**CSE 404: Introduction to Machine Learning (Fall 2019)**
Homework #1
Due 9/16/2017 (Mon) before class

# Linear Algebra

Note: Please upload PDF to D2L. Scanned copy is accepted for this assignment.

1. (10 points) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}, C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, E = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}.$$

   If possible, compute the following:

   (a) $(2A)^T$
   (b) $(A - B)^T$
   (c) $(3B^T - A)^T$
   (d) $(-A)^T E$
   (e) $(C + 2D^T + E)^T$

2. (10 points) Let

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}.$$

   Is $AB = BA$? Justify your answer.

3. (10 points) Given the three vectors $v_1 = (-2, 0, 1), v_2 = (0, 1, 0)$ and $v_3 = (2, 0, 4)$ in $\mathbb{R}^3$.

   - Show that they form an orthogonal set under the standard Euclidean inner product for $\mathbb{R}^3$ but not an orthonormal set.
   - Turn them into a set of vectors that will form an orthonormal set of vectors under the standard Euclidean inner product for $\mathbb{R}^3$.

4. (10 points) Given $x \in \mathbb{R}^m$, $y \in \mathbb{R}^n$, show that the rank of matrix $xy^T$ is one.

5. (10 points) Given $X = [x_1, x_2, \cdots, x_n] \in \mathbb{R}^{m \times n}$ where $x_i \in \mathbb{R}^m$ for all $i$, and $Y^T = [y^1, y^2, \cdots, y^n] \in \mathbb{R}^{p \times n}$ where $y^i \in \mathbb{R}^p$ for all $i$. Show that

$$XY = \sum_{i=1}^{n} x_i (y^i)^T.$$

6. (10 points) Given $X \in \mathbb{R}^{m \times n}$, show that the matrix $X^T X$ is symmetric and positive semi-definite. When is it positive definite?

7. (10 points) Given $g(x, y) = e^x + e^{y^2} + e^{2xy}$, compute $\frac{\partial g}{\partial y}$.

8. (30 points) Consider the matrix

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 3 & 2 & 5 \end{pmatrix},$$

(a) Compute the eigenvalues and corresponding eigenvectors of $A$. You are allowed to use Matlab/Python to compute the eigenvectors (but not the eigenvalues). Please include the code that you used for computing eigenvectors.

(b) What is the eigen-decomposition of $A$?

(c) What is the rank of $A$?

(d) Is $A$ positive definite?

(e) Is $A$ positive semi-definite?

(f) Is $A$ singular?