more efficient than binary encoding.

* A1 — 0.4 — 0 — 1 — 0

A2 — 0.3 — 0 — 0.6 — 1 — 10

A3 — 0.2 — 0 — 0.3 — 1 — 110

A4 — 0.1 — 1 — 111

~~Huffman coding~~

~~H = 0.4 log (0.4) + 0.3 log~~

$L = (0.4 \times 1) + (0.3 \times 2) + (0.2 \times 3) + (0.1 \times 3)$

$= 0.4 + 0.6 + 0.6 + 0.3$

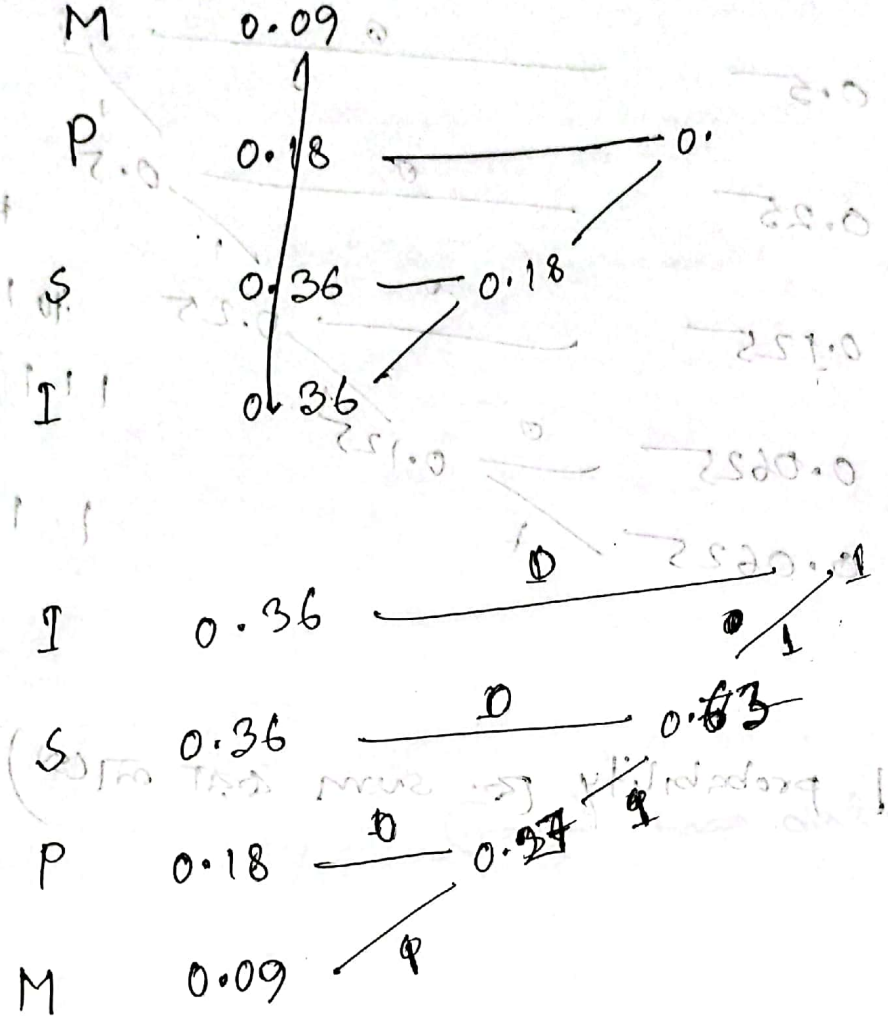
Code length $L = 1.9$

* 'MISSISSIPPI'

M = ? P = S = I = ?

$M = \frac{1}{11}$ $P = \frac{2}{11}$ $S = \frac{4}{11}$ $I = \frac{4}{11}$

$= 0.09$ $= 0.18$ $= 0.36$ $= 0.36$



0 0

1 0

1 1 0

1 1 1

~~Not~~ prefix free

কোন code word এর prefix হতে পারবে না।

Q: Exam:

Is it a prefix free coding?

→ Huffman coding?

→ Efficiency

→ Binary coding and efficiency.

→ Which one is more efficient?

optimal Huffman coding.

a	0.5	0	1	0
b	0.25	0	0.5	10
c	0.125	0	0.25	110
d	0.0625	0	0.125	1110
e	0.0625	1		1111

(Nearest দুইটা probability কে sum করা লাগে)

Assignment:

for entropy.

Jensen's Inequality entropy তে কি explain.

like 'coin toss'

Redundancy তে data বার বার আসছে.

6-11-2023

Promy Mrs

Huffman Encoding (Example)

Step1: Sort all symbols according to their probabilities (left to right) from smallest to largest.

Step2: Build a binary tree from left to right.

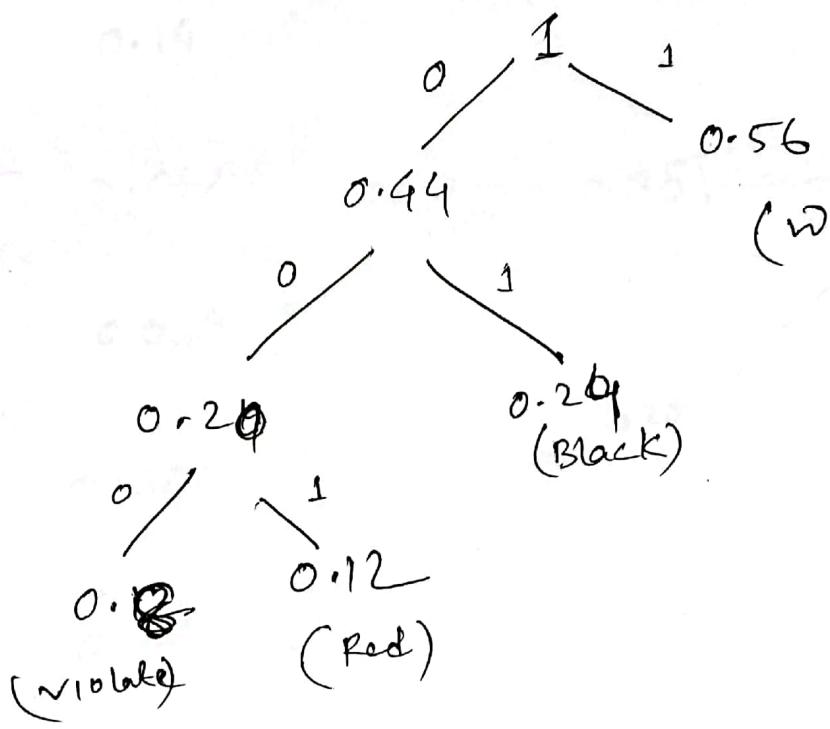
Step3: Label left branches of the tree with zero and right branches of the tree with 1.

Step4:

CamScanner

Whi $\frac{14}{25} = 0.56$ Red $\frac{3}{25} = 0.12$

Black $\frac{6}{25} = 0.24$ violate $\frac{2}{25} = 0.08$.



white – 1

Black – 01

violate – 000

Red – 001

(num × bit) + ..

Total bit = $(14 \times 1) + (2 \times 6) + (3 \times 3) + 2 \times 3$

= 41 bit.

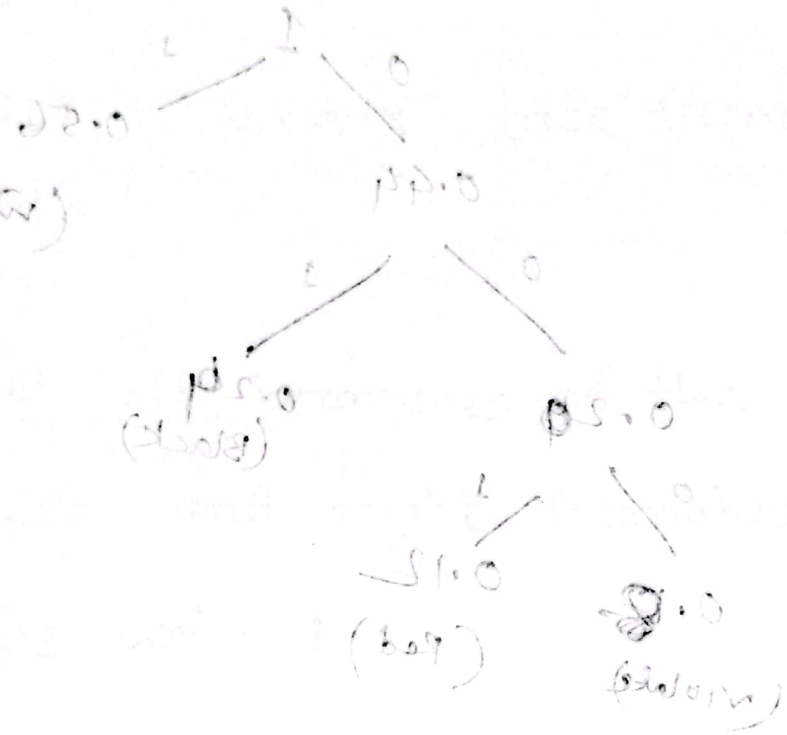
= 5 byte.

* Diff. between Huffman and shanon fano encoding. Compare between bit length.

Shanon fano coding:

step1: List the source symbols (message) in the order of decreasing probability.

step2:



Diff. কম অনুযায়ী add করতে লাগবে. .21 এর .0.29 কম থাকবে।

m_1	0.25			
m_2	0.21			
m_3	0.15	0.29		
m_4	0.14			
m_5	0.0625	0.125	0.25	
m_6	0.0625			
m_7	0.0625	0.125		
m_8	0.0625			

9-11-2023

Promy MTCS

Arithmetic Encoding

a → 001

b → 01

c → 010

d = 011

~~probab~~

B A d

01 001 011

Individualy encode না করে একসাথে encode করা

Bad সম্পূর্ণ একসাথে

→ Probability গুলোকে একটা নির্দিষ্ট range এ নিয়ে আসে, এর মাধ্যমে সম্পূর্ণ সম্পূর্ণ encode করে.

0 ← range → 1

b a b b c b a b b b

a → 2

b → 7

c → 1

Frequency table generate করা লাগবে,

$a = \frac{2}{10} = 0.2$

$b = \frac{7}{10} = 0.7$

$c = \frac{1}{10} = 0.1$

লক্ষ্য: upper / lower seq. অনুযায়ী সাজাতে হবে।

a b c → sequency অনুযায়ী সাজাতে হবে।

Step:

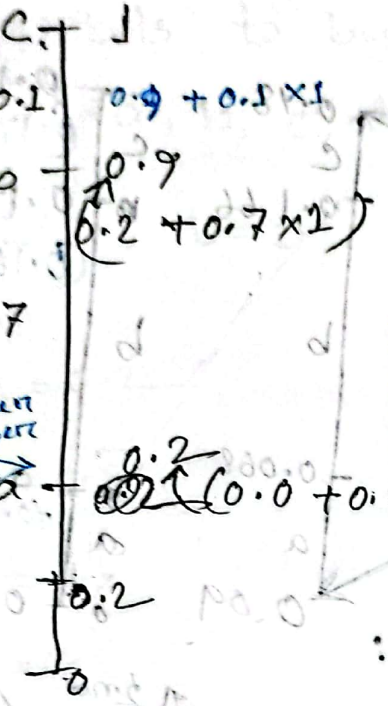
① Freq table gen

② 0 → 1 range G divide করা

c go lower limit

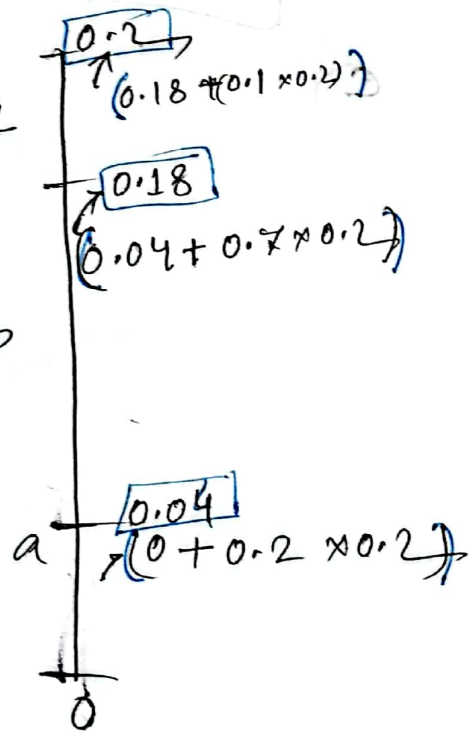
b go upper

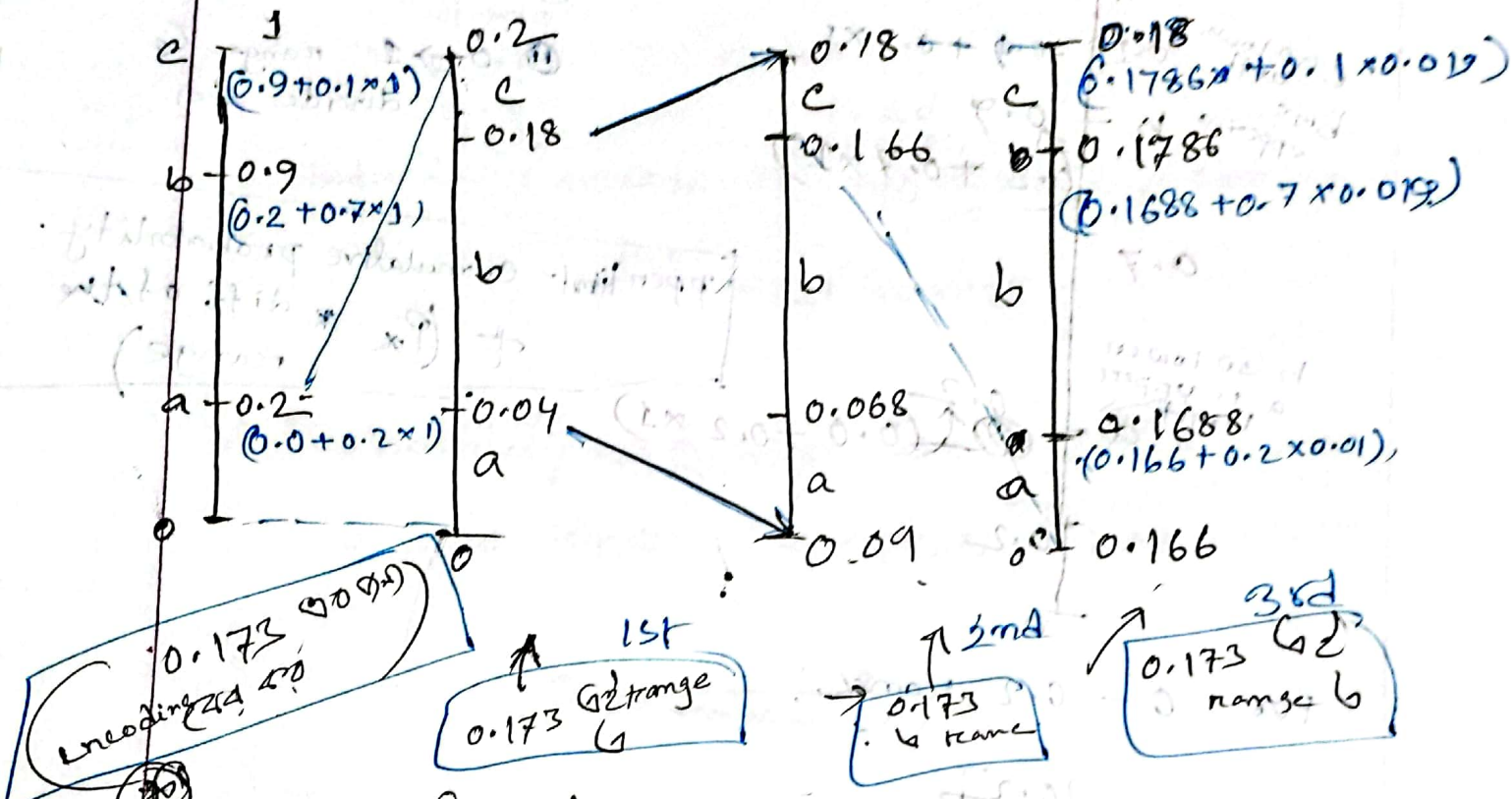
b Go lower
a " upper limit



upper limit = cumulative probability + (P_x * diff. of the range)

For 0 – 0.2 range:




1
c
(0.9+0.1×1)
b
0.9
(0.2+0.7×1)
a
0.2
(0.0+0.2×1)
0
0.2
c
0.18
b
0.04
a
0
0.18
c
0.166
b
0.068
a
0.09
0.18
c
(0.1786+0.1×0.01)
b
0.1786
(0.1688+0.7×0.01)
a
0.1688
(0.166+0.2×0.01)
a
0
0.166
0.173
(encoding 0.0)
1st
0.173 (2 range b)
2nd
0.173 b range
3rd
0.173 (2 range b
Seq: a b c

(*) 4 symbols to build a message

a, b, c, d

$a = 2$, $b = 3$, $c = 1$, $d = 4$

$P_a = \frac{2}{10} = 0.2$

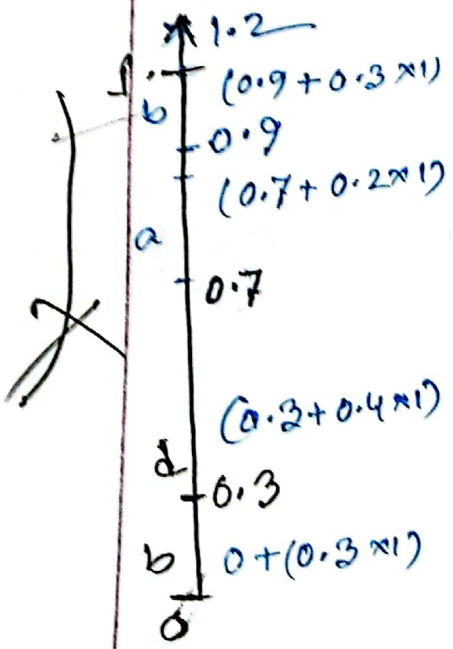
$P_b = \frac{3}{10} = 0.3$

$P_c = \frac{1}{10} = 0.1$

$P_d = \frac{4}{10} = 0.4$

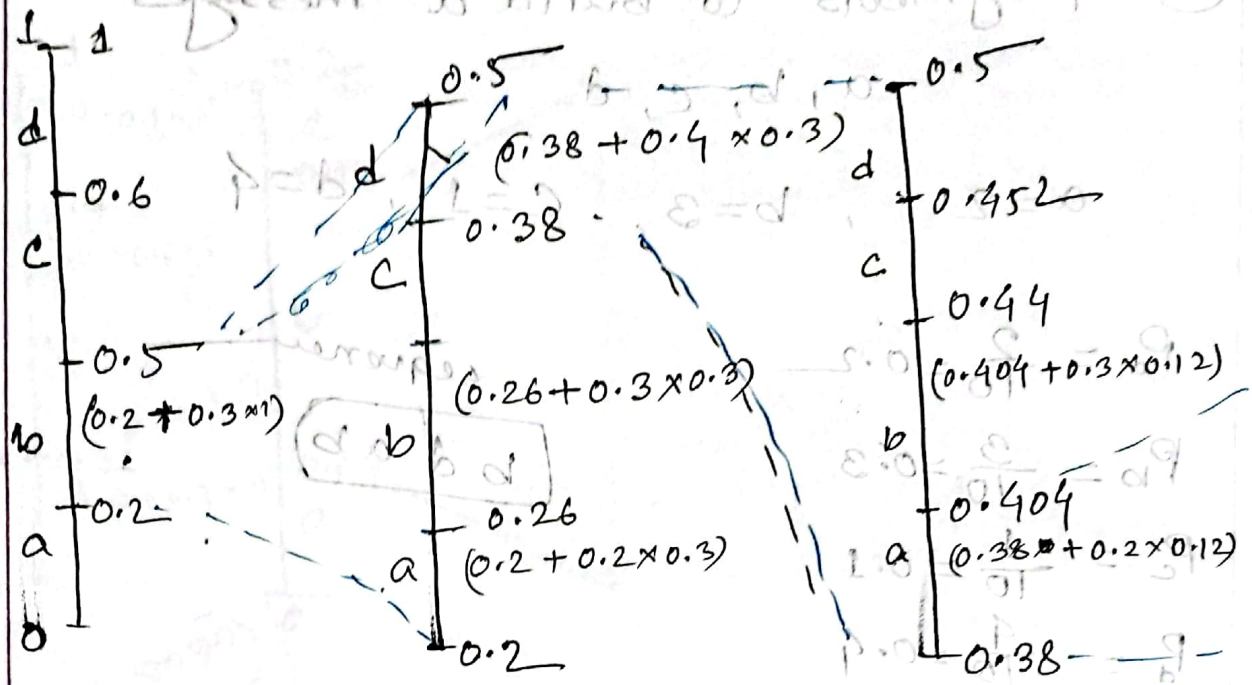
sequence

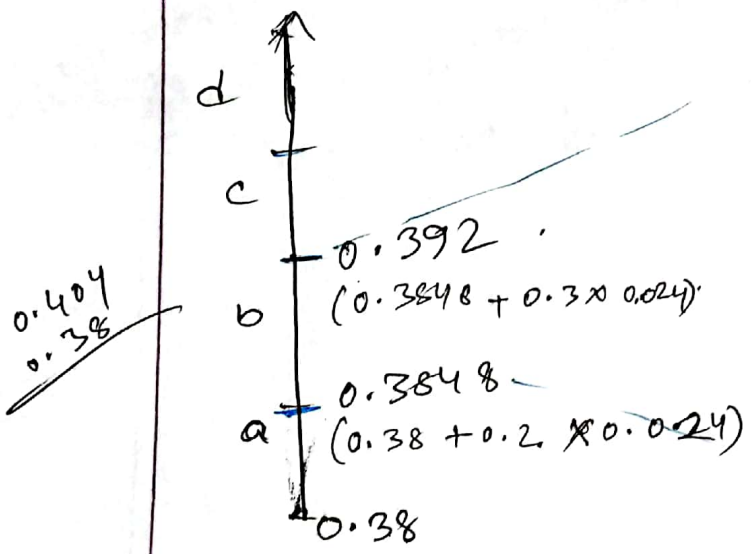
$\boxed{b\ d\ a\ b}$



b d ab b

0.12


1
d
0.6
c
0.5
(0.2 + 0.3×1)
b
0.2
a
0
0.5
(0.38 + 0.4 × 0.3)
d
0.38
c
(0.26 + 0.3 × 0.3)
b
0.26
(0.2 + 0.2 × 0.3)
a
0.2
0.5
d
0.452
c
0.44
(0.404 + 0.3 × 0.12)
b
0.404
(0.38 + 0.2 × 0.12)
a
0.38


d
c
0.392
(0.3848 + 0.3 × 0.024)
b
0.3848
(0.38 + 0.2 × 0.024)
a
0.38
0.404
0.38

Assignment: কিভাবে Arithmatic encoding Huffman encoding এর respect এ bit save করে? (Example এর সাহায্যে প্রমাণ)

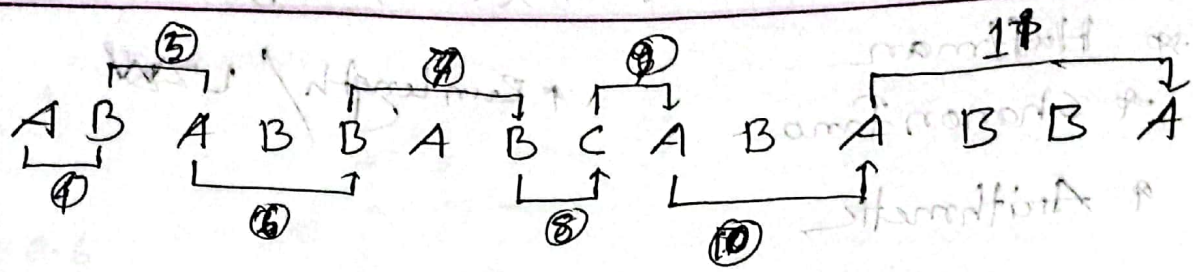
- Huffman
- Shanon fano
- Arithmetic
- Run length / LZW

Run length Encoding:

Example:

a a a a b b b b b b c d d d d d

LZW: Dictionary Table করে

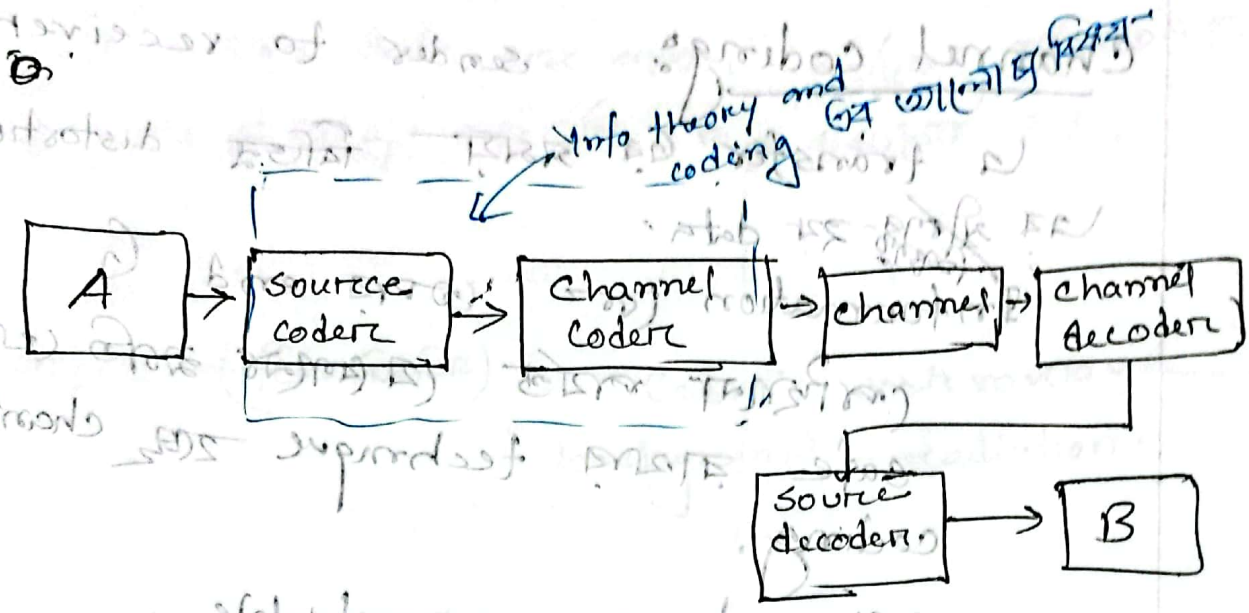


1	A
2	B
3	C
4	A B
5	B A
6	A B B
7	B A B
8	B C
9	C A
10	A B A
11	A B B A

16-11-2023

Arif Sir

Info. theory and coding → Error correction coding.



→ Source coding: Unnecessary information বাদ দেওয়ার technique.

50 GB data কে 10 MB তে convert করতে পারে,
, 99% information ও বাদ দিয়ে দিতে পারে.

→ BW বাঁচায়.

- Time ~~বাঁ~~ ও save করে.

$$1920 \times 1080 \times 30 \times \underbrace{3 \times 8}_{RGB} \times \underbrace{60 \times 5}_{5\ min}$$

$$= 4.97 \times 10^{11}$$

বড় size এর data কে ছোট সাইজে convert করতে হয়।

DCT → coding (Discrete Cosine Transform)

Channel coding: sender to receiver

↳ transfer এর সময় বিভিন্ন distortion এর শিকার হয় data.

Information কে প্রেরক end ↳ গন্তব্য পর্যন্ত যে কোনো গতি থেকে safe রাখার technique হচ্ছে channel coding.

→ Channel হয় unreliable.

Reliable communication over an unreliable channel.

→ To recover data and avoid corruption.

→ Hamming code.

DCT [1 0 1 1] [1 1 0]

→ Video এর background change না হলে বারবার একই data পাঠানোর দরকার নেই। একবার পাঠালেই হয়। ফলে data size কমে। যতটুকু পরিবর্তন হয় কেবল সে অংশের পরিবর্তন শুধু পাঠালেই হয়।

CMDR – (ফিচি জিরফিরে) – Cosmic microwave background radiation.

ECC ram → Error correction encoding ram.

Parity: Even – বেশি সংখ্যক 1 → 1 (tx code)

Odd – বেজোড় no. 1 → 0

10 11 0 1 0 — even parity

↑ parity bit

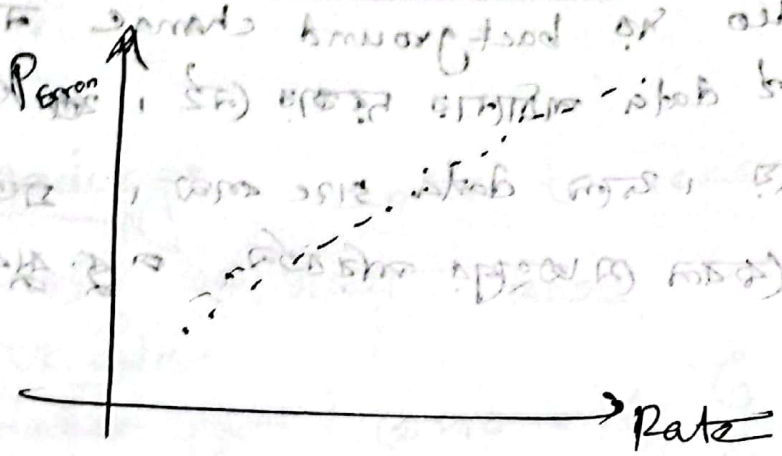
0, parity '1' মানে বেশি সংখ্যক 1

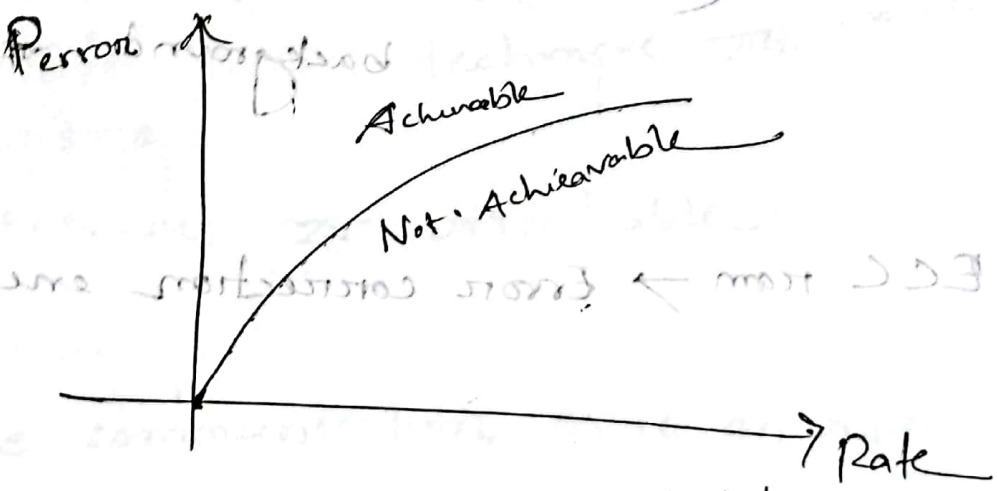
0 → বেজোড় "1 transfer হয়েছে

1 টি parity বিট 1 bit error correct করতে পারবে, multiple bit correction এর জন্য বেশি parity bit লাগবে

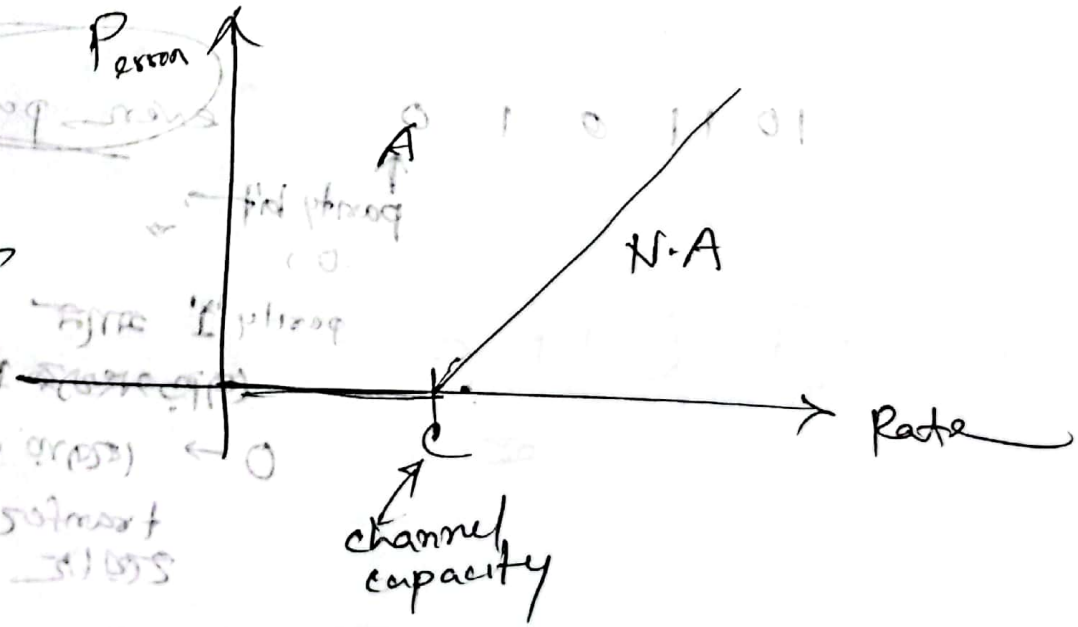
Source coding – Compression

~~Error~~ Channel coding – Error correction





Bit rate কম হলে probability of error কম হবে



Originally

Shannon - second theorem:

Tx rate channel capacity এর কম হলে almost no error এ data পাঠানো যাবে।
(কেউ না কেউ এমন algorithm বানাতে পারবে) theory তে বলেছি শুধু সম্ভাবনা সম্পর্কে।

18-12-2023
Arif Sir

Error control Coding:

* multipath receiver: একাধিক method এ data আসতে পারে, কেননা distance ভিন্ন হতে পারে/ phase error হতে পারে.

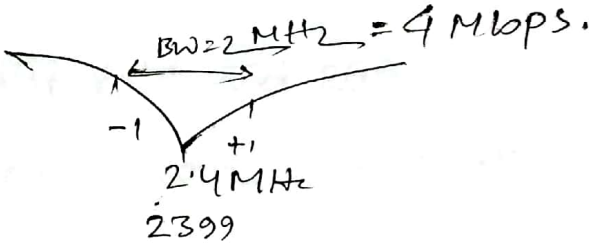
phase error correct করে - PLO

~~Voltage~~ frequency " - VCO (voltage contr osc)

Every real life system is a LPF.

But amplitude error correct করার জন্য extra bit

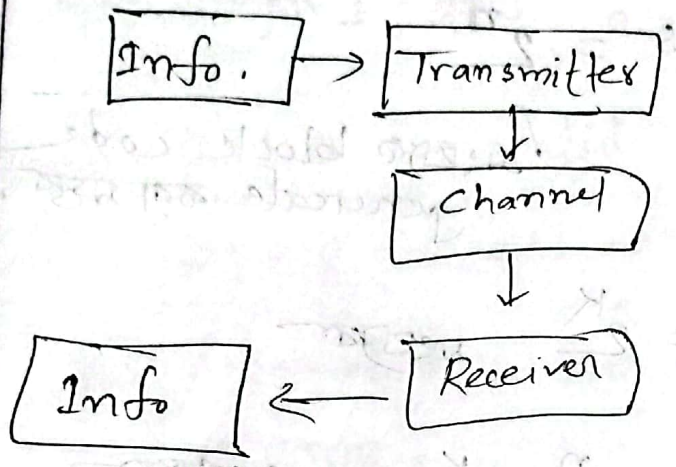
* 1Hz b.w এ per second এ 2 bit পাঠানো যায়, Rising edge, falling edge.



* প্রতি second এ যত সংখ্যক bit generate হতে পারে না, channel capacity নির্ভর.

* Coding যত complex হবে encode এ or decode করতে তত সময় লাগবে.

Basic structure:



Basic of Block code:

* Info bits = K → original bit error হতে পারে
  $i = [i_1, i_2 \cdots i_k]$ (তখন) Extra R সংখ্যক bit পাঠানো হয়ে error control.

parity / Redundant = r

∴ Total bit ⇒ $n = r + K$.

* Block code scheme of size (n, K).

$(4, 3)$

পাঠানো হয়েছে — actual bit

Extra $4 - 3 = 1$, redundant bit.

* total code words required as per n block codes $= 2^n$.

সংখ্যক block code generate করা সম্ভব।

* Total info. $= 2^K$ সংখ্যক

$\therefore$ Parity $= 2^n - 2^K$ সংখ্যক

- Code rate $r = \frac{K}{n}$

- Code rate maximum 1 হতে পারে।
যখন কোনো redundancy use করা যাবে না।

K – info bit, r = redundant bits (parity bits)

* (4, 3) block code

info	parity	codeword	
0 0 0	0	0 0 0 0	✓ even (0) সংখ্যক 1
0 0 0	1	0 0 0 1	
0 0 1	0	0 0 1 0	Even parity check
0 0 1	1	0 0 1 1	✓ even
⋮	⋮	⋮	
1 1 1	0	1 1 1 0	
1 1 1	1	1 1 1 1	✓ even

b-টি valid
b-টি invalid

→ এই ধরনের valid code গুলোকে use করে বলে block code বলে।

Even parity check করবে, Even

জোড় সংখ্যক 1 হলে accept করবে, অন্যথায় করবে না।

0110 → valid

0100 → odd → receive করবে না (ভুল)।

ভুল original ও হতে পারে, Parity bit ও হতে পারে।

* Code word $c = [i, P]$

$$c = [i_1, i_2 \ldots, i_k, P_1, P_2 \ldots P_r]$$

Error code word

$$e = (e_1, e_2, e_3 \ldots e_n)$$

where, $e_j = 1$ → error, $e_j = 0$ → no error.

(RSA = public, private key word)

private key দ্বারা image লুকানো হয়,

Receiver পুনরায় key দ্বারা ডিকোড করলে original image ফিরে পাবে।

Decode

Ex: Given the (5, 4) even parity block code, find a codewords corresponding to I = (1011) and (1010)

$(5,4) = (n, K)$

Info bits $= K = 4$

parity bit $r = n - K = 5 - 4 = 1$

$\Rightarrow$ for info : $I = 1011 = I_1 I_2 I_3 I_4$

$\Rightarrow$ for even parity:

$$P = I_1 \oplus I_2 \oplus I_3 \oplus I_4$$

$$P = 1 \oplus 0 \oplus 1 \oplus 1$$

$$P = 1$$

$\therefore$ Codeword is = [info, parity]

$$c = [i_1, i_2, i_3, i_4, P]$$

$$c = [1, 0, 1, 1, 1]$$

CamScanner

Hridesh Dolapren

- For info. $I = 1010 = I_1 I_2 I_3 I_4$

- for even parity:

$$P = I_1 \oplus I_2 \oplus I_3 \oplus I_4$$

$$P = 1 \oplus 0 \oplus 1 \oplus 0$$

$$P = 0$$

$\therefore$ Code word is = [Info, parity].

Ex 2:

$N_1 = 10110110$ ✗

✓ $N_2 = 01101001 = 0$

জোড় সংখ্যক 1 হলে even parity তে sum 0 হবে। তাহলে code ঠিক.

বিজোড় সংখ্যক 1 হলে sum 1 হবে, ফলে ভুল আছে, তখন 1 add করা লাগবে. in the end জোড় সংখ্যক 1 থাকা লাগবে,

$(8,7) = (n, K)$

- info bit $K = 7$
- parity bit $r = n - K = 8 - 7 = 1$
- for $V_1 = (10110110)$
- Checksum S will be.

$$S = D_1 \oplus D_2 \oplus D_3 \oplus D_4 \oplus D_5 \oplus D_6 \oplus D_7 \oplus D_8$$

$$= 1 \oplus 0 \oplus 1 \oplus 1 \oplus 0 \oplus 1 \oplus 1 \oplus 0$$

$$S = 1$$

- For even received codeword, it is parity failure
- Means there is error in received data.

* for V_2 = 01101001)

Error correction:

এক bit error এর, কি সম্ভব.

এই bit এ error

Data bits			Row parity check	Row parity	
1	1	0	1	1	Extra 1
0	1	1	0	0	
1	0	0	0	1	
0	0	0	1	0	এই row তে ধরল
1	1	1	0	1	
1	0	0	1	0	
Column parity ch.	0	1	0	0	1	Overall parity

(No extra 1)

এই column এ ধরল

Row wise - যেকোনো সংখ্যক 1 হলে extra 1 অন্যথায় 0

Column " " " 1 হলে Extra 1 " 0

দুই bit error দুইটি Comparison দ্বারা সম্ভব না.

* Enough redundancy থাকলে কিছু ভুল থাকলেও info পাওয়া যায়, ভুল correct করা যায়

CODNG THEORY IS AN INTRSTNG SUBJCT

- Redundancy বেশি হলে error correction capability বেশি হয়।

* Word is a set of symbols.

ABDE = ANT → word

↑ Code word

অনেকগুলো word কে একত্রে code বলে।

* Hamming weight: No of non zero element in a code word.

0 1 1 0 → weight 2

* দুইটি code word এর মধ্যে Hamming distance হচ্ছে তাদের মধ্যেকার mismatch.

0 1 1 0
0 1 0 0 → H.D = 1.

$d(c_1, c_2) = w(c_1 - c_2)$

0 1 1 0 → 2 — Code word weight

0 1 0 0 → 1 — Code word "

(diff)(same)

0 0 1 0 → 1 (H.D)

- Distance is always positive.
- Distance can be 0 but can't be negative.

$$\boxed{H.D \geq 0}$$

$$\boxed{d(c_1, c_2) = d(c_2, c_1)}$$

(1 থেকে 2) ও (2-3)

- 1, থেকে 3 এর distance দুইটার যোগফলের বড় অথবা সমান হবে

- $C = \{0100, 1111\}$

H.D = 3 = Diff of weight

H.W = 1, 4 ∴ Diff of weight = (4-1) = 3.

* A block code consist of a set of fixed length codeword.

Ex 3.4

The code $C = \{00000, 10100, 11110, 11001\}$ is a block code of block length equal to 5.

← code length = 4 (4টি word)

Here, $M = 4$ (Block code length → কতটি codeword আছে)

$n = 5$ → codeword এর data সংখ্যা

$K = 2$ (২টি math বসানো actual info.)

$\therefore r = 5 - 2 = 3$ (redundant bit)

$2^n = 2^r = 4$

$q = 2$ (2টি binary digit (bits) আছে)

$\boxed{M = q^K}$

$= 2^n$

$= 4$ টি codeword

Ans : 4

0	0	→	00000
0	1	→	10100
1	0	→	11110
1	1	→	11001

(Easy Engineering Classes - Youtube)

21-12-2023
Arif Sir

8 bit - symbol

Multiple symbol - word

* Meaningful একটি word কে অন্য কিছু দ্বারা represent করলে তাকে codeword বলে,

* Multiple codeword মিলে code block হয়।

3.5: The code rate, r, of an (n, K) code is defined as the ratio (K/n). and reflects the fraction of the codeword that consists of the info. symbols.

$$\text{ratio} = \frac{K}{n}$$

কতটুকু actual, কতটুকু redundant data তা represent করে।

* Code rate কম ↓ Info কম ↓ Redundant Info বেশি ↑ Error correction capability বেশি ↑ and vice-versa.

Redundancy বাড়লে error correction capability বাড়ে।

$$n = r + k$$

r ↑ ECC, k ↑ Imp

3.6: Minimum distance: (d^*)

d^* দ্বারা represent করা হয়।

একটা code এ 10টা codeword থাকলে প্রত্যেকটা codeword কে নিজেদের মধ্যে distance বের করে সর্বনিম্ন যে distance হবে তাই minimum distance.

1 2 3 4 5 6 7 8 9 10

প্রত্যেকের সাথে প্রত্যেকের comparison.

Minimum weight: Among all code words there might be any zero codeword (0000000). Avoiding this, among all other code words, ~~those~~ that which has the minimum weight is minimum weight of the codeword.

∞ A linear code has the following properties.

→ কোনো code block linear হওয়ার শর্ত.

প্রত্যেকটা codeword compare করলে তাদের minimum distance এবং সেখানের code word এর minimum weight same হবে,

→ Linear block code এর যেকোনো দুইটি codeword যোগ করলে তাদের যোগফল এ একটি codeword যা same to code block এ থাকবে.

→ All zero codword is always a codeword. (00000000) এ ধরনের একটি codeword সবসময় থাকবে.

eg 10110 → min dis = 3
00000 → weight = 3

∴ min dis = Min weight.

Galois Field-Th...

* The code $C = \{0000, 1010, 0101, 1111\}$

All zero code word

Error coding - Block codes!

Repetition code:

These are the codes that repeat info bits two or more times. They are block codes in which the parity bits are set equal to a single info bit and if the no of parity bits is '$n-1$' then the code is referred to as $(n,1)$ repetition code.

* Info এর সুরুতে meta data (header) থাকে, যা Actual data সম্পর্কে কিছু তথ্য দেয়া থাকে; source decoder সে তথ্য অনুযায়ী main data decode করে।

এক bit info এর জন্য multiple bits of parity লাগাতে পারব। Data যা হবে parity ও তাই হবে (same)

(3, 1) → redundancy (3-1) = 2 bit

↑ info

1 1 1

0 0 0

↑ info (k) — red. (r)

Input	Decoding Decision	Output Codeword	Info bit
000	No error	000	0
001, 010, 100, 100 →	one (1) error	000	0
	No error	111	1
101, 110, 011	one (1) error	111	1

~~দুই bit error~~

বেশি সংখ্যক bit error correction এর জন্য redundancy বাড়াতে হবে,

0 0 0 0 0
K red

0 1 1 0 0 → 0 0 0 0 0 (Correction)

Error.
(2 bit) (0, 3টি)

Redundancy বাড়ার সাথে info কমে গেছে, কিন্তু Error correction amount বেড়ে যাবে,

* system এ error কতো তা আগে থেকে ধারণা করা সম্ভব নয়, Its variable, depend on the circumstance.

* Same info বারবার repeat (repetation) এ run length coding use হয়,

Q: Why linear code?

Error coding - linear codes:

⇒ A block code is said to be linear code if its codewords satisfy the condition that the sum of any codewords gives another codeword.

★★ Q: Show that (4,3) Even-parity code is a linear and (4,3) odd parity is not linear.

Even parity (4,3) codes:

(0000) (0011) (0110) (1100)

(0101) (1010) (1001) (1111)

sum:

$$\begin{array}{r} 0000 \\ 0011 \\ \hline 0011 \end{array} \qquad \begin{array}{r} 0101 \\ 1001 \\ \hline 1100 \end{array}$$

← No carry

Distance: 1010
1001 ⇒ 2

Weight = 2 ∴ linear code.

⇒ coding scheme এ single bit হলে error বোঝানো carry বহন bit এ সাহায্য নাই!

Hamming Code (7,4)

* They are the first class of linear codes devised for error control

Hamming codes are (n, k) codes with following properties.

(i) $n = 2^q - 1$ $\quad$ q = no of parity bits $(n-k)$

(ii) info $k = 2^q - q - 1$ $\quad$ ← (no of module) $\quad$ k = no of info bits.

(iii) $q \geq 3$; i.e minimum no of parity bits = 3 $\quad$ $i = (i_1 + i_2 + i_3, i_4)$

when

$q = 3$

$n = 2^3 - 1 = 7$ $\quad$ → parity 3 → Act Info → 4 $\quad$ Total info = 7

$k = 2^3 - 3 - 1 = 8 - 4 = 4$

Parity bits:

(i) $P_1 = i_1 + i_2 + i_3$

(ii) $P_2 = i_2 + i_3 + i_4$

(iii) $P_3 = i_1 + i_2 + i_4$

Hamming coding scheme.

→ Info (2)
parity (4)
সংরক্ষণ

(7, 4) (7,4) Hamming Coding scheme:

to $C \Rightarrow (i_1, i_2, i_3, i_4, P_1, P_2, P_3)$

$$P_1 = i_1 + i_2 + i_3$$

$$P_2 = i_2 + i_3 + i_4$$

$$P_3 = i_1 + i_2 + i_4$$

$$\begin{array}{ccccccc} 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ i_1 & i_2 & i_3 & i_4 & P_1 & P_2 & P_3 \end{array}$$

Decoding

Received codeword

$$V = (v_1, v_2, v_3, v_4, v_5, v_6, v_7)$$

For error checking, 3 parity check sum is

i) $S_1 = (v_1 + v_2 + v_3) + v_5$

ii) $S_2 = (v_2 + v_3 + v_4) + v_6$

iii) $S_3 = (v_1 + v_2 + v_4) + v_7$.

(+) compare parity

1	1	0	0	sum = 0
1	0	0	1	sum = 0
0	0	0	0	sum = 0

parity

* Bit stuffing: Extra bit add করা

$$S_1 = (V_1 + V_2 + V_3) + V_5$$

(msg)

যোগফল 0 হলে ঠিক

1 হলে ভুল

info bit (+) করে তাকের sum এর সমান হবে parity.

sum + parity যোগ করলে 0 হবে।

0 হলে ঠিক আছে,

নয়তো ভুল হয়েছে।

$$S_2 = (V_2 + V_3 + V_4) + V_6$$

$$S_3 = (V_1 + V_2 + V_4) + V_7$$

Errore pattern								Errore Syndrome (S) (S_1, S_2, S_3)
e_1	e_2	e_3	e_4	e_5	e_6	e_7		
0	0	0	0	0	0	0	$\longrightarrow$	0 0 0 $\rightarrow$ 0
0	0	0	0	0	0	1	$\longrightarrow$	0 0 1 $\rightarrow$ 1
0	0	0	0	0	1	0	$\longrightarrow$	
0	0	0	0	1	1	1		
0	0	0	1	1	1	1		
0	0	1	1	1	1	1		
0	1	1	1	1	1	1		
0	1	1	1	1	1	1		

I_1	I_2	I_3	I_4	P_1	P_2	P_3
v_1	v_2	v_3	v_4	v_5	v_6	v_7
0	1	1	1	0	1	0

$P_1 = 0 + 1 + 1 = 0$

$P_2 = 1 + 1 + 1 = 1$

$P_3 = 0 + 1 + 1 = 0$

V_1	V_2	V_3	V_4	P_1	P_2	P_3
0	1	1	1	0	1	0

error

0	1	0	1	0	1	0

$\overset{0}{V_1} + \overset{1}{V_2} + \overset{0}{V_3} = P_1$ $(0+1)$ ✓ 1

$\overset{1}{V_2} + \overset{0}{V_3} + \overset{1}{V_4} = P_2$ $(0+1) = 1$ ✓ 1

$\overset{0}{V_1} + \overset{1}{V_2} + \overset{1}{V_4} = P_3$ $(0+0) = 0$ 0

∴ Error pattern = 1 1 0

∴ V_3 has error.

∴ Correction 0 1 ~~0~~ 1 1 0 1 0 1 0

* (7,4) G 1 bit error correction করার যাবে।

$\boxed{\text{(data/parity) bit}}$

→ GPU G code অনেকগুলো, এগুলো সুন, যোগ করে।
- Display, মনিটরের pixel color up করে।
Normally Gaming G use হয়।

→ fast computation G GPU use হয়।

* Matrix অনেকবার fetch, decode execute করে।
(CNN এর মতো)

specific
ASIC — Circuit
Integration
Application

} GPU এর মতো কাজ করে। Graphic Data এগুলো process করে।

Generator matrix for (7,4) code :

$$G = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

parity matrix

Identity matrix $(K \times K)$
(I_4)

$$[1\ 1\ 1\ 0]\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} = [1\ 1\ 1\ 0\ 1\ 0\ 0]$$

(Ans)

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Arif Sir

$C = j\,G \leftarrow$ Generation matrix

$(7, 4) \rightarrow G = [I_k \mid P]$

n, k

column

row

7

column
3 bit ZIP
error correction code info

i_k

4 →

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & \end{bmatrix}$$

$\leftarrow v_1$
$\leftarrow v_2$
$\leftarrow v_3$
$\leftarrow v_4$

4 →

4 bit info

4×4 → Identity matrix

$(v_1+v_2+v_3)$ $(v_2+v_3+v_4)$ $(v_1+v_2+v_4)$

Example: $i = (1\ 1\ 1\ 0)$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$\cancel{[1\ 1\ 1\ 0]}$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$1+1$

$1+1+0+0$

(6, 3)
(5, 3) } Generator matrix সুবিধা

100 101 ← G দ্বারা গুণ করে convert করা হয়

110110 101101

উই লেভেল পার্থক্য হয়

110001 কে (6, 3) codeword হিসেবে transmit করা

[1 0 0 0 1] দ্বারা G দ্বারা গুণ করে

Systemetic Code

~~Linear code~~

Systematic Generator matrix :

→ Code word এর যদি 1st portion যদি info এবং বাকি portion parity হলে তাকে systemetic code বলে।

110 → info

systematic code → 110 110 → codeword

* 1st portion info এর সমান না হলে তাকে non systematic code বলে।

* let,

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

identity matrix না।
এজন্য non systematic codeword হবে।

011 → 00111

110 → 11101

Non systematic codeword.

public, private key encryption

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$ Non systematic

$$\downarrow$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$ $r_3 =$

$$\downarrow$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$ → systematic

Non systematic [to systematic] করতে 2নং

এর bit info এর size এর identity matrix

আনতে হবে।

* Hamming ~~Gx~~ decoding value always zero.

Decoding:

Parity checker Matrices:

$$H = \left[P^T \right] \vdots \left[I_{n-k} \right]$$

$\hookrightarrow$ n-k by n-k identity matrix

Transpose of p(n-k) by k matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

① $P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ $\quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$H(7,4) = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Ans

$$\underset{\text{info}}{3} \times \underset{\text{Generator}}{10} = 30 \rightarrow 30 \times \underset{\text{parity checker (H)}}{\frac{1}{10}} = 3$$

* prove that GH^T and $CH^T = 0$

$$GH^T = \left[I_k \mid P \right] \cdot \left[P^T \mid I_{n-k} \right]^T$$

$$= \left[I \mid P \right] \left[\begin{array}{c} P \\ \hline I_{n-k} \end{array} \right]$$

(7,4)

$$= I_n P + P I_{n-k}$$

$$= P + P$$

$$= 0$$

$$\left[P \rightarrow 0 \rightarrow 0+0=0 \quad 1 \rightarrow 1+1=0 \right]$$

$$\left[GH^T = 0 \right]$$

Again,

$$CH^T = (iG)H^T$$

$$= i(GH^T)$$

$$= i \cdot 0$$

$$\therefore CH^T = 0$$

(*) Determine the parity check matrix H for the (5, 3) code. Show that $GH^T = 0$ and $CH^T = 0$ for (11010)

$$G = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right] \qquad H = \left[\begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$GH^T = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1+1), & 1+1 \\ 0 & 1+1 \\ (1+1) & 1+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \therefore GH^T = 0$$

0 হলে codeword Valid.

0 না হলে codeword valid নয়।

$$CH^T = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 0$$

* $VH^T = 0 \rightarrow$ valid codeword received (V)

$\neq 0 \rightarrow$ invalid codeword (V)

* we know.

$$V = C + e \quad [\text{multiplying by } H^T]$$

$$VH^T = (C+e)H^T$$

$$VH^T = CH^T + eH^T \quad [CH^T = 0]$$

$$\therefore VH^T = eH^T$$

$\therefore$ Error syndrome(s) $= VH^T$

$V = 1\ 1\ 0\ 1\ 1\ 0\ 1$

Error syndrome:

$$S_1 = V_1 H^T$$

$$= [1\ 1\ 0\ 1\ 1\ 0\ 1] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = (1\ 0\ 0)$$

$$S_2 = V_2 H^T$$

$$S_2 = 0\ 1\ 0$$

$$V = c + e \quad \rightarrow c = V - e$$

$$VH^T = (c+e) H^T$$

$$= cH^T + eH^T$$

$$= eH^T$$

Shannon-Fano Coding

Algorithm:

Step1: Write the messages in the order by decreasing probability.

Step2: Divide the set into 2 parts such that both parts have equal probability.

Step3: Assign '0' to the upper set and '1' to the lower set of equiprobable messages.

Step4: Continue step 3 until all the message are coded and no further partition of set is possible.

Step5: We get the code words for each symbol.

$$\eta = \frac{H}{L} \text{ where, } L = \sum_{i=1}^{n} l_i P_i$$

							Codeword	Code length
A	$\frac{1}{2}$	0					0	1
B	$\frac{1}{4}$	1	0				10	2
C	$\frac{1}{16}$	1	1	0	0		1100	4
D	$\frac{1}{16}$	1	1	0	1		1101	4
E	$\frac{1}{32}$	1	1	1	0	0	11100	5
F	$\frac{1}{32}$	1	1	1	0	1	11101	5
G	$\frac{1}{32}$	1	1	1	1	0	11110	5
H	$\frac{1}{32}$	1	1	1	1	1	11111	5

Avg Code length $= \sum l_i \cdot P_i$

$$= \left(\frac{1}{2} \times 1\right) + \left(\frac{1}{4} \times 2\right) + 2\left(\frac{1}{16} \times 4\right) + 4\left(\frac{1}{32} \times 5\right)$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{2}{4} + \frac{4 \times 5}{32}$$

$$= 2.125$$

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Arif Sir

Cyclic Code

* Burst error এর ক্ষেত্রে cyclic code use হয়,

(মানে একসাথে অনেক error)

কোনো codeword কে left or right এ rotate করলেও resultant code টা একটা codeword হবে,

Info এর possibility 4 টা → Info – 2 bit.

$$\begin{matrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{matrix}$$

Rotate left.

Rotate right

$$C = (C_{n-1}, C_{n-2}, \cdots C_1, C_0)$$

left shift

Right shift.

$$C' = (C_{n-2}, C_{n-3}, \cdots C_1, C_0, C_{n-1})$$

It is also a codeword.

$$C' = (C_2, C_1, C_0, \cdots C_4, C_3)$$

→ codeword ..

prove that (7, 4) ~~cyclic~~ Hamming code is cyclic code

Example: (7,4) Hamming code

↳ linear code; ↳ cyclic

$C_3 = (0011101) \rightarrow$ cyclic code

↓ left shift by 2 place

$c'' = (1110100) \rightarrow C_{14}$

↓ L.S by 3 place

$c''' = (0100111) \rightarrow C_{10}$

(5, 2) linear codes

Polynomials - Encoding (Decoding).

An n-bit word $(a_{n-1}, a_{n-2}, \dots a_1, a_0)$ can be represented by the polynomial

$$P(x) = a_{n-1} x^n + a_{n-2} x^{n-2} + \dots a_1 x + a_0$$

word.

$$= \begin{array}{ccccccc} (0 & 1 & 1 & 0 & 0 & 1 & 0) \\ a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 \end{array}$$

$$P_1(x) = a_6 \cdot x^6 + a_5 x^5 + \dots a_0 x^0$$

$$= 0 \cdot x^6 + 1 \cdot x^5 + 1 \cdot x^4 + x$$

$$= x^5 + x^4 + x.$$

* Find the value of $P_1(x) \div P_2(x)$

$$
\begin{array}{l}
x^3+x+1 \,\big)\, x^5+x^4+x \,\big(\, x^2+x+1 \\
\qquad\qquad\quad \underline{x^5+x^3+x^2} \\
\qquad\qquad\quad x^4+x^3+x^2+x \\
\qquad\qquad\quad \underline{x^4+x^2+x} \\
\qquad\qquad\qquad x^3 \\
\qquad\qquad\qquad \underline{x^3+x+1} \\
\qquad\qquad\qquad\quad x+1
\end{array}
$$

modular arithmetic
(+, − একই same)
same প্রতিটির একই অংশে এলে (0).

* Q

Generator polynomials:

$$\overbrace{0\ 0\ 0\ 1}^{\text{info}}\ \overbrace{0\ 1\ 1}^{\text{parity}}$$

$x^3 + x^1 + x^0$

Codeword polynomial:

$x^3 + x + 1$ → সর্বচেয়ে nonzero codeword এর polynomial G নিলে যে eqn generate হয় তাকে generator polynomial বলে.

$x(x^3 + x + 1)$

$= x^4 + x^2 + x$

$0\ 0\ 1\ 0\ 1\ 1\ 0$ codeword.

1 bit left rotate shift → Cyclic code.

→ For an (n, k) binary cyclic code

$$g(x) = g_{n-k}\, x^{n-k} + g_{n-k-1}\, x^{n-k-1} + \cdots g_1 x + g_0]$$

Que : $i = (0\ 1\ 0\ 1)$, $g(x) = x^3 + x + 1$

Convert this (7, 4) Hamming cyclic Code.

~~polynomial = $x^2 +$~~

Codeword : ~~0 1 0 1 1 0~~

$\therefore i(x) = 0 \cdot x^3 + 1 \cdot x^2 + 0 \cdot x + 1$

$= x^2 + 1$.

$c(x) = i(x)\, g(x)$

$= (x^2 + 1) \cdot (x^3 + x + 1)$

$= x^5 + x^3 + x^2 + x^3 + x + 1$

$c(x) = x^5 + 0x^3 + x^2 + x + 1$

$\therefore \overbrace{0\ 1\ 0\ 0}^{i}\ \underbrace{1\ 1\ 1}_{p}$

This doesn't match to $i\ (0\ 1\ 0\ 1)$

$\therefore$ This is not a systematic

codeword.

* linear code G systematic তানত easier.
  * Cyclic " G non systematic তান easier.

Systematic code generation:

→ Info G to polynomial কে x^{n-k} দ্বারা গুণ করা লাগবে.

→ গুণফলকে $g(x)$ (generator polynomial) দ্বারা ভাগ করে যে ভাগশেষ পাওয়া যাবে তাকে পুনরায় $g(x)$ দ্বারা গুণ

Info 0 1 0 1 , $g(x) = x^3 + x + 1$

Generate codeword using Hamming (7,4).

⇒ poly : $x^2 + 1$

$n = 7$
$k = 4$
$\therefore n-k = 7-4 = 3$

multiply: $x^3(x^2+1)$
$= x^5 + x^3$

$$x^3+x+1 \;\big)\; x^5+x^3 \;\big(\; x^2 \cancel{+x+1}$$

$$x^5+x^3+x^2$$

$$x^4 \quad 0$$

$$x^4+x^3+x$$

$$x^3+x$$

$$x^3+x^2+x$$

$$x^2$$

$$x^2+x+1$$

$$x+1$$

Decoding cyclic codes:

$R_g(x) \rightarrow$ Remainder of $g(x)$

Decoding cyclic codes:

If received word $= V(x)$

first syndrome Polynomial is calculated.

$S(x) = R_{g(x)}[V(x)]$ $\left\{ \frac{V(x)}{g(x)} \text{ is devided by } g(x). \right\}$

For a codeword polynomia $C(x)$

R_g

e	$e(x)$	$s(x)$
0000000	0	0
0000001	1	1
0000010	x	x
100	x^2	x^2
1000	x^3	$x+1$
	~~x^4~~	~~x^2+1~~
10000	x^4	x^2+x
01000000	x^5	x^2+x+1
10000000	x^6	x^2+1

← position G

← error (1) 2(n-r)

info to error

← GZ2M4 1 2(n-r) parity error.

$(c = 1010011)$, $e = 1000000$

Consider decoding when the codeword

$c = (1010011)$ belonging to the (7, 4) code.

$e(x) = x^6 + x^4 + x + 1$

$e = 1000000$ $\quad e(x) = x^6$

$\therefore V(x) = c(x) + e(x)$

$= x^6 + x^4 + x + 1 + x^6$

$= x^4 + x + 1$ $\longrightarrow$ Received.

<u>Decoding</u>:

$S(x) = R_{g(x)}[V(x)]$

$g(x) = x^3 + x + 1$

$$
\begin{array}{l}
x^3 + x + 1 \,\big)\, x^4 + x + 1 \,\big(\, x \\
\qquad\qquad\quad x^4 + x^2 + x \\
\hline
\qquad\qquad\qquad x^2 + 1
\end{array}
$$

$\Leftarrow$ error syndrom.

x^2+1

$e \rightarrow 1\ 0\ 1$ → error syndrom

∴ parity 1 ৪০০ ৫ 3 G হয়

∴ Codeword G0, 1 no bit G

error এর

$e \rightarrow 1\ 0\ 0\ 0\ 0\ 0\ 0 \rightarrow x^6$

$e(x) = x^6$

$\hat{e}(x) = x^6$ for $S(x) \rightarrow x^2+1$

$\hat{c}(x) = v(x) + \hat{e}(x)$

$= x^4+x+1+x^6$

$c(x) = x^6+x^4+x+1$

$\Rightarrow 1\ 0\ 1\ 0\ 0\ 1\ 1$