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HW 2

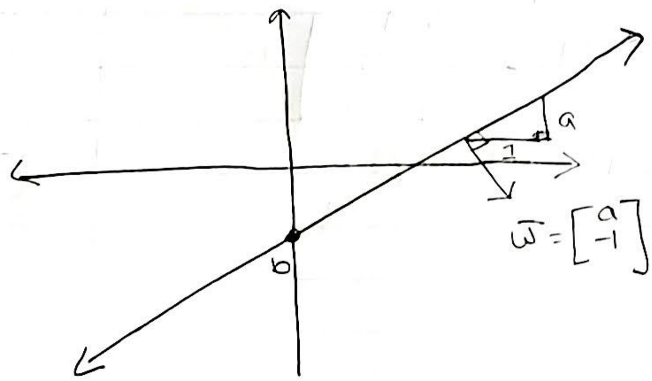
Problem 1.2 from LB

Consider the perceptron in 2 dimensions ⇒
$h(x) = \text{sign}(w^T x)$ where
$\bar{w} = [w_0, w_1, w_2]^T$ $\quad \bar{x} = [1, x_1, x_2]^T$

a) $h(x) = \text{sign}(w^T \bar{x})$, $\bar{w} \in \{\mathbb{R}^3 | \{0\}\}$ $\quad x_2 = ax_1 + b$

For $a = 0 \longrightarrow$ the line is horizontal.
For $a > 0 \longrightarrow$ it is increasing. For this case to be true, b is the offset. In here, we cross the x_2 axis at $x_2 = b$ & the line runs through the origin $\Longleftrightarrow b = 0$.

$x_2 = ax_1 + b$
$\Rightarrow -ax_1 + x_2 = b$
$\Rightarrow -ax_1 + x_2 - b = 0$
$\Rightarrow [a, -1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + b = 0$
$\Rightarrow [b, -a, 1] \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix} = 0$



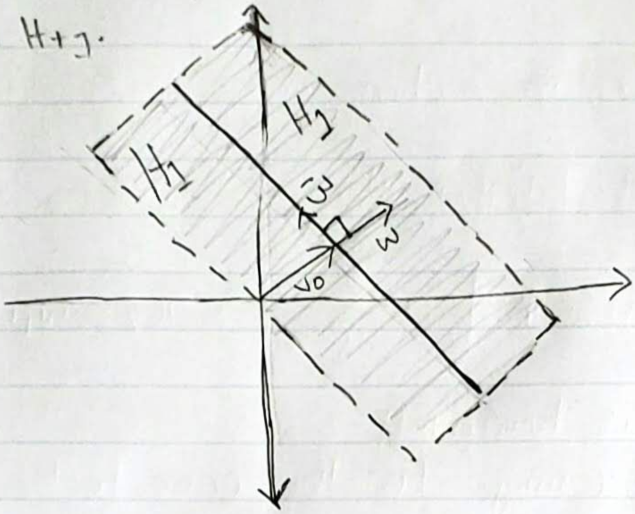
In $\mathbb{R}^3$ $w^T \bar{x}$ is the hyper-plane.

The geometrical line of a linear discriminant function $y(x) = \bar{w}^T \bar{x}$.
Defined by $y(x) = 0 \Rightarrow (w_0, w_1, w_2) \cdot \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow w_0 + w_1 x_1 + w_2 x_2 = 0$

is the hyperplane with normal vector $\bar{w}$.
The hyperplane seperates the null space $\mathbb{R}^2$, into two half space. Hence it will be defined by $h(x) = 1$ & $h(x) = -1$. That is,
$H_{+1} = \{x | y \; y(x) > 0\}$ & $H_{-1} = \{x | y(x) < 0\}$

then $h(x) = \text{sign}(y(x)) = 1$ for $x \in H_{+1}$
& $h(x) = \text{sign}(y(x)) = -1$ for $x \in H_{-1}$.

The normal vector $\bar{w}$, started from any-where on the hyperplane, points into H_{+1}.



Separating hyperplane in $\mathbb{R}^2$.

b) $y(x) = 0$

$\Rightarrow w^T \bar{x} = 0$

$\Rightarrow (1,2,3) \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix} = 0$

$\Rightarrow 1 + 2x_1 + 3x_2 = 0$

$\Rightarrow 2x_1 + 3x_2 = -1$

$y(x) = 0$

$\Rightarrow w^T \bar{x} = 0$

$\Rightarrow (-1,-2,-3) \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix} = 0$

$\Rightarrow -1 - 2x_1 - 3x_2 = 0$

$\Rightarrow 2x_1 + 3x_2 + 1 = 0$

$\Rightarrow 2x_1 + 3x_2 = -1$

$\Rightarrow [2,3] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 1 = 0$