ncert_classXII(12)_chapter1_electric_charges_and_fields: Chapter One ELECTRIC CHARGES AND FIELDS 1.1 I NTRODUCTION All of us have the experience of seeing a spark or hearing a crackle when we take of f our synthetic clothes or sweater , particularly in dry weather . Have you ever tried to find any explanation for this phenomenon? Another common example of electric discharge is the lightning that we see in the sky during thunderstor ms. W e also experience a sensation of an electric shock either while opening the door of a car or holding the iron bar of a bus after sliding from our seat. The reason for these experiences is discharge of electric charges through our body, which were accumulated due to rubbing of insulating sur faces. Y ou might have also hear d that this is due to generation of static electricity. This is precisely the topic we are going to discuss in this and the next chapter . Static means anything that does not move or change with time. Electrostatics deals with the study of forces, fields and potentials arising from static charges . 1.2 E LECTRIC CHARGE Historically the credit of discovery of the fact that amber rubbed with wool or silk cloth attracts light objects goes to Thales of Miletus, Greece, around 600 BC. The name electricity is coined from the Greek word Rationalised 2023-24 2Physics elektr on meaning amber . Many such pairs of materials wer e known which on rubbing could attract light objects like straw, pith balls and bits of papers. It was observed that if two glass rods rubbed with wool or silk cloth are brought close to each other , they r epel each other [Fig. 1.1(a)]. The two strands of wool or two pieces of silk cloth, with which the rods were rubbed, also r epel each other . However , the glass r od and wool attracted each other . Similarly, two plastic r ods rubbed with cat's fur r epelled each other [Fig. 1.1(b)] but attracted the fur . On the other hand, the plastic rod attracts the glass rod [Fig. 1.1(c)] and repel the silk or wool with which the glass r od is rubbed. The glass r od repels the fur . These seemingly simple facts were established from years of efforts and careful experiments and their analyses. It was concluded, after many careful studies by different scientists, that there were only two kinds of an entry which is called the electric char ge. We say that the bodies like glass or plastic rods, silk, fur and pith balls are electrified. They acquire an electric charge on rubbing. T here are two kinds of electrification and we find that (i) like charges repel and (ii) unlike charges attract each other . The pr operty which dif ferentiates the two kinds of char ges is called the polarity of charge. When a glass rod is rubbed with silk, the rod acquires one kind of charge and the silk acquires the second kind of charge. This is true for any pair of objects that are rubbed to be electrified. Now if the electrified glass rod is brought in contact with silk, with which it was rubbed, they no longer attract each other . They also do not attract or r epel other light objects as they did on being electrified. Thus, the charges acquired after rubbing are lost when the charged bodies are brought in contact. What can you conclude from these observations? It just tells us that unlike charges acquired by the objects neutralise or nullify each other's effect. Therefore, the charges were named as positive and negative by the American scientist Benjamin Franklin. By convention, the charge on glass rod or cat's fur is called positive and that on plastic rod or silk is termed negative. If an object possesses an electric charge, it is said to be electrified or charged. When it has no charge it is said to be electrically neutral.FIGURE 1.1 Rods: like char ges repel and unlike char ges attract each other . Rationalised 2023-24 Electric Charges and Fields 3A simple apparatus to detect charge on a body is the gold-leaf electroscope [Fig. 1.2(a)]. It consists of a vertical metal rod housed in a box, with two thin gold leaves attached to its bottom end. When a charged object touches the metal knob at the top of the rod, charge flows on to the leaves and they diverge. The degree of divergance is an indicator of the amount of charge. Try to understand why material bodies acquir e char ge. You know that all matter is made up of atoms and/or molecules. Although normally the materials are electrically neutral, they do contain charges; but their charges are exactly balanced. For ces that hold the molecules together , forces that hold atoms together in a solid, the adhesive force of glue, forces associated with surface tension, all are basically electrical in nature, arising from the forces between charged particles. Thus the electric force is all pervasive and it encompasses almost each and every field associated with our life. It is therefore essential that we learn more about such a force. To electrify a neutral body, we need to add or remove one kind of charge. When we say that a body is charged, we always refer to this excess charge or deficit of charge. In solids, some of the electrons, being less tightly bound in the atom, are the charges which are transferred from one body to the other . A body can thus be char ged positively by losing some of its electrons. Similarly, a body can be charged negatively by gaining electrons. When we rub a glass rod with silk, some of the electrons from the rod are transferred to the silk cloth. Thus the rod gets positively charged and the silk gets negatively charged. No new charge is created in the process of rubbing. Also the number of electrons, that are transferred, is a very small fraction of the total number of electrons in the material body. 1.3 C ONDUCTORS AND INSULA TORS Some substances readily allow passage of electricity through them, others do not. Those which allow electricity to pass through them easily are called conductors . They have electric charges (electrons) that are comparatively free to move inside the material. Metals, human and animal bodies and earth are conductors. Most of the non-metals like glass, porcelain, plastic, nylon, wood offer high resistance to the passage of electricity through them. They are called insulators . Most substances fall into one of the two classes stated above *. When some char ge is transferr ed to a conductor , it r eadily gets distributed over the entir e sur face of the conductor . In contrast, if some charge is put on an insulator , it stays at the same place. Y ou will lear n why this happens in the next chapter . This property of the materials tells you why a nylon or plastic comb gets electrified on combing dry hair or on rubbing, but a metal article *There is a third category called semiconductors , which offer resistance to the movement of charges which is intermediate between the conductors and insulators. Rationalised 2023-24 4Physics like spoon does not. The charges on metal leak through our body to the ground as both are conductors of electricity. However , if a metal r od with a wooden or plastic handle is rubbed without touching its metal part, it shows signs of charging. 1.4 B ASIC PROPERTIES OF ELECTRIC CHARGE We have seen that ther e are two types of char ges, namely positive and negative and their effects tend to cancel each other . Here, we shall now describe some other pr operties of the electric charge. If the sizes of charged bodies are very small as compared to the distances between them, we treat them as point charges . All the charge content of the body is assumed to be concentrated at one point in space . 1.4.1 Additivity of charges We have not as yet given a quantitative definition of a charge; we shall follow it up in the next section. W e shall tentatively assume that this can be done and proceed. If a system contains two point charges q1 and q2, the total charge of the system is obtained simply by adding algebraically q1 and q2 , i.e., charges add up like real numbers or they are scalars like the mass of a body. If a system contains n charges q1, q2, q3, …, qn, then the total charge of the system is q1 + q2 + q3 + … + qn . Char ge has magnitude but no dir ection, similar to mass. However , there is one difference between mass and charge. Mass of a body is always positive whereas a charge can be either positive or negative. Proper signs have to be used while adding the charges in a system. For example, the total charge of a system containing five charges +1, +2, –3, +4 and –5, in some arbitrary unit, is (+1) + (+2) + (–3) + (+4) + (–5) = –1 i n the same unit. 1.4.2 Charge is conserved We have alr eady hinted to the fact that when bodies ar e char ged by rubbing, there is transfer of electrons from one body to the other; no new charges are either created or destroyed. A picture of particles of electric charge enables us to understand the idea of conservation of charge. When we rub two bodies, what one body gains in charge the other body loses. Within an isolated system consisting of many charged bodies, due to interactions among the bodies, charges may get redistributed but it is found that the total charge of the isolated system is always conserved . Conservation of charge has been established experimentally. It is not possible to create or destroy net charge carried by any isolated system although the charge carrying particles may be created or destroyedFIGURE 1.2 Electroscopes: (a) The gold leaf electroscope, (b) Schematics of a simple electroscope. Rationalised 2023-24 Electric Charges and Fields 5in a process. Sometimes nature creates charged particles: a neutron turns into a proton and an electron. The proton and electron thus create d have equal and opposite charges and the total charge is zero before and after the creation. 1.4.3 Quantisation of charge Experimentally it is established that all free charges are integral multiples of a basic unit of charge denoted by e. Thus charge q on a body is always given by q = ne wher e n is any integer , positive or negative. This basic unit of char ge is the charge that an electron or proton carries. By convention, the charge on an electron is taken to be negative; therefore charge on an electron is written as –e and that on a proton as +e. The fact that electric charge is always an integral multiple of e is termed as quanti sation of charge . There are a large number of situations in physics where certain physical quantities are quanti sed. The quanti sation of charge was first suggested by the experimental laws of electrolysis discovered by English experimentalist Faraday. It was experimentally demonstrated by Millikan in 1912. In the International System (SI) of Units, a unit of charge is called a coulomb and is denoted by the symbol C. A coulomb is defined in terms the unit of the electric current which you are going to learn in a subsequent chapter . In ter ms of this definition, one coulomb is the char ge flowing through a wire in 1 s if the current is 1 A (ampere), (see Chapter 1 of Class XI, Physics Textbook , Part I). In this system, the value of the basic unit of charge is e = 1.602192 × 10–19 C Thus, there are about 6 × 1018 electrons in a charge of –1C. In electrostatics, charges of this large magnitude are seldom encountered and hence we use smaller units 1 mC (micro coulomb) = 10–6 C or 1 mC (milli coulomb) = 10–3 C. If the protons and electrons are the only basic charges in the universe, all the observable charges have to be integral multiples of e. Thus, if a body contains n1 electrons and n2 protons, the total amount of charge on the body is n2 × e + n1 × (–e) = (n2 – n1) e. Since n1 and n2 are integers, their dif ference is also an integer . Thus the char ge on any body is always an integral multiple of e and can be increased or decreased also in steps of e. The step size e is, however , very small because at the macr oscopic level, we deal with charges of a few mC. At this scale the fact that charge of a body can increase or decrease in units of e is not visible. In this respect, the grainy nature of the charge is lost and it appears to be continuous. This situation can be compared with the geometrical concepts of points and lines. A dotted line viewed from a distance appears continuous to us but is not continuous in reality. As many points very close to Rationalised 2023-24 6Physics

EXAMPLE 1.2 EXAMPLE 1.1each other normally give an impression of a continuous line, many small charges taken together appear as a continuous charge distribution. At the macroscopic level, one deals with charges that are enormous compared to the magnitude of charge e . Since e = 1.6 × 10–19 C, a charge of magnituOde, say 1 mC, contains something like 1013 times the electronic charge. At this scale, the fact that charge can increase or decrease only in units of e is not very different from saying that charge can take continuous values. Thus, at the macroscopic level, the quantisation of charge has no practical consequence and can be ignor ed. However , at the micr oscopic level, where the charges involved are of the order of a few tens or hundreds of e, i.e., they can be counted, they appear in discrete lumps and quantisation of charge cannot be ignored. It is the magnitude of scale involved that is very important. Example 1.1 If 109 electrons move out of a body to another body every second, how much time is required to get a total charge of 1 C on the other body? Solution In one second 109 electrons move out of the body. Therefore the charge given out in one second is 1.6 × 10–19 × 109 C = 1.6 × 10–10 C. The time required to accumulate a charge of 1 C can then be estimated to be 1 C ÷ (1.6 × 10–10 C/s) = 6.25 × 109 s = 6.25 × 109 ÷ (365 × 24 × 3600) years = 198 years. Thus to collect a charge of one coulomb, from a body from which 109 electrons move out every second, we will need approximately 200 years. One coulomb is, therefore, a very large unit for many practical purposes. It is, however , also important to know what is r oughly the number of electrons contained in a piece of one cubic centimetre of a material. A cubic piece of copper of side 1 cm contains about 2.5 × 1024 electrons. Example 1.2 How much positive and negative charge is there in a cup of water? Solution Let us assume that the mass of one cup of water is 250 g. The molecular mass of water is 18g. Thus, one mole (= 6.02 × 1023 molecules) of water is 18 g. Therefore the number of molecules in one cup of water is (250/18) × 6.02 × 1023. Each molecule of water contains two hydrogen atoms and one oxygen atom, i.e., 10 electrons and 10 protons. Hence the total positive and total negative charge has the same magnitude. It is equal to (250/18) × 6.02 × 1023 × 10 × 1.6 × 10–19 C = 1.3 4 × 107 C. 1.5 C OULOMB 'S LAW Coulomb's law is a quantitative statement about the force between two point charges. When the linear size of charged bodies are much smaller than the distance separating them, the size may be ignored and the charged bodies are treated as point charges . Coulomb measured the force between two point charges and found that it varied inversely as the square of the distance between the charges and was directly proportional to the product of the magnitude of the two charges and Rationalised 2023-24 Electric Charges and Fields 7acted along the line joining the two charges . Thus, if two point charges q1, q2 are separated by a distance r in vacuum, the magnitude of the force (F) between them is given by 2 1 2q qF kr= (1.1) How did Coulomb arrive at this law from his experiments? Coulomb used a torsion balance * for measuring the force between two charged metallic spheres. When the separation between two spheres is much larger than the radius of each sphere, the charged spheres may be regarded as point charges. However , the char ges on the spher es wer e unknown, to begin with. How then could he discover a relation like Eq. (1.1)? Coulomb thought of the following simple way: Suppose the charge on a metallic sphere is q. If the sphere is put in contact with an identical uncharged sphere, the charge will spread over the two spheres. By symmetry, the charge on each sphere will be q/2*. Repeating this process, we can get charges q/2, q/4, etc. Coulomb varied the distance for a fixed pair of charges and measured the force for different separations. He then varied the charges in pairs, keeping the distance fixed for each pair . Comparing forces for different pairs of charges at different distances, Coulomb arrived at the relation, Eq. (1.1). Coulomb's law, a simple mathematical statement, was initially experimentally arrived at in the manner described above. While the original experiments established it at a macroscopic scale, it has also been established down to subatomic level (r ~ 10–10 m). Coulomb discovered his law without knowing the explicit magnitude of the charge. In fact, it is the other way round: Coulomb's law can now be employed to furnish a definition for a unit of charge. In the relation, Eq. (1.1), k is so far arbitrary. W e can choose any positive value of k. The choice of k determines the size of the unit of charge. In SI units, the value of k is about 9 × 109 2 2Nm C. The unit of charge that results from this choice is called a coulomb which we defined earlier in Section 1.4. Putting this value of k in Eq. (1.1), we see that for q1 = q2 = 1 C, r = 1 m F = 9 × 109 N That is, 1 C is the charge that when placed at a distance of 1 m from another charge of the same magnitude in vacuum experiences an electrical force of repulsion of magnitude *A torsion balance is a sensitive device to measure force. It was also used later by Cavendish to measure the very feeble gravitational force between two objects, to verify Newton's Law of Gravitation. *Implicit in this is the assumption of additivity of charges and conservation: two charges (q/2 each) add up to make a total charge q.Charles Augustin de Coulomb (1736 – 1806) Coulomb, a French physicist, began his career as a military engineer in the W est Indies. In 1776, he returned to Paris and retired to a small estate to do his scientific research. He invented a torsion balance to measure the quantity of a force and used it for determination of forces of electric attraction or repulsion between small charged spheres. He thus arrived in 1785 at the inverse square law relation, now known as Coulomb's law. The law had been anticipated by Priestley and also by Cavendish earlier , though Cavendish never published his results. Coulomb also found the inverse square law of force between unlike and like magnetic poles. CHARLES AUGUSTIN DE COULOMB (1736 –1806) Rationalised 2023-24 8Physics 9 × 109 N. One coulomb is evidently too big a unit to be used. In practice, in electrostatics, one uses smaller units like 1 mC or 1 mC. The constant k in Eq. (1.1) is usually put as k = 1/4 pe0 for later convenience, so that Coulomb's law is written as 01 2 21 4q qFrε=π(1.2) e0 is called the permittivity of fr ee space . The value of e0 in SI units is 0= 8.854 × 10–12 C2 N–1m–2 Since for ce is a vector , it is better to write Coulomb's law in the vector notation. Let the position vectors of charges q1 and q2 be r1 and r2 respectively [see Fig.1.3(a)]. W e denote for ce on q1 due to q2 by F12 and force on q2 due to q1 by F21. The two point charges q1 and q2 have been numbered 1 and 2 for convenience and the vector leading from 1 to 2 is denoted by r21: r21 = r2 – r1 In the same way, the vector leading from 2 to 1 is denoted by r12: r12 = r1 – r2 = – r21 The magnitude of the vectors r21 and r12 is denoted by r21 and r12, respectively (r12 = r21). The direction of a vector is specified by a unit vector along the vector . To denote the dir ection fr om 1 to 2 (or fr om 2 to 1), we define the unit vectors: ɵ 2121 21=rrr, ɵɵ ɵ 1212 21 12 12,= −rr r r r Coulomb's force law between two point charges q1 and q2 located at r1 and r2, respectively is then expressed as ɵ 1 22121 2 211 4ε=πF r oq q r(1.3) Some remarks on Eq. (1.3) are relevant: ·Equation (1.3) is valid for any sign of q1 and q2 whether positive or negative. If q1 and q2 are of the same sign (either both positive or both negative), F21 is along ˆr21, which denotes repulsion, as it should be for like charges. If q1 and q2 are of opposite signs, F21 is along –ɵr21(=ɵr12), which denotes attraction, as expected for unlike charges. Thus, we do not have to write separate equations for the cases of like and unlike charges. Equation (1.3) takes care of both cases correctly [Fig. 1. 3(b)].FIGURE 1.3 (a) Geometry and (b) Forces between charges. Rationalised 2023-24 Electric Charges and Fields 9 EXAMPLE 1.3Interactive animation on Coulomb's law: http://webphysics.davidson.edu/physlet_resources/bu_semester2/menu_semester2.html·The force F12 on charge q1 due to charge q2, is obtained from Eq. (1.3), by simply interchanging 1 and 2, i.e., 1 2 12 12 21 2 0 121ˆ 4ε= = −πF r Fq q r Thus, Coulomb's law agrees with the Newton's third law. ·Coulomb's law [Eq. (1.3)] gives the force between two charges q1 and q2 in vacuum. If the charges are placed in matter or the intervening space has matter , the situation gets complicated due to the pr esence of char ged constituents of matter . We shall consider electr ostatics in matter in the next chapter . Example 1.3 Coulomb's law for electrostatic force between two point charges and Newton's law for gravitational force between two stationary point masses, both have inverse-square dependence on the distance between the charges and masses respectively. (a) Compare the strength of these forces by determining the ratio of their magnitudes (i) for an electron and a proton and (ii) for two protons. (b) Estimate the accelerations of electron and proton due to the electrical force of their mutual attraction when they are 1 Å (= 10-10 m) apart? (mp = 1.67 × 10–27 kg, me = 9.11 × 10–31 kg) Solution (a)(i) The electric force between an electron and a proton at a distance r apart is: 2 2 01 4eeFrε= −π where the negative sign indicates that the force is attractive. The corresponding gravitational force (always attractive) is: 2p e Gm m F Gr= − where mp and me are the masses of a proton and an electron respectively. 2 39 02.4 104e G p eF e F Gm m ε= = ×π (ii) On similar lines, the ratio of the magnitudes of electric force to the gravitational force between two protons at a distance r apart is: F Fe Gm me G p p= =2 04πε 1.3 × 1036 However , it may be mentioned her e that the signs of the two for ces are different. For two protons, the gravitational force is attractive in nature and the Coulomb force is repulsive. The actual values of these forces between two protons inside a nucleus (distance between two protons is ~ 10-15 m inside a nucleus) are Fe ~ 230 N, whereas, FG ~ 1.9 × 10–34 N. The (dimensionless) ratio of the two forces shows that electrical forces are enormously stronger than the gravitational forces. Rationalised 2023-24 10Physics (b) The electric force F exerted by a proton on an electron is same in magnitude to the for ce exerted by an electr on on a pr oton; however , the masses of an electron and a proton are different. Thus, the magnitude of force is |F| = 1 402 2πεe r = 8.987 × 109 Nm2/C2 × (1.6 ×10–19C)2 / (10–10m)2 = 2.3 × 10–8 N Using Newton's second law of motion, F = ma, the acceleration that an electron will undergo is a = 2.3×10–8 N / 9.11 ×10–31 kg = 2.5 × 1022 m/s2 Comparing this with the value of acceleration due to gravity, we can conclude that the effect of gravitational field is negligible on the motion of electron and it undergoes very large accelerations under the action of Coulomb force due to a proton. The value for acceleration of the proton is 2.3 × 10–8 N / 1.67 × 10–27 kg = 1.4 × 1019 m/s2 EXAMPLE 1.3 FIGURE 1.4Example 1.4 A charged metallic sphere A is suspended by a nylon thread. Another charged metallic sphere B held by an insulating EXAMPLE 1.4 Rationalised 2023-24 Electric Charges and Fields 11 EXAMPLE 1.4handle is brought close to A such that the distance between their centres is 10 cm, as shown in Fig. 1.4(a). The resulting repulsion of A is noted (for example, by shining a beam of light and measuring the deflection of its shadow on a screen). Spheres A and B are touched by uncharged spheres C and D respectively, as shown in Fig. 1.4(b). C and D are then removed and B is brought closer to A to a distance of 5.0 cm between their centres, as shown in Fig. 1.4(c). What is the expected repulsion of A on the basis of Coulomb's law? Spheres A and C and spheres B and D have identical sizes. Ignore the sizes of A and B in comparison to the separation between their centres. Solution Let the original charge on sphere A be q and that on B be q¢. At a distance r between their centres, the magnitude of the electrostatic force on each is given by Fqq r=′ 1 402πε neglecting the sizes of spher es A and B in comparison to r. When an identical but uncharged sphere C touches A, the charges redistribute on A and C and, by symmetry, each sphere carries a charge q/2. Similarly, after D touches B, the redistributed charge on each is q¢/2. Now, if the separation between A and B is halved, the magnitude of the electrostatic force on each is ′=′=′= Fq q rqq rF1 42 2 21 402 02π π ε ε(/)(/) (/)( ) Thus the electrostatic force on A, due to B, remains unaltered. 1.6 F ORCES BETWEEN MULTIPLE C HARGES The mutual electric force between two charges is given by Coulomb's law. How to calculate the force on a charge where there are not one but several charges around? Consider a system of n stationary charges q1, q2, q3, ..., qn in vacuum. What is the force on q1 due to q2, q3, ..., qn? Coulomb's law is not enough to answer this question. Recall that forces of mechanical origin add according to the parallelogram law of addition. Is the same true for forces of electrostatic origin? Experimentally, it is verified that force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges . This is termed as the principle of superposition . To better understand the concept, consider a system of three charges q1, q2 and q3, as shown in Fig. 1.5(a) . The force on one charge, say q1, due to two other charges q2, q3 can therefore be obtained by performing a vector addition of the forces due to each one of these charges. Thus, if the force on q1 due to q2 is denoted by F12, F12 is given by Eq. (1.3) even though other charges are present. Thus, F12 =1 401 2 122 12πεq q rˆrFIGURE 1.5 A system of (a) three charges (b) multiple charges. Rationalised 2023-24 12Physics EXAMPLE 1.5In the same way, the force on q1 due to q3, denoted by F13, is given by F r13 01 3 132 131 4=πεq q rˆ which again is the Coulomb force on q1 due to q3, even though other charge q2 is present. Thus the total force F1 on q1 due to the two charges q2 and q3 is given as F F F r r1 12 13 01 2 122 12 01 3 1 32 131 41 4= + = +π π ε εq q rq q rˆ ˆ(1.4) The above calculation of force can be generalised to a system of charges more than three, as shown in Fig. 1.5(b). The principle of superposition says that in a system of charges q1, q2, ..., qn, the force on q1 due to q2 is the same as given by Coulomb's law, i.e., it is unaffected by the presence of the other charges q3, q4, ..., qn. The total force F1 on the charge q1, due to all other charges, is then given by the vector sum of the forces F12, F13, ..., F1n: i.e., F F F F r r1 12 13 1n = + + ...+ = +1 401 2 122 121 3 1 32 13πεq q rq q rˆ ˆ + + +   ... ˆq q rn nn1 12 1r = =∑q q ri i in i1 0 12 214πεˆr(1.5) The vector sum is obtained as usual by the parallelogram law of addition of vectors. All of electrostatics is basically a consequence of Coulomb's law and the superposition principle. Example 1.5 Consider three charges q1, q2, q3 each equal to q at the vertices of an equilateral triangle of side l. What is the force on a charge Q (with the same sign as q) placed at the centroid of the triangle, as shown in Fig. 1.6? FIGURE 1.6 Solution In the given equilateral triangle ABC of sides of length l, if we draw a perpendicular AD to the side BC, AD = AC cos 30º = (3 /2) l and the distance AO of the centroid O from A is (2/3) AD = (1/ 3) l. By symmatry AO = BO = CO. Rationalised 2023-24 Electric Charges and Fields 13 EXAMPLE 1.5Thus, Force F1 on Q due to charge q at A = 3 402πεQq l along AO Force F2 on Q due to charge q at B = 3 402πεQq l along BO Force F3 on Q due to charge q at C = 3 402πεQq l along CO The resultant of forces F2 and F3 is 3 402πεQq l along OA, by the parallelogram law. Therefore, the total force on Q = 3 402πεQq lˆ ˆr r−( ) = 0, where ˆris the unit vector along OA. It is clear also by symmetry that the three forces will sum to zero. Suppose that the resultant force was non-zero but in some direction. Consider what would happen if the system was rotated through 60° about O. Example 1.6 Consider the charges q, q, and – q placed at the vertices of an equilateral triangle, as shown in Fig. 1.7. What is the force on each charge? FIGURE 1.7 Solution The forces acting on charge q at A due to charges q at B and –q at C are F12 along BA and F13 along AC respectively, as shown in Fig. 1.7. By the parallelogram law, the total force F1 on the charge q at A is given by F1 = F 1ˆr where 1ˆr is a unit vector along BC. The force of attraction or repulsion for each pair of charges has the same magnitude Fq=2 024πεl The total force F2 on charge q at B is thus F2 = F ˆr2, where ˆr2 is a unit vector along AC. EXAMPLE 1.6 Rationalised 2023-24 14Physics EXAMPLE 1.6Similarly the total force on charge – q at C is F3 = 3 F ˆn, where ˆnis the unit vector along the direction bisecting the ÐBCA. It is interesting to see that the sum of the forces on the three charges is zero, i.e., F1 + F2 + F3 = 0 The result is not at all surprising. It follows straight from the fact that Coulomb's law is consistent with Newton's third law. The proof is left to you as an exercise. 1.7 E LECTRIC FIELD Let us consider a point charge Q placed in vacuum, at the origin O. If we place another point char ge q at a point P , wher e OP = r, then the char ge Q will exert a for ce on q as per Coulomb's law. W e may ask the question: If charge q is removed, then what is left in the surrounding? Is there nothing? If ther e is nothing at the point P , then how does a for ce act when we place the char ge q at P. In or der to answer such questions, the early scientists introduced the concept of field. According to this, we say that the charge Q produces an electric field everywhere in the surrounding. When another char ge q is brought at some point P , the field ther e acts on it and produces a force. The electric field produced by the charge Q at a point r is given as E r r r ()= =1 41 402 02π πε εQ rQ rˆ ˆ(1.6) where ˆ=r r/r, is a unit vector fr om the origin to the point r. Thus, Eq.(1.6) specifies the value of the electric field for each value of the position vector r. The wor d "field" signifies how some distributed quantity (which could be a scalar or a vector) varies with position. The effect of the charge has been incorporated in the existence of the electric field. W e obtain the force F exerted by a charge Q on a charge q, as F r =1 402πεQq rˆ(1.7) Note that the charge q also exerts an equal and opposite force on the charge Q. The electrostatic force between the charges Q and q can be looked upon as an interaction between charge q and the electric field of Q and vice versa. If we denote the position of char ge q by the vector r, it experiences a force F equal to the charge q multiplied by the electric field E at the location of q. Thus, F(r) = q E(r) (1.8) Equation (1.8) defines the SI unit of electric field as N/C *. Some important remarks may be made here: (i)From Eq. (1.8), we can infer that if q is unity, the electric field due to a charge Q is numerically equal to the force exerted by it. Thus, the electric field due to a charge Q at a point in space may be defined as the force that a unit positive charge would experience if placed *An alter nate unit V/m will be intr oduced in the next chapter .FIGURE 1.8 Electric field (a) due to a charge Q, (b) due to a charge –Q. Rationalised 2023-24 Electric Charges and Fields 15at that point . The charge Q, which is producing the electric field, is called a source charge and the charge q, which tests the effect of a source charge, is called a test charge . Note that the source charge Q must r emain at its original location. However , if a char ge q is brought at any point around Q, Q itself is bound to experience an electrical force due to q and will tend to move. A way out of this difficulty is to make q negligibly small. The force F is then negligibly small but the ratio F/q is finite and defines the electric field: EF=   →lim q q0(1.9) A practical way to get around the problem (of keeping Q undisturbed in the presence of q) is to hold Q to its location by unspecified forces! This may look strange but actually this is what happens in practice. When we are considering the electric force on a test charge q due to a charged planar sheet (Section 1.14), the charges on the sheet are held to their locations by the forces due to the unspecified charged constituents inside the sheet. (ii)Note that the electric field E due to Q, though defined operationally in terms of some test charge q, is independent of q. This is because F is proportional to q, so the ratio F/q does not depend on q. The force F on the charge q due to the charge Q depends on the particular location of charge q which may take any value in the space around the charge Q. Thus, the electric field E due to Q is also dependent on the space coor dinate r. For dif ferent positions of the char ge q all over the space , we get different values of electric field E. The field exists at every point in three-dimensional space. (iii)For a positive charge, the electric field will be directed radially outwards from the charge. On the other hand, if the source charge is negative, the electric field vector , at each point, points radially inwar ds. (iv)Since the magnitude of the force F on charge q due to charge Q depends only on the distance r of the charge q from charge Q, the magnitude of the electric field E will also depend only on the distance r. Thus at equal distances fr om the char ge Q, the magnitude of its electric field E is same. The magnitude of electric field E due to a point charge is thus same on a sphere with the point charge at its centre; in other words, it has a spherical symmetry. 1.7.1 Electric field due to a system of charges Consider a system of charges q1, q2, ..., qn with position vectors r1, r2, ..., rn relative to some origin O. Like the electric field at a point in space due to a single charge, electric field at a point in space due to the system of charges is defined to be the force experienced by a unit test charge placed at that point, without disturbing the original positions of char ges q1, q2, ..., qn. We can use Coulomb's law and the superposition principle to determine this field at a point P denoted by position vector r. Rationalised 2023-24 16Physics Electric field E1 at r due to q1 at r1 is given by E1 = 1 401 12πεq rP1P ˆr wher e 1Pˆr is a unit vector in the dir ection fr om q1 to P, and r1P is the distance between q1 and P . In the same manner , electric field E2 at r due to q2 at r2 is E2 = 1 402 22πεq rP2Pˆr where 2Pˆr is a unit vector in the direction from q2 to P and r2P is the distance between q2 and P . Similar expressions hold good for fields E3, E4, ..., En due to charges q3, q4, ..., qn. By the superposition principle, the electric field E at r due to the system of charges is (as shown in Fig. 1.9) E(r) = E1 (r) + E2 (r) + … + En(r) =1 41 41 401 121 02 222 02π π π ε ε εq rq rq rn nn PP PP PPˆ ˆ... ˆ r r r + + + E(r) = =∑ 1 40 Pi Pπεq ri i in 2 1ˆr (1.10) E is a vector quantity that varies from one point to another point in space and is determined from the positions of the source charges. 1.7.2 Physical significance of electric field You may wonder why the notion of electric field has been intr oduced here at all. After all, for any system of charges, the measurable quantity is the force on a charge which can be directly determined using Coulomb's law and the superposition principle [Eq. (1.5)]. Why then introduce this intermediate quantity called the electric field? For electrostatics, the concept of electric field is convenient, but not really necessary. Electric field is an elegant way of characterising the electrical environment of a system of charges. Electric field at a point in the space around a system of charges tells you the force a unit positive test charge would experience if placed at that point (without disturbing the system). Electric field is a characteristic of the system of charges and is independent of the test charge that you place at a point to determine the field. The term field in physics generally refers to a quantity that is defined at every point in space and may vary from point to point. Electric field is a vector field, since force is a vector quantity. The true physical significance of the concept of electric field, however , emerges only when we go beyond electrostatics and deal with time- dependent electromagnetic phenomena. Suppose we consider the force between two distant charges q1, q2 in accelerated motion. Now the greatest speed with which a signal or information can go from one point to another is c, the speed of light. Thus, the effect of any motion of q1 on q2 cannotFIGURE 1.9 Electric field at a point due to a system of charges is the vector sum of the electric fields at the point due to individual charges. Rationalised 2023-24 Electric Charges and Fields 17arise instantaneously. There will be some time delay between the effect (force on q2) and the cause (motion of q1). It is precisely here that the notion of electric field (strictly, electromagnetic field) is natural and very useful. The field picture is this: the accelerated motion of charge q1 produces electromagnetic waves, which then propagate with the speed c, reach q2 and cause a force on q2. The notion of field elegantly accounts for the time delay. Thus, even though electric and magnetic fields can be detected only by their effects (forces) on charges, they are regarded as physical entities, not merely mathematical constructs. They have an independent dynamics of their own, i.e., they evolve according to laws of their own. They can also transport energy. Thus, a source of time-dependent electromagnetic fields, turned on for a short interval of time and then switched off, leaves behind propagating electromagnetic fields transporting energy. The concept of field was first introduced by Faraday and is now among the central concepts in physics. Example 1.7 An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude 2.0 × 104 N C–1 [Fig. 1.10(a)]. The direction of the field is reversed keeping its magnitude unchanged and a proton falls through the same distance [Fig. 1.10(b)]. Compute the time of fall in each case. Contrast the situation with that of 'free fall under gravity'. FIGURE 1.10 Solution In Fig. 1.10(a) the field is upward, so the negatively charged electron experiences a downward force of magnitude eE where E is the magnitude of the electric field. The acceleration of the electron is ae = eE/me where me is the mass of the electron. Starting from rest, the time required by the electron to fall through a distance h is given by 2 2 ee eh m hta e E= = For e = 1.6 × 10–19C, me = 9.11 × 10–31 kg, E = 2.0 × 104 N C–1, h = 1.5 × 10–2 m, te = 2.9 × 10–9s In Fig. 1.10 (b), the field is downward, and the positively charged proton experiences a downward force of magnitude eE. The acceleration of the proton is ap = eE/mp where mp is the mass of the proton; mp = 1.67 × 10–27 kg. The time of fall for the proton is EXAMPLE 1.7 Rationalised 2023-24 18Physics EXAMPLE 1.8 EXAMPLE 1.7–72 21 3 10 sp p ph m ht .a e E= == × Thus, the heavier particle (proton) takes a greater time to fall through the same distance. This is in basic contrast to the situation of 'free fall under gravity' where the time of fall is independent of the mass of the body. Note that in this example we have ignored the acceleration due to gravity in calculating the time of fall. To see if this is justified, let us calculate the acceleration of the proton in the given electric field: p pe Eam= 19 4 1 27(1 6 10 C) (2 0 10 N C) 1 67 10 kg. . .− − −× × ×=× 12 –21 9 10 m s.= × which is enormous compared to the value of g (9.8 m s–2), the acceleration due to gravity. The acceleration of the electron is even greater . Thus, the ef fect of acceleration due to gravity can be ignor ed in this example. Example 1.8 Two point charges q1 and q2, of magnitude +10–8 C and –10–8 C, respectively, are placed 0.1 m apart. Calculate the electric fields at points A, B and C shown in Fig. 1.11. FIGURE 1.11 Solution The electric field vector E1A at A due to the positive charge q1 points towards the right and has a magnitude 9 2 -2 8 1A 2(9 10 Nm C) (10 C) (0.05m)E−× ×= = 3.6 × 104 N C–1 The electric field vector E2A at A due to the negative charge q2 points towards the right and has the same magnitude. Hence the magnitude of the total electric field EA at A is EA = E1A + E2A = 7.2 × 104 N C–1 EA is directed toward the right. Rationalised 2023-24 Electric Charges and Fields 19The electric field vector E1B at B due to the positive charge q1 points towards the left and has a magnitude 9 2 –2 8 1B 2(9 10 Nm C) (10 C) (0.05 m)E−× ×= = 3.6 × 104 N C–1 The electric field vector E2B at B due to the negative charge q2 points towards the right and has a magnitude 9 2 –2 8 2B 2(9 10 Nm C) (10 C) (0.15 m)E−× ×= = 4 × 103 N C–1 The magnitude of the total electric field at B is EB = E1B – E2B = 3.2 × 104 N C–1 EB is directed towards the left. The magnitude of each electric field vector at point C, due to charge q1 and q2 is 9 2 –2 8 1C 2C 2(9 10 Nm C) (10 C) (0.10 m)E E−× ×= = = 9 × 103 N C–1 The directions in which these two vectors point are indicated in Fig. 1.11. The resultant of these two vectors is 1 2 cos cos3 3π π= +C c c E E E = 9 × 103 N C–1 EC points towards the right. 1.8 E LECTRIC F IELD LINES We have studied electric field in the last section. It is a vector quantity and can be represented as we represent vectors. Let us try to represent E due to a point charge pictorially. Let the point charge be placed at the origin. Draw vectors pointing along the direction of the electric field with their lengths proportional to the strength of the field at each point. Since the magnitude of electric field at a point decreases inversely as the square of the distance of that point from the charge, the vector gets shorter as one goes away from the origin, always pointing radially outward. Figure 1.12 shows such a picture. In this figure, each arrow indicates the electric field, i.e., the force acting on a unit positive charge, placed at the tail of that arrow. Connect the arrows pointing in one direction and the r esulting figur e represents a field line. W e thus get many field lines, all pointing outwards from the point charge. Have we lost the information about the strength or magnitude of the field now, because it was contained in the length of the arrow? No. Now the magnitude of the field is represented by the density of field lines. E is strong near the charge, so the density of field lines is more near the char ge and the lines ar e closer . Away fr om the char ge, the field gets weaker and the density of field lines is less, resulting in well-separated lines. Another person may draw more lines. But the number of lines is not important. In fact, an infinite number of lines can be drawn in any region.FIGURE 1.12 Field of a point charge. EXAMPLE 1.8 Rationalised 2023-24 20Physics It is the relative density of lines in different regions which is important. We draw the figur e on the plane of paper , i.e., in two- dimensions but we live in three-dimensions. So if one wishes to estimate the density of field lines, one has to consider the number of lines per unit cross-sectional area, perpendicular to the lines. Since the electric field decreases as the square of the distance from a point charge and the area enclosing the charge increases as the square of the distance, the number of field lines crossing the enclosing area remains constant, whatever may be the distance of the area from the charge. We started by saying that the field lines carry infor mation about the direction of electric field at different points in space. Having drawn a certain set of field lines, the relative density (i.e., closeness) of the field lines at different points indicates the relative strength of electric field at those points. The field lines crowd where the field is strong and are spaced apart wher e it is weak. Figur e 1.13 shows a set of field lines. W e can imagine two equal and small elements of area placed at points R and S normal to the field lines there. The number of field lines in our picture cutting the area elements is proportional to the magnitude of field at these points. The picture shows that the field at R is stronger than at S. To understand the dependence of the field lines on the area, or rather the solid angle subtended by an area element, let us try to relate the area with the solid angle, a generalisation of angle to three dimensions. Recall how a (plane) angle is defined in two-dimensions. Let a small transverse line element Dl be placed at a distance r from a point O. Then the angle subtended by D l at O can be appr oximated as Dq = Dl/r. Likewise, in three-dimensions the solid angle * subtended by a small perpendicular plane ar ea DS, at a distance r, can be written as DW = DS/r2. We know that in a given solid angle the number of radial field lines is the same. In Fig. 1.13, for two points P1 and P2 at distances r1 and r2 from the charge, the element of area subtending the solid angle DW is 2 1rDW at P1 and an element of area 2 2rDW at P2, respectively. The number of lines (say n) cutting these area elements are the same. The number of field lines, cutting unit area element is therefore n/(2 1rDW) at P1 and n/(2 2rDW) at P2, respectively. Since n and DW are common, the strength of the field clearly has a 1/ r2 dependence. The picture of field lines was invented by Faraday to develop an intuitive non-mathematical way of visualising electric fields around charged configurations. Faraday called them lines of for ce. This ter m is somewhat misleading, especially in case of magnetic fields. The more appr opriate ter m is field lines (electric or magnetic) that we have adopted in this book. Electric field lines are thus a way of pictorially mapping the electric field around a configuration of charges. An electric field line is, in general,FIGURE 1.13 Dependence of electric field strength on the distance and its relation to the number of field lines. *Solid angle is a measure of a cone. Consider the intersection of the given cone with a sphere of radius R. The solid angle DW of the cone is defined to be equal to DS/R2, where DS is the area on the sphere cut out by the cone. Rationalised 2023-24 Electric Charges and Fields 21a curve drawn in such a way that the tangent to it at each point is in the direction of the net field at that point. An arrow on the curve is obviously necessary to specify the direction of electric field from the two possible directions indicated by a tangent to the curve. A field line is a space curve, i.e., a curve in three dimensions. Figure 1.14 shows the field lines around some simple charge configurations. As mentioned earlier , the field lines are in 3-dimensional space, though the figure shows them only in a plane. The field lines of a single positive charge are radially outward while those of a single negative charge are radially inward. The field lines around a system of two positive charges (q, q) give a vivid pictorial description of their mutual repulsion, while those around the configuration of two equal and opposite charges (q, –q), a dipole, show clearly the mutual attraction between the charges. The field lines follow some important general properties: (i)Field lines start from positive charges and end at negative charges. If there is a single charge, they may start or end at infinity. (ii)In a charge-free region, electric field lines can be taken to be continuous curves without any breaks. (iii)Two field lines can never cr oss each other . (If they did, the field at the point of intersection will not have a unique direction, which is absurd.) (iv)Electrostatic field lines do not form any closed loops. This follows from the conservative nature of electric field (Chapter 2). 1.9 E LECTRIC FLUX Consider flow of a liquid with velocity v, through a small flat surface d S, in a direction normal to the surface. The rate of flow of liquid is given by the volume crossing the area per unit time v dS and represents the flux of liquid flowing across the plane. If the normal to the surface is not parallel to the direction of flow of liquid, i.e., to v , but makes an angle q with it, the projected area in a plane perpendicular to v is δdS cos q. Therefore, the flux going out of the surface d S is v.ˆndS. For the case of the electric field, we define an analogous quantity and call it electric flux. We should, however , note that ther e is no flow of a physically observable quantity unlike the case of liquid flow. In the picture of electric field lines described above, we saw that the number of field lines crossing a unit area, placed normal to the field at a point is a measure of the strength of electric field at that point. This means that ifFIGURE 1.14 Field lines due to some simple charge configurations. Rationalised 2023-24 22Physics we place a small planar element of area DS normal to E at a point, the number of field lines crossing it is proportional * to E DS. Now suppose we tilt the area element by angle q. Clearly, the number of field lines crossing the area element will be smaller . The pr ojection of the area element normal to E is DS cos q. Thus, the number of field lines crossing DS is proportional to E DS cos q. When q = 90°, field lines will be parallel to DS and will not cross it at all (Fig. 1.15). The orientation of area element and not merely its magnitude is important in many contexts. For example, in a stream, the amount of water flowing through a ring will naturally depend on how you hold the ring. If you hold it normal to the flow, maximum water will flow through it than if you hold it with some other orientation. This shows that an area element should be tr eated as a vector . It has a magnitude and also a direction. How to specify the direction of a planar area? Clearly, the normal to the plane specifies the orientation of the plane. Thus the direction of a planar area vector is along its normal. How to associate a vector to the ar ea of a curved sur face? W e imagine dividing the surface into a large number of very small area elements. Each small area element may be treated as planar and a vector associated with it, as explained before. Notice one ambiguity here. The direction of an area element is along its normal. But a normal can point in two directions. Which direction do we choose as the direction of the vector associated with the area element? This problem is resolved by some convention appropriate to the given context. For the case of a closed surface, this convention is very simple. The vector associated with every area element of a closed surface is taken to be in the direction of the outward normal. This is the convention used in Fig. 1.16. Thus, the area element vector DS at a point on a closed surface equals DS ˆn where D S is the magnitude of the area element and ˆn is a unit vector in the direction of outward normal at that point. We now come to the definition of electric flux. Electric flux Df through an area element DS is defined by Df = E.DS = E DS cos q (1.11) which, as seen before, is proportional to the number of field lines cutting the area element. The angle q here is the angle between E and DS. For a closed surface, with the convention stated already, q is the angle between E and the outward normal to the area element. Notice we could look at the expression E DS cos q in two ways: E (DS cos q) i.e., E times theFIGURE 1.15 Dependence of flux on the inclination q between E and ˆn. FIGURE 1.16 Convention for defining normal ˆn and DS. *It will not be proper to say that the number of field lines is equal to E DS. The number of field lines is after all, a matter of how many field lines we choose to draw. What is physically significant is the relative number of field lines crossing a given area at different points. Rationalised 2023-24 Electric Charges and Fields 23projection of area normal to E, or E^ DS, i.e., component of E along the normal to the area element times the magnitude of the area element. The unit of electric flux is N C–1 m2. The basic definition of electric flux given by Eq. (1.11) can be used, in principle, to calculate the total flux through any given surface. All we have to do is to divide the surface into small area elements, calculate the flux at each element and add them up. Thus, the total flux f through a surface S is f ~ S E.DS (1.12) The approximation sign is put because the electric field E is taken to be constant over the small area element. This is mathematically exact only when you take the limit DS ® 0 and the sum in Eq. (1.12) is written as an integral. 1.10 E LECTRIC DIPOLE An electric dipole is a pair of equal and opposite point charges q and – q, separated by a distance 2 a. The line connecting the two charges defines a direction in space. By convention, the direction from – q to q is said to be the direction of the dipole. The mid-point of locations of – q and q is called the centre of the dipole. The total charge of the electric dipole is obviously zero. This does not mean that the field of the electric dipole is zero. Since the charge q and –q are separated by some distance, the electric fields due to them, when added, do not exactly cancel out. However , at distances much lar ger than the separation of the two charges forming a dipole (r >> 2 a), the fields due to q and – q nearly cancel out. The electric field due to a dipole therefore falls off, at large distance, faster than like 1/ r2 (the dependence on r of the field due to a single charge q). These qualitative ideas are borne out by the explicit calculation as follows: 1.10.1 The field of an electric dipole The electric field of the pair of charges (– q and q) at any point in space can be found out from Coulomb's law and the superposition principle. The results are simple for the following two cases: (i) when the point is on the dipole axis, and (ii) when it is in the equatorial plane of the dipole, i.e., on a plane perpendicular to the dipole axis through its centre. The electric field at any general point P is obtained by adding the electric fields E–q due to the charge – q and E+q due to the charge q, by the parallelogram law of vectors. (i) For points on the axis Let the point P be at distance r from the centre of the dipole on the side of the charge q, as shown in Fig. 1.17(a). Then E p−= − +qq r a402πε( )□[1.13(a)] where ˆp is the unit vector along the dipole axis (from – q to q). Also E p+=−qq r a 402πε( )□[1.13(b)] Rationalised 2023-24 24Physics The total field at P is E E E p = + = −− +□ □□□ □□ + −q qq r a r a 41 1 02 2πε( ) ( )□ =−q a r r ao44 2 2 2πε( )□p (1.14) For r >> a E p=4 403q a rπεˆ (r >> a) (1.15) (ii) For points on the equatorial plane The magnitudes of the electric fields due to the two charges + q and – q are given by Eq r aq+=+ 41 02 2πε[1.16(a)] Eq r aq–=+ 41 02 2πε[1.16(b)] and are equal. The directions of E+q and E–q are as shown in Fig. 1.17(b). Clearly, the components normal to the dipole axis cancel away. The components along the dipole axis add up. The total electric field is opposite to ˆp. We have E = – (E +q + E –q) cos q ˆp = −+2 42 2 3 2q a r aoπε( )□/p (1.17) At large distances (r >> a), this reduces to E p = − >>2 43q a rr a oπεˆ ( ) (1.18) From Eqs. (1.15) and (1.18), it is clear that the dipole field at large distances does not involve q and a separately; it depends on the product qa. This suggests the definition of dipole moment. The dipole moment vector p of an electric dipole is defined by p = q × 2a ˆp (1.19) that is, it is a vector whose magnitude is charge q times the separation 2a (between the pair of charges q, –q) and the direction is along the line from – q to q. In terms of p, the electric field of a dipole at large distances takes simple forms: At a point on the dipole axis Ep=2 43πεor(r >> a) (1.20) At a point on the equatorial plane Ep= −43πεor(r >> a) (1.21)FIGURE 1.17 Electric field of a dipole at (a) a point on the axis, (b) a point on the equatorial plane of the dipole. p is the dipole moment vector of magnitude p = q × 2a and directed from – q to q. Rationalised 2023-24 Electric Charges and Fields 25 EXAMPLE 1.9 Notice the important point that the dipole field at large distances falls of f not as 1/ r2 but as1/ r3. Further , the magnitude and the dir ection of the dipole field depends not only on the distance r but also on the angle between the position vector r and the dipole moment p. We can think of the limit when the dipole size 2 a appr oaches zer o, the charge q approaches infinity in such a way that the product p = q × 2a is finite. Such a dipole is referred to as a point dipole . For a point dipole, Eqs. (1.20) and (1.21) ar e exact, true for any r. 1.10.2 Physical significance of dipoles In most molecules, the centres of positive charges and of negative charges * lie at the same place. Therefore, their dipole moment is zero. CO2 and CH4 are of this type of molecules. However , they develop a dipole moment when an electric field is applied. But in some molecules, the centres of negative charges and of positive charges do not coincide. Therefore they have a permanent electric dipole moment, even in the absence of an electric field. Such molecules are called polar molecules. W ater molecules, H2O, is an example of this type. V arious materials give rise to inter esting properties and important applications in the presence or absence of electric field. Example 1.9 Two charges ±10 mC are placed 5.0 mm apart. Determine the electric field at (a) a point P on the axis of the dipole 15 cm away from its centre O on the side of the positive charge, as shown in Fig. 1.18(a), and (b) a point Q, 15 cm away from O on a line passing through O and normal to the axis of the dipole, as shown in Fig. 1.18(b). FIGURE 1.18 *Centre of a collection of positive point charges is defined much the same way as the centre of mass: rr cm=∑ ∑q qi ii ii. Rationalised 2023-24 26Physics EXAMPLE 1.9Solution (a) Field at P due to charge +10 mC = 5 12 2 1 210 C 4 (8.854 10 C N m)− − − −π ×2 4 21 (15 0.25) 10 m−× − × = 4.13 × 106 N C–1 along BP Field at P due to charge –10 mC –5 12 2 1 210 C 4 (8.854 10 C N m)− − −=π × 2 4 21 (15 0.25) 10 m−×+ × = 3.86 × 106 N C–1 along PA The resultant electric field at P due to the two charges at A and B is = 2.7 × 105 N C–1 along BP . In this example, the ratio OP/OB is quite large (= 60). Thus, we can expect to get approximately the same result as above by directly using the for mula for electric field at a far -away point on the axis of a dipole. For a dipole consisting of charges ± q, 2a distance apart, the electric field at a distance r from the centre on the axis of the dipole has a magnitude Ep r=2 403πε(r/a >> 1) where p = 2a q is the magnitude of the dipole moment. The direction of electric field on the dipole axis is always along the direction of the dipole moment vector (i.e., from – q to q). Here, p =10–5 C × 5 × 10–3 m = 5 × 10–8 C m Therefore, E =8 12 2 1 22 5 10 Cm 4 (8.854 10 C N m)− − − −× × π ×3 6 31 (15) 10 m−× × = 2.6 × 105 N C–1 along the dipole moment direction AB, which is close to the result obtained earlier . (b) Field at Q due to charge + 10 mC at B =5 12 2 1 210 C 4 (8.854 10 C N m)− − − −π ×2 2 4 21 [15 (0.25)] 10 m−+ ×× = 3.99 × 106 N C–1 along BQ Field at Q due to charge –10 mC at A =5 12 2 1 210 C 4 (8.854 10 C N m)− − − −π ×2 2 4 21 [15 (0.25)] 10 m−+ ×× = 3.99 × 106 N C–1 along QA. Clearly, the components of these two forces with equal magnitudes cancel along the direction OQ but add up along the direction parallel to BA. Therefore, the resultant electric field at Q due to the two charges at A and B is = 2 × 6 –1 2 20.253.99 10 N C 15 (0.25)× × +along BA = 1.33 × 105 N C–1 along BA. As in (a), we can expect to get approximately the same result by directly using the formula for dipole field at a point on the normal to the axis of the dipole: Rationalised 2023-24 Electric Charges and Fields 27 EXAMPLE 1.9Ep r=43π0ε(r/a >> 1) 8 12 2 –1 –25 10 Cm 4 (8.854 10 C N m)− −×=π ×3 6 31 (15) 10 m−×× = 1.33 × 105 N C–1. The direction of electric field in this case is opposite to the direction of the dipole moment vector . Again, the r esult agr ees with that obtained before. 1.11 D IPOLE IN A UNIFORM EXTERNAL FIELD Consider a permanent dipole of dipole moment p in a uniform external field E, as shown in Fig. 1.19. (By permanent dipole, we mean that p exists irrespective of E; it has not been induced by E.) There is a force qE on q and a force – qE on – q. The net force on the dipole is zer o, since E is unifor m. However , the char ges ar e separated, so the forces act at different points, resulting in a torque on the dipole. When the net force is zero, the torque (couple) is independent of the origin. Its magnitude equals the magnitude of each force multiplied by the arm of the couple (perpendicular distance between the two antiparallel forces). Magnitude of torque = q E × 2 a sinq = 2 q a E sinq Its dir ection is nor mal to the plane of the paper , coming out of it. The magnitude of p × E is also p E sinq and its direction is nor mal to the paper , coming out of it. Thus, t tt tt = p × E (1.22) This torque will tend to align the dipole with the field E. When p is aligned with E, the torque is zero. What happens if the field is not uniform? In that case, the net force will evidently be non-zero. In addition there will, in general, be a torque on the system as before. The general case is involved, so let us consider the simpler situations when p is parallel to E or antiparallel to E. In either case, the net torque is zero, but there is a net force on the dipole if E is not uniform. Figure 1.20 is self-explanatory. It is easily seen that when p is parallel to E, the dipole has a net force in the direction of increasing field. When p is antiparallel to E, the net force on the dipole is in the direction of decreasing field. In general, the force depends on the orientation of p with respect to E. This brings us to a common observation in frictional electricity. A comb run through dry hair attracts pieces of paper . The comb, as we know, acquir es char ge thr ough friction. But the paper is not charged. What then explains the attractive force? Taking the clue from the precedingFIGURE 1.19 Dipole in a uniform electric field. FIGURE 1.20 Electric force on a dipole: (a) E parallel to p, (b) E antiparallel to p. Rationalised 2023-24 28Physics discussion, the char ged comb 'polarises' the piece of paper , i.e., induces a net dipole moment in the dir ection of field. Further , the electric field due to the comb is not uniform. This non-uniformity of the field makes a dipole to experience a net force on it. In this situation, it is easily seen that the paper should move in the direction of the comb! 1.12 C ONTINUOUS CHARGE DISTRIBUTION We have so far dealt with char ge configurations involving discr ete char ges q1, q2, ..., qn. One reason why we restricted to discrete charges is that the mathematical treatment is simpler and does not involve calculus. For many purposes, however , it is impractical to work in ter ms of discr ete charges and we need to work with continuous charge distributions. For example, on the sur face of a char ged conductor , it is impractical to specify the charge distribution in terms of the locations of the microscopic charged constituents. It is more feasible to consider an area element DS (Fig. 1.21) on the surface of the conductor (which is very small on the macroscopic scale but big enough to include a very large number of electrons) and specify the char ge DQ on that element. W e then define a surface char ge density s at the area element by Q SσΔ=Δ(1.23) We can do this at dif ferent points on the conductor and thus arrive at a continuous function s, called the surface charge density. The surface charge density s so defined ignores the quantisation of charge and the discontinuity in charge distribution at the microscopic level *. s represents macroscopic surface charge density, which in a sense, is a smoothed out average of the microscopic charge density over an area element DS which, as said before, is large microscopically but small macroscopically. The units for s are C/m2. Similar considerations apply for a line charge distribution and a volume charge distribution. The linear charge density l of a wire is defined by Q lλΔ=Δ(1.24) where Dl is a small line element of wire on the macroscopic scale that, however , includes a lar ge number of micr oscopic char ged constituents, and DQ is the charge contained in that line element. The units for l are C/m. The volume charge density (sometimes simply called charge density) is defined in a similar manner: Q VρΔ=Δ(1.25) where DQ is the charge included in the macroscopically small volume element DV that includes a large number of microscopic charged constituents. The units for r are C/m3. The notion of continuous charge distribution is similar to that we adopt for continuous mass distribution in mechanics. When we refer toFIGURE 1.21 Definition of linear , surface and volume charge densities. In each case, the element (Dl, DS, DV) chosen is small on the macroscopic scale but contains a very large number of microscopic constituents. *At the microscopic level, charge distribution is discontinuous, because they are discrete charges separated by intervening space where there is no charge. Rationalised 2023-24 Electric Charges and Fields 29the density of a liquid, we are referring to its macroscopic density. We regard it as a continuous fluid and ignore its discrete molecular constitution. The field due to a continuous charge distribution can be obtained in much the same way as for a system of discrete charges, Eq. (1.10). Suppose a continuous char ge distribution in space has a char ge density r. Choose any convenient origin O and let the position vector of any point in the charge distribution be r. The char ge density r may vary fr om point to point, i.e., it is a function of r. Divide the char ge distribution into small volume elements of size DV. The char ge in a volume element DV is rDV. Now, consider any general point P (inside or outside the distribution) with position vector R (Fig. 1.21). Electric field due to the charge rDV is given by Coulomb's law: 2 01ˆ 4V'r'ρ εΔΔ =πE r (1.26) wher e r¢ is the distance between the char ge element and P , and ˆr¢ is a unit vector in the dir ection fr om the char ge element to P . By the superposition principle, the total electric field due to the charge distribution is obtained by summing over electric fields due to different volume elements: 2 01ˆ 4all VV'r'ρ εΔΔ≅ ΣπE r (1.27) Note that r, r¢, ˆr all can vary fr om point to point. In a strict mathematical method, we should let DV®0 and the sum then becomes an integral; but we omit that discussion here, for simplicity. In short, using Coulomb's law and the superposition principle, electric field can be determined for any charge distribution, discrete or continuous or part discrete and part continuous. 1.13 G AUSS'S LAW As a simple application of the notion of electric flux, let us consider the total flux thr ough a spher e of radius r, which encloses a point char ge q at its centre. Divide the sphere into small area elements, as shown in Fig. 1.22. The flux through an area element DS is 2 0ˆ 4q rφεΔ = Δ = ΔπE S r Sii (1.28) where we have used Coulomb's law for the electric field due to a single charge q. The unit vector ˆr is along the radius vector from the centre to the area element. Now, since the normal to a sphere at every point is along the radius vector at that point, the area element DS and ˆr have the same direction. Therefore, 2 04qSrφεΔ = Δπ(1.29) since the magnitude of a unit vector is 1. The total flux through the sphere is obtained by adding up flux through all the different area elements:FIGURE 1.22 Flux through a sphere enclosing a point charge q at its centre. Rationalised 2023-24 30Physics 2 04all SqSrφεΔ= Σ Δπ Since each area element of the sphere is at the same distance r from the charge, 2 2 0 4 4all Soq qS Sr rφε εΔ= Σ Δ =π π Now S, the total area of the sphere, equals 4 pr2. Thus, 2 2 0 044q qrrφε ε= × π =π(1.30) Equation (1.30) is a simple illustration of a general result of electrostatics called Gauss's law. We state Gauss's law without pr oof: Electric flux through a closed surface S = q/ e0(1.31) q = total charge enclosed by S. The law implies that the total electric flux through a closed surface is zero if no char ge is enclosed by the sur face. W e can see that explicitly in the simple situation of Fig. 1.23. Here the electric field is uniform and we are considering a closed cylindrical surface, with its axis parallel to the uniform field E . The total flux f through the surface is f = f1 + f2 + f3, where f1 and f2 represent the flux through the surfaces 1 and 2 (of circular cross-section) of the cylinder and f3 is the flux through the curved cylindrical part of the closed surface. Now the normal to the surface 3 at every point is perpendicular to E, so by definition of flux, f3 = 0. Further , the outwar d normal to 2 is along E while the outward normal to 1 is opposite to E. Therefore, f1 = –E S1, f2 = +E S2 S1 = S2 = S where S is the area of circular cross-section. Thus, the total flux is zero, as expected by Gauss's law. Thus, whenever you find that the net electric flux through a closed surface is zero, we conclude that the total charge contained in the closed surface is zero. The great significance of Gauss's law Eq. (1.31), is that it is true in general, and not only for the simple cases we have considered above. Let us note some important points regarding this law: (i)Gauss's law is true for any closed surface, no matter what its shape or size. (ii)The term q on the right side of Gauss's law, Eq. (1.31), includes the sum of all charges enclosed by the surface. The charges may be located anywhere inside the surface. (iii)In the situation when the surface is so chosen that there are some charges inside and some outside, the electric field [whose flux appears on the left side of Eq. (1.31)] is due to all the charges, both inside and outside S. The ter m q on the right side of Gauss's law, however , represents only the total charge inside S. FIGURE 1.23 Calculation of the flux of uniform electric field through the sur face of a cylinder . Rationalised 2023-24 Electric Charges and Fields 31 EXAMPLE 1.10(iv)The surface that we choose for the application of Gauss's law is called the Gaussian sur face. Y ou may choose any Gaussian sur face and apply Gauss's law. However , take car e not to let the Gaussian sur face pass through any discrete charge. This is because electric field due to a system of discrete charges is not well defined at the location of any charge. (As you go close to the charge, the field grows without any bound.) However , the Gaussian sur face can pass thr ough a continuous charge distribution. (v)Gauss's law is often useful towards a much easier calculation of the electrostatic field when the system has some symmetry. This is facilitated by the choice of a suitable Gaussian surface. (vi) Finally, Gauss's law is based on the inverse square dependence on distance contained in the Coulomb's law. Any violation of Gauss's law will indicate departure from the inverse square law. Example 1.10 The electric field components in Fig. 1.24 are Ex = ax1/2, Ey = Ez = 0, in which a = 800 N/C m1/2. Calculate (a) the flux through the cube, and (b) the charge within the cube. Assume that a = 0.1 m. FIGURE 1.24 Solution (a)Since the electric field has only an x component, for faces perpendicular to x direction, the angle between E and DS is ± p/2. Therefore, the flux f = E.DS is separately zero for each face of the cube except the two shaded ones. Now the magnitude of the electric field at the left face is EL = ax1/2 = aa1/2 (x = a at the left face). The magnitude of electric field at the right face is ER = a x1/2 = a (2a)1/2 (x = 2a at the right face). The corresponding fluxes are fL= EL.DS = ˆL LSΔE n⋅ =EL DS cos q = –EL DS, since q = 180° = – ELa2 fR= ER.DS = ER DS cos q = ER DS, since q = 0° = ERa2 Net flux through the cube Rationalised 2023-24 32Physics EXAMPLE 1.11 EXAMPLE 1.10= fR + fL = ERa2 – ELa2 = a2 (ER – EL) = aa2 [(2a)1/2 – a1/2] = aa5/2 ()2 –1 = 800 (0.1)5/2 ()2 –1 = 1.05 N m2 C–1 (b) W e can use Gauss's law to find the total char ge q inside the cube. We have f = q/ e0 or q = fe0. Ther efore, q = 1.05 × 8.854 × 10–12 C = 9.27 × 10–12 C. Example 1.11 An electric field is uniform, and in the positive x direction for positive x, and uniform with the same magnitude but in the negative x direction for negative x. It is given that E = 200 ˆi N/C for x > 0 and E = –200 ˆi N/C for x < 0. A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the x-axis so that one face is at x = +10 cm and the other is at x = –10 cm (Fig. 1.25). (a) What is the net outward flux through each flat face? (b) What is the flux through the side of the cylinder? (c) What is the net outward flux through the cylinder? (d) What is the net charge inside the cylinder? Solution (a)We can see fr om the figur e that on the left face E and DS are parallel. Therefore, the outward flux is fL= E.DS = – 200 ˆΔi Si = + 200 DS, since ˆΔi Si= – DS = + 200 × p (0.05)2 = + 1.57 N m2 C–1 On the right face, E and DS are parallel and therefore fR = E.DS = + 1.57 N m2 C–1. (b)For any point on the side of the cylinder E is perpendicular to DS and hence E.DS = 0. Therefore, the flux out of the side of the cylinder is zero. (c)Net outward flux through the cylinder f = 1.57 + 1.57 + 0 = 3.14 N m2 C–1 FIGURE 1.25 (d)The net charge within the cylinder can be found by using Gauss's law which gives q = e0f = 3.14 × 8.854 × 10–12 C = 2.78 × 10–11 C Rationalised 2023-24 Electric Charges and Fields 331.14 A PPLICA TIONS OF GAUSS'S LAW The electric field due to a general charge distribution is, as seen above, given by Eq. (1.27). In practice, except for some special cases, the summation (or integration) involved in this equation cannot be carried out to give electric field at every point in space. For some symmetric charge configurations, however , it is possible to obtain the electric field in a simple way using the Gauss's law. This is best understood by some examples. 1.14.1 Field due to an infinitely long straight uniformly charged wire Consider an infinitely long thin straight wire with uniform linear charge density l. The wire is obviously an axis of symmetry. Suppose we take the radial vector from O to P and rotate it around the wir e. The points P , P¢, P¢¢ so obtained are completely equivalent with respect to the charged wire. This implies that the electric field must have the same magnitude at these points. The direction of electric field at every point must be radial (outward if l > 0, inward if l < 0). This is clear from Fig. 1.2 6. Consider a pair of line elements P1 and P2 of the wire, as shown. The electric fields produced by the two elements of the pair when summed give a resultant electric field which is radial (the components normal to the radial vector cancel). This is true for any such pair and hence the total field at any point P is radial. Finally, since the wire is infinite, electric field does not depend on the position of P along the length of the wire. In short, the electric field is everywhere radial in the plane cutting the wire normally, and its magnitude depends only on the radial distance r. To calculate the field, imagine a cylindrical Gaussian surface, as shown in the Fig. 1.2 6(b). Since the field is everywhere radial, flux through the two ends of the cylindrical Gaussian surface is zero. At the cylindrical part of the surface, E is normal to the surface at every point, and its magnitude is constant, since it depends only on r. The surface ar ea of the curved part is 2 prl, where l is the length of the cylinder .FIGURE 1.26 (a) Electric field due to an infinitely long thin straight wire is radial, (b) The Gaussian surface for a long thin wire of uniform linear charge density. Rationalised 2023-24 34Physics Flux through the Gaussian surface =flux through the curved cylindrical part of the surface =E × 2prl The surface includes charge equal to l l. Gauss's law then gives E × 2prl = ll/e0 i.e.,E = 02rλ επ Vectorially, E at any point is given by 0ˆ 2rλ ε=πE n (1.32) where ˆn is the radial unit vector in the plane normal to the wire passing through the point. E is directed outward if l is positive and inward if l is negative. Note that when we write a vector A as a scalar multiplied by a unit vector , i.e., as A = A ˆa, the scalar A is an algebraic number . It can be negative or positive. The direction of A will be the same as that of the unit vector ˆaif A > 0 and opposite to ˆa if A < 0. When we want to restrict to non-negative values, we use the symbol Aand call it the modulus of A. Thus, 0≥A . Also note that though only the charge enclosed by the surface (ll) was included above, the electric field E is due to the charge on the entire wire. Further , the assumption that the wir e is infinitely long is crucial. Without this assumption, we cannot take E to be normal to the curved part of the cylindrical Gaussian sur face. However , Eq. (1.32) is approximately true for electric field around the central portions of a long wire, where the end effects may be ignored. 1.14.2 Field due to a uniformly charged infinite plane sheet Let s be the uniform surface charge density of an infinite plane sheet (Fig. 1.27). W e take the x-axis nor mal to the given plane. By symmetry, the electric field will not depend on y and z coordinates and its direction at every point must be parallel to the x-direction. We can take the Gaussian sur face to be a rectangular parallelepiped of cross-sectional area A, as shown. (A cylindrical surface will also do.) As seen from the figure, only the two faces 1 and 2 will contribute to the flux; electric field lines are parallel to the other faces and they, therefore, do not contribute to the total flux. The unit vector normal to surface 1 is in – x direction while the unit vector normal to surface 2 is in the + x direction. Therefore, flux E.DS through both the surfaces are equal and add up. Therefore the net flux through the Gaussian surface is 2 EA. The charge enclosed by the closed surface is sA. Therefore by Gauss's law,FIGURE 1.27 Gaussian surface for a uniformly charged infinite plane sheet. Rationalised 2023-24 Electric Charges and Fields 352 EA = sA/e0 or, E = s/2e0 Vectorically, 0ˆ 2σ ε=E n (1.33) where ˆn is a unit vector normal to the plane and going away from it. E is directed away from the plate if s is positive and toward the plate if s is negative. Note that the above application of the Gauss' law has brought out an additional fact: E is independent of x also. For a finite large planar sheet, Eq. (1.33) is approximately true in the middle regions of the planar sheet, away from the ends. 1.14.3 Field due to a uniformly charged thin spherical shell Let s be the uniform surface charge density of a thin spherical shell of radius R (Fig. 1.28). The situation has obvious spherical symmetry. The field at any point P , outside or inside, can depend only on r (the radial distance from the centre of the shell to the point) and must be radial (i.e., along the radius vector). (i)Field outside the shell: Consider a point P outside the shell with radius vector r. To calculate E at P, we take the Gaussian surface to be a sphere of radius r and with centre O, passing thr ough P . All points on this spher e are equivalent relative to the given charged configuration. (That is what we mean by spherical symmetry.) The electric field at each point of the Gaussian surface, therefore, has the same magnitude E and is along the radius vector at each point. Thus, E and DS at every point are parallel and the flux through each element is E DS. Summing over all DS, the flux through the Gaussian surface is E × 4 p r2. The charge enclosed is s × 4 p R2. By Gauss's law E × 4 p r2 = 2 04Rσ επ Or, 2 2 2 0 04R qEr rσ ε ε= =π where q = 4 p R2 s is the total charge on the spherical shell. Vectorially, 2 0ˆ 4q rε=πE r (1.34) The electric field is directed outward if q > 0 and inward if q < 0. This, however , is exactly the field pr oduced by a char ge q placed at the centre O. Thus for points outside the shell, the field due to a uniformly charged shell is as if the entire charge of the shell is concentrated at its centre. (ii)Field inside the shell: In Fig. 1.28(b), the point P is inside the shell. The Gaussian surface is again a sphere through P centred at O.FIGURE 1.28 Gaussian surfaces for a point with (a) r > R, (b) r < R. Rationalised 2023-24 36Physics EXAMPLE 1.12The flux through the Gaussian surface, calculated as before, is E × 4 p r2. However , in this case, the Gaussian sur face encloses no charge. Gauss's law then gives E × 4 p r2 = 0 i.e., E = 0 (r < R) (1.35) that is, the field due to a uniformly charged thin shell is zero at all points inside the shell *. This important result is a direct consequence of Gauss's law which follows from Coulomb's law. The experimental verification of this result confirms the 1/ r2 dependence in Coulomb's law. Example 1.12 An early model for an atom considered it to have a positively charged point nucleus of charge Ze, surrounded by a uniform density of negative charge up to a radius R. The atom as a whole is neutral. For this model, what is the electric field at a distance r from the nucleus? FIGURE 1.29 Solution The charge distribution for this model of the atom is as shown in Fig. 1.29. The total negative charge in the uniform spherical charge distribution of radius R must be – Z e, since the atom (nucleus of charge Z e + negative charge) is neutral. This immediately gives us the negative char ge density r, since we must have 340–3RZeρπ= or 33 4Ze Rρ=−π To find the electric field E(r) at a point P which is a distance r away from the nucleus, we use Gauss's law. Because of the spherical symmetry of the charge distribution, the magnitude of the electric field E(r) depends only on the radial distance, no matter what the direction of r. Its dir ection is along (or opposite to) the radius vector r from the origin to the point P . The obvious Gaussian sur face is a spherical sur face centr ed at the nucleus. W e consider two situations, namely, r < R and r > R. (i) r < R : The electric flux f enclosed by the spherical surface is f = E (r) × 4 p r2 *Compare this with a uniform mass shell discussed in Section 7.5 of Class XI Textbook of Physics. Rationalised 2023-24 Electric Charges and Fields 37 EXAMPLE 1.12wher e E (r) is the magnitude of the electric field at r. This is because the field at any point on the spherical Gaussian surface has the same direction as the normal to the surface there, and has the same magnitude at all points on the surface. The charge q enclosed by the Gaussian surface is the positive nuclear charge and the negative char ge within the spher e of radius r, i.e., 34 3rq Z e ρπ= + Substituting for the char ge density r obtained earlier , we have 3 3rq Z e Z e R= − Gauss's law then gives, 2 3 01( ) ;4Z e rE r r Rr Rε = − < π The electric field is directed radially outward. (ii) r > R: In this case, the total charge enclosed by the Gaussian spherical surface is zero since the atom is neutral. Thus, from Gauss's law, E (r) × 4 p r2 = 0 or E (r) = 0; r > R At r = R, both cases give the same result: E = 0. SUMMARY 1.Electric and magnetic forces determine the properties of atoms, molecules and bulk matter . 2.From simple experiments on frictional electricity, one can infer that there are two types of charges in nature; and that like charges repel and unlike charges attract. By convention, the charge on a glass rod rubbed with silk is positive; that on a plastic rod rubbed with fur is then negative. 3.Conductors allow movement of electric charge through them, insulators do not. In metals, the mobile charges are electrons; in electrolytes both positive and negative ions are mobile. 4.Electric charge has three basic properties: quantisation, additivity and conservation. Quantisation of electric charge means that total charge (q) of a body is always an integral multiple of a basic quantum of charge (e) i.e., q = n e, where n = 0, ±1, ±2, ±3, ... Proton and electron have charges +e, –e, respectively. For macroscopic charges for which n is a very large number , quantisation of char ge can be ignor ed. Additivity of electric charges means that the total charge of a system is the algebraic sum (i.e., the sum taking into account proper signs) of all individual charges in the system. Conservation of electric charges means that the total charge of an isolated system remains unchanged with time. This means that when Rationalised 2023-24 38Physics bodies are charged through friction, there is a transfer of electric charge from one body to another , but no cr eation or destruction of charge. 5.Coulomb's Law : The mutual electrostatic force between two point charges q1 and q2 is proportional to the product q1q2 and inversely proportional to the square of the distance r21 separating them. Mathematically, F21 = force on q2 due to 1 2 1 21 2 21ˆk (q q)qr= r where 21ˆr is a unit vector in the direction from q1 to q2 and k = 01 4επ is the constant of proportionality. In SI units, the unit of charge is coulomb. The experimental value of the constant e0 is e0 = 8.854 × 10–12 C2 N–1 m–2 The approximate value of k is k = 9 × 109 N m2 C–2 6.The ratio of electric force and gravitational force between a proton and an electron is 2 392 4 10 e pk e.G m m≅ × 7.Superposition Principle : The principle is based on the property that the forces with which two charges attract or repel each other are not affected by the presence of a third (or more) additional charge(s). For an assembly of charges q1, q2, q3, ..., the force on any charge, say q1, is the vector sum of the force on q1 due to q2, the force on q1 due to q3, and so on. For each pair , the for ce is given by the Coulomb's law for two char ges stated earlier . 8.The electric field E at a point due to a charge configuration is the force on a small positive test charge q placed at the point divided by the magnitude of the charge. Electric field due to a point charge q has a magnitude | q|/4pe0r2; it is radially outwards from q, if q is positive, and radially inwards if q is negative. Like Coulomb force, electric field also satisfies superposition principle. 9.An electric field line is a curve drawn in such a way that the tangent at each point on the curve gives the direction of electric field at that point. The relative closeness of field lines indicates the relative strength of electric field at different points; they crowd near each other in regions of strong electric field and are far apart where the electric field is weak. In regions of constant electric field, the field lines are uniformly spaced parallel straight lines. 10.Some of the important properties of field lines are: (i) Field lines are continuous curves without any breaks. (ii) Two field lines cannot cross each other . (iii) Electr ostatic field lines start at positive char ges and end at negative charges —they cannot form closed loops. 11.An electric dipole is a pair of equal and opposite charges q and – q separated by some distance 2 a. Its dipole moment vector p has magnitude 2 qa and is in the direction o f the dipole axis from – q to q. Rationalised 2023-24 Electric Charges and Fields 3912.Field of an electric dipole in its equatorial plane (i.e., the plane perpendicular to its axis and passing through its centre) at a distance r from the centre: 2 2 3/21 4 ( )oa rε−=π +pE 3,4ofor r arε−≅ >>πp Dipole electric field on the axis at a distance r from the centre: 2 2 2 02 4 ( )r r aε=π −pE 3 02 4for r arε≅ >>πp The 1/ r3 dependence of dipole electric fields should be noted in contrast to the 1/ r2 dependence of electric field due to a point charge. 13.In a uniform electric field E, a dipole experiences a torque τ given by τ = p × E but experiences no net force. 14.The flux Df of electric field E through a small area element DS is given by Df = E.DS The vector area element DS is DS = DS ˆn where DS is the magnitude of the area element and ˆn is normal to the area element, which can be considered planar for sufficiently small DS. For an area element of a closed surface, ˆn is taken to be the direction of outward normal, by convention. 15.Gauss's law : The flux of electric field through any closed surface S is 1/e0 times the total charge enclosed by S. The law is especially useful in determining electric field E , when the source distribution has simple symmetry: (i) Thin infinitely long straight wire of uniform linear charge density l 0ˆ 2 rλ ε=πE n where r is the perpendicular distance of the point from the wire and ˆnis the radial unit vector in the plane normal to the wire passing through the point. (ii) Infinite thin plane sheet of uniform surface charge density s 0ˆ 2σ ε=E n where ˆn is a unit vector normal to the plane, outward on either side. Rationalised 2023-24 40Physics (iii) Thin spherical shell of uniform surface charge density s 2 0ˆ ( )4qr Rrε= ≥πE r E = 0 (r < R) where r is the distance of the point from the centre of the shell and R the radius of the shell. q is the total charge of the shell: q = 4 pR2s. The electric field outside the shell is as though the total charge is concentrated at the centre. The same result is true for a solid sphere of uniform volume charge density. The field is zero at all points inside the shell. Physical quantity Symbol Dimensions Unit Remarks Vector ar ea element DS [L2] m2DS = DS ˆn Electric field E [MLT–3A–1] V m–1 Electric flux f [ML3 T–3A–1] V m Df = E.DS Dipole moment p [LTA] C m Vector directed from negative to positive charge Charge density: linear l [L–1 TA] C m–1 Charge/length surface s [L–2 TA] C m–2Charge/area volume r [L–3 TA] C m–3Charge/volume POINTS TO PONDER 1.You might wonder why the pr otons, all carrying positive char ges, are compactly residing inside the nucleus. Why do they not fly away? You will lear n that ther e is a thir d kind of a fundamental for ce, called the str ong for ce which holds them together . The range of distance wher e this for ce is ef fective is, however , very small ~10-14 m. This is precisely the size of the nucleus. Also the electrons are not allowed to sit on top of the protons, i.e. inside the nucleus, due to the laws of quantum mechanics. This gives the atoms their structure as they exist in na ture. 2.Coulomb force and gravitational force follow the same inverse-square law. But gravitational force has only one sign (always attractive), while Rationalised 2023-24 Electric Charges and Fields 41Coulomb force can be of both signs (attractive and repulsive), allowing possibility of cancellation of electric forces. This is how gravity, despite being a much weaker force, can be a dominating and more pervasive force in nature. 3.The constant of proportionality k in Coulomb's law is a matter of choice if the unit of charge is to be defined using Coulomb's law. In SI units, however , what is defined is the unit of curr ent (A) via its magnetic effect (Ampere's law) and the unit of charge (coulomb) is simply defined by (1C = 1 A s). In this case, the value of k is no longer arbitrary; it is approximately 9 × 109 N m2 C–2. 4.The rather large value of k, i.e., the large size of the unit of charge (1C) from the point of view of electric effects arises because (as mentioned in point 3 already) the unit of charge is defined in terms of magnetic forces (forces on current–carrying wires) which are generally much weaker than the electric forces. Thus while 1 ampere is a unit of reasonable size for magnetic effects, 1 C = 1 A s, is too big a unit for electric effects. 5.The additive property of charge is not an 'obvious' property. It is related to the fact that electric charge has no direction associated with it; charge is a scalar . 6.Charge is not only a scalar (or invariant) under rotation; it is also invariant for frames of reference in relative motion. This is not always true for every scalar . For example, kinetic ener gy is a scalar under rotation, but is not invariant for frames of reference in relative motion. 7.Conservation of total charge of an isolated system is a property independent of the scalar nature of charge noted in point 6. Conservation refers to invariance in time in a given frame of reference. A quantity may be scalar but not conserved (like kinetic energy in an inelastic collision). On the other hand, one can have conserved vector quantity (e.g., angular momentum of an isolated system). 8.Quantisation of electric charge is a basic (unexplained) law of nature; interestingly, there is no analogous law on quantisation of mass. 9.Superposition principle should not be regarded as 'obvious', or equated with the law of addition of vectors. It says two things: force on one charge due to another charge is unaffected by the presence of other charges, and there are no additional three-body, four-body, etc., for ces which arise only when ther e are mor e than two charges. 10.The electric field due to a discrete charge configuration is not defined at the locations of the discrete charges. For continuous volume charge distribution, it is defined at any point in the distribution. For a surface charge distribution, electric field is discontinuous across the surfac e. 11.The electric field due to a charge configuration with total charge zero is not zero; but for distances large compared to the size of the configuration, its field falls off faster than 1/ r2, typical of field due to a single charge. An electric dipole is the simplest example of this fa ct. Rationalised 2023-24 42Physics EXERCISES 1.1 What is the force between two small charged spheres having charges of 2 × 10–7C and 3 × 10–7C placed 30 cm apart in air? 1.2 The electrostatic force on a small sphere of charge 0.4 mC due to another small sphere of charge –0.8 mC in air is 0.2 N. (a) What is the distance between the two spheres? (b) What is the force on the second sphere due to the first? 1.3 Check that the ratio ke2/G memp is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify? 1.4 (a)Explain the meaning of the statement 'electric charge of a body is quantised'. (b)Why can one ignore quantisation of electric charge when dealing with macroscopic i.e., large scale charges? 1.5 When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge. 1.6 Four point charges qA = 2 mC, qB = –5 mC, qC = 2 m C, and qD = –5 mC are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 mC placed at the centre of the square? 1.7 (a)An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not? (b)Explain why two field lines never cross each other at any point? 1.8 Two point charges qA = 3 mC and qB = –3 mC are located 20 cm apart in vacuum. (a)What is the electric field at the midpoint O of the line AB joining the two charges? (b)If a negative test charge of magnitude 1.5 × 10–9 C is placed at this point, what is the force experienced by the test charge? 1.9 A system has two charges qA = 2.5 × 10–7 C and qB = –2.5 × 10–7 C located at points A: (0, 0, –15 cm) and B: (0,0, +15 cm), respectively. What are the total charge and electric dipole moment of the system? 1.10 An electric dipole with dipole moment 4 × 10–9 C m is aligned at 30° with the direction of a uniform electric field of magnitude 5 × 104 NC–1. Calculate the magnitude of the torque acting on the dipole. 1.11 A polythene piece rubbed with wool is found to have a negative charge of 3 × 10–7 C. (a)Estimate the number of electrons transferred (from which to which?) (b)Is there a transfer of mass from wool to polythene? 1.12 (a)Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5 × 10–7 C? The radii of A and B are negligible compared to the distance of separation. (b)What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved? 1.13 Figure 1.30 shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio? Rationalised 2023-24 Electric Charges and Fields 43FIGURE 1.30 1.14 Consider a uniform electric field E = 3 × 103 î N/C. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane? (b) What is the flux through the same square if the normal to its plane makes a 60° angle with the x-axis? 1.15 What is the net flux of the uniform electric field of Exercise 1.14 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes? 1.16 Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8.0 × 103 Nm2/C. (a) What is the net charge inside the box?

(b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or Why not? 1.17 A point charge +10 mC is a distance 5 cm directly above the centre of a square of side 10 cm, as shown in Fig. 1.31. What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge 10 cm.) FIGURE 1.31 1.18 A point charge of 2.0 mC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface? 1.19 A point charge causes an electric flux of –1.0 × 103 Nm2/C to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) What is the value of the point charge? 1.20 A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5 × 103 N/C and points radially inward, what is the net charge on the sphere? Rationalised 2023-24 44Physics 1.21 A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 mC/m2. (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere? 1.22 An infinite line charge produces a field of 9 × 104 N/C at a distance of 2 cm. Calculate the linear charge density. 1.23 Two lar ge, thin metal plates ar e parallel and close to each other . On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0 × 10–22 C/m2. What is E: (a) in the outer region of the first plate, (b) in the outer region of the second plate, and (c) between the plates? Rationalised 2023-24 ncert_classXII(12)_chapter2_electrostatic_potential_and_capacitance: 2.1 I NTRODUCTION In Chapters 6 and 8 (Class XI), the notion of potential energy was introduced. When an external force does work in taking a body from a point to another against a force like spring force or gravitational force, that work gets stored as potential energy of the body. When the external force is removed, the body moves, gaining kinetic energy and losing an equal amount of potential energy. The sum of kinetic and potential energies is thus conserved. Forces of this kind are called conservative forces. Spring force and gravitational force are examples of conservative forces. Coulomb force between two (stationary) charges is also a conservative force. This is not surprising, since both have inverse-square dependence on distance and differ mainly in the proportionality constants – the masses in the gravitational law are replaced by charges in Coulomb's law. Thus, like the potential energy of a mass in a gravitational field, we can define electrostatic potential energy of a charge in an electrostatic field. Consider an electrostatic field E due to some charge configuration. First, for simplicity, consider the field E due to a charge Q placed at the origin. Now, imagine that we bring a test charge q from a point R to a point P against the repulsive force on it due to the charge Q. With referenceChapter Two ELECTROSTATIC POTENTIAL AND CAPACITANCE Rationalised 2023-24 Physics 46to Fig. 2.1, this will happen if Q and q are both positive or both negative. For definiteness, let us take Q, q > 0. Two remarks may be made here. First, we assume that the test charge q is so small that it does not disturb the original configuration, namely the charge Q at the origin (or else, we keep Q fixed at the origin by some unspecified force). Second, in bringing the charge q from R to P , we apply an exter nal for ce Fext just enough to counter the repulsive electric force FE (i.e, Fext= –FE). This means there is no net force on or acceleration of the char ge q when it is br ought fr om R to P , i.e., it is brought with infinitesimally slow constant speed. In this situation, work done by the external force is the negative of the work done by the electric force, and gets fully stored in the form of potential energy of the char ge q. If the exter nal for ce is r emoved on r eaching P, the electric force will take the charge away from Q – the stored energy (potential energy) at P is used to provide kinetic energy to the charge q in such a way that the sum of the kinetic and potential energies is conserved. Thus, work done by external forces in moving a charge q from R to P is WRP = = – (2.1) This work done is against electrostatic repulsive force and gets stored as potential energy. At every point in electric field, a particle with charge q possesses a certain electrostatic potential energy, this work done increases its potential energy by an amount equal to potential energy difference between points R and P . Thus, potential energy difference P R RP U U U WΔ = − = (2.2) (Note here that this displacement is in an opposite sense to the electric force and hence work done by electric field is negative, i.e. , –WRP.) Therefore, we can define electric potential energy difference between two points as the work required to be done by an external force in moving (without acceleratin g) charge q from one point to another for electric field of any arbitrary charge configuration. Two important comments may be made at this stage: (i)The right side of Eq. (2.2) depends only on the initial and final positions of the charge. It means that the work done by an electrostatic field in moving a charge from one point to another depends only on the initial and the final points and is independent of the path taken to go from one point to the other . This is the fundamental characteristic of a conservative force. The concept of the potential energy would not be meaningful if the work depended on the path. The path-independence of work done by an electrostatic field can be proved using the Coulomb's law. W e omit this pr oof her e.FIGURE 2.1 A test charge q (> 0) is moved from the point R to the point P against the repulsive force on it by the charge Q (> 0) placed at the origin. Rationalised 2023-24 Electrostatic Potential and Capacitance 47(ii)Equation (2.2) defines potential energy difference in terms of the physically meaningful quantity work . Clearly, potential energy so defined is undetermined to within an additive constant.What this means is that the actual value of potential energy is not physically significant; it is only the dif ference of potential ener gy that is significant. W e can always add an arbitrary constant a to potential energy at every point, since this will not change the potential energy difference: ( ) ( )P R P R U U U Uα α+ − + = − Put it differently, there is a freedom in choosing the point where potential energy is zero. A convenient choice is to have electrostatic potential energy zero at infinity. With this choice, if we take the point R at infinity, we get from Eq. (2.2) P P P W U U U∞ ∞ = − = (2.3) Since the point P is arbitrary, Eq. (2.3) provides us with a definition of potential energy of a charge q at any point. Potential energy of charge q at a point (in the presence of field due to any charge configuration) is the work done by the exter nal for ce (equal and opposite to the electric for ce) in bringing the charge q from infinity to that point . 2.2 E LECTROST ATIC POTENTIAL Consider any general static char ge configuration. W e define potential energy of a test charge q in terms of the work done on the charge q. This work is obviously proportional to q , since the force at any point is qE, where E is the electric field at that point due to the given charge configuration. It is, therefore, convenient to divide the work by the amount of charge q, so that the resulting quantity is independent of q . In other words, work done per unit test charge is characteristic of the electric field associated with the charge configuration. This leads to the idea of electrostatic potential V due to a given charge configuration. From Eq. (2.1), we get: Work done by exter nal for ce in bringing a unit positive charge from point R to P = VP – VR =− U U qP R(2.4) where VP and VR are the electrostatic potentials at P and R, respectively. Note, as before, that it is not the actual value of potential but the potential difference that is physically significant. If, as before, we choose the potential to be zero at infinity, Eq. (2.4) implies: Work done by an exter nal for ce in bringing a unit positive char ge from infinity to a point = electrostatic potential (V) at that point.COUNT ALESSANDRO VOLTA (1745 –1827) Count Alessandr o Volta (1745 – 1827) Italian physicist, professor at Pavia. V olta established that the animal electri- city observed by Luigi Galvani, 1737–1798, in experiments with frog muscle tissue placed in contact with dissimilar metals, was not due to any exceptional property of animal tissues but was also generated whenever any wet body was sandwiched between dissimilar metals. This led him to develop the first voltaic pile, or battery, consisting of a large stack of moist disks of cardboard (electro- lyte) sandwiched between disks of metal (electrodes). Rationalised 2023-24 Physics 48In other words, the electrostatic potential (V) at any point in a region with electrostatic field is the work done in bringing a unit positive charge (without acceleration) from infinity to that point. The qualifying remarks made earlier regarding potential energy also apply to the definition of potential. To obtain the work done per unit test charge, we should take an infinitesimal test charge dq, obtain the work done dW in bringing it from infinity to the point and determine the ratio dW/dq. Also, the external force at every point of the path is to be equal and opposite to the electrostatic force on the test charge at that point. 2.3 P OTENTIAL DUE TO A POINT CHARGE Consider a point charge Q at the origin (Fig. 2.3). For definiteness, take Q to be positive. W e wish to deter mine the potential at any point P with position vector r from the origin. For that we must calculate the work done in bringing a unit positive test char ge from infinity to the point P . For Q > 0, the work done against the repulsive force on the test charge is positive. Since work done is independent of the path, we choose a convenient path – along the radial direction from infinity to the point P . At some intermediate point P ¢ on the path, the electrostatic force on a unit positive charge is 2 01ˆ4 'Q rεx'πr (2.5) wher e ˆ'ris the unit vector along OP ¢. Work done against this force from r¢ to r¢ + Dr¢ is 2 04 'QW rrεΔ = − Δ 'π(2.6) The negative sign appears because for Dr¢ < 0, DW is positive. Total work done (W) by the external force is obtained by integrating Eq. (2.6) from r¢ = ¥ to r¢ = r, WQ rdrQ rQ rrr = −''=' ∞∞∫4 4 402 0 0 π π πε ε ε(2.7) This, by definition is the potential at P due to the charge Q 0( )4QV rrε=π(2.8)FIGURE 2.2 Work done on a test char ge q by the electrostatic field due to any given charge configuration is independent of the path, and depends only on its initial and final positions. FIGURE 2.3 Work done in bringing a unit positive test charge from infinity to the point P , against the r epulsive for ce of charge Q (Q > 0), is the potential at P due to the charge Q. Rationalised 2023-24 Electrostatic Potential and Capacitance 49 EXAMPLE 2.1Equation (2.8) is true for any sign of the charge Q, though we considered Q > 0 in its derivation. For Q < 0, V < 0, i.e., work done (by the external force) per unit positive test charge in bringing it from infinity to the point is negative. This is equivalent to saying that work done by the electrostatic force in bringing the unit positive charge form infinity to the point P is positive. [This is as it should be, since for Q < 0, the force on a unit positive test charge is attractive, so that the electrostatic force and the displacement (from infinity to P) are in the same direction.] Finally, we note that Eq. (2.8) is consistent with the choice that potential at infinity be zero. Figure (2.4) shows how the electrostatic potential (1/r) and the electr ostatic field (1/r2) varies with r. Example 2.1 (a)Calculate the potential at a point P due to a charge of 4 × 10–7C located 9 cm away. (b)Hence obtain the work done in bringing a charge of 2 × 10–9 C from infinity to the point P . Does the answer depend on the path along which the charge is brought? Solution (a) = 4 × 104 V (b)W = qV = 2 × 10–9C × 4 × 104V = 8 × 10–5 J No, work done will be path independent. Any arbitrary infinitesimal path can be resolved into two perpendicular displacements: One along r and another perpendicular to r. The work done corresponding to the later will be zero. 2.4 P OTENTIAL DUE TO AN ELECTRIC DIPOLE As we lear nt in the last chapter , an electric dipole consists of two char ges q and – q separated by a (small) distance 2 a. Its total charge is zero. It is characterised by a dipole moment vector p whose magnitude is q × 2a and which points in the dir ection fr om –q to q (Fig. 2.5). W e also saw that the electric field of a dipole at a point with position vector r depends not just on the magnitude r, but also on the angle between r and p. Further ,FIGURE 2.4 Variation of potential V with r [in units of (Q/4pe0) m-1] (blue curve) and field with r [in units of (Q/4pe0) m-2] (black curve) for a point charge Q. Rationalised 2023-24 Physics 50the field falls off, at large distance, not as 1/r2 (typical of field due to a single charge) but as 1/ r3. We, now, deter mine the electric potential due to a dipole and contrast it with the potential due to a single charge. As before, we take the origin at the centre of the dipole. Now we know that the electric field obeys the superposition principle. Since potential is related to the work done by the field, electrostatic potential also follows the superposition principle. Thus, the potential due to the dipole is the sum of potentials due to the charges q and – q Vq rq r= −   1 40 1 2πε(2.9) where r1 and r2 are the distances of the point P from q and – q, respectively. Now, by geometry, 2 2 2 1 2 r r a ar= + − cosq 2 2 2 2 2 r r a ar= + + cosq (2.10) We take r much gr eater than a (ar) and r etain ter ms only upto the first order in a/r r ra ra r12 2 22 12= − +   cosθ ≅ −   ra r212 cos θ (2.11) Similarly, r ra r22 212≅ +   cosθ (2.12) Using the Binomial theorem and retaining terms upto the first order in a/r ; we obtain, 1 112 11 11 2 r ra r ra r≅ −   +   −coscos/θθ [2.13(a)] 1 112 11 21 2 r ra r ra r≅ +   −   −coscos/θθ [2.13(b)] Using Eqs. (2.9) and (2.13) and p = 2qa, we get Vq a rp r= =4 402 02π π εθ θ ε2 cos cos (2.14) Now, p cos q = p.rˆFIGURE 2.5 Quantities involved in the calculation of potential due to a dipole. Rationalised 2023-24 Electrostatic Potential and Capacitance 51where ˆr is the unit vector along the position vector OP. The electric potential of a dipole is then given by Vr=1 402πεp.rˆ ; (r >> a) (2.15) Equation (2.15) is, as indicated, approximately true only for distances large compared to the size of the dipole, so that higher order terms in a/r are negligible. For a point dipole p at the origin, Eq. (2.15) is, however , exact. From Eq. (2.15), potential on the dipole axis (q = 0, p) is given by 2 01 4pVrε= ±π(2.16) (Positive sign for q = 0, negative sign for q = p.) The potential in the equatorial plane (q = p/2) is zero. The important contrasting features of electric potential of a dipole from that due to a single charge are clear from Eqs. (2.8) and (2.15): (i)The potential due to a dipole depends not just on r but also on the angle between the position vector r and the dipole moment vector p. (It is, however , axially symmetric about p. That is, if you r otate the position vector r about p, keeping q fixed, the points corresponding to P on the cone so generated will have the same potential as at P .) (ii)The electric dipole potential falls off, at large distance, as 1/ r2, not as 1/r, characteristic of the potential due to a single char ge. (You can refer to the Fig. 2.5 for graphs of 1/ r2 versus r and 1/ r versus r, drawn there in another context.) 2.5 P OTENTIAL DUE TO A SYSTEM OF CHARGES Consider a system of charges q1, q2,…, qn with position vectors r1, r2,…, rn relative to some origin (Fig. 2.6). The potential V1 at P due to the charge q1 is 1 1 0 1P1 4qVrε=π wher e r1P is the distance between q1 and P . Similarly, the potential V2 at P due to q2 and V3 due to q3 are given by 2 2 0 2P1 4qVrε=π, 3 3 0 3P1 4qVrε=π where r2P and r3P are the distances of P from charges q2 and q3, respectively; and so on for the potential due to other charges. By the superposition principle, the potential V at P due to the total charge configuration is the algebraic sum of the potentials due to the individual charges V = V1 + V2 + ... + Vn(2.17)FIGURE 2.6 Potential at a point due to a system of charges is the sum of potentials due to individual charges. Rationalised 2023-24 Physics 52 EXAMPLE 2.2= + + +   1 401 12 2 πεq rq rq rn n P P P...... (2.18) If we have a continuous charge distribution characterised by a charge density r(r), we divide it, as before, into small volume elements each of size Dv and carrying a char ge rDv. W e then deter mine the potential due to each volume element and sum (strictly speaking , integrate) over all such contributions, and thus determine the potential due to the entire distribution. We have seen in Chapter 1 that for a unifor mly char ged spherical shell, the electric field outside the shell is as if the entire charge is concentrated at the centre. Thus, the potential outside the shell is given by 01 4qVrε=π ( )r R≥ [2.19(a)] where q is the total charge on the shell and R its radius. The electric field inside the shell is zero. This implies (Section 2.6) that potential is constant inside the shell (as no work is done in moving a charge inside the shell), and, therefore, equals its value at the surface, which is 01 4qVRε=π[2.19(b)] Example 2.2 Two charges 3 × 10–8 C and –2 × 10–8 C are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero. Solution Let us take the origin O at the location of the positive charge. The line joining the two charges is taken to be the x-axis; the negative charge is taken to be on the right side of the origin (Fig. 2.7). FIGURE 2.7 Let P be the required point on the x-axis where the potential is zero. If x is the x-coor dinate of P , obviously x must be positive. (There is no possibility of potentials due to the two charges adding up to zero for x < 0.) If x lies between O and A, we have 1 43 10 102 10 15 100 08 28 2πε× ×−× − ×=   – — –( ) x x where x is in cm. That is, 3 2015x x− =− which gives x = 9 cm. If x lies on the extended line OA, the required condition is 3 2015 x x− =− Rationalised 2023-24 Electrostatic Potential and Capacitance 53 EXAMPLE 2.2which gives x = 45 cm Thus, electric potential is zero at 9 cm and 45 cm away from the positive charge on the side of the negative charge. Note that the formula for potential used in the calculation required choosing potential to be zero at infinity. Example 2.3 Figures 2.8 (a) and (b) show the field lines of a positive and negative point charge respectively. FIGURE 2.8 (a)Give the signs of the potential dif ference VP – VQ; VB – VA. (b)Give the sign of the potential energy difference of a small negative charge between the points Q and P; A and B. (c)Give the sign of the work done by the field in moving a small positive char ge from Q to P . (d)Give the sign of the work done by the external agency in moving a small negative charge from B to A. (e)Does the kinetic energy of a small negative charge increase or decrease in going from B to A? Solution (a)As 1Vr∝, VP > VQ. Thus, (VP – VQ) is positive. Also VB is less negative than VA . Thus, VB > VA or (VB – VA) is positive. (b)A small negative charge will be attracted towards positive charge. The negative charge moves from higher potential energy to lower potential energy. Therefore the sign of potential energy difference of a small negative charge between Q and P is positive. Similarly, (P .E.)A > (P .E.)B and hence sign of potential ener gy differences is positive. (c)In moving a small positive charge fr om Q to P , work has to be done by an external agency against the electric field. Therefore, work done by the field is negative. (d)In moving a small negative charge from B to A work has to be done by the external agency. It is positive. (e)Due to force of repulsion on the negative charge, velocity decreases and hence the kinetic energy decreases in going from B to A. EXAMPLE 2.3Electric potential, equipotential surfaces: http://video.mit.edu/watch/4-electrostatic-potential-elctric-energy-ev-conservative-field-equipotential-sufaces-12584/ Rationalised 2023-24 Physics 54FIGURE 2.10 Equipotential surfaces for a uniform electric field.2.6 E QUIPOTENTIAL SURFACES An equipotential surface is a surface with a constant value of potential at all points on the surface. For a single charge q, the potential is given by Eq. (2.8): 1 4oqVrε=π This shows that V is a constant if r is constant. Thus, equipotential surfaces of a single point charge are concentric spherical surfaces centred at the charge. Now the electric field lines for a single charge q are radial lines starting from or ending at the charge, depending on whether q is positive or negative. Clearly, the electric field at every point is normal to the equipotential surface passing through that point. This is true in general: for any charge configuration , equipotential sur face thr ough a point is nor mal to the electric field at that point . The proof of this statement is simple. If the field were not normal to the equipotential surface, it would have non-zero component along the surface. To move a unit test charge against the direction of the component of the field, work would have to be done. But this is in contradiction to the definition of an equipotential surface: there is no potential difference between any two points on the surface and no work is required to move a test charge on the surface. The electric field must, therefore, be normal to the equipotential surface at every point. Equipotential surfaces offer an alternative visual picture in addition to the picture of electric field lines around a charge configuration.FIGURE 2.9 For a single charge q (a) equipotential surfaces are spherical surfaces centred at the charge, and (b) electric field lines are radial, starting from the charge if q > 0. For a uniform electric field E, say, along the x-axis, the equipotential surfaces are planes normal to the x-axis, i.e., planes parallel to the y-z plane (Fig. 2.10). Equipotential surfaces for (a) a dipole and (b) two identical positive charges are shown in Fig. 2.11. FIGURE 2.11 Some equipotential surfaces for (a) a dipole, (b) two identical positive charges. Rationalised 2023-24 Electrostatic Potential and Capacitance 552.6.1 Relation between field and potential Consider two closely spaced equipotential surfaces A and B (Fig. 2.12) with potential values V and V + d V, where d V is the change in V in the direction of the electric field E. Let P be a point on the surface B. d l is the perpendicular distance of the surface A fr om P. Imagine that a unit positive char ge is moved along this perpendicular from the surface B to surface A against the electric field. The work done in this process is | E|d l. This work equals the potential difference VA–VB. Thus, |E|d l = V – (V + dV)= – dV i.e., |E|=−δ δV l(2.20) Since dV is negative, dV = – |dV|. we can rewrite Eq (2.20) as E= − = +δ δδ δV lV l(2.21) We thus arrive at two important conclusions concer ning the r elation between electric field and potential: (i)Electric field is in the direction in which the potential decreases steepest. (ii)Its magnitude is given by the change in the magnitude of potential per unit displacement nor mal to the equipotential sur face at the point. 2.7 P OTENTIAL E NERGY OF A SYSTEM OF CHARGES Consider first the simple case of two charges q1and q2 with position vector r1 and r2 relative to some origin. Let us calculate the work done (externally) in building up this configuration. This means that we consider the charges q1 and q2 initially at infinity and determine the work done by an external agency to bring the charges to the given locations. Suppose, first the charge q1 is brought from infinity to the point r1. There is no external field against which work needs to be done, so work done in bringing q1 from infinity to r1 is zero. This charge produces a potential in space given by Vq r1 01 11 4=πεP where r1P is the distance of a point

P in space from the location of q1. From the definition of potential, work done in bringing charge q2 from infinity to the point r2 is q2 times the potential at r2 due to q1: work done on q2 = 1 401 2 12πεq q rFIGURE 2.12 From the potential to the field. Rationalised 2023-24 Physics 56where r12 is the distance between points 1 and 2. Since electrostatic force is conservative, this work gets stored in the form of potential energy of the system. Thus, the potential energy of a system of two charges q1 and q2 is Uq q r=1 401 2 12πε(2.22) Obviously, if q2 was brought first to its present location and q1 brought later , the potential ener gy U would be the same. More generally, the potential energy expression, Eq. (2.22), is unaltered whatever way the charges are brought to the specified locations, because of path -independence of work for electrostatic force. Equation (2.22) is true for any sign of q1and q2. If q1q2 > 0, potential energy is positive. This is as expected, since for like charges (q1q2 > 0), electrostatic force is repulsive and a positive amount of work is needed to be done against this force to bring the charges from infinity to a finite distance apart. For unlike charges (q1 q2 < 0), the electrostatic force is attractive. In that case, a positive amount of work is needed against this force to take the charges from the given location to infinity. In other words, a negative amount of work is needed for the reverse path (from infinity to the present locations), so the potential energy is negative. Equation (2.22) is easily generalised for a system of any number of point charges. Let us calculate the potential energy of a system of three charges q1, q2 and q3 located at r1, r2, r3, respectively. To bring q1 first from infinity to r1, no work is required. Next we bring q2 from infinity to r2. As before, work done in this step is 1 2 2 1 2 0 121( )4q qq Vrε=πr (2.23) The charges q1 and q2 produce a potential, which at any point P is given by Vq rq r1 2 01 12 21 4,= + πεP P(2.24) Work done next in bringing q3 from infinity to the point r3 is q3 times V1, 2 at r3 q Vq q rq q r3 1 2 3 01 3 132 3 231 4,( )r= + πε(2.25) The total work done in assembling the charges at the given locations is obtained by adding the work done in different steps [Eq. (2.23) and Eq. (2.25)], Uq q rq q rq q r= + + 1 401 2 121 3 132 3 23 πε(2.26) Again, because of the conservative nature of the electrostatic force (or equivalently, the path independence of work done), the final expression for U, Eq. (2.26), is independent of the manner in which the configuration is assembled. The potential energyFIGURE 2.13 Potential energy of a system of charges q1 and q2 is directly proportional to the product of charges and inversely to the distance between them. FIGURE 2.14 Potential energy of a system of three charges is given by Eq. (2.26), with the notation given in the figure. Rationalised 2023-24 Electrostatic Potential and Capacitance 57 EXAMPLE 2.4is characteristic of the present state of configuration, and not the way the state is achieved . Example 2.4 Four charges are arranged at the corners of a square ABCD of side d, as shown in Fig. 2.15.(a) Find the work required to put together this arrangement. (b) A charge q0 is brought to the centre E of the square, the four charges being held fixed at its corners. How much extra work is needed to do this? FIGURE 2.15 Solution (a) Since the work done depends on the final arrangement of the charges, and not on how they ar e put together , we calculate work needed for one way of putting the charges at A, B, C and D. Suppose, first the charge + q is brought to A, and then the charges – q, +q, and –q are brought to B, C and D, respectively. The total work needed can be calculated in steps: (i)Work needed to bring char ge +q to A when no char ge is pr esent elsewhere: this is zero. (ii)Work needed to bring – q to B when + q is at A. This is given by (charge at B) × (electrostatic potential at B due to charge + q at A) = − × = − qq dq d 4 402 0 π πε ε (iii)Work needed to bring char ge +q to C when + q is at A and – q is at B. This is given by (charge at C) × (potential at C due to charges at A and B) = +++− qq dq d 4 2 40 0 π π ε ε =−− q d2 0411 2 πε (iv)Work needed to bring –q to D when +q at A, –q at B, and +q at C. This is given by (charge at D) × (potential at D due to charges at A, B and C) = −++−+ qq dq dq d 4 4 2 40 0 0 π π π ε ε ε =−− q d2 0421 2 πε Rationalised 2023-24 Physics 58 EXAMPLE 2.4Add the work done in steps (i), (ii), (iii) and (iv). The total work required is =−+ + − + − q d2 040 1 11 221 2 πε( ) ( ) =−−( )q d2 044 2πε The work done depends only on the arrangement of the charges, and not how they are assembled. By definition, this is the total electrostatic energy of the charges. (Students may try calculating same work/energy by taking charges in any other order they desire and convince themselves that the energy will remain the same.) (b) The extra work necessary to bring a charge q0 to the point E when the four charges are at A, B, C and D is q0 × (electrostatic potential at E due to the charges at A, B, C and D). The electrostatic potential at E is clearly zero since potential due to A and C is cancelled by that due to B and D. Hence, no work is required to bring any charge to point E. 2.8 P OTENTIAL E NERGY IN AN EXTERNAL F IELD 2.8.1 Potential energy of a single charge In Section 2.7, the source of the electric field was specified – the charges and their locations - and the potential energy of the system of those charges was determined. In this section, we ask a related but a distinct question. What is the potential energy of a charge q in a given field? This question was, in fact, the starting point that led us to the notion of the electrostatic potential (Sections 2.1 and 2.2). But here we address this question again to clarify in what way it is different from the discussion in Section 2.7. The main difference is that we are now concerned with the potential energy of a char ge (or char ges) in an exter nal field. The exter nal field E is not produced by the given charge(s) whose potential energy we wish to calculate. E is produced by sources external to the given charge(s).The external sources may be known, but often they are unknown or unspecified; what is specified is the electric field E or the electrostatic potential V due to the exter nal sour ces. W e assume that the char ge q does not significantly affect the sources producing the external field. This is true if q is very small, or the external sources are held fixed by other unspecified forces. Even if q is finite, its influence on the external sources may still be ignored in the situation when very strong sources far away at infinity produce a finite field E in the region of interest. Note again that we are interested in determining the potential energy of a given charge q (and later , a system of char ges) in the exter nal field; we ar e not inter ested in the potential energy of the sources producing the external electric field. The external electric field E and the corresponding external potential V may vary from point to point. By definition, V at a point P is the work done in bringing a unit positive char ge from infinity to the point P . Rationalised 2023-24 Electrostatic Potential and Capacitance 59 EXAMPLE 2.5(We continue to take potential at infinity to be zer o.) Thus, work done in bringing a char ge q from infinity to the point P in the exter nal field is q V. This work is stored in the form of potential energy of q . If the point P has position vector r relative to some origin, we can write: Potential energy of q at r in an external field = qV(r) (2.27) where V(r) is the exter nal potential at the point r. Thus, if an electron with charge q = e = 1.6×10–19 C is accelerated by a potential difference of DV = 1 volt, it would gain energy of qDV = 1.6 × 10–19J. This unit of energy is defined as 1 electron volt or 1eV, i.e., 1 eV=1.6 × 10–19J. The units based on eV are most commonly used in atomic, nuclear and particle physics, (1 keV = 103eV = 1.6 × 10–16J, 1 MeV = 106eV = 1.6 × 10–13J, 1 GeV = 109eV = 1.6 × 10–10J and 1 TeV = 1012eV = 1.6 × 10–7J). [This has already been defined on Page 117, XI Physics Part I, Table 6.1.] 2.8.2 Potential energy of a system of two charges in an external field Next, we ask: what is the potential energy of a system of two charges q1 and q2 located at r1and r2, respectively, in an external field? First, we calculate the work done in bringing the charge q1 from infinity to r1. Work done in this step is q1 V(r1), using Eq. (2.27). Next, we consider the work done in bringing q2 to r2. In this step, work is done not only against the external field E but also against the field due to q1. Work done on q2 against the exter nal field = q2 V (r2) Work done on q2 against the field due to q1 1 2 124oq q rε=π wher e r12 is the distance between q1 and q2. We have made use of Eqs. (2.27) and (2.22). By the superposition principle for fields, we add up the work done on q2 against the two fields (E and that due to q1): Work done in bringing q2 to r2 1 2 2 2 12( )4oq qq Vrε= +πr (2.28) Thus, Potential energy of the system = the total work done in assembling the configuration 1 2 1 1 2 2 0 12( ) ( )4q qq V q Vrε= + +πr r (2.29) Example 2.5 (a)Determine the electrostatic potential energy of a system consisting of two charges 7 mC and –2 mC (and with no external field) placed at (–9 cm, 0, 0) and (9 cm, 0, 0) respectively. (b)How much work is required to separate the two charges infinitely away from each other? Rationalised 2023-24 Physics 60 EXAMPLE 2.5(c)Suppose that the same system of charges is now placed in an external electric field E = A (1/r2); A = 9 × 105 NC–1 m2. What would the electrostatic energy of the configuration be? Solution (a)12 9 1 2 01 7 (2) 109 104 0.18q qUrε−× − ×= = × ×π= –0.7 J. (b)W = U2 – U1 = 0 – U = 0 – (–0.7) = 0.7 J. (c)The mutual interaction energy of the two charges remains unchanged. In addition, there is the energy of interaction of the two charges with the external electric field. We find, ( ) ( )1 1 2 27 C 2 C 0.09m 0.09mq V q V A Aμ − μ+ = +r r and the net electrostatic energy is ( ) ( )1 2 1 1 2 2 0 127 C 2 C0.7 J4 0.09m 0.09mq qq V q V A Arεμ − μ+ + = + −πr r 70 20 0.7 49.3 J= − − = 2.8.3 Potential energy of a dipole in an external field Consider a dipole with charges q1 = +q and q2 = –q placed in a uniform electric field E, as shown in Fig. 2.16. As seen in the last chapter, in a uniform electric field, the dipole experiences no net force; but experiences a torque t t t t t given by t = t = t = t = t = p × E (2.30) which will tend to rotate it (unless p is parallel or antiparallel to E). Suppose an external torque ttt ttext is applied in such a manner that it just neutralises this torque and rotates it in the plane of paper from angle q0 to angle q1 at an infinitesimal angular speed and without angular acceleration. The amount of work done by the external torque will be given by ( ) cos cospEθ θ0 1 = − (2.31) This work is stored as the potential energy of the system. We can then associate potential energy U(q) with an inclination q of the dipole. Similar to other potential energies, there is a freedom in choosing the angle where the potential energy U is taken to be zero. A natural choice is to take q0 = p / 2. (A n explanation for it is provided towards the end of discussion.) We can then write, (2.32)FIGURE 2.16 Potential energy of a dipole in a uniform external field. Rationalised 2023-24 Electrostatic Potential and Capacitance 61 EXAMPLE 2.6This expression can alternately be understood also from Eq. (2.29). We apply Eq. (2.29) to the present system of two charges + q and – q. The potential energy expression then reads ()( ) ( )2 1 2[ ]4 2qU q V Vaθε0= − −′π ×r r (2.33) Here, r1 and r2 denote the position vectors of + q and – q. Now, the potential difference between positions r1 and r2 equals the work done in bringing a unit positive charge against field from r2 to r1. The displacement parallel to the force is 2 a cosq. Thus, [V(r1)–V (r2)] = –E × 2a cosq . We thus obtain, ()2 2 cos4 2 4 2θ θε ε0 0= − − = − − ′π × π ×p.Eq qU pEa a(2.34) We note that U¢ (q) differs fr om U(q) by a quantity which is just a constant for a given dipole. Since a constant is insignificant for potential energy, we can drop the second term in Eq. (2.34) and it then reduces to Eq. (2.32). We can now understand why we took q0=p/2. In this case, the work done against the exter nal field E in bringing + q and – q are equal and opposite and cancel out, i.e., q [V (r1) – V (r2)]=0. Example 2.6 A molecule of a substance has a permanent electric dipole moment of magnitude 10–29 C m. A mole of this substance is polarised (at low temperature) by applying a strong electrostatic field of magnitude 106 V m–1. The direction of the field is suddenly changed by an angle of 60º. Estimate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity, assume 100% polarisation of the sample. Solution Here, dipole moment of each molecules = 10–29 C m As 1 mole of the substance contains 6 × 1023 molecules, total dipole moment of all the molecules, p = 6 × 1023 × 10–29 C m = 6 × 10–6 C m Initial potential energy, Ui = –pE cos q = –6×10–6×106 cos 0° = –6 J Final potential energy (when q = 60°), Uf = –6 × 10–6 × 106 cos 60° = –3 J Change in potential energy = –3 J – (–6J) = 3 J So, there is loss in potential energy. This must be the energy released by the substance in the form of heat in aligning its dipoles. 2.9 E LECTROST ATICS OF CONDUCTORS Conductors and insulators were described briefly in Chapter 1. Conductors contain mobile charge carriers. In metallic conductors, these charge carriers are electrons. In a metal, the outer (valence) electrons part away from their atoms and are free to move. These electrons are free within the metal but not free to leave the metal. The free electrons form a kind of 'gas'; they collide with each other and with the ions, and move randomly in different directions. In an external electric field, they drift against the direction of the field. The positive ions made up of the nuclei and the bound electrons remain held in their fixed positions. In electrolytic conductors, the charge carriers are both positive and negative ions; but Rationalised 2023-24 Physics 62the situation in this case is more involved – the movement of the charge carriers is affected both by the external electric field as also by the so-called chemical for ces (see Chapter 3). W e shall r estrict our discussion to metallic solid conductors. Let us note important results regarding electrostatics of conductors. 1. Inside a conductor , electr ostatic field is zer o Consider a conductor , neutral or char ged. Ther e may also be an exter nal electrostatic field. In the static situation, when there is no current inside or on the sur face of the conductor , the electric field is zer o everywher e inside the conductor . This fact can be taken as the defining pr operty of a conductor . A conductor has fr ee electr ons. As long as electric field is not zero, the free charge carriers would experience force and drift. In the static situation, the free charges have so distributed themselves that the electric field is zero everywhere inside. Electrostatic field is zero inside a conductor . 2.At the sur face of a char ged conductor , electr ostatic field must be normal to the surface at every point If E were not normal to the surface, it would have some non-zero component along the surface. Free charges on the surface of the conductor would then experience force and move. In the static situation, therefore, E should have no tangential component. Thus electrostatic field at the surface of a char ged conductor must be nor mal to the sur face at every point. (For a conductor without any surface charge density, field is zero even at the surface.) See result 5. 3.The interior of a conductor can have no excess charge in the static situation A neutral conductor has equal amounts of positive and negative charges in every small volume or surface element. When the conductor is charged, the excess charge can reside only on the surface in the static situation. This follows from the Gauss's law. Consider any arbitrary volume element v inside a conductor . On the closed sur face S bounding the volume element v, electrostatic field is zero. Thus the total electric flux through S is zero. Hence, by Gauss's law, there is no net charge enclosed by S. But the surface S can be made as small as you like, i.e., the volume v can be made vanishingly small. This means there is no net charge at any point inside the conductor , and any excess char ge must r eside at the sur face. 4.Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface This follows from results 1 and 2 above. Since E = 0 inside the conductor and has no tangential component on the surface, no work is done in moving a small test charge within the conductor and on its surface. That is, there is no potential difference between any two points inside or on the sur face of the conductor . Hence, the r esult. If the conductor is char ged, Rationalised 2023-24 Electrostatic Potential and Capacitance 63electric field normal to the surface exists; this means potential will be different for the surface and a point just outside the surface. In a system of conductors of arbitrary size, shape and charge configuration, each conductor is characterised by a constant value of potential, but this constant may differ from one conductor to the other . 5.Electric field at the surface of a charged conductor 0ˆσ ε=E n (2.35) where s is the surface charge density and ˆn is a unit vector normal to the surface in the outward direction. To derive the result, choose a pill box (a short cylinder) as the Gaussian surface about any point P on the surface, as shown in Fig. 2.17. The pill box is partly inside and partly outside the surface of the conductor. It has a small area of cross section d S and negligible height. Just inside the surface, the electrostatic field is zero; just outside, the field is normal to the surface with magnitude E. Thus, the contribution to the total flux through the pill box comes only from the outside (circular) cross-section of the pill box. This equals ± EdS (positive for s > 0, negative for s < 0), since over the small area dS, E may be considered constant and E and dS are parallel or antiparallel. The charge enclosed by the pill box is sdS. By Gauss's law EdS = 0Sσ δ ε E = 0σ ε(2.36) Including the fact that electric field is normal to the surface, we get the vector relation, Eq. (2.35), which is true for both signs of s. For s > 0, electric field is normal to the surface outward; for s < 0, electric field is normal to the surface inward. 6.Electrostatic shielding Consider a conductor with a cavity, with no charges inside the cavity. A remarkable result is that the electric field inside the cavity is zero, whatever be the size and shape of the cavity and whatever be the charge on the conductor and the external fields in which it might be placed. We have proved a simple case of this result already: the electric field inside a charged spherical shell is zero. The proof of the result for the shell makes use of the spherical symmetry of the shell (see Chapter 1). But the vanishing of electric field in the (charge-free) cavity of a conductor is, as mentioned above, a very general result. A related result is that even if the conductorFIGURE 2.17 The Gaussian surface (a pill box) chosen to derive Eq. (2.35) for electric field at the surface of a charged conductor . Rationalised 2023-24 Physics 64 EXAMPLE 2.7FIGURE 2.18 The electric field inside a cavity of any conductor is zero. All charges reside only on the outer surface of a conductor with cavity. (There are no charges placed in the cavity.)is charged or charges are induced on a neutral conductor by an external field, all charges reside only on the outer surface of a conductor with cavity. The proofs of the results noted in Fig. 2.18 are omitted here, but we note their important implication. Whatever be the charge and field configuration outside, any cavity in a conductor remains shielded from outside electric influence: the field inside the cavity is always zero . This is known as electrostatic shielding . The effect can be made use of in protecting sensitive instruments from outside electrical influence. Figure 2.19 gives a summary of the important electrostatic properties of a conductor. Example 2.7 (a)A comb run through one's dry hair attracts small bits of paper. Why? What happens if the hair is wet or if it is a rainy day? (Remember , a paper does not conduct electricity.) (b)Ordinary rubber is an insulator . But special rubber tyr es of aircraft are made slightly conducting. Why is this necessary? (c)Vehicles carrying inflammable materials usually have metallic ropes touching the ground during motion. Why? (d)A bird perches on a bare high power line, and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why? Solution (a)This is because the comb gets charged by friction. The molecules in the paper gets polarised by the charged comb, resulting in a net force of attraction. If the hair is wet, or if it is rainy day, friction between hair and the comb reduces. The comb does not get charged and thus it will not attract small bits of paper .FIGURE 2.19 Some important electr ostatic pr operties of a conductor . Rationalised 2023-24 Electrostatic Potential and Capacitance 65 EXAMPLE 2.7(b)To enable them to conduct charge (produced by friction) to the ground; as too much of static electricity accumulated may result in spark and result in fire. (c)Reason similar to (b). (d)Current passes only when there is difference in potential. 2.10 D IELECTRICS AND P OLARISATION Dielectrics are non-conducting substances. In contrast to conductors, they have no (or negligible number o f) charge carriers. Recall from Section 2.9 what happens when a conductor is placed in an external electric field. The free charge carriers move and charge distribution in the conductor adjusts itself in such a way that the electric field due to induced charges opposes the external field within the conductor. This happens until, in the static situation, the two fields cancel each other and the net electrostatic field in the conductor is zero. In a dielectric, this free movement of charges is not possible. It turns out that the external field induces dipole moment by stretching or re-orienting molecules of the dielectric. The collective effect of all the molecular dipole moments is net charges on the surface of the dielectric which produce a field that opposes the external field. Unlike in a conductor, however, the opposing field so induced does not exactly cancel the external field. It only reduces it. The extent of the effect depends on the nature of the dielectric. To understand the effect, we need to look at the charge distribution of a dielectric at the molecular level. The molecules of a substance may be polar or non-polar. In a non-polar molecule, the centres of positive and negative charges coincide. The molecule then has no permanent (or intrinsic) dipole moment. Examples of non-polar molecules are oxygen (O2) and hydrogen (H2) molecules which, because of their symmetry, have no dipole moment. On the other hand, a polar molecule is one in which the centres of positive and negative charges are separated (even when there is no external field). Such molecules have a permanent dipole moment. An ionic molecule such as HCl or a molecule of water (H2O) are examples of polar molecules.FIGURE 2.20 Difference in behaviour of a conductor and a dielectric in an external electric field. FIGURE 2.21 Some examples of polar and non-polar molecules. Rationalised 2023-24 Physics 66In an external electric field, the positive and negative charges of a non- polar molecule are displaced in opposite directions. The displacement stops when the external force on the constituent charges of the molecule is balanced by the restoring force (due to internal fields in the molecule). The non-polar molecule thus develops an induced dipole moment. The dielectric is said to be polarised by the exter nal field. W e consider only the simple situation when the induced dipole moment is in the direction of the field and is proportional to the field strength. (Substances for which th is assumption is true are called linear isotropic dielectrics.) The induced dipole moments of different molecules add up giving a net dipole moment of the dielectric in the presence of the external field. A dielectric with polar molecules also develops a net dipole moment in an external field, but for a different reason. In the absence of any external field, the different permanent dipoles are oriented randomly due to thermal agitation; so the total dipole moment is zero. When an external field is applied, the individual dipole moments tend to align with the field. When summed overall the molecules, there is then a net dipole moment in the direction of the external field, i.e., the dielectric is polarised. The extent of polarisation depends on the relative strength of two mutually opposite factors: the dipole potential energy in the external field tending to align the dipoles with the field and thermal energy tending to disrupt the alignment. There may be, in addition, the 'induced dipole moment' effect as for non-polar molecules, but generally the alignment effect is more important for polar molecules. Thus in either case, whether polar or non-polar , a dielectric develops a net dipole moment in the presence of an external field. The dipole moment per unit volume is called polarisation and is denoted by P. For linear isotropic dielectrics, 0ε χ=P Ee (2.37) where ce is a constant characteristic of the dielectric and is known as the electric susceptibility of the dielectric medium. It is possible to relate ce to the molecular properties of the substance, but we shall not pursue that here. The question is: how does the polarised dielectric modify the original external field inside it? Let us consider, for simplicity, a rectangular dielectric slab placed in a uniform external field E0 parallel to two of its faces. The field causes a uniform polarisation P of the dielectric. ThusFIGURE 2.22 A dielectric develops a net dipole moment in an external electric field. (a) Non-polar molecules, (b) Polar molecules. Rationalised 2023-24 Electrostatic Potential and Capacitance 67every volume element Dv of the slab has a dipole moment P Dv in the direction of the field. The volume element Dv is macroscopically small but contains a very large number of molecular dipoles. Anywhere inside the dielectric, the volume element Dv has no net charge (though it has net dipole moment). This is, because, the positive charge of one dipole sits close to the negative charge of the adjacent dipole. However , at the sur faces of the dielectric nor mal to the electric field, there is evidently a net charge density. As seen in Fig 2.23, the positive ends of the dipoles remain unneutralised at the right surface and the negative ends at the left surface. The unbalanced charges are the induced charges due to the external field. Thus, the polarised dielectric is equivalent to two charged surfaces with induced surface charge densities, say sp and – sp. Clearly, the field produced by these surface charges opposes the external field. The total field in the dielectric is, thereby, reduced from the case when no dielectric is present. We should note that the surface charge density ±sp arises from bound (not free charges) in the dielectric. 2.11 C APACITORS AND C APACITANCE A capacitor is a system of two conductors separated by an insulator (Fig. 2.24). The conductors have charges, say Q1 and Q2, and potentials V1 and V2. Usually, in practice, the two conductors have charges Q and – Q, with potential difference V = V1 – V2 between them. We shall consider only this kind of charge configuration of the capacitor. (Even a single conductor can be used as a capacitor by assuming the other at infinity.) The conductors may be so charged by connecting them to the two terminals of a battery. Q is called the charge of the capacitor, though this, in fact, is the charge on one of the conductors – the total charge of the capacitor is zero. The electric field in the region between the conductors is proportional to the charge Q. That is, if the charge on the capacitor is, say doubled, the electric field will also be doubled at every point. (This follows from the direct proportionality between field and charge implied by Coulomb's law and the superposition principle.) Now, potential difference V is the work done per unit positive charge in taking a small test charge from the conductor 2 to 1 against the field. Consequently, V is also proportio nal to Q, and the ratio Q/V is a constant: QCV= (2.38) The constant C is called the capacitance of the capacitor. C is independent of Q or V, as stated above. The capacitance C depends only on theFIGURE 2.23 A uniformly polarised dielectric amounts to induced surface charge density, but no volume charge density. FIGURE 2.24 A system of two conductors separated by an insulator for ms a capacitor . Rationalised 2023-24 Physics 68geometrical configuration (shape, size, separation) of the system of two conductors. [As we shall see later, it also depends on the nature of the insulator (dielectric) separating the two conductors.] The SI unit of capacitance is 1 farad (=1 coulomb volt-1) or 1 F = 1 C V–1. A capacitor with fixed capacitance is symbolically shown as - --||---, while the one with variable capacitance is shown as . Equation (2.38) shows that for large C, V is small for a given Q. This means a capacitor with large capacitance can hold large amount of charge Q at a r elatively small V. This is of practical importance. High potential difference implies strong electric field around the conductors. A strong electric field can ionise the surrounding air and accelerate the charges so produced to the oppositely charged plates, thereby neutralising the charge on the capacitor plates, at least partly. In other words, the charge of the capacitor leaks away due to the reduction in insulating power of the intervening medium. The maximum electric field that a dielectric medium can withstand without break-down (of its insulating property) is called its dielectric strength ; for air it is about 3 × 106 Vm–1. For a separation between conductors of the order of 1 cm or so, this field corresponds to a potential difference of 3 × 104 V between the conductors. Thus, for a capacitor to store a large amount of charge without leaking, its capacitance should be high enough so that the potential difference and hence the electric field do not exceed the break-down limits. Put differently, there is a limit to the amount of charge that can be stored on a given capacitor without significant leaking. In practice, a farad is a very big unit; the most common units are its sub-multiples 1 mF = 10–6 F, 1 nF = 10–9 F, 1 pF = 10–12 F, etc. Besides its use in storing charge, a capacitor is a key element of most ac circuits with important functions, as described in Chapter 7. 2.12 T HE PARALLEL PLATE CAPACITOR A parallel plate capacitor consists of two large plane parallel conducting plates separated by a small distance (Fig. 2.25). We first take the intervening medium between the plates to be vacuum. The effect of a dielectric medium between the plates is discussed in the next section. Let A be the area of each plate and d the separation between them. The two plates have charges Q and – Q. Since d is much smaller than the linear dimension of the plates (d2 << A), we can use the result on electric field by an infinite plane sheet of uniform surface charge density (Section 1.15). Plate 1 has surface charge density s = Q/A and plate 2 has a surface charge density – s. Using Eq. (1.33), the electric field in different regions is: Outer region I (region above the plate 1), 0 002 2Eσ σ ε ε= − = (2.39)FIGURE 2.25 The parallel plate capacitor . Rationalised 2023-24 Electrostatic Potential and Capacitance 69Outer region II (region below the plate 2), 0 002 2Eσ σ ε ε= − = (2.40) In the inner region between the plates 1 and 2, the electric fields due to the two charged plates add up, giving 0 0 0 02 2QEAσ σ σ ε ε ε ε= + = = (2.41) The direction of electric field is from the positive to the negative plate. Thus, the electric field is localised between the two plates and is uniform throughout. For plates with finite area, this will not be true near the outer boundaries of the plates. The field lines bend outward at the edges — an effect called 'fringing of the field'. By the same token, s will not be strictly uniform on the entire plate. [E and s are related by Eq. (2.35).] However , for d2 << A, these ef fects can be ignor ed in the r egions sufficiently far from the edges, and the field there is given by Eq. (2.41). Now for uniform electric field, potential difference is simply the electric field times the distance between the plates, that is, 01QdV E dAε= = (2.42) The capacitance C of the parallel plate capacitor is then QCV= = 0A dε= (2.43) which, as expected, depends only on the geometry of the system. For typical values like A = 1 m2, d = 1 mm, we get 12 2 –1 –2 2 9 38.85 10 C N m 1m8.85 10 F10 mC− − −× ×= = × (2.44) (You can check that if 1F= 1C V–1 = 1C (NC–1m)–1 = 1 C2 N–1m–1.) This shows that 1F is too big a unit in practice, as r emarked earlier . Another way of seeing the 'bigness' of 1F is to calculate the area of the plates needed to have C = 1F for a separation of, say 1 cm: 0CdAε= =2 9 2 12 2 –1 –21F 10 m10 m8.85 10 C N m− −×=×(2.45) which is a plate about 30 km in length and breadth! 2.13 E FFECT OF DIELECTRIC ON CAPACITANCE With the understanding of the behaviour of dielectrics in an external field developed in Section 2.10, let us see how the capacitance of a parallel plate capacitor is modified when a dielectric is present. As before, we have two large plates, each of area A, separated by a distance d. The charge on the plates is ± Q, corresponding to the charge density ± s (with s = Q/A). When there is vacuum between the plates, 0 0Eσ ε=Factors affecting capacitance, capacitors in action Interactive Java tutorial http://micro.magnet.fsu.edu/electromag/java/capacitance/ Rationalised 2023-24 Physics 70and the potential difference V0 is V0 = E0d The capacitance C0 in this case is 0 0 0Q ACV dε= = (2.46) Consider next a dielectric inserted between the plates fully occupying the intervening region. The dielectric is polarised by the field and, as explained in Section 2.10, the effect is equivalent to two charged sheets (at the surfaces of the dielectric normal to the field) with surface charge densities sp and – sp. The electric field in the dielectric then corresponds to the case when the net surface charge density on the plates is ±(s – sp). That is, 0PEσ σ ε−= (2.47) so that the potential difference across the plates is 0PV E d dσ σ ε−= = (2.48) For linear dielectrics, we expect sp to be proportional to E0, i.e., to s. Thus, (s – sp) is proportional to s and we can write PKσσ σ− = (2.49) wher e K is a constant characteristic of the dielectric. Clearly, K > 1. W e then have 0 0d QdVK A Kσ ε ε= = (2.50) The capacitance C, with dielectric between the plates, is then 0KA QCV dε= = (2.51) The pr oduct e0K is called the permittivity of the medium and is denoted by e e = e0 K (2.52) For vacuum K = 1 and e = e0; e0 is called the permittivity of the vacuum . The dimensionless ratio 0Kε ε= (2.53) is called the dielectric constant of the substance. As remarked before, from Eq. (2.49), it is clear that K is greater than 1. From Eqs. (2.46) and (2. 51) 0CKC= (2.54) Thus, the dielectric constant of a substance is the factor (>1) by which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor . Though we arrived at Rationalised 2023-24 Electrostatic Potential and Capacitance 71 EXAMPLE 2.8Eq. (2.54) for the case of a parallel plate capacitor , it holds good for any type of capacitor and can, in fact, be viewed in general as a definition of the dielectric constant of a substance. Example 2.8 A slab of material of dielectric constant K has the same area as the plates of a parallel-plate capacitor but has a thickness (3/4) d, where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates? Solution Let E0 = V0/d be the electric field between the plates when there is no dielectric and the potential difference is V0. If the dielectric is now inserted, the electric field in the dielectric will be E = E0/K. The potential difference will then be 0 01 3( ) ( )4 4EV E d dK= + 0 01 3 3( )4 4 4KE d VK K+= + = The potential difference decreases by the factor (K + 3)/4 K while the free charge Q0 on the plates remains unchanged. The capacitance thus increases 0 0 0 04 4 3 3Q Q K KC CV K V K= = =+ + 2.14 C OMBINA TION OF CAPACITORS We can combine several capacitors of capacitance C1, C2,…, Cn to obtain a system with some effective capacitance C. The effective capacitance depends on the way the individual capacitors are combined. Two simple possibilities are discussed below. 2.14.1 Capacitors in series Figure 2.26 shows capacitors C1 and C2 combined in series. The left plate of C1 and the right plate of C2 are connected to two terminals of a battery and have charges Q and – Q , respectively. It then follows that the right plate of C1 has charge – Q and the left plate of C2 has charge Q. If this was not so, the net charge on each capacitor would not be zero. This would result in an electric field in the conductor connecting C1and C2. Charge would flow until the net charge on both C1 and C2 is zero and there is no electric field in the conductor connecting C1 and C2. Thus, in the series combination, charges on the two plates (±Q) are the same on each capacitor . The totalFIGURE 2.26 Combination of two capacitors in series. FIGURE 2.27 Combination of n capacitors in series. Rationalised 2023-24 Physics 72potential drop V across the combination is the sum of the potential drops V1 and V2 across C1 and C2, respectively . V = V1 + V2 = 1 2Q Q C C+ (2.55) i.e., 1 21 1 V Q C C= + , (2.56) Now we can regard the combination as an effective capacitor with charge Q and potential dif ference V. The effective capacitance of the combination is QCV= (2.57) We compar e Eq. (2.57) with Eq. (2.56), and obtain 1 21 1 1 C C C= + (2.58) The proof clearly goes through for any number of capacitors arranged in a similar way. Equation (2.55), for n capacitors arranged in series, generalises to 1 2 n 1 2 n... ...Q Q QV V V VC C C= + + + = + + + (2.59) Following the same steps as for the case of two capacitors, we get the general formula for effective capacitance of a series combination of n capacitors: 1 2 3 n1 1 1 1 1...C C C C C= + + + + (2.60) 2.14.2 Capacitors in parallel Figure 2.28 (a) shows two capacitors arranged in parallel. In this case, the same potential difference is applied across both the capacitors. But the plate charges (±Q1) on capacitor 1 and the plate charges (± Q2) on the capacitor 2 are not necessarily the same: Q1 = C1V, Q2 = C2V (2.61) The equivalent capacitor is one with charge Q = Q1 + Q2(2.62) and potential dif ference V. Q = CV = C1V + C2V (2.63) The effective capacitance C is, from Eq. (2.63), C = C1 + C2(2.64) The general formula for effective capacitance C for parallel combination of n capacitors [Fig. 2.28 (b)] follows similarly, Q = Q1 + Q2 + ... + Qn(2.65) i.e., CV = C1V + C2V + ... CnV(2.66) which gives C = C1 + C2 + ... Cn(2.67)FIGURE 2.28 Parallel combination of (a) two capacitors, (b) n capacitors. Rationalised 2023-24 Electrostatic Potential and Capacitance 73 EXAMPLE 2.9FIGURE 2.29Example 2.9 A network of four 10 mF capacitors is connected to a 500 V supply, as shown in Fig. 2.29. Determine (a) the equivalent capacitance of the network and (b) the charge on each capacitor . (Note, the charge on a capacitor is the charge on the plate with higher potential, equal and opposite to the charge on the plate with lower potential.) Solution (a)In the given network, C1, C2 and C3 are connected in series. The effective capacitance C ¢ of these three capacitors is given by 1 2 31 1 1 1 C C C C= + +′ For C1 = C2 = C3 = 10 mF, C¢ = (10/3) mF. The network has C¢ and C4 connected in parallel. Thus, the equivalent capacitance C of the network is C = C¢ + C4 = 10 310+ mF =13.3 mF (b)Clearly, from the figure, the charge on each of the capacitors, C1, C2 and C3 is the same, say Q. Let the charge on C4 be Q¢. Now, since the potential difference across AB is Q/C1, across BC is Q/C2, across CD is Q/C3 , we have 1 2 3500 VQ Q Q C C C+ + = . Also, Q¢/C4 = 500 V . This gives for the given value of the capacitances, 3 10500 F 1.7 10 C3Q V−= × μ = × and 3500 10 F 5.0 10 CQ V−= × μ = ×′ 2.15 E NERGY S TORED IN A CAPACITOR A capacitor , as we have seen above, is a system of two conductors with charge Q and – Q. To determine the energy stored in this configuration, consider initially two uncharged conductors 1 and 2. Imagine next a process of transferring charge from conductor 2 to conductor 1 bit by Rationalised 2023-24 Physics 74bit, so that at the end, conductor 1 gets charge Q. By charge conservation, conductor 2 has charge – Q at the end (Fig 2.30) . In transferring positive charge from conductor 2 to conductor 1, work will be done externally, since at any stage conductor 1 is at a higher potential than conductor 2. To calculate the total work done, we first calculate the work done in a small step involving transfer of an infinitesimal (i.e., vanishingly small) amount of charge. Consider the intermediate situation when the conductors 1 and 2 have charges Q¢ and –Q¢ respectively. At this stage, the potential difference V¢ between conductors 1 to 2 is Q¢/C, where C is the capacitance of the system. Next imagine that a small charge d Q¢ is transferr ed from conductor 2 to 1. W ork done in this step (d W), resulting in charge Q¢ on conductor 1 increasing to Q¢+ d Q¢, is given by QW V Q QCδ δ δ′= =′ ′ ′ (2.68) Integrating eq. (2.68) WQ CQCQ Q CQ Q =′=′= ∫ 02 021 2 2δ′ We can write the final r esult, in dif ferent ways 2 21 1 2 2 2QW CV QVC= = = (2.69) Since electrostatic force is conservative, this work is stored in the form of potential energy of the system. For the same reason, the final result for potential energy [Eq. (2.69)] is independent of the manner in which the charge configuration of the capacitor is built up. When the capacitor discharges, this stored-up energy is released. It is possible to view the potential energy of the capacitor as 'stored' in the electric field between the plates. To see this, consider for simplicity, a parallel plate capacitor [of area A (of each plate) and separation d between the plates]. Energy stored in the capacitor = 2 2 01 ( ) 2 2Q A d C Aσ ε= × (2.70) The surface charge density s is related to the electric field E between the plates, 0Eσ ε= (2.71) From Eqs. (2.70) and (2.71) , we get Energy stored in the capacitor U = ()2 0 1/2 E A dε× (2.72)FIGURE 2.30 (a) W ork done in a small step of building charge on conductor 1 from Q¢ to Q¢ + d Q¢. (b) Total work done in charging the capacitor may be viewed as stored in the energy of electric field between the plates. Rationalised 2023-24 Electrostatic Potential and Capacitance 75 EXAMPLE 2.10Note that Ad is the volume of the region between the plates (where electric field alone exists). If we define energy density as energy stored per unit volume of space , Eq (2.72) shows that Energy density of electric field, u =(1/2)e0E2(2.73) Though we derived Eq. (2.73) for the case of a parallel plate capacitor , the r esult on ener gy density of an electric field is, in fact, very general and holds true for electric field due to any configuration of charges. Example 2.10 (a) A 900 pF capacitor is charged by 100 V battery [Fig. 2.31(a)]. How much electrostatic energy is stored by the capacitor? (b) The capacitor is disconnected from the battery and connected to another 900 pF capacitor [Fig. 2.31(b)]. What is the electrostatic energy stored by the system? FIGURE 2.31 Solution (a)The charge on the capacitor is Q = CV = 900 × 10–12 F × 100 V = 9 × 10–8 C The energy stored by the capacitor is = (1/2) CV2 = (1/2) QV = (1/2) × 9 × 10–8C × 100 V = 4.5 × 10–6 J (b)In the steady situation, the two capacitors have their positive plates at the same potential, and their negative plates at the same potential. Let the common potential difference be V ¢. The Rationalised 2023-24 Physics 76charge on each capacitor is then Q¢ = CV¢. By charge conservation, Q¢ = Q/2. This implies V¢ = V/2. The total energy of the system is 6 1 12 ' ' 2.25 10 J2 4Q V QV−= × = = × Thus in going from (a) to (b), though no charge is lost; the final energy is only half the initial energy. Where has the remaining energy gone? There is a transient period before the system settles to the situation (b). During this period, a transient current flows from the first capacitor to the second. Energy is lost during this time in the form of heat and electromagnetic radiation. EXAMPLE 2.10 SUMMARY 1.Electr ostatic for ce is a conservative for ce. W ork done by an exter nal force (equal and opposite to the electrostatic force) in bringing a charge q from a point R to a point P is q(VP–VR), which is the difference in potential energy of charge q between the final and initial points. 2.Potential at a point is the work done per unit charge (by an external agency) in bringing a charge from infinity to that point. Potential at a point is arbitrary to within an additive constant, since it is the potential difference between two points which is physically significant. If potential at infinity is chosen to be zero; potential at a point with position vector r due to a point charge Q placed at the origin is given is given by 1( )4oQVrε=πr 3.The electrostatic potential at a point with position vector r due to a point dipole of dipole moment p placed at the origin is 2ˆ 1( )4ε=πp.rr oVr The result is true also for a dipole (with charges – q and q separated by 2a) for r >> a. 4.For a charge configuration q1, q2, ..., qn with position vectors r1, r2, ... rn, the potential at a point P is given by the superposition principle 1 2 0 1P 2P P1(...)4n nq q qVr r rε= + + +π wher e r1P is the distance between q1 and P , as and so on. 5.An equipotential surface is a surface over which potential has a constant value. For a point charge, concentric spheres centred at a location of the charge are equipotential surfaces. The electric field E at a point is perpendicular to the equipotential surface through the point. E is in the direction of the steepest decrease of potential. Rationalised 2023-24 Electrostatic Potential and Capacitance 776.Potential energy stored in a system of charges is the work done (by an external agency) in assembling the charges at their locations. Potential energy of two charges q1, q2 at r1, r2 is given by 1 2 0 121 4q q Urε=π where r12 is distance between q1 and q2. 7.The potential energy of a charge q in an external potential V(r) is qV(r). The potential energy of a dipole moment p in a uniform electric field E is –p.E. 8.Electrostatics field E is zero in the interior of a conductor; just outside the sur face of a char ged conductor , E is nor mal to the sur face given by 0ˆσ ε=E n where ˆn is the unit vector along the outward normal to the surface and s is the surface charge density. Charges in a conductor can reside only at its surface. Potential is constant within and on the surface of a conductor . In a cavity within a conductor (with no char ges), the electric field is zero. 9. A capacitor is a system of two conductors separated by an insulator . Its capacitance is defined by C = Q/V , where Q and –Q are the charges on the two conductors and V is the potential difference between them. C is determined purely geometrically, by the shapes, sizes and relative positions of the two conductors. The unit of capacitance is farad:, 1 F = 1 C V–1. For a parallel plate capacitor (with vacuum between the plates), C = 0A dε where A is the area of each plate and d the separation between them. 10.If the medium between the plates of a capacitor is filled with an insulating substance (dielectric), the electric field due to the charged plates induces a net dipole moment in the dielectric. This effect, called polarisation, gives rise to a field in the opposite direction. The net electric field inside the dielectric and hence the potential difference between the plates is thus reduced. Consequently, the capacitance C increases from its value C0 when there is no medium (vacuum), C = KC0 where K is the dielectric constant of the insulating substance. 11.For capacitors in the series combination, the total capacitance C is given by 1 2 31 1 1 1...C C C C= + + + In the parallel combination, the total capacitance C is: C = C1 + C2 + C3 + ... where C1, C2, C3... are individual capacitances. Rationalised 2023-24 Physics 7812.The energy U stored in a capacitor of capacitance C, with charge Q and voltage V is U QV CVQ C= = =1 21 21 222 The electric energy density (energy per unit volume) in a region with electric field is (1/2) e0E2. Physical quantity Symbol Dimensions Unit Remark Potential or V [M1 L2 T–3 A–1] V Potential difference is physically significant Capacitance C [M–1 L–2 T–4 A2] F Polarisation P [L–2 AT] C m-2Dipole moment per unit volume Dielectric constant K [Dimensionless] POINTS TO PONDER 1.Electrostatics deals with forces between charges at rest. But if there is a force on a charge, how can it be at rest? Thus, when we are talking of electrostatic force between charges, it should be understood that each charge is being kept at rest by some unspecified force that opposes the net Coulomb force on the charge. 2.A capacitor is so configured that it confines the electric field lines within a small region of space. Thus, even though field may have considerable strength, the potential difference between the two conductors of a capacitor is small. 3.Electric field is discontinuous across the surface of a spherical charged shell. It is zero inside and σε0ˆn outside. Electric potential is, however continuous across the surface, equal to q/4pe0R at the surface. 4.The torque p × E on a dipole causes it to oscillate about E. Only if there is a dissipative mechanism, the oscillations are damped and the dipole eventually aligns with E. 5.Potential due to a charge q at its own location is not defined – it is infinite. 6.In the expression qV(r) for potential energy of a charge q, V(r) is the potential due to external charges and not the potential due to q. As seen in point 5, this expression will be ill-defined if V(r) includes potential due to a charge q itself. Rationalised 2023-24 Electrostatic Potential and Capacitance 797.A cavity inside a conductor is shielded from outside electrical influences. It is worth noting that electrostatic shielding does not work the other way round; that is, if you put charges inside the cavity, the exterior of the conductor is not shielded from the fields by the inside charges. EXERCISES 2.1 Two charges 5 × 10–8 C and –3 × 10–8 C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero. 2.2 A regular hexagon of side 10 cm has a charge 5 mC at each of its vertices. Calculate the potential at the centre of the hexagon. 2.3 Two charges 2 mC and –2 mC are placed at points A and B 6 cm apart. (a) Identify an equipotential surface of the system. (b) What is the direction of the electric field at every point on this surface? 2.4 A spherical conductor of radius 12 cm has a charge of 1.6 × 10–7C distributed uniformly on its surface. What is the electric field (a) inside the sphere (b) just outside the sphere (c) at a point 18 cm from the centre of the sphere? 2.5 A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1pF = 10–12 F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6? 2.6 Three capacitors each of capacitance 9 pF are connected in series. (a) What is the total capacitance of the combination? (b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply? 2.7 Three capacitors of capacitances 2 pF , 3 pF and 4 pF ar e connected in parallel. (a) What is the total capacitance of the combination? (b) Determine the charge on each capacitor if the combination is connected to a 100 V supply. 2.8 In a parallel plate capacitor with air between the plates, each plate has an area of 6 × 10–3 m2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor . If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor? Rationalised 2023-24 Physics 802.9 Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates, (a) while the voltage supply remained connected. (b) after the supply was disconnected. 2.10 A 12pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor? 2.11 A 600pF capacitor is charged by a 200V supply. It is then disconnected from the supply and is connected to another unchar ged 600 pF capacitor . How much electr ostatic ener gy is lost in the process? Rationalised 2023-24 Electrostatics_wiki: An electrostatic effect: foam peanuts clinging to a cat's fur due to static electricity. The triboelectric effect causes an electrostatic charge to build up on the surface of the fur due to the cat's motions. The electric field of the charge causes polarization of the molecules of the foam due to electrostatic induction, resulting in a slight attraction of the light plastic pieces to the charged fur. [1][2][3][4] This effect is also the cause of static cling in clothes.Electrostatics Electrostatics is a branch of physics that studies electric charges at rest (static electricity). Since classical times, it has been know n that some materials, such as amber, attract lightweight particles after rubbing. The Greek word for amber, ἤλεκτρον (ḗlektron), was thus the source of the word 'electricity'. Electrostatic phenomena arise from the forces that electric charges exert on each other. Such forces are described by Coulomb's law. Even though electrostatically induc ed forces seem to be rather weak, some electrostatic forces are relatively large. The force between an electron and a proton, which together make up a hydrogen atom, is about 36 orders of magnitude stronge r than the gravitational force acting be tween them. There are many examples of electrostatic phenomena, from those as simple as the attraction of plastic wrap to one's hand after it is removed from a package, to the apparently spontaneous explosion of grain silos, the damage of electronic compone nts during manufacturing, and photocopier & laser printer operation. Electrostatics involves the buildup of charge on the surface of objects due to contact with other surfaces. Although charge exchange happens whenever any two surfaces contact and separate, the effects of charge exchange are usually noticed only when at least one of the surfaces has a high resistance to electrical flow, because the charges that transfer are trapped there for a long enough time for their effects to be observed. These charges then remain on the object until they either bleed off to ground, or are quickly neutralized by a discharge. The familiar phenomenon of a static "shock" is caused by t he neutralization of charge built up in the body f rom contact with insulated surfaces. Coulomb's law states that: 'The magnitude of the electrostatic force of attraction or repulsion between two point charges is directly proportional to the produc t of the magnitudes of charges and inversely proportional to the square of the distance between them.' The force is along the straight line joining them. If the two charges have the same sign, the electrostatic force between them is repulsive; if they have different signs, the force between them is attractive. If is the distance (in meters) between two charges, then the force (in newtons) between two point charges and (in coulombs) is:Coulomb's lawThe electrostatic field (lines with arrows) of a nearby positive charge (+) causes the mobile charges in conductive objects to separate due to electrostatic induction. Negative charges (blue) are attracted and move to the surface of the object facing the external charge. Positive charges (red) are repelled and move to the surface facing away. These induced surface charges are exactly the right size and shape so their opposing electric field cancels the electric field of the external charge throughout the interior of the metal. Therefore, the electrostatic field everywhere inside a conductive object is zero, and the electrostatic potential is constant.where ε0 is the vacuum permittivity, or permittivity of free space:[5] The SI units of ε0 are equivalently A2⋅s4 ⋅kg−1⋅m−3 or C2⋅N−1⋅m−2 or F ⋅m−1. The Coulomb constant is: A single proton has a charge of e, and the electron has a charge of −e, where, These physical constants (ε0, ke, e) are currently defined so that e is exactly defined, and ε0 and ke are measured qua ntities. The electric field, , in units of newtons per coulomb or volts per meter, is a vector field that can be defined everywhere, except at the location of point charges (where it diverges to infinity).[6] It is defined as the electrostatic force in newtons on a hypot hetical small test charge at the point due to Coulomb's Law, divided by the magnitude of the charge in coulombs Electric field lines are useful for visualizing the electric field. Field lines begin on positive charge and terminate on negative charge. They are parallel to the direction of the electric field at each point, and the density of these field lines is a measure of the magnitude of the electric field at any gi ven poi nt.Electric fieldConsider a collection of particles of charge , located at points (called source points), the electric field at (called the field poi nt) is:[6] where is the displacement vector from a source point to the field point , and is a unit vector that indicates the direction of the field. For a single point charge at the origin, the magnitude of this electric field is and points away from that charge if it is positive. The fact that the force (and hence the field) can be calculated by summing over all the contributions due to individual source particles is an example of the superposition principle. The electric field produc ed by a distribution of charges is given by the volume charge density and can be obtained by conve rting this sum into a triple integral: Gauss' law states that "the total electric flux through any closed surface in free space of any shape drawn in an electric field is proportional to the total electric charge enclosed by the surface." Mathematically, Gauss's law takes the form of an integral equation: where is a volume element. If the charge is distributed over a surface or along a line, replace by or . The divergence theorem allows Gauss's Law to be written in differential form: where is the divergence operator. The definition of electrostatic potential, combined with the differential form of Gauss's law (above), provides a relationship between the potential Φ and the charge density ρ: This relationship is a form of Poisson's equation. In the absence of unpa ired electric charge, the equation becomes Laplace's equation:Gauss' law Poisson and Laplace equationsThe validity of the electrostatic approximation rests on the assumption that the electric field is irrotational: From Faraday's law, this assumption implies the absence or near-absence of time-varying m agnetic fields: In other words, electrostatics does not require the absence of magnetic fields or electric currents. Rather, if magnetic fields or electric currents do exist, they must not change with time, or in the worst-case, they must change with time only very slowly. In some problems, both electrostatics and magnetostatics may be required for accurate predictions, but the coupling between the two can still be ignored. Electrostatics and magnetostatics can bot h be seen as Galilean limits for electromagnetism.[7] As the electric field is irrotational, it is possible to express the electric field as the gradient of a scalar function, , called the electrostatic potential (also know n as the voltage). An electric field, , points from regions of high electric potential to regions of low electric potential, expressed mathematically as The gradient theorem can be used to establish that the electrostatic potential is the amount of work per unit charge required to move a charge from point to poi nt with the following line integral: From these equations, we see that the electric potential is constant in any region for which the electric field vanishes (such as occurs inside a conduc ting obj ect). A test particle's potential energy, , can be calculated from a line integral of the work, . We integrate from a point at infinity, and assume a collection of particles of charge , are already situated at the points . This potential energy (in Joules) is:Electrostatic approximation Electrostatic potential Electrostatic energywhere is the distance of each charge from the test charge , which situated at the point , and is the electric potential that would be at if the test charge were not present. If only two charges are present, the potential energy is . The total electric potential energy due a collection of N charges is calculating by a ssembling these particles one at a time: where the following sum from, j = 1 to N, excludes i = j: This electric potential, is what would be measured at if the charge were missing. This formula obviously excludes the (infinite) energy that would be required to assemble each point charge from a disperse cloud of charge. The sum over charges can be conve rted into an integral over charge density using the prescription : This second expression for electrostatic energy uses the fact that the electric field is the negative gradient of the electric potential, as well as vector calculus identities in a way that resembles integration by parts. These two integrals for electric field energy seem to indicate two mutually exclusive formulas for electrostatic energy density, namely and ; they yield equal values for the total electrostatic energy only if both are integrated ove r all space. On a conduc tor, a surface charge will experience a force in the presence of an electric field. This force is the average of the discontinuous electric field at the surface charge. This average in terms of the field just outside the surface amounts to: This pressure tends to draw the conduc tor into the field, regardless of the sign of the surface charge. The triboelectric effect is a type of contact electrification in which certain materials become electrically charged when they are brought into contact with a different material and then separated. One of the materials acquires a positive charge, and the other acquires an equal negative charge. The polarity and strength of the charges produc ed differ according to the materials, surface roughne ss, temperature, strain, and other properties. Amber, for example, can acquire an electric charge by friction with a material likeElectrostatic pressure Triboelectric effectwool. This property, first recorded by Thales of Miletus, was the first electrical phenomenon investigated by humans. Other examples of materials that can acquire a significant charge when rubbe d together include glass rubbe d with silk, and ha rd rubbe r rubbe d with fur. The presence of surface charge imbalance means that the objects will exhibit attractive or repulsive forces. This surface charge imbalance, which yields static electricity, can be generated by touching two differing surfaces together and then separating them due to the phenomena of contact electrification and the triboelectric effect. Rubbing two nonc onduc tive objects generates a great amount of static electricity. This is not just the result of friction; two nonc onductive surfaces can become charged by just being placed one on top of the other. Since most surfaces have a rough texture, it takes longe r to achieve charging through contact than through rubbing. Rubbing objects together increases the amount of adhesive contact between the two surfaces. Usually insulators, i.e., substances that do not conduc t electricity, are good at both generating, and holding, a surface charge. Some examples of these substances are rubbe r, plastic, glass, and pith. Conduc tive objects rarely generate charge imbalance, except when a metal surface is impacted by solid or liquid nonc onduc tors. The charge that is transferred during contact electrification is stored on the surface of each object. Electrostatic generators, devices which produc e very high voltage at very low current and us ed for classroom phys ics demonstrations, rely on t his effect. The presence of electric current does not detract from the electrostatic forces nor from the sparking, from the corona discharge, or other phenomena. Both phe nomena can exist simultaneously in the same system. See also: Wimshurst machine, and Van de Graaff generator. The most familiar natural electrostatic phenomenon, often regarded as an occasional annoya nce in seasons of low humidity, is Static electricity. Static electricity is generally harmless, but it can be destructive and harmful in some situations (e.g. electronics manufacturing), when working in direct contact with integrated circuit electronics (especially delicate MOSFETs). In the presence of flammable gas, care must be taken to avoid accumulating and sudde nly discharging a static charge (see Electrostatic discharge), because an electrostatic spark can ignite the gas. Electrostatic induc tion, discovered by British scientist John Canton in 1753 and Swedish professor Johan Carl Wilcke in 1762[8][9][10] is a redistribution of charges in an object caused by the electric field of a nearby charge. For example, if a positively charged object is brought near an uncharged metal object, the mobile negatively-charged electrons in the metal will be attracted by the external charge, and move to the side of the metal facing it, creating a negative charge on the surface. When the electrons move out of an area they leave a positive charge due to the metal atoms' nuclei, so the side of the metal object facing away from the charge acquires a positive charge. These induc ed charges disappear when the external charge is removed. Induc tion is also responsible for the attraction of light objects, such as balloons, paper scraps and foam packing peanuts to static charges. The surface charges induc ed in conduc tive objects exactly cancel external electric fields inside the conduc tor, so there is no electric field inside a metal object. This is theElectrostatic generators Charge neutralization Electrostatic inductionLightning over Oradea in Romaniabasis for the electric field shielding action of a Faraday cage. Since the electric field is the gradient of the voltage, electrostatic induc tion is also responsible for making the electric potential (voltage) constant throughout a conduc tive object. Before the year 1832, when Michael Faraday published the results of his experiment on the identity of electricities, physicists thought "static electricity" was somehow different from other electrical charges. Michael Faraday proved that the electricity induc ed from the magnet, voltaic electricity produc ed by a battery, and static electricity are all the same. Static electricity is usually caused when certain materials are rubbe d against each other, like wool on plastic or the soles of shoes on carpet. The process causes electrons to be pulled from the surface of one material and relocated on t he surface of the other material. A static shock occurs when the surface of the second material, negatively charged with electrons, touches a (relatively) positively charged conduc tor, or vice versa. Static electricity is commonly used in xerography, air filters, and some coating processes used in manufacturing. Static electricity is a build-up of electric charges on two objects that have become separated from each other. Small electrical compone nts can be damaged by static electricity, and compone nt manufacturers use a number of antistatic devices to avoid this. When different materials are brought together and then separated, an accumulation of electric charge can occur which leaves one material positively charged while the other becomes negatively charged. The mild shock that you receive when touching a grounde d object after walking on carpet is an example of excess electrical charge accumulating in your body from frictional charging between your shoes and the carpet. The resulting charge build-up upon your body can generate a strong electrical discharge. Although experimenting with static electricity may be fun, similar sparks create severe hazards in those industries dealing with flammable substances, where a small electrical spark may ignite explosive mixtures with devastating consequences. A similar charging mechanism can occur within low conduc tivity fluids flowing through pipelines—a process called flow electrification. F luids which have low electrical conduc tivity (below 50 pi cosiemens per meter), are called accumulators. Fluids having conduc tivities above 50 pS/m are called non-accumulators. In non-accumulators, charges recombine as fast as they are separated and hence electrostatic charge generation is not significant. In the petrochemical industry, 50 pS/m is the recommended minimum value of electrical conduc tivity for adequate removal of charge from a fluid. An important concept for insulating fluids is the static relaxation time. This is similar to the time constant (tau) within an RC circuit. For insulating materials, it is the ratio of the static dielectric constant divided by the electrical conduc tivity of the material. For hydrocarbon fluids, this is sometimes approximated by dividing the number 18 by the electrical conduc tivity of the fluid. Thus a fluid that has an electricalStatic electricity Static electricity and chemical industryconduc tivity of 1 pS/cm (100 pS/m) will have an estimated relaxation time of about 18 seconds. The excess charge within a fluid will be almost completely dissipated after 4 to 5 times the relaxation time, or 90 seconds for the fluid in the above example. Charge generation increases at higher fluid velocities and larger pipe diameters, becoming quite significant in pipes 8 inches (200 mm) or larger. Static charge generation in these systems is best controlled by limiting fluid velocity. The British standard BS PD CLC/TR 50404: 2003 (formerly BS-5958- Part 2) Code of Practice for Control of Undesirable Static Electricity prescribes velocity limits. Because of its large impact on dielectric constant, the recommended velocity for hydrocarbon fluids containing water should be limited to 1 m /s. Bonding and earthing are the usual ways by which charge buildup can be prevented. For fluids with electrical conduc tivity below 10 pS/m, bonding and earthing are not adequate for charge dissipation, and anti-static additives may be required. BS PD CLC/TR 50404:2003 Code of Practice for Control of Undesirable Static Electricity NFPA 77 (2007) Recommended Practice on Static Electricity API RP 2003 (1998) Protection Against Ignitions Arising Out of Static, Lightning, and Stray Currents Electrostatic induc tion was used in the past to build high-voltage generators know n as influence machines. The main compone nt that emerged in these times is the capacitor. Electrostatic induc tion is also used for electro-mechanic precipitation or projection. In such technologies, charged particles of small sizes are collected or deposited intentionally on surfaces. Applications range from electrostatic precipitator to electrostatic coating and inkjet printing. Electrostatic actuators have recently been attracting interest in the soft robotics research area. Electrostatic actuators can be employed as clutches for wearable devices which can exhibit mechanical impedance tuning and improved energy efficiency.[11][12][13] Other relevant applications include but not limited to multimode hydraulically amplified electrostatic actuators for wearable haptics [14] and robots driven by electrostatic actuator.[15][16] Electromagnetism Electronegativity Electrostatic discharge Electrostatic separator Electrostatic voltmeter Ionic bond Permittivity and relative permittivity Quantisation of chargeApplicable standards Electrostatic induction in commercial applications See also Footnotes1. Ling, Samuel J.; Moebs, William; Sanny, Jeff (2019). University Physics, Vol. 2 (https://opent extbc.ca/universityphysicsv2openstax/chapter/conductors-insulators-and-charging-by-induct ion/). OpenStax. ISBN

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chapterElectric Fields PUZZLERSoft contact lenses are comfortable towear because they attract the proteins inthe wearer's tears, incorporating thecomplex molecules right into the lenses.They become, in a sense, part of thewearer. Some types of makeup exploitthis same attractive force to adhere tothe skin. What is the nature of this force?(Charles D. Winters) Chapter Outline23.1Properties of Electric Charges23.2Insulators and Conductors23.3Coulomb's Law23.4The Electric Field23.5Electric Field of a ContinuousCharge Distribution23.6Electric Field Lines23.7Motion of Charged Particles in aUniform Electric FieldPUZZLER 708 23.1Properties of Electric Charges709he electromagnetic force between charged particles is one of the fundamen-tal forces of nature. We begin this chapter by describing some of the basicproperties of electric forces. We then discuss Coulomb's law, which is the fun-damental law governing the force between any two charged particles. Next, we in-troduce the concept of an electric field associated with a charge distribution anddescribe its effect on other charged particles. We then show how to useCoulomb's law to calculate the electric field for a given charge distribution. Weconclude the chapter with a discussion of the motion of a charged particle in auniform electric field.PROPERTIES OF ELECTRIC CHARGESA number of simple experiments demonstrate the existence of electric forces andcharges. For example, after running a comb through your hair on a dry day, youwill find that the comb attracts bits of paper. The attractive force is often strongenough to suspend the paper. The same effect occurs when materials such as glassor rubber are rubbed with silk or fur.Another simple experiment is to rub an inflated balloon with wool. The bal-loon then adheres to a wall, often for hours. When materials behave in this way,they are said to be electrified,or to have become electrically charged.You can eas-ily electrify your body by vigorously rubbing your shoes on a wool rug. The electriccharge on your body can be felt and removed by lightly touching (and startling) afriend. Under the right conditions, you will see a spark when you touch, and bothof you will feel a slight tingle. (Experiments such as these work best on a dry daybecause an excessive amount of moisture in the air can cause any charge you buildup to "leak" from your body to the Earth.)In a series of simple experiments, it is found that there are two kinds of elec-tric charges, which were given the names positive and negative by BenjaminFranklin (1706–1790). To verify that this is true, consider a hard rubber rod thathas been rubbed with fur and then suspended by a nonmetallic thread, as shownin Figure 23.1. When a glass rod that has been rubbed with silk is brought near therubber rod, the two attract each other (Fig. 23.1a). On the other hand, if twocharged rubber rods (or two charged glass rods) are brought near each other, asshown in Figure 23.1b, the two repel each other. This observation shows that therubber and glass are in two different states of electrification. On the basis of theseobservations, we conclude that like charges repel one another and unlikecharges attract one another.Using the convention suggested by Franklin, the electric charge on the glassrod is called positive and that on the rubber rod is called negative. Therefore, anycharged object attracted to a charged rubber rod (or repelled by a charged glassrod) must have a positive charge, and any charged object repelled by a chargedrubber rod (or attracted to a charged glass rod) must have a negative charge.Attractive electric forces are responsible for the behavior of a wide variety ofcommercial products. For example, the plastic in many contact lenses, etafilcon,ismade up of molecules that electrically attract the protein molecules in humantears. These protein molecules are absorbed and held by the plastic so that thelens ends up being primarily composed of the wearer's tears. Because of this, thewearer's eye does not treat the lens as a foreign object, and it can be worn com-fortably.

Many cosmetics also take advantage of electric forces by incorporatingmaterials that are electrically attracted to skin or hair, causing the pigments orother chemicals to stay put once they are applied.23.1T 11.2 QuickLabRub an inflated balloon against yourhair and then hold the balloon near athin stream of water running from afaucet. What happens? (A rubbedplastic pen or comb will also work.) 710CHAPTER 23Electric Fields Another important aspect of Franklin's model of electricity is the implicationthat electric charge is always conserved.That is, when one object is rubbedagainst another, charge is not created in the process. The electrified state is due toa transfer of charge from one object to the other. One object gains some amount ofnegative charge while the other gains an equal amount of positive charge. For ex-ample, when a glass rod is rubbed with silk, the silk obtains a negative charge thatis equal in magnitude to the positive charge on the glass rod. We now know fromour understanding of atomic structure that negatively charged electrons are trans-ferred from the glass to the silk in the rubbing process. Similarly, when rubber isrubbed with fur, electrons are transferred from the fur to the rubber, giving therubber a net negative charge and the fur a net positive charge. This process is con-sistent with the fact that neutral, uncharged matter contains as many positivecharges (protons within atomic nuclei) as negative charges (electrons).If you rub an inflated balloon against your hair, the two materials attract each other, asshown in Figure 23.2. Is the amount of charge present in the balloon and your hair afterrubbing (a) less than, (b) the same as, or (c) more than the amount of charge present be-fore rubbing?In 1909, Robert Millikan (1868–1953) discovered that electric charge alwaysoccurs as some integral multiple of a fundamental amount of charge e. In modernterms, the electric charge qis said to be quantized,where qis the standard symbolused for charge. That is, electric charge exists as discrete "packets," and we canwrite where Nis some integer. Other experiments in the same periodshowed that the electron has a charge /H11002eand the proton has a charge of equalmagnitude but opposite sign /H11001e. Some particles, such as the neutron, have nocharge. A neutral atom must contain as many protons as electrons.Because charge is a conserved quantity, the net charge in a closed region re-mains the same. If charged particles are created in some process, they are alwayscreated in pairs whose members have equal-magnitude charges of opposite sign.q/H11005Ne,Quick Quiz 23.1RubberRubber (a)FF(b)FFRubber––––––––––––––––++++++Glass–+Figure 23.1(a) A negatively charged rubber rod suspended by a thread is attracted to a posi-tively charged glass rod. (b) A negatively charged rubber rod is repelled by another negativelycharged rubber rod. Figure 23.2Rubbing a balloonagainst your hair on a dry daycauses the balloon and your hair to become charged.Charge is conserved Charge is quantized23.2Insulators and Conductors711From our discussion thus far, we conclude that electric charge has the follow-ing important properties:•Two kinds of charges occur in nature, with the property that unlike chargesattract one another and like charges repel one another.•Charge is conserved.•Charge is quantized.Properties of electric chargeINSULATORS AND CONDUCTORSIt is convenient to classify substances in terms of their ability to conduct electriccharge:23.2Electrical conductors are materials in which electric charges move freely,whereas electrical insulators are materials in which electric charges cannotmove freely.Materials such as glass, rubber, and wood fall into the category of electrical insula-tors. When such materials are charged by rubbing, only the area rubbed becomescharged, and the charge is unable to move to other regions of the material.In contrast, materials such as copper, aluminum, and silver are good electricalconductors. When such materials are charged in some small region, the chargereadily distributes itself over the entire surface of the material. If you hold a cop-per rod in your hand and rub it with wool or fur, it will not attract a small piece ofpaper. This might suggest that a metal cannot be charged. However, if you attach awooden handle to the rod and then hold it by that handle as you rub the rod, therod will remain charged and attract the piece of paper. The explanation for this isas follows: Without the insulating wood, the electric charges produced by rubbingreadily move from the copper through your body and into the Earth. The insulat-ing wooden handle prevents the flow of charge into your hand.Semiconductors are a third class of materials, and their electrical propertiesare somewhere between those of insulators and those of conductors. Silicon andgermanium are well-known examples of semiconductors commonly used in thefabrication of a variety of electronic devices, such as transistors and light-emittingdiodes. The electrical properties of semiconductors can be changed over many or-ders of magnitude by the addition of controlled amounts of certain atoms to thematerials.When a conductor is connected to the Earth by means of a conducting wire orpipe, it is said to be grounded.The Earth can then be considered an infinite"sink" to which electric charges can easily migrate. With this in mind, we can un-derstand how to charge a conductor by a process known as induction.To understand induction, consider a neutral (uncharged) conducting sphereinsulated from ground, as shown in Figure 23.3a. When a negatively charged rub-ber rod is brought near the sphere, the region of the sphere nearest the rod ob-tains an excess of positive charge while the region farthest from the rod obtains anequal excess of negative charge, as shown in Figure 23.3b. (That is, electrons inthe region nearest the rod migrate to the opposite side of the sphere. This occurseven if the rod never actually touches the sphere.) If the same experiment is per-formed with a conducting wire connected from the sphere to ground (Fig. 23.3c),some of the electrons in the conductor are so strongly repelled by the presence of11.3 Metals are good conductors Charging by induction712CHAPTER 23Electric Fields ++++++++––––––––––––(b) ++++++++––––(c) ++++++++––––(d) ++++++ (e)++++++++––––––––––(a) ++Figure 23.3Charging a metallic object by induction(that is, the two objects never touch eachother). (a) A neutral metallic sphere, with equal numbers of positive and negative charges. (b) The charge on the neutral sphere is redistributed when a charged rubber rod is placed nearthe sphere. (c) When the sphere is grounded, some of its electrons leave through the groundwire. (d) When the ground connection is removed, the sphere has excess positive charge that isnonuniformly distributed. (e) When the rod is removed, the excess positive charge becomes uni-formly distributed over the surface of the sphere.23.3Coulomb's Law713 the negative charge in the rod that they move out of the sphere through theground wire and into the Earth. If the wire to ground is then removed (Fig.23.3d), the conducting sphere contains an excess of induced positive charge. Whenthe rubber rod is removed from the vicinity of the sphere (Fig. 23.3e), this in-duced positive charge remains on the ungrounded sphere. Note that the chargeremaining on the sphere is uniformly distributed over its surface because of the re-pulsive forces among the like charges. Also note that the rubber rod loses none ofits negative charge during this process.Charging an object by induction requires no contact with the body inducingthe charge. This is in contrast to charging an object by rubbing (that is, by conduc-tion), which does require contact between the two objects.A process similar to induction in conductors takes place in insulators. In mostneutral molecules, the center of positive charge coincides with the center of nega-tive charge. However, in the presence of a charged object, these centers insideeach molecule in an insulator may shift slightly, resulting in more positive chargeon one side of the molecule than on the other. This realignment of charge withinindividual molecules produces an induced charge on the surface of the insulator,as shown in Figure 23.4. Knowing about induction in insulators, you should beable to explain why a comb that has been rubbed through hair attracts bits of elec-trically neutral paper and why a balloon that has been rubbed against your cloth-ing is able to stick to an electrically neutral wall.Object A is attracted to object B. If object B is known to be positively charged, what can wesay about object A? (a) It is positively charged. (b) It is negatively charged. (c) It is electri-cally neutral. (d) Not enough information to answer.COULOMB'S LAWCharles Coulomb (1736–1806) measured the magnitudes of the electric forces be-tween charged objects using the torsion balance, which he invented (Fig. 23.5).23.3Quick Quiz 23.2QuickLabTear some paper into very smallpieces. Comb your hair and thenbring the comb close to the paperpieces. Notice that they are acceler-ated toward the comb. How does themagnitude of the electric force com-pare with the magnitude of the gravi-tational force exerted on the paper?Keep watching and you might see afew pieces jump away from the comb.They don't just fall away; they are re-pelled. What causes this? +++++++–+–+–+–+–+–Insulator InducedchargesChargedobject(a) Figure 23.4(a) The charged object on the left induces charges on the surface of an insulator.(b) A charged comb attracts bits of paper because charges are displaced in the paper .(b) 11.4 Charles Coulomb(1736 – 1806)Coulomb's major contribution to sci-ence was in the field of electrostaticsand magnetism. During his lifetime, healso investigated the strengths of ma-terials and determined the forces thataffect objects on beams, thereby con-tributing to the field of structural me-chanics. In the field of ergonomics,his research provided a fundamentalunderstanding of the ways in whichpeople and animals can best do work.(Photo courtesy of AIP Niels BohrLibrary/E. Scott Barr Collection) 714CHAPTER 23Electric FieldsCoulomb confirmed that the electric force between two small charged spheres isproportional to the inverse square of their separation distance r—that is,The operating principle of the torsion balance is the same as that of theapparatus used by Cavendish to measure the gravitational constant (see Section14.2), with the electrically neutral spheres replaced by charged ones. The electricforce between charged spheres A and B in Figure 23.5 causes the spheres to eitherattract or repel each other, and the resulting motion causes the suspended fiber totwist. Because the restoring torque of the twisted fiber is proportional to the anglethrough which the fiber rotates, a measurement of this angle provides a quantita-tive measure of the electric force of attraction or repulsion. Once the spheres arecharged by rubbing, the electric force between them is very large compared withthe gravitational attraction, and so the gravitational force can be neglected.Coulomb's experiments showed that the electric force between two stationarycharged particles•is inversely proportional to the square of the separation rbetween the particlesand directed along the line joining them;•is proportional to the product of the charges q1and q2on the two particles;•is attractive if the charges are of opposite sign and repulsive if the charges havethe same sign.From these observations, we can express Coulomb's law as an equation givingthe magnitude of the electric force (sometimes called the Coulomb force) betweentwo point charges:(23.1)where keis a constant called the Coulomb constant.In his experiments, Coulombwas able to show that the value of the exponent of rwas 2 to within an uncertaintyof a few percent. Modern experiments have shown that the exponent is 2 to withinan uncertainty of a few parts in

1016.The value of the Coulomb constant depends on the choice of units. The SIunit of charge is the coulomb (C). The Coulomb constant ke in SI units has thevalueThis constant is also written in the formwhere the constant /H92800(lowercase Greek epsilon) is known as the permittivity of freespaceand has the value The smallest unit of charge known in nature is the charge on an electron orproton,1which has an absolute value ofTherefore, 1 C of charge is approximately equal to the charge of 6.24/H110031018elec-trons or protons. This number is very small when compared with the number of/H20841e/H20841/H110051.602 19/H1100310/H1100219 C8.854 2/H1100310/H1100212 C2/N/H11080m2.ke/H1100514/H9266/H92800ke/H110058.987 5/H11003109 N/H11080m2/C2Fe/H11005ke /H20841q1/H20841/H20841q2/H20841r2Fe/H110081/r2. Coulomb constant Charge on an electron or proton1No unit of charge smaller than ehas been detected as a free charge; however, recent theories proposethe existence of particles called quarks having charges e/3 and 2e/3. Although there is considerable ex-perimental evidence for such particles inside nuclear matter, free quarks have never been detected. Wediscuss other properties of quarks in Chapter 46 of the extended version of this text. SuspensionheadFiber BAFigure 23.5Coulomb's torsionbalance, used to establish the in-verse-square law for the electricforce between two charges.23.3Coulomb's Law715 free electrons2in 1 cm3of copper, which is of the order of 1023. Still, 1 C is a sub-stantial amount of charge. In typical experiments in which a rubber or glass rod ischarged by friction, a net charge of the order of 10/H110026C is obtained. In otherwords, only a very small fraction of the total available charge is transferred be-tween the rod and the rubbing material.The charges and masses of the electron, proton, and neutron are given inTable 23.1.The Hydrogen Atom EXAMPLE23.1 The ratio Thus, the gravitational force be-tween charged atomic particles is negligible when comparedwith the electric force. Note the similarity of form of New-ton's law of gravitation and Coulomb's law of electric forces.Other than magnitude, what is a fundamental difference be-tween the two forces?Fe/Fg/H110152/H110031039.3.6/H1100310/H1100247 N /H11005 /H11005/H11003(9.11/H1100310/H1100231 kg)(1.67/H1100310/H1100227 kg)(5.3/H1100310/H1100211 m)2 /H11005/H208986.7/H1100310/H1100211 N/H11080m2kg2/H20899 Fg/H11005G mempr2 The electron and proton of a hydrogen atom are separated(on the average) by a distance of approximately 5.3/H1100310/H1100211m. Find the magnitudes of the electric force and thegravitational force between the two particles.SolutionFrom Coulomb's law, we find that the attractiveelectric force has the magnitudeUsing Newton's law of gravitation and Table 23.1 for theparticle masses, we find that the gravitational force has themagnitude8.2/H1100310/H110028 N/H11005Fe/H11005ke /H20841e/H208412r2/H11005/H208988.99/H11003109 N/H11080m2C2/H20899 (1.60/H1100310/H1100219 C)2(5.3/H1100310/H1100211 m)2 When dealing with Coulomb's law, you must remember that force is a vectorquantity and must be treated accordingly. Thus, the law expressed in vector formfor the electric force exerted by a charge q1on a second charge q2, written F12, is(23.2)where is a unit vector directed from q1to q2, as shown in Figure 23.6a. Becausethe electric force obeys Newton's third law, the electric force exerted by q2on q1isrˆF12/H11005ke q1q2r2 rˆ 2A metal atom, such as copper, contains one or more outer electrons, which are weakly bound to thenucleus. When many atoms combine to form a metal, the so-called free electrons are these outer elec-trons, which are not bound to any one atom. These electrons move about the metal in a manner simi-lar to that of gas molecules moving in a container.TABLE 23.1Charge and Mass of the Electron, Proton, andNeutronParticle Charge (C) Mass (kg)Electron (e)/H110021.602 191 7/H1100310/H11002199.109 5/H1100310/H1100231Proton (p)/H110011.602 191 7/H1100310/H11002191.672 61/H1100310/H1100227Neutron (n) 0 1.674 92/H1100310/H1100227716CHAPTER 23Electric Fields equal in magnitude to the force exerted by q1on q2and in the opposite direction;that is, Finally, from Equation 23.2, we see that if q1and q2have thesame sign, as in Figure 23.6a, the product q1q2is positive and the force is repulsive.If q1and q2are of opposite sign, as shown in Figure 23.6b, the product q1q2is neg-ative and the force is attractive. Noting the sign of the product q1q2is an easy wayof determining the direction of forces acting on the charges.Object A has a charge of /H110012/H9262C, and object B has a charge of /H110016/H9262C. Which statement istrue?(a) . (b) . (c) .When more than two charges are present, the force between any pair of themis given by Equation 23.2. Therefore, the resultant force on any one of themequals the vector sum of the forces exerted by the various individual charges. Forexample, if four charges are present, then the resultant force exerted by particles2, 3, and 4 on particle 1 isF1/H11005F21/H11001F31/H11001F413FAB/H11005/H11002FBAFAB/H11005/H11002FBAFAB/H11005/H110023FBAQuick Quiz 23.3F21/H11005/H11002F12 .–+r(a)F21F12q1q2 (b)F21F12q1q2rˆ ++Figure 23.6Two point charges separated by a distance rex-ert a force on each other that is given by Coulomb's law. Theforce F21exerted by q2on q1is equal in magnitude and oppo-site in direction to the force F12exerted by q1on q2. (a) Whenthe charges are of the same sign, the force is repulsive. (b) When the charges are of opposite signs, the force isattractive. Find the Resultant Force EXAMPLE23.2The magnitude of F23is Note that because q3and q2have opposite signs, F23is to theleft, as shown in Figure 23.7. /H110059.0 N /H11005/H208988.99/H11003109 N/H11080m2C2/H20899 (2.0/H1100310/H110026 C)(5.0/H1100310/H110026 C)(0.10 m)2F23/H11005ke /H20841q2/H20841/H20841q3/H20841a2 Consider three point charges located at the corners of a righttriangle as shown in Figure 23.7, where and Find the resultant force ex-erted on q3.SolutionFirst, note the direction of the individual forcesexerted by q1and q2on q3. The force F23exerted by q2on q3is attractive because q2and q3have opposite signs. The forceF13exerted by q1on q3is repulsive because both charges arepositive.a/H110050.10 m. q2/H11005/H110022.0 /H9262C,q1/H11005q3/H110055.0 /H9262C,23.3Coulomb's Law717F13q3 q1q2aay x– ++F232a√ The magnitude of the force exerted by q1on q3isF13/H11005ke /H20841q1/H20841/H20841q3/H20841(!2a)2 The force F13is repulsive and makes an angle of 45° with thexaxis. Therefore, the xand ycomponents of F13are equal,with magnitude given by F13cos 45°/H110057.9 N.The force F23is in the negative xdirection. Hence, the xand ycomponents of the resultant force acting on q3areWe can also express the resultant force acting on q3in unit-vector form asExerciseFind the magnitude and direction of the resultantforce F3.Answer8.0 N at an angle of 98° with the xaxis.(/H110021.1i/H110017.9j) NF3/H11005F3y/H11005F13y/H110057.9 N F3x/H11005F13x/H11001F23/H110057.9 N/H110029.0 N/H11005/H110021.1 N /H1100511 N /H11005/H208988.99/H11003109 N/H11080m2C2/H20899 (5.0/H1100310/H110026 C)(5.0/H1100310/H110026 C)2(0.10 m)2 Figure 23.7The force exerted by q1on q3is F13. The force ex-erted by q2on q3is F23. The resultant force F3exerted on q3is thevector sum F13/H11001F23 . Where Is the Resultant Force Zero? EXAMPLE23.3Solving this quadratic equation for x, we find that Why is the negative root not acceptable? x/H110050.775 m.(4.00/H110024.00x/H11001x2)(6.00/H1100310/H110026 C)/H11005x2(15.0/H1100310/H110026 C)(2.00/H11002x)2/H20841q2/H20841/H11005x2/H20841q1/H20841 Three point charges lie along the xaxis as shown in Figure23.8. The positive charge q1/H1100515.0 /H9262C is at x/H110052.00 m, thepositive charge q2/H110056.00 /H9262C is at the origin, and the resul-tant force acting on q3is zero. What is the xcoordinate of q3?SolutionBecause q3is negative and q1and q2are positive,the forces F13and F23are both attractive, as indicated in Fig-ure 23.8. From Coulomb's law, F13and F23have magnitudesFor the resultant force on q3to be zero, F23must be equal inmagnitude and opposite in direction to F13, orNoting that keand q3are common to both sides and so can bedropped, we solve for xand find thatke /H20841q2/H20841/H20841q3/H20841x2/H11005ke /H20841q1/H20841/H20841q3/H20841(2.00/H11002x)2F13/H11005ke /H20841q1/H20841/H20841q3/H20841(2.00/H11002x)2 F23/H11005ke /H20841q2/H20841/H20841q3/H20841x22.00 mxq1xq3–q2F13F232.00 – x++Figure 23.8Three point charges are placed along the xaxis. Ifthe net force acting on q3is zero, then the force F13exerted by q1onq3must be equal in magnitude and opposite in direction to the forceF23exerted by q2on q3.Find the Charge on the Spheres EXAMPLE23.4we see that sin /H9258/H11005a/L. Therefore,The separation of the spheres is The forces acting on the left sphere are shown in Figure23.9b. Because the sphere is in equilibrium, the forces in the2a/H110050.026 m.a/H11005L sin /H9258/H11005(0.15 m)sin 5.0/H11034/H110050.013 mTwo identical small charged spheres, each having a mass of3.0/H1100310/H110022kg, hang in equilibrium as shown in Figure 23.9a.The length of each string is 0.15 m, and the angle /H9258is 5.0°.Find the magnitude of the charge on each sphere.SolutionFrom the right triangle shown in Figure 23.9a,718CHAPTER 23Electric Fields QuickLabFor this experiment you need two 20-cm strips of transparent tape (mass of each/H1101565 mg). Fold about1c m o f t a p e o v e r a t o n e e n d o f e a c h s t r i p t o c r e a t e a h a n d l e . P r e s s b o t h p i e c e s o f t a p e s i d e b y s i d e o n t oa table top, rubbing your finger back and forth across the strips. Quickly pull the strips off the surfaceso that they become charged. Hold the tape handles together and the strips will repel each other, form-ing an inverted "V" shape. Measure the angle between the pieces, and estimate the excess charge oneach strip. Assume that the charges act as if they were located at the center of mass of each strip. Figure 23.9(a) Two identical spheres, each carrying the samecharge q, suspended in equilibrium. (b) The free-body diagram forthe sphere on the left.(a) (b)mgLLθθ L = 0.15 mθ = 5.0°qaqθTT cos θT sin θθFeθθθ THE ELECTRIC FIELDTwo field forces have been introduced into our discussions so far — the gravita-tional force and the electric force. As pointed out earlier, field forces can actthrough space, producing an effect even when no physical contact between the ob-jects occurs. The gravitational field gat a point in space was defined in Section14.6 to be equal to the gravitational force Fgacting on a test particle of mass mdi-vided by that mass: A similar approach to electric forces was developedby Michael Faraday and is of such practical value that we shall devote much atten-tion to it in the next several chapters. In this approach, an electric fieldis said toexist in the region of space around a charged object. When another charged ob-ject enters this electric field, an electric force acts on it. As an example, considerFigure 23.10, which shows a small positive test charge q0placed near a second ob-ject carrying a much greater positive charge Q. We define the strength (in otherwords, the magnitude) of the electric field at the location of the test charge to bethe electric force per unit charge,or to be more specificg/H11013Fg/m.23.4horizontal and vertical directions must separately add up tozero:(1)(2)From Equation (2), we see that

/H9258; thus, Tcan beT/H11005mg/cos/H9018Fy/H11005T cos /H9258/H11002mg/H110050/H9018Fx/H11005T sin /H9258/H11002Fe/H110050eliminated from Equation (1) if we make this substitution.This gives a value for the magnitude of the electric force Fe: (3)From Coulomb's law (Eq. 23.1), the magnitude of the elec-tric force iswhere r/H110052a/H110050.026 m and is the magnitude of thecharge on each sphere. (Note that the term arises herebecause the charge is the same on both spheres.) This equa-tion can be solved for to giveExerciseIf the charge on the spheres were negative, howmany electrons would have to be added to them to yield a netcharge of /H110024.4/H1100310/H110028C?Answer2.7/H110031011electrons.4.4/H1100310/H110028 C /H20841q/H20841q/H11005/H20841q/H208412/H11005Fer2ke/H11005(2.6/H1100310/H110022 N)(0.026 m)28.99/H11003109 N/H11080m2/C2/H20841q/H208412/H20841q/H208412/H20841q/H20841Fe/H11005ke /H20841q/H208412r2 /H110052.6/H1100310/H110022 N /H11005(3.0/H1100310/H110022 kg)(9.80 m/s2)tan 5.0/H11034Fe/H11005mg tan /H9258 +++++++++++++++Qq0EFigure 23.10A small positivetest charge q0placed near an objectcarrying a much larger positivecharge Qexperiences an electricfield Edirected as shown. 11.5 23.4The Electric Field719 This dramatic photograph captures a lightning bolt striking a tree near some rural homes.the electric field Eat a point in space is defined as the electric force Feactingon a positive test charge q0placed at that point divided by the magnitude of thetest charge: (23.3)E/H11013Feq0Note that Eis the field produced by some charge external to the test charge—it isnot the field produced by the test charge itself. Also, note that the existence of anelectric field is a property of its source. For example, every electron comes with itsown electric field. The vector Ehas the SI units of newtons per coulomb (N/C), and, as Figure23.10 shows, its direction is the direction of the force a positive test charge experi-ences when placed in the field. We say that an electric field exists at a point if atest charge at rest at that point experiences an electric force.Once the mag-nitude and direction of the electric field are known at some point, the electricforce exerted on anycharged particle placed at that point can be calculated fromDefinition of electric field720CHAPTER 23Electric Fields Equation 23.3. Furthermore, the electric field is said to exist at some point (evenempty space) regardless of whether a test charge is located at that point.(This is analogous to the gravitational field set up by any object, which is said toexist at a given point regardless of whether some other object is present at thatpoint to "feel" the field.) The electric field magnitudes for various field sourcesare given in Table 23.2.When using Equation 23.3, we must assume that the test charge q0is smallenough that it does not disturb the charge distribution responsible for the electricfield. If a vanishingly small test charge q0is placed near a uniformly charged metal-lic sphere, as shown in Figure 23.11a, the charge on the metallic sphere, whichproduces the electric field, remains uniformly distributed. If the test charge isgreat enough , as shown in Figure 23.11b, the charge on the metallicsphere is redistributed and the ratio of the force to the test charge is different:. That is, because of this redistribution of charge on the metallicsphere, the electric field it sets up is different from the field it sets up in the pres-ence of the much smaller q0.To determine the direction of an electric field, consider a point charge qlo-cated a distance rfrom a test charge q0located at a point P, as shown in Figure23.12. According to Coulomb's law, the force exerted by qon the test charge iswhere is a unit vector directed from qtoward q0. Because the electric field at P,the position of the test charge, is defined by we find that at P, the elec-tric field created by qis(23.4)If qis positive, as it is in Figure 23.12a, the electric field is directed radially outwardfrom it. If qis negative, as it is in Figure 23.12b, the field is directed toward it.To calculate the electric field at a point Pdue to a group of point charges, wefirst calculate the electric field vectors at Pindividually using Equation 23.4 andthen add them vectorially. In other words,E/H11005ke qr2 rˆE/H11005Fe/q0 ,rˆFe/H11005ke qq0r2 rˆ(F/H11032e/q/H110320/HS11005Fe/q0)(q/H110320Wq0) at any point P, the total electric field due to a group of charges equals the vec-tor sum of the electric fields of the individual charges.TABLE 23.2Typical Electric Field ValuesSourceE(N/C)Fluorescent lighting tube 10Atmosphere (fair weather) 100Balloon rubbed on hair 1 000Atmosphere (under thundercloud) 10 000Photocopier 100 000Spark in air/H110223 000 000Near electron in hydrogen atom 5/H110031011 (a)(b)q0+q′0>>q0+——————————————Figure 23.11(a) For a smallenough test charge q0, the chargedistribution on the sphere is undis-turbed. (b) When the test chargeis greater, the charge distribu-tion on the sphere is disturbed asthe result of the proximity of q/H110320 .q/H110320 Figure 23.12A test charge q0atpoint Pis a distance rfrom a pointcharge q. (a) If qis positive, thenthe electric field at Ppoints radiallyoutward from q. (b) If qis nega-tive, then the electric field at Ppoints radially inward toward q.(a)Eqq0rPr –(b)Eqq0Prˆˆ+ This superposition principle applied to fields follows directly from the superposi-tion property of electric forces. Thus, the electric field of a group of charges can23.4The Electric Field721 be expressed as(23.5)where riis the distance from the ith charge qito the point P(the location of thetest charge) and is a unit vector directed from qitoward P.A charge of /H110013/H9262C is at a point Pwhere the electric field is directed to the right and has amagnitude of 4/H110031O6N/C. If the charge is replaced with a /H110023-/H9262C charge, what happens tothe electric field at P?Quick Quiz 23.4rˆiE/H11005ke /H20858i qiri 2 rˆi This metallic sphere is charged by agenerator so that it carries a net elec-tric charge. The high concentration ofcharge on the sphere creates a strongelectric field around the sphere. Thecharges then leak through the gas sur-rounding the sphere, producing apink glow. Electric Field Due to Two Charges EXAMPLE23.5A charge q1/H110057.0/H9262C is located at the origin, and a secondcharge q2/H11005/H110025.0/H9262C is located on the xaxis, 0.30 m fromthe origin (Fig. 23.13). Find the electric field at the point P,which has coordinates (0, 0.40) m.SolutionFirst, let us find the magnitude of the electricfield at Pdue to each charge. The fields E1due to the 7.0-/H9262Ccharge and E2due to the /H110025.0-/H9262C charge are shown in Fig-ure 23.13. Their magnitudes are The vector E1has only a ycomponent. The vector E2has anxcomponent given by and a negative ycompo-nent given by Hence, we can express thevectors as/H11002E2 sin /H9258/H11005/H1100245E2 .E2 cos /H9258/H1100535E2 /H110051.8/H11003105 N/CE2/H11005ke /H20841q2/H20841r2 2/H11005/H208988.99/H11003109 N/H11080m2C2/H20899 (5.0/H1100310/H110026 C)(0.50 m)2 /H110053.9/H11003105 N/CE1/H11005ke /H20841q1/H20841r1 2/H11005/H208988.99/H11003109 N/H11080m2C2/H20899 (7.0/H1100310/H110026 C)(0.40 m)20.40 mPθEE20.50 mE1y θxq2q10.30 m–φ +Figure 23.13The total electric field Eat Pequals the vector sumwhere E1is the field due to the positive charge q1and E2isthe field due to the negative charge q2.E1/H11001E2 ,722CHAPTER 23Electric Fields Electric Field of a Dipole EXAMPLE23.6variation in Efor the dipole also is obtained for a distantpoint along the xaxis (see Problem 21) and for any generaldistant point.The electric dipole is a good model of many molecules,such as hydrochloric acid (HCl). As we shall see in laterchapters, neutral atoms and molecules behave as dipoleswhen placed in an external electric field. Furthermore, manymolecules, such as HCl, are permanent dipoles. The effect ofsuch dipoles on the behavior of materials subjected to elec-tric fields is discussed in Chapter 26.An electric dipole is defined as a positive charge qand anegative charge /H11002qseparated by some distance. For the di-pole shown in Figure 23.14, find the electric field Eat Pdueto the charges, where Pis a distance from the origin.SolutionAt P, the fields E1and E2due to the two chargesare equal in magnitude because Pis equidistant from thecharges. The total field is whereThe ycomponents of E1and E2cancel each other, and the xcomponents add because they are both in the positive xdirection. Therefore, Eis parallel to the xaxis and has amagnitude equal to 2E1cos /H9258. From Figure 23.14 we see that cos Therefore,Because we can neglect a2and writeThus, we see that, at distances far from a dipole but along theperpendicular bisector of the line joining the two charges,the magnitude of the electric field created by the dipolevaries as 1/r3, whereas the more slowly varying field of apoint charge varies as 1/r2(see Eq. 23.4). This is because atdistant points, the fields of the two charges of equal magni-tude and opposite sign almost cancel each other. The 1/r3E/H11015ke 2qay3yWa, /H11005ke 2qa(y2/H11001a2)3/2 E/H110052E1 cos /H9258/H110052ke q(y2/H11001a2) a(y2/H11001a2)1/2/H9258/H11005a/r/H11005a/(y2/H11001a2)1/2.E1/H11005E2/H11005ke qr2/H11005ke qy2/H11001a2E/H11005E1/H11001E2 ,yWaThe resultant field Eat Pis the superposition of E1and E2:(1.1/H11003105i/H110012.5/H11003105j) N/CE/H11005E1/H11001E2/H11005E2/H11005(1.1/H11003105i/H110021.4/H11003105j) N/CE1/H110053.9/H11003105j N/C From this result, we find that Ehas a magnitude of 2.7/H11003105N/C and makes an angle /H9278of 66° with the positive xaxis.ExerciseFind the electric force exerted on a charge of 2.0/H1100310/H110028C located at P.Answer5.4/H1100310/H110023N in the same direction as E. PEθθyE1 E2yrθaqθa–q–x+Figure 23.14The total electric field Eat Pdue to two charges ofequal magnitude and opposite sign (an electric dipole) equals thevector sum The field E1is due to the positive charge q,and E2is the field due to the negative charge /H11002q.E1/H11001E2 .ELECTRIC FIELD OF A CONTINUOUSCHARGE DISTRIBUTIONVery often the distances between charges in a group of charges are much smallerthan the distance from the group to some point of interest (for example, a pointwhere the electric field is to be calculated). In such situations, the system of23.523.5Electric Field of a Continuous Charge Distribution723charges is smeared out, or continuous.That is, the system of closely spaced chargesis equivalent to a total charge that is continuously distributed along some line,over some surface, or throughout some volume.To evaluate the electric field created by a continuous charge distribution, weuse the following procedure: First, we divide the charge distribution into small ele-ments, each of which contains a small charge /H9004q, as shown in Figure 23.15. Next,we use Equation 23.4 to calculate the electric field due to one of these elements ata point P. Finally, we evaluate the total field at Pdue to the charge distribution bysumming the contributions of all the charge elements (that is, by applying the su-perposition principle).The electric field at Pdue to one element carrying charge /H9004qiswhere ris the distance from the element to point Pand is a unit vector directedfrom the charge element toward P. The total electric field at Pdue to all elementsin the charge distribution is approximatelywhere the index irefers to the ith element in the distribution. Because the chargedistribution is approximately continuous, the total field at Pin the limit is(23.6)where the integration is over the entire charge distribution. This is a vector operation and must be treated appropriately.We illustrate this type of calculation with several examples, in which we assumethe charge is uniformly distributed on a line, on a surface, or throughout a vol-ume. When performing such calculations, it is convenient to use the concept of acharge density along with the following notations:•If a charge Qis uniformly distributed throughout a volume V, the volumecharge density/H9267is defined bywhere /H9267has units of coulombs per cubic meter (C/m3).•If a charge Qis uniformly distributed on a surface of area A, the surface chargedensity/H9268(lowercase Greek sigma) is defined bywhere /H9268has units of coulombs per square meter (C/m2).•If a charge Qis uniformly distributed along a line of length , the linear chargedensity/H9261is defined bywhere /H9261has units of coulombs per meter (C/m)./H9261/H11013Q/H5129/H5129/H9268/H11013QA/H9267/H11013QVE/H11005ke l i m/H9004qi:0 /H20858i /H9004qiri 2 rˆi/H11005ke /H20885 dqr2 rˆ/H9004qi:0E/H11015ke /H20858i /H9004qiri 2 rˆir̂/H9004E/H11005ke /H9004qr2 rˆA continuous charge distribution Electric field of a continuouscharge distribution Volume charge density Surface charge densityr∆qrˆ P∆EFigure 23.15The electric fieldat Pdue to a continuous charge dis-tribution is the vector sum of thefields /H9004Edue to all the elements/H9004qof the charge distribution. Linear charge density724CHAPTER 23Electric Fields•If the charge is nonuniformly distributed over a volume, surface, or line, wehave to express the charge densities aswhere dQis the amount of charge in a small volume, surface, or length element./H9267/H11005dQdV /H9268/H11005dQdA /H9261/H11005dQd/H5129The Electric Field Due to a Charged Rod EXAMPLE23.7 where we have used the fact that the total charge Q/H11005/H9261.If Pis far from the rod then the in the denomi-nator can be neglected, and This is just the formyou would expect for a point charge. Therefore, at large val-ues of a/, t h e c h a r g e d i s t r i b u t i o n a p p e a r s t o b e a p o i n tcharge of magnitude Q. The use of the limiting techniqueoften is a good method for checking a theoreticalformula.

(a//H5129:/H11009)/H5129E/H11015keQ/a2./H5129(aW/H5129),/H5129keQa(/H5129/H11001a) /H11005ke/H9261 /H208981a/H110021/H5129/H11001a/H20899/H11005E/H11005ke /H9261 /H20885/H5129/H11001aa dxx2/H11005ke/H9261/H20900/H110021x/H20901/H5129/H11001aa A rod of length /H5133has a uniform positive charge per unitlength /H9261and a total charge Q. Calculate the electric field at apoint Pthat is located along the long axis of the rod and adistance afrom one end (Fig. 23.16).SolutionLet us assume that the rod is lying along the xaxis, that dxis the length of one small segment, and that dqisthe charge on that segment. Because the rod has a chargeper unit length /H9261, the charge dqon the small segment isThe field dEdue to this segment at Pis in the negative xdirection (because the source of the field carries a positivecharge Q), and its magnitude isBecause every other element also produces a field in the neg-ative xdirection, the problem of summing their contribu-tions is particularly simple in this case. The total field at Pdue to all segments of the rod, which are at different dis-tances from P, is given by Equation 23.6, which in this casebecomes3where the limits on the integral extend from one end of therod to the other The constants keand /H9261can be removed from the integral to yield(x/H11005/H5129/H11001a).(x/H11005a)E/H11005/H20885/H5129/H11001aake/H9261 dxx2dE/H11005ke dqx2/H11005ke/H9261 dxx2dq/H11005/H9261 dx. The Electric Field of a Uniform Ring of Charge EXAMPLE23.8This field has an xcomponent cos /H9258along the axisand a component dE/H11036perpendicular to the axis. As we see inFigure 23.17b, however, the resultant field at Pmust lie alongthe xaxis because the perpendicular components of all thedEx/H11005dEdE/H11005ke dqr2A ring of radius acarries a uniformly distributed positive totalcharge Q. Calculate the electric field due to the ring at apoint Plying a distance xfrom its center along the centralaxis perpendicular to the plane of the ring (Fig. 23.17a).SolutionThe magnitude of the electric field at Pdue tothe segment of charge dqis3It is important that you understand how to carry out integrations such as this. First, express thecharge element dqin terms of the other variables in the integral (in this example, there is one variable,x, and so we made the change The integral must be over scalar quantities; therefore, youmust express the electric field in terms of components, if necessary. (In this example the field has onlyan xcomponent, so we do not bother with this detail.) Then, reduce your expression to an integralover a single variable (or to multiple integrals, each over a single variable). In examples that havespherical or cylindrical symmetry, the single variable will be a radial coordinate.dq/H11005/H9261 dx).xy/H5129aPxdxdq = λdxdEλFigure 23.16The electric field at Pdue to a uniformly chargedrod lying along the xaxis. The magnitude of the field at Pdue to thesegment of charge dqis kedq/x2. The total field at Pis the vector sumover all segments of the rod.23.5Electric Field of a Continuous Charge Distribution725 The Electric Field of a Uniformly Charged Disk EXAMPLE23.9butions of all rings making up the disk. By symmetry, the fieldat an axial point must be along the central axis.The ring of radius rand width drshown in Figure 23.18has a surface area equal to 2/H9266r dr. The charge dqon this ringis equal to the area of the ring multiplied by the surfacecharge density: Using this result in the equa-tion given for Exin Example 23.8 (with areplaced by r), wehave for the field due to the ringTo obtain the total field at P, we integrate this expressionover the limits r/H110050 to r/H11005R, noting that xis a constant. Thisgives /H110052/H9266ke/H9268 /H20898x/H20841x/H20841/H11002x(x2/H11001R2)1/2/H20899 /H11005kex/H9266/H9268 /H20900(x2/H11001r2)/H110021/2/H110021/2/H20901R0 /H11005kex/H9266/H9268 /H20885R0 (x2/H11001r2)/H110023/2 d(r2)E/H11005kex/H9266/H9268 /H20885R0 2r dr(x2/H11001r2)3/2 dE/H11005kex(x2/H11001r2)3/2 (2/H9266/H9268r dr)dq/H110052/H9266/H9268r dr.A disk of radius Rhas a uniform surface charge density /H9268.Calculate the electric field at a point Pthat lies along the cen-tral perpendicular axis of the disk and a distance xfrom thecenter of the disk (Fig. 23.18).SolutionIf we consider the disk as a set of concentricrings, we can use our result from Example 23.8—which givesthe field created by a ring of radius a—and sum the contri-various charge segments sum to zero. That is, the perpen-dicular component of the field created by any charge ele-ment is canceled by the perpendicular component created byan element on the opposite side of the ring. Becauseand cos /H9258/H11005x/r, we find thatAll segments of the ring make the same contribution to thefield at Pbecause they are all equidistant from this point.Thus, we can integrate to obtain the total field at P:dEx/H11005dE cos /H9258/H11005/H20898ke dqr2/H20899xr/H11005kex(x2/H11001a2)3/2 dqr/H11005(x2/H11001a2)1/2This result shows that the field is zero at x/H110050. Does this find-ing surprise you?ExerciseShow that at great distances from the ring the electric field along the axis shown in Figure 23.17 ap-proaches that of a point charge of magnitude Q.(xWa)kex(x2/H11001a2)3/2 Q/H11005Ex/H11005/H20885 kex(x2/H11001a2)3/2 dq/H11005kex(x2/H11001a2)3/2 /H20885 dq (a)+++++++++++++++++θPdExdEdE⊥xrdqa (b)+++++++++++++++++θdE21 dE12Figure 23.17A uniformly charged ring of radius a. (a) The field at Pon the xaxis due to an ele-ment of charge dq. (b) The total electric field at Pis along the xaxis. The perpendicular component ofthe field at Pdue to segment 1 is canceled by the perpendicular component due to segment 2. Figure 23.18A uniformly charged disk of radius R. The electricfield at an axial point Pis directed along the central axis, perpendic-ular to the plane of the disk.PxrRdqdr726CHAPTER 23Electric Fields ELECTRIC FIELD LINESA convenient way of visualizing electric field patterns is to draw lines that followthe same direction as the electric field vector at any point. These lines, called elec-tric field lines,are related to the electric field in any region of space in the fol-lowing manner:•The electric field vector Eis tangent to the electric field line at each point.•The number of lines per unit area through a surface perpendicular to the linesis proportional to the magnitude of the electric field in that region. Thus, Eisgreat when the field lines are close together and small when they are far apart.These properties are illustrated in Figure 23.19. The density of lines throughsurface A is greater than the density of lines through surface B. Therefore, theelectric field is more intense on surface A than on surface B. Furthermore, the factthat the lines at different locations point in different directions indicates that thefield is nonuniform.Representative electric field lines for the field due to a single positive pointcharge are shown in Figure 23.20a. Note that in this two-dimensional drawing weshow only the field lines that lie in the plane containing the point charge. Thelines are actually directed radially outward from the charge in all directions; thus,instead of the flat “wheel” of lines shown, you should picture an entire sphere oflines. Because a positive test charge placed in this field would be repelled by thepositive point charge, the lines are directed radially away from the positive point23.611.5 This result is valid for all values of x. We can calculate thefield close to the disk along the axis by assuming that ;thus, the expression in parentheses reduces to unity:/H92682/H92800E/H110152/H9266ke/H9268/H11005RWxwhere is the permittivity of free space. As weshall find in the next chapter, we obtain the same result forthe field created by a uniformly charged infinite sheet./H92800/H110051/(4/H9266ke) BAFigure 23.19Electric field linespenetrating two surfaces. The mag-nitude of the field is greater on sur-face A than on surface B. Figure 23.20The electric field lines for a point charge. (a) For a positive point charge, thelines are directed radially outward. (b) For a negative point charge, the lines are directed radiallyinward. Note that the figures show only those field lines that lie in the plane containing thecharge. (c) The dark areas are small pieces of thread suspended in oil, which align with the elec-tric field produced by a small charged conductor at the center.(a)+q(b)—q (c)23.6Electric Field Lines727 Is this visualization of the electric field in terms of field lines consistent withEquation 23.4, the expression we obtained for Eusing Coulomb’s law? To answerthis question, consider an imaginary spherical surface of radius rconcentric with apoint charge. From symmetry, we see that the magnitude of the electric field is thesame everywhere on the surface of the sphere. The number of lines Nthat emergefrom the charge is equal to the number that penetrate the spherical surface.Hence, the number of lines per unit area on the sphere is N/4/H9266r2(where the sur-face area of the sphere is 4/H9266r2). Because Eis proportional to the number of linesper unit area, we see that Evaries as 1/r2; this finding is consistent with Equation23.4.As we have seen, we use electric field lines to qualitatively describe the electricfield. One problem with this model is that we always draw a finite number of linesfrom (or to) each charge. Thus, it appears as if the field acts only in certain direc-tions; this is not true. Instead, the field is continuous—that is, it exists at everypoint. Another problem associated with this model is the danger of gaining thewrong impression from a two-dimensional drawing of field lines being used to de-scribe a three-dimensional situation. Be aware of these shortcomings every timeyou either draw or look at a diagram showing electric field lines.We choose the number of field lines starting from any positively charged ob-ject to be C/H11032qand the number of lines ending on any negatively charged object tobe where C/H11032is an arbitrary proportionality constant. Once C/H11032is chosen, thenumber of lines is fixed. For example, if object 1 has charge Q1and object 2 hascharge Q2, then the ratio of number of lines is The electric field lines for two point charges of equal magnitude but oppositesigns (an electric dipole) are shown in Figure 23.21. Because the charges are ofequal magnitude, the number of lines that begin at the positive charge must equalthe number that terminate at the negative charge. At points very near the charges,the lines are nearly radial. The high density of lines between the charges indicatesa region of strong electric field.Figure 23.22 shows the electric field lines in the vicinity of two equal positivepoint charges. Again, the lines are nearly radial at points close to either charge,and the same number of lines emerge from each charge because the charges areequal in magnitude. At great distances from the charges, the field is approximatelyequal to that of a single point charge of magnitude 2q.Finally, in Figure 23.23 we sketch the electric field lines associated with a posi-tive charge /H110012qand a negative charge /H11002q. In this case, the number of lines leav-ing /H110012qis twice the number terminating at /H11002q. Hence, only half of the lines thatleave the positive charge reach the negative charge. The remaining half terminateN2/N1/H11005Q2/Q1 .C/H11032/H20841q/H20841,•The lines must begin on a positive charge and terminate on a negativecharge.•The number of lines drawn leaving a positive charge or approaching a nega-tive charge is proportional to the magnitude of the charge.•No two field lines can cross.charge. The electric field lines representing the field due to a single negative pointcharge are directed toward the charge (Fig. 23.20b). In either case, the lines arealong the radial direction and extend all the way to infinity. Note that the lines be-come closer together as they approach the charge; this indicates that the strengthof the field increases as we move toward the source charge.The rules for drawing electric field lines are as follows:Rules for drawing electric fieldlines (a)–+ Figure 23.21(a) The electricfield lines for two point charges ofequal magnitude and opposite sign(an electric dipole). The numberof lines leaving the positive chargeequals the number terminating atthe negative charge. (b) The darklines are small pieces of thread sus-pended in oil, which align with theelectric field of a dipole.(b)728CHAPTER 23Electric Fields on a negative charge we assume to be at infinity. At distances that are muchgreater than the charge separation, the electric field lines are equivalent to thoseof a single charge /H11001q.Rank the magnitude of the electric field at points A, B, and Cshown in Figure 23.22a(greatest magnitude first).MOTION OF CHARGED PARTICLES IN AUNIFORM ELECTRIC FIELDWhen a particle of charge qand mass mis placed in an electric field E, the electricforce exerted on the charge is qE. If this is the only force exerted on the particle,it must be the net force and so must cause the particle to accelerate. In this case,Newton's second law applied to the particle givesThe acceleration of the particle is therefore(23.7)If Eis uniform (that is, constant in magnitude and direction), then the accelera-tion is constant. If the particle has a positive charge, then its acceleration is in thedirection of the electric field. If the particle has a negative charge, then its acceler-ation is in the direction opposite the electric field.a/H11005qEmFe/H11005qE/H11005ma23.7Quick Quiz 23.5 (a)++CAB Figure 23.22(a) The electric field lines for two positive point charges. (The locations A, B,and Care discussed in Quick Quiz 23.5.) (b) Pieces of thread suspended in oil, which align withthe electric field created by two equal-magnitude positive charges. Figure 23.23The electric fieldlines for a point charge /H110012qand asecond point charge /H11002q. Note thattwo lines leave /H110012qfor every onethat terminates on /H11002q.+2q+—q(b) An Accelerating Positive Charge EXAMPLE23.10SolutionThe acceleration is constant and is given byqE/m. The motion is simple linear motion along the xaxis.Therefore, we can apply the equations of kinematics in oneA positive point charge qof mass mis released from rest in auniform electric field Edirected along the xaxis, as shown inFigure 23.24. Describe its motion.23.7Motion of Charged Particles in a Uniform Electric Field729 The electric field in the region between two oppositely charged flat metallicplates is approximately uniform (Fig. 23.25). Suppose an electron of charge /H11002eisprojected horizontally into this field with an initial velocity vii. Because the electricfield Ein Figure 23.25 is in the positive ydirection, the acceleration of the elec-tron is in the negative ydirection. That is,(23.8)Because the acceleration is constant, we can apply the equations of kinematics intwo dimensions (see Chapter 4) with and After the electron hasbeen in the electric field for a time t, the components of its velocity are(23.9)(23.10)vy/H11005ayt/H11005/H11002eEm tvx/H11005vi/H11005constantvyi/H110050.vxi/H11005via/H11005/H11002eEm j———++++++Evv = 0qx++Figure 23.24A positive point charge qin a uniform electric fieldEundergoes constant acceleration in the direction of the field. (0, 0)/H5129 E–(x,y)–vxy————————++++++++++++viiFigure 23.25An electron is pro-jected horizontally into a uniformelectric field produced by twocharged plates. The electron under-goes a downward acceleration (op-posite E), and its motion is para-bolic while it is between the plates.theorem because the work done by the electric force isand W/H11005/H9004K.Fex/H11005qExdimension (see Chapter 2):Taking and , we have The kinetic energy of the charge after it has moved a distanceisWe can also obtain this result from the work – kinetic energyK/H1100512mv2/H1100512m /H208982qEm/H20899x/H11005qExx/H11005xf/H11002xivxf 2/H110052axxf/H11005/H208982qEm/H20899xf vxf/H11005axt/H11005qEm t xf/H1100512axt2/H11005qE2m t2 vxi/H110050xi/H110050vxf 2 /H11005vxi 2/H110012ax(xf/H11002xi) vxf/H11005vxi/H11001axt xf/H11005xi/H11001vxit/H1100112axt2 730CHAPTER 23Electric FieldsIts coordinates after a time tin the field are(23.11)(23.12)Substituting the value from Equation 23.11 into Equation 23.12, we seethat yis proportional to x2. Hence, the trajectory is a parabola. After the electronleaves the field, it continues to move in a straight line in the direction of vin Fig-ure 23.25, obeying Newton's first law, with a speed Note that we have neglected the gravitational force acting on the electron.This is a good approximation when we are dealing with atomic particles. For anelectric field of 104N/C, the ratio of the magnitude of the electric force eEto themagnitude of the gravitational force mgis of the order of 1014for an electron andof the order of 1011for a proton.v/H11022vi .t/H11005x/vi y/H1100512ayt2/H11005/H1100212 eEm t2x/H11005vit An Accelerated Electron EXAMPLE23.11(c) What is the vertical displacement yof the electronwhile it is in the field?SolutionUsing Equation 23.12 and the results from parts(a) and (b), we find thatIf the separation between the plates is less than this, the elec-tron will strike the positive plate.ExerciseFind the speed of the electron as it emerges fromthe field.Answer3.22/H11003106m/s./H110021.95 cm/H11005/H110020.019 5 m/H11005y/H1100512ayt2/H1100512(/H110023.51/H110031013 m/s2)(3.33/H1100310/H110028 s)23.33/H1100310/H110028 st/H11005/H5129vi/H110050.100 m3.00/H11003106 m/s/H11005An electron enters the region of a uniform electric field asshown in Figure 23.25, with andN/C. The horizontal length of the plates is /H110050.100 m. (a) Find the acceleration of the electron while it isin the electric field.SolutionThe charge on the electron has an absolutevalue of 1.60/H1100310/H1100219C, and There-fore, Equation 23.8 gives(b) Find the time it takes the electron to travel throughthe field.SolutionThe horizontal distance across the field is /H110050.100 m. Using Equation 23.11 with x/H11005, we find that thetime spent in the electric field is/H5129/H5129/H110023.51/H110031013j m/s2/H11005a/H11005/H11002eEm j/H11005/H11002(1.60/H1100310/H1100219 C)(200 N/C)9.11/H1100310/H1100231 kg jm/H110059.11/H1100310/H1100231 kg./H5129E/H11005200vi/H110053.00/H11003106 m/s The Cathode Ray TubeThe example we just worked describes a portion of a cathode ray tube (CRT). Thistube, illustrated in Figure 23.26, is commonly used to obtain a visual display ofelectronic information in oscilloscopes, radar systems, television receivers, andcomputer monitors. The CRT is a vacuum tube in which a beam of electrons is ac-celerated and deflected under the influence of electric or magnetic fields. Theelectron beam is produced by an assembly called an electron gunlocated in theneck of the tube. These electrons, if left undisturbed, travel in a straight-line pathuntil they strike the front of the CRT, the "screen," which is coated with a materialthat emits visible light when bombarded with electrons.In an oscilloscope, the electrons are deflected in various directions by two setsof plates placed at right angles to each other in the neck of the tube. (A televisionSummary731 CRT steers the beam with a magnetic field, as discussed in Chapter 29.) An exter-nal electric circuit is used to control the amount of charge present on the plates.The placing of positive charge on one horizontal plate and negative charge on theother creates an electric field between the plates and allows the beam to besteered from side to side. The vertical deflection plates act in the same way, exceptthat changing the charge on them deflects the beam vertically.SUMMARYElectric chargeshave the following important properties:•Unlike charges attract one another, and like charges repel one another.•Charge is conserved.•Charge is quantized—that is, it exists in discrete packets that are some integralmultiple of the electronic charge.Conductorsare materials in which charges move freely. Insulatorsare mate-rials in which charges do not move freely.Coulomb's lawstates that the electric force exerted by a charge q1on a sec-ond charge q2is(23.2)where ris the distance between the two charges and is a unit vector directedfrom q1to q2. The constant ke, called the Coulomb constant, has the valueThe smallest unit of charge known to exist in nature is the charge on an elec-tron or proton, The electric field Eat some point in space is defined as the electric force Fethat acts on a small positive test charge placed at that point divided by the magni-tude of the test charge q0:(23.3)At a distance rfrom a point charge q, the electric field due to the charge is givenby(23.4)where is a unit vector directed from the charge to the point in question. Therˆ E/H11005ke qr2 rˆ E/H11013Feq0/H20841e/H20841/H110051.602 19/H1100310/H1100219 C.ke/H110058.99/H11003109 N/H11080m2/C2.rˆ F12/H11005ke q1q2r2 rˆElectrongunVerticaldeflectionplatesHorizontaldeflectionplatesElectronbeamFluorescentscreenHorizontalinputVerticalinputCAFigure 23.26Schematic diagram of acathode ray tube. Electrons leaving thehot cathode C are accelerated to the an-ode A. In addition to accelerating elec-trons, the electron gun is also used to fo-cus the beam of electrons, and the platesdeflect the beam.732CHAPTER 23Electric Fieldselectric field is directed radially outward from a positive charge and radially in-ward toward a negative charge.The electric field due to a group of point charges can be obtained by usingthe superposition principle. That is, the total electric field at some point equalsthe vector sum of the electric fields of all the charges:(23.5)The electric field at some point of a continuous charge distribution is(23.6)where dqis the charge on one element of the charge distribution and ris the dis-tance from the element to the point in question.Electric field linesdescribe an electric field in any region of space. The num-ber of lines per unit area through a surface perpendicular to the lines is propor-tional to the magnitude of Ein that region.A charged particle of mass mand charge qmoving in an electric field Ehas anacceleration(23.7)a/H11005qEmE/H11005ke /H20885 dqr2 rˆE/H11005ke /H20858i qiri 2 rˆi Problem-Solving HintsFinding the Electric Field•Units:In calculations using the Coulomb constant chargesmust be expressed in coulombs and distances in meters.•Calculating the electric field of point charges:To find the total electricfield at a given point, first calculate the electric field at the point due toeach individual charge. The resultant field at the point is the vector sum ofthe fields due to the individual charges.•Continuous charge distributions:When you are confronted with prob-lems that involve a continuous distribution of charge, the vector sums forevaluating the total electric field at some point must be replaced by vectorintegrals. Divide the charge distribution into infinitesimal pieces, and calcu-late the vector sum by integrating over the entire charge distribution. Youshould review Examples 23.7 through 23.9.•Symmetry:With both distributions of point charges and continuouscharge distributions, take advantage of any symmetry in the system to sim-plify your calculations.ke (/H110051/4/H9266/H92800), QUESTIONSclings to a wall. Does this mean that the wall is positivelycharged? Why does the balloon eventually fall?4.A light, uncharged metallic sphere suspended from athread is attracted to a charged rubber rod. After touch-ing the rod, the sphere is repelled by the rod. Explain.1.Sparks are often observed (or heard) on a dry day whenclothes are removed in the dark. Explain.2.Explain from an atomic viewpoint why charge is usuallytransferred by electrons.3.A balloon is negatively charged by rubbing and thenProblems7335.Explain what is meant by the term “a neutral atom.”6.Why do some clothes cling together and to your body af-ter they are removed from a dryer?7.A large metallic sphere insulated from ground is chargedwith an electrostatic generator while a person standing onan insulating stool holds the sphere. Why is it safe to dothis? Why wouldn’t it be safe for another person to touchthe sphere after it has been charged?8.What are the similarities and differences between New-ton’s law of gravitation, and Coulomb’slaw, 9.Assume that someone proposes a theory that states that people are bound to the Earth by electric forcesrather than by gravity. How could you prove this theorywrong?10.How would you experimentally distinguish an electricfield from a gravitational field?11.Would life be different if the electron were positivelycharged and the proton were negatively charged? Doesthe choice of signs have any bearing on physical andchemical interactions? Explain.12.When defining the electric field, why is it necessary tospecify that the magnitude of the test charge be verysmall (that is, why is it necessary to take the limit of Fe/qas 13.Two charged conducting spheres, each of radius a, areseparated by a distance r/H110222a. Is the force on eithersphere given by Coulomb’s law? Explain. (Hint:Refer toChapter 14 on gravitation.)14.When is it valid to approximate a charge distribution by apoint charge?15.Is it possible for an electric field to exist in empty space?Explain.16.Explain why electric field lines never cross. (Hint:Emusthave a unique direction at all points.)17.A free electron and free proton are placed in an identicalq:0)?Fe/H11005keq1q2/r2?Fg/H11005Gm1m2/r2,electric field. Compare the electric forces on each parti-cle. Compare their accelerations.18.Explain what happens to the magnitude of the electricfield of a point charge as rapproaches zero.19.A negative charge is placed in a region of space where theelectric field is directed vertically upward. What is the di-rection of the electric force experienced by this charge?20.A charge 4qis a distance rfrom a charge /H11002q. Comparethe number of electric field lines leaving the charge 4qwith the number entering the charge /H11002q.21.In Figure 23.23, where do the extra lines leaving thecharge /H110012qend?22.Consider two equal point charges separated by some dis-tance d. At what point (other than /H11009) would a third testcharge experience no net force?23.A negative point charge /H11002qis placed at the point Pnearthe positively charged ring shown in Figure 23.17. Ifdescribe the motion of the point charge if it is re-leased from rest.24.Explain the differences between linear, surface, and vol-ume charge densities, and give examples of when eachwould be used.25.If the electron in Figure 23.25 is projected into the elec-tric field with an arbitrary velocity vi(at an angle to E),will its trajectory still be parabolic? Explain.26.It has been reported that in some instances people nearwhere a lightning bolt strikes the Earth have had theirclothes thrown off. Explain why this might happen.27.Why should a ground wire be connected to the metallicsupport rod for a television antenna?28.A light strip of aluminum foil is draped over a woodenrod. When a rod carrying a positive charge is broughtclose to the foil, the two parts of the foil stand apart.Why? What kind of charge is on the foil?29.Why is it more difficult to charge an object by rubbing ona humid day than on a dry day?xVa, PROBLEMSforce compare with the magnitude of the gravitationalforce between the two protons? (c) What must be thecharge-to-mass ratio of a particle if the magnitude of thegravitational force between two of these particles equalsthe magnitude of the electric force between them?3.Richard Feynman once said that if two persons stood atarm’s length from each other and each person had 1%more electrons than protons, the force of repulsion be-tween them would be enough to lift a “weight” equal to that of the entire Earth. Carry out an order-of-magnitude calculation to substantiate this assertion.4.Two small silver spheres, each with a mass of 10.0 g, areseparated by 1.00 m. Calculate the fraction of the elec-Section 23.1Properties of Electric ChargesSection 23.2Insulators and ConductorsSection 23.3Coulomb’s Law1.(a) Calculate the number of electrons in a small, electri-cally neutral silver pin that has a mass of 10.0 g. Silverhas 47 electrons per atom, and its molar mass is 107.87 g/mol. (b) Electrons are added to the pin untilthe net negative charge is 1.00 mC. How many electronsare added for every 109electrons already present?2.(a) Two protons in a molecule are separated by a distanceof 3.80/H1100310/H1100210m. Find the electric force exerted by oneproton on the other. (b) How does the magnitude of this1, 2, 3= straightforward, intermediate, challenging = full solution available in the Student Solutions Manual and Study GuideWEB= solution posted at http://www.saunderscollege.com/physics/= Computer useful in solving problem = Interactive Physics= paired numerical/symbolic problems WEB734CHAPTER 23Electric Fieldstrons in one sphere that must be transferred to theother to produce an attractive force of 1.00/H11003104N(about 1 ton) between the spheres. (The number ofelectrons per atom of silver is 47, and the number ofatoms per gram is Avogadro’s number divided by themolar mass of silver, 107.87 g/mol.)5.Suppose that 1.00 g of hydrogen is separated into elec-trons and protons. Suppose also that the protons areplaced at the Earth’s north pole and the electrons areplaced at the south pole. What is the resulting compres-sional force on the Earth?6.Two identical conducting small spheres are placedwith their centers 0.300 m apart. One is given a charge of 12.0 nC, and the other is given a charge of/H1100218.0 nC. (a) Find the electric force exerted on onesphere by the other. (b) The spheres are connected bya conducting wire. Find the electric force between thetwo after equilibrium has occurred.7.Three point charges are located at the corners of anequilateral triangle, as shown in Figure P23.7. Calculatethe net electric force on the 7.00-/H9262C charge. 14.An airplane is flying through a thundercloud at aheight of 2 000 m. (This is a very dangerous thing to dobecause of updrafts, turbulence, and the possibility ofelectric discharge.) If there are charge concentrationsof /H1100140.0 C at a height of 3 000 m within the cloud andof /H1100240.0 C at a height of 1 000 m, what is the electricfield Eat the aircraft?Section 23.4The Electric Field11.What are the magnitude and direction of the electricfield that will balance the weight of (a) an electron and(b) a proton? (Use the data in T able 23.1.)12.An object having a net charge of 24.0/H9262C is placed in auniform electric field of 610 N/C that is directed verti-cally. What is the mass of this object if it “floats” in thefield?13.In Figure P23.13, determine the point (other than in-finity) at which the electric field is zero.10.Review Problem.Two identical point charges eachhaving charge /H11001qare fixed in space and separated by adistance d. A third point charge /H11002Qof mass mis free tomove and lies initially at rest on a perpendicular bisec-tor of the two fixed charges a distance xfrom the mid-point of the two fixed charges (Fig. P23.10). (a) Showthat if xis small compared with d, the motion of /H11002Qissimple harmonic along the perpendicular bisector. De-termine the period of that motion. (b) How fast will thecharge /H11002Qbe moving when it is at the midpoint be-tween the two fixed charges, if initially it is released at adistance from the midpoint? x/H11005aVd 9.Review Problem.In the Bohr theory of the hydrogenatom, an electron moves in a circular orbit about a pro-ton, where the radius of the orbit is 0.529/H1100310/H1100210m.(a) Find the electric force between the two. (b) If thisforce causes the centripetal acceleration of the electron,what is the speed of the electron?8.Two small beads having positive charges 3qand qarefixed at the opposite ends of a horizontal insulating rodextending from the origin to the point x/H11005d. As shownin Figure P23.8, a third small charged bead is free toslide on the rod. At what position is the third bead inequilibrium? Can it be in stable equilibrium?0.500 m7.00 µC 2.00 µC –4.00 µC60.0°xyµ µµ–++ Figure P23.7Problems 7 and 15. Figure P23.8Figure P23.10 d+3q+q+q +q–Qxyd/2d/2x 1.00 m–2.50 µC 6.00 µCµµFigure P23.13Problems73515.Three charges are at the corners of an equilateral trian-gle, as shown in Figure P23.7. (a) Calculate the electricfield at the position of the 2.00-/H9262C charge due to the7.00-/H9262C and /H110024.00-/H9262C charges. (b) Use your answer topart (a) to determine the force on the 2.00-/H9262C charge.16.Three point charges are arranged as shown in FigureP23.16. (a) Find the vector electric field that the 6.00-nC and /H110023.00-nC charges together create at theorigin. (b) Find the vector force on the 5.00-nC charge. 22.Consider nequal positive point charges each of magni-tude Q/nplaced symmetrically around a circle of ra-dius R. (a) Calculate the magnitude of the electric fieldEat a point a distance xon the line passing through thecenter of the circle and perpendicular to the plane ofthe circle. (b) Explain why this result is identical to theone obtained in Example 23.8.23.Consider an infinite number of identical charges (eachof charge q) placed along the xaxis at distances a, 2a,3a, 4a,...from the origin. What is the electric fieldat the origin due to this distribution? Hint:Use the factthatSection 23.5Electric Field of a Continuous Charge

Distribution24.A rod 14.0 cm long is uniformly charged and has a totalcharge of /H1100222.0/H9262C. Determine the magnitude and di-rection of the electric field along the axis of the rod at apoint 36.0 cm from its center.1/H11001122/H11001132/H11001142/H11001/H11080 /H11080 /H11080/H11005/H926626nents of the electric field at point (x, y) due to thischarge qare21.Consider the electric dipole shown in Figure P23.21.Show that the electric field at a distantpoint along the xaxis is Ex/H110614keqa/x3.Ey/H11005keq(y/H11002y0)[(x/H11002x0)2/H11001(y/H11002y0)2]3/2Ex/H11005keq(x/H11002x0)[(x/H11002x0)2/H11001(y/H11002y0)2]3/2 18.Two 2.00-/H9262C point charges are located on the xaxis.One is at x/H110051.00 m, and the other is at x/H11005/H110021.00 m.(a) Determine the electric field on the yaxis at y/H110050.500 m. (b) Calculate the electric force on a /H110023.00-/H9262Ccharge placed on the yaxis at y/H110050.500 m.19.Four point charges are at the corners of a square of sidea, as shown in Figure P23.19. (a) Determine the magni-tude and direction of the electric field at the location ofcharge q. (b) What is the resultant force on q?20.A point particle having charge qis located at point (x0, y0) in the xyplane. Show that the xand ycompo-17.Three equal positive charges qare at the corners of anequilateral triangle of side a, as shown in Figure P23.17.(a) Assume that the three charges together create anelectric field. Find the location of a point (other than/H11009) where the electric field is zero. (Hint:Sketch thefield lines in the plane of the charges.) (b) What arethe magnitude and direction of the electric field at Pdue to the two charges at the base? Figure P23.17Figure P23.19 Figure P23.21Figure P23.160.100 mx–3.00 nC5.00 nC0.300 m6.00 nCy qqaqaaP+++aaa aq 3q4q2q 2ax–qqyWEB736CHAPTER 23Electric Fields25.A continuous line of charge lies along the xaxis, extend-ing from x/H11005/H11001x0to positive infinity. The line carries auniform linear charge density /H92610. What are the magni-tude and direction of the electric field at the origin?26.A line of charge starts at x/H11005/H11001x0and extends to posi-tive infinity. If the linear charge density is /H9261/H11005/H92610x0/x,determine the electric field at the origin.27.A uniformly charged ring of radius 10.0 cm has a totalcharge of 75.0/H9262C. Find the electric field on the axis ofthe ring at (a) 1.00 cm, (b) 5.00 cm, (c) 30.0 cm, and(d) 100 cm from the center of the ring.28.Show that the maximum field strength Emaxalong theaxis of a uniformly charged ring occurs at (see Fig. 23.17) and has the value 29.A uniformly charged disk of radius 35.0 cm carries acharge density of 7.90/H1100310/H110023C/m2. Calculate theelectric field on the axis of the disk at (a) 5.00 cm, (b) 10.0 cm, (c) 50.0 cm, and (d) 200 cm from the cen-ter of the disk.30.Example 23.9 derives the exact expression for the elec-tric field at a point on the axis of a uniformly chargeddisk. Consider a disk of radius cm having auniformly distributed charge of /H110015.20/H9262C. (a) Usingthe result of Example 23.9, compute the electric field ata point on the axis and 3.00 mm from the center. Com-pare this answer with the field computed from the near-field approximation (b) Using the result ofExample 23.9, compute the electric field at a point onthe axis and 30.0 cm from the center of the disk. Com-pare this result with the electric field obtained by treat-ing the disk as a /H110015.20-/H9262C point charge at a distance of30.0 cm.31.The electric field along the axis of a uniformly chargeddisk of radius Rand total charge Qwas calculated in Ex-ample 23.9. Show that the electric field at distances xthat are great compared with Rapproaches that of apoint charge (Hint:First show thatand use the bino-mial expansion when 32.A piece of Styrofoam having a mass mcarries a netcharge of /H11002qand floats above the center of a very largehorizontal sheet of plastic that has a uniform chargedensity on its surface. What is the charge per unit areaon the plastic sheet?33.A uniformly charged insulating rod of length 14.0 cm isbent into the shape of a semicircle, as shown in FigureP23.33. The rod has a total charge of /H110027.50/H9262C. Findthe magnitude and direction of the electric field at O,the center of the semicircle.34.(a) Consider a uniformly charged right circular cylin-drical shell having total charge Q, radius R, and heighth. Determine the electric field at a point a distance dfrom the right side of the cylinder, as shown in FigureP23.34. (Hint:Use the result of Example 23.8 and treatthe cylinder as a collection of ring charges.) (b) Con-sider now a solid cylinder with the same dimensions and/H9254V1.)(1/H11001/H9254)n/H110151/H11001n/H9254x/(x2/H11001R2)1/2/H11005(1/H11001R2/x2)/H110021/2,Q/H11005/H9268/H9266R2.E/H11005/H92.R/H110053.00Q/(6!3/H9266/H92800a2).x/H11005a/!2 WEB 36.Three solid plastic cylinders all have a radius of 2.50 cmand a length of 6.00 cm. One (a) carries charge withcarrying the same charge, which is uniformly distrib-uted through its volume. Use the result of Example 23.9to find the field it creates at the same point.35.A thin rod of length and uniform charge per unitlength /H9261lies along the xaxis, as shown in Figure P23.35. (a) Show that the electric field at P, a distance yfromthe rod, along the perpendicular bisector has no xcom-ponent and is given by (b) Usingyour result to part (a), show that the field of a rod of in-finite length is (Hint:First calculate thefield at Pdue to an element of length dx, which has acharge /H9261dx. Then change variables from xto /H9258, usingthe facts that x/H11005ytan /H9258and sec2/H9258d/H9258, and inte-grate over /H9258.)dx/H11005yE/H110052ke/H9261/y.E/H110052ke/H9261 sin /H92580/y./H5129OFigure P23.33 Figure P23.34 Figure P23.35Rddxh yy dxxP O/H5129θθ0θProblems737uniform density 15.0 nC/m2everywhere on its surface.Another (b) carries charge with the same uniform den-sity on its curved lateral surface only. The third (c) car-ries charge with uniform density 500 nC/m3through-out the plastic. Find the charge of each cylinder.37.Eight solid plastic cubes, each 3.00 cm on each edge,are glued together to form each one of the objects (i, ii,iii, and iv) shown in Figure P23.37. (a) If each objectcarries charge with a uniform density of 400 nC/m3throughout its volume, what is the charge of each ob-ject? (b) If each object is given charge with a uniformdensity of 15.0 nC/m2everywhere on its exposed sur-face, what is the charge on each object? (c) If charge isplaced only on the edges where perpendicular surfacesmeet, with a uniform density of 80.0 pC/m, what is thecharge of each object? Section 23.7Motion of Charged Particles in a Uniform Electric Field41.An electron and a proton are each placed at rest in anelectric field of 520 N/C. Calculate the speed of eachparticle 48.0 ns after being released.42.A proton is projected in the positive xdirection into a region of uniform electric field N/C.The proton travels 7.00 cm before coming to rest. Deter-mine (a) the acceleration of the proton, (b) its initialspeed, and (c) the time it takes the proton to come torest.43.A proton accelerates from rest in a uniform electricfield of 640 N/C. At some later time, its speed hasreached 1.20/H11003106m/s (nonrelativistic, since vismuch less than the speed of light). (a) Find the acceler-ation of the proton. (b) How long does it take the pro-ton to reach this speed? (c) How far has it moved in thistime? (d) What is its kinetic energy at this time?44.The electrons in a particle beam each have a kinetic en-ergy of 1.60/H1100310/H1100217J. What are the magnitude and di-rection of the electric field that stops these electrons ina distance of 10.0 cm?45.The electrons in a particle beam each have a kinetic en-ergy K. What are the magnitude and direction of theelectric field that stops these electrons in a distance d?46.A positively charged bead having a mass of 1.00 g fallsfrom rest in a vacuum from a height of 5.00 m in auniform vertical electric field with a magnitude of 1.00/H11003104N/C. The bead hits the ground at a speed of 21.0 m/s. Determine (a) the direction of theelectric field (up or down) and (b) the charge on thebead.47.A proton moves at 4.50/H11003105m/s in the horizontaldirection. It enters a uniform vertical electric field witha magnitude of 9.60/H11003103N/C. Ignoring any gravita-tional effects, find (a) the time it takes the proton totravel 5.00 cm horizontally, (b) its vertical displacementafter it has traveled 5.00 cm horizontally, and (c) thehorizontal and vertical components of its velocity afterit has traveled 5.00 cm horizontally.48.An electron is projected at an angle of 30.0° above thehorizontal at a speed of 8.20/H11003105m/s in a regionwhere the electric field is N/C. Neglectingthe effects of gravity, find (a) the time it takes the elec-tron to return to its initial height, (b) the maximumheight it reaches, and (c) its horizontal displacementwhen it reaches its maximum height.49.Protons are projected with an initial speedm/s into a region where a uniformelectric field N/C is present, as shown inFigure P23.49. The protons are to hit a target that lies ata horizontal distance of 1.27 mm from the point wherethe protons are launched. Find (a) the two projectionangles /H9258that result in a hit and (b) the total time offlight for each trajectory.E/H11005(/H11002720j)vi/H110059.55/H11003103E/H11005390jE/H11005/H110026.00/H11003105i WEBSection 23.6Electric Field Lines38.A positively charged disk has a uniform charge per unitarea as described in Example 23.9. Sketch the electricfield lines in a plane perpendicular to the plane of thedisk passing through its center.39.A negatively charged rod of finite length has a uniformcharge per unit length. Sketch the electric field lines ina plane containing the rod.40.Figure P23.40 shows the electric field lines for two pointcharges separated by a small distance. (a) Determinethe ratio q1/q2. (b) What are the signs of q1and q2?Figure P23.37 Figure P23.40(i) (ii) (iii)(iv) q2q1738CHAPTER 23Electric Fields ADDITIONAL PROBLEMS50.Three point charges are aligned along the xaxis asshown in Figure P23.50. Find the electric field at (a) theposition (2.00, 0) and (b) the position (0, 2.00).makes a 15.0° angle with the vertical, what is the netcharge on the ball?53.A charged cork ball of mass 1.00 g is suspended on a light string in the presence of a uniform electricfield, as shown in Figure P23.53. When N/C, the ball is in equilibrium at /H9258/H1100537.0°. Find (a) the charge on the ball and (b) the tension in the string.54.A charged cork ball of mass mis suspended on a lightstring in the presence of a uniform electric field, asshown in Figure P23.53. When N/C,where Aand Bare positive numbers, the ball is in equi-librium at the angle /H9258. Find (a) the charge on the balland (b) the tension in the string.Bj)E/H11005(Ai/H110015.00j)/H11003105E/H11005(3.00i/H11001 WEB 56.Three identical small Styrofoam balls aresuspended from a fixed point by three nonconductingthreads, each with a length of 50.0 cm and with negligi-(m/H110052.00 g)55.Four identical point charges arelocated on the corners of a rectangle, as shown inFigure P23.55. The dimensions of the rectangle arecm and cm. Calculate the magni-tude and direction of the net electric force exerted onthe charge at the lower left corner by the other threecharges.W/H1100515.0L/H1100560.0(q/H11005/H1100110.0 /H9262C)51.A uniform electric field of magnitude 640 N/C existsbetween two parallel plates that are 4.00 cm apart. Aproton is released from the positive plate at the same in-stant that an electron is released from the negativeplate. (a) Determine the distance from the positiveplate at which the two pass each other. (Ignore the elec-trical attraction between the proton and electron.) (b) Repeat part (a) for a sodium ion (Na/H11001) and a chlo-rine ion (Cl/H11002).52.A small, 2.00-g plastic ball is suspended by a 20.0-cm-long string in a uniform electric field, as shown in Fig-ure P23.52. If the ball is in equilibrium when the stringθvi1.27 mmTargetE = (–720j) N/C×ProtonbeamFigure P23.49 Figure P23.50 Figure P23.52Figure P23.53Problems 53 and 54. Figure P23.550.800 my3.00 nC5.00 nC0.500 m– 4.00 nCx yx15.0°20.0 cmm = 2.00 gE = 1.00 × 103i N/CxyEqθ qqqqy xLWProblems739ble mass. At equilibrium the three balls form an equilat-eral triangle with sides of 30.0 cm. What is the commoncharge qcarried by each ball?57.Two identical metallic blocks resting on a frictionlesshorizontal surface are connected by a light metallicspring having the spring constant N/m and anunstretched length of 0.300 m, as shown in FigureP23.57a. A total charge of Qis slowly placed on the sys-tem, causing the spring to stretch to an equilibriumlength of 0.400 m, as shown in Figure P23.57b. Deter-mine the value of Q, assuming that all the charge re-sides on the blocks and that the blocks are like pointcharges.58.Two identical metallic blocks resting on a frictionlesshorizontal surface are connected by a light metallicspring having a spring constant kand an unstretchedlength Li, as shown in Figure P23.57a. A total charge ofQis slowly placed on the system, causing the spring tostretch to an equilibrium length L, as shown in FigureP23.57b. Determine the value of Q, assuming that allthe charge resides on the blocks and that the blocks arelike point charges.k/H110051001 N/C. Will the charged particle remain nonrelativisticfor a shorter or a longer time in a much larger electricfield?61.A line of positive charge is formed into a semicircle ofradius cm, as shown in Figure P23.61. Thecharge per unit length along the semicircle is describedby the expression The total charge on thesemicircle is 12.0/H9262C. Calculate the total force on acharge of 3.00/H9262C placed at the center of curvature./H9261/H11005/H92610 cos /H9258.R/H1100560.0 62.Two small spheres, each of mass 2.00 g, are suspendedby light strings 10.0 cm in length (Fig. P23.62). A uni-form electric field is applied in the xdirection. Thespheres have charges equal to /H110025.00/H1100310/H110028C and/H110015.00/H1100310/H110028C. Determine the electric field that en-ables the spheres to be in equilibrium at an angle of/H9258/H1100510.0/H11034.59.Identical thin rods of length 2acarry equal charges,/H11001Q, uniformly distributed along their lengths. Therods lie along the xaxis with their centers separated bya distance of (Fig. P23.59). Show that the magni-tude of the force exerted by the left rod on the rightone is given by60.A particle is said to be nonrelativistic as long as its speedis less than one-tenth the speed of light, or less than3.00/H11003107m/s. (a) How long will an electron remainnonrelativistic if it starts from rest in a region of anelectric field of 1.00 N/C? (b) How long will a protonremain nonrelativistic in the same electric field? (c) Electric fields are commonly much larger than F/H11005/H20898keQ24a2/H20899ln/H20898b2b2/H110024a2/H20899b/H110222aFigure P23.57Problems 57 and 58.(a) (b)mmkmmkbya–a b –a b + axFigure P23.59 Figure P23.61 Figure P23.62yRxθ θEθ–+740CHAPTER 23Electric Fields63.Two small spheres of mass mare suspended from stringsof length that are connected at a common point. Onesphere has charge Q; the other has charge 2Q. Assumethat the angles /H92581and /H92582that the strings make with thevertical are small. (a) How are /H92581and /H92582related? (b) Show that the distance rbetween the spheres is64.Three charges of equal magnitude qare fixed in posi-tion at the vertices of an equilateral triangle (Fig.P23.64). A fourth charge Qis free to move along thepositive xaxis under the influence of the forces exertedby the three fixed charges. Find a value for sfor whichQis in equilibrium. You will need to solve a transcen-dental equation.r/H11061/H208984keQ2/H5129mg/H208991/3/H5129 66.Review Problem.A 1.00-g cork ball with a charge of2.00/H9262C is suspended vertically on a 0.500-m-long lightstring in the presence of a uniform, downward-directedelectric field of magnitude N/C. If theball is displaced slightly from the vertical, it oscillateslike a simple pendulum. (a) Determine the period ofthis oscillation. (b) Should gravity be included in thecalculation for part (a)? Explain.67.Three charges of equal magnitude qreside at the cor-ners of an equilateral triangle of side length a(Fig.P23.67). (a) Find the magnitude and direction of theelectric field at point P, midway between the negativecharges, in terms of ke, q, and a. (b) Where must a /H110024qcharge be placed so that any charge located at Pexperi-ences no net electric force? In part (b), let Pbe the ori-gin and let the distance between the /H11001qcharge and Pbe 1.00 m.E/H110051.00/H11003105 68.Two identical beads each have a mass mand charge q.When placed in a hemispherical bowl of radius Rwithfrictionless, nonconducting walls, the beads move, andat equilibrium they are a distance Rapart (Fig. P23.68).Determine the charge on each bead.65.Review Problem.Four identical point charges, eachhaving charge /H11001q, are fixed at the corners of a squareof side L. A fifth point charge /H11002Qlies a distance zalongthe line perpendicular to the plane of the square andpassing through the center of the square (Fig. P23.65).(a) Show that the force exerted on /H11002Qby the otherfour charges isNote that this force is directed toward the center of thesquare whether zis positive (/H11002Qabove the square) ornegative (/H11002Qbelow the square). (b) If zis small com-pared with L, the above expression reduces toWhy does this imply that the mo-tion of /H11002Qis simple harmonic, and what would be theperiod of this motion if the mass of /H11002Qwere m?F/H11015/H11002(constant)zk.F/H11005/H110024keqQ z/H20898z2/H11001L22/H208993/2 kFigure P23.64Figure P23.65 Figure P23.67x+Q+q +q–qa2–a2–sraθy a 32√LL+q+qz–Qz+q+q +q–q–qaaa/2a/2P Problems741 This charge distribution, which is essentially that of twoelectric dipoles, is called an electric quadrupole.Note thatEvaries as r/H110024for the quadrupole, compared with varia-tions of r/H110023for the dipole and r/H110022for the monopole (asingle charge).73.Review Problem.A negatively charged particle /H11002qis placed at the center of a uniformly charged ring,where the ring has a total positive charge Q,as shownin Example 23.8. The particle, confined to move alongthe xaxis, is displaced a small distance xalong the axis(where and released. Show that the particle os-cillates with simple harmonic motion with a frequency74.Review Problem.An electric dipole in a uniform elec-tric field is displaced slightly from its equilibrium posi-tion, as shown in Figure P23.74, where /H9258is small andthe charges are separated by a distance 2a. The momentof inertia of the dipole is I. If the dipole is released fromthis position, show that its angular orientation exhibitssimple harmonic motion with a frequencyf/H1100512/H9266 !2qaEIf/H1100512/H9266 /H20898keqQma3/H208991/2xVa) 70.Consider the charge distribution shown in FigureP23.69. (a) Show that the magnitude of the electricfield at the center of any face of the cube has a value of2.18keq/s2. (b) What is the direction of the electricfield at the center of the top face of the cube?71.A line of charge with a uniform density of 35.0 nC/mlies along the line y/H11005/H1100215.0 cm, between the pointswith coordinates x/H110050 and x/H1100540.0 cm. Find the elec-tric field it creates at the origin.72.Three point charges q, /H110022q, and qare located along thexaxis, as shown in Figure P23.72. Show that the electricfield at Palong the yaxis isE/H11005/H11002ke 3qa2y4 j(yWa)69.Eight point charges, each of magnitude q, are locatedon the corners of a cube of side s, as shown in FigureP23.69. (a) Determine the x, y, and zcomponents of theresultant force exerted on the charge located at point Aby the other charges. (b) What are the magnitude anddirection of this resultant force?Figure P23.68 Figure P23.69Problems 69 and 70.Figure P23.72 Figure P23.74RRmRm PointAxyzqqq qqqqqsssPy xq–2qqaay Eθq+—q2a742CHAPTER 23Electric FieldsANSWERS TOQUICKQUIZZES23.3(b). From Newton's third law, the electric force exertedby object B on object A is equal in magnitude to theforce exerted by object A on object B and in the oppo-site direction—that is, 23.4Nothing, if we assume that the source charge producingthe field is not disturbed by our actions. Remember thatthe electric field is created not by the /H110013-/H9262C charge orby the /H110023-/H9262C charge but by the source charge (unseenin this case).23.5A, B, and C. The field is greatest at point Abecause thisis where the field lines are closest together. The absenceof lines at point Cindicates that the electric field there iszero.FAB/H11005/H11002FBA .23.1(b). The amount of charge present after rubbing is thesame as that before; it is just distributed differently.23.2(d). Object A might be negatively charged, but it alsomight be electrically neutral with an induced chargeseparation, as shown in the following figure: ++++++++++BA+++——2.2This is the Nearest One Head743 chapterGauss's Law PUZZLERSome railway companies are planning tocoat the windows of their commutertrains with a very thin layer of metal.(The coating is so thin you can seethrough it.) They are doing this in re-sponse to rider complaints about otherpassengers' talking loudly on cellulartelephones. How can a metallic coatingthat is only a few hundred nanometersthick overcome this problem?(ArthurTilley/FPG International) Chapter Outline24.1Electric Flux24.2Gauss's Law24.3Application of Gauss's Law toCharged Insulators24.4Conductors in ElectrostaticEquilibrium24.5(Optional)ExperimentalVerification of Gauss's Law andCoulomb's Law24.6(Optional)Formal Derivation ofGauss's LawPUZZLER 743744CHAPTER 24Gauss's Lawn the preceding chapter we showed how to use Coulomb's law to calculate theelectric field generated by a given charge distribution. In this chapter, we de-scribe Gauss's lawand an alternative procedure for calculating electric fields.The law is based on the fact that the fundamental electrostatic force between pointcharges exhibits an inverse-square behavior. Although a consequence ofCoulomb's law, Gauss's law is more convenient for calculating the electric fields ofhighly symmetric charge distributions and makes possible useful qualitative rea-soning when we are dealing with complicated problems.ELECTRIC FLUXThe concept of electric field lines is described qualitatively in Chapter 23. We nowuse the concept of electric flux to treat electric field lines in a more quantitativeway.Consider an electric field that is uniform in both magnitude and direction, asshown in Figure 24.1. The field lines penetrate a rectangular surface of area A,which is perpendicular to the field. Recall from Section 23.6 that the number oflines per unit area (in other words, the line density) is proportional to the magni-tude of the electric field. Therefore, the total number of lines penetrating the sur-face is proportional to the product EA. This product of the magnitude of the elec-tric field Eand surface area Aperpendicular to the field is called the electric flux/H9021E(uppercase Greek phi):(24.1)From the SI units of Eand A, we see that /H9021Ehas units of newton–meters squaredper coulomb Electric flux is proportional to the number of elec-tric field lines penetrating some surface.(N/H11080m2/C)./H9021E/H11005EA24.1 Flux Through a Sphere EXAMPLE24.1perpendicular to the surface of the sphere. The flux throughthe sphere (whose surface area is thusExerciseWhat would be the (a) electric field and (b) fluxthrough the sphere if it had a radius of 0.500 m?

Answer(a) N/C; (b) 1.13/H11003105 N/H11080m2/C.3.60/H110031041.13/H11003105 N/H11080m2/C/H11005/H9021E/H11005EA/H11005(8.99/H11003103 N/C)(12.6 m2)A/H110054/H9266r2/H1100512.6 m2)What is the electric flux through a sphere that has a radius of1.00 m and carries a charge of /H110011.00/H9262C at its center?SolutionThe magnitude of the electric field 1.00 m fromthis charge is given by Equation 23.4,The field points radially outward and is therefore everywhere /H110058.99/H11003103 N/CE/H11005ke qr2/H11005(8.99/H11003109 N/H11080m2/C2) 1.00/H1100310/H110026 C(1.00 m)2I 11.6 Area = AEFigure 24.1Field lines repre-senting a uniform electric fieldpenetrating a plane of area Aper-pendicular to the field. The electricflux /H9021Ethrough this area is equalto EA. If the surface under consideration is not perpendicular to the field, the fluxthrough it must be less than that given by Equation 24.1. We can understand thisby considering Figure 24.2, in which the normal to the surface of area Ais at anangle /H9258to the uniform electric field. Note that the number of lines that cross thisarea Ais equal to the number that cross the area A/H11032, which is a projection of area Aaligned perpendicular to the field. From Figure 24.2 we see that the two areas arerelated by cos /H9258. Because the flux through Aequals the flux through A/H11032, weA/H11032/H11005A24.1Electric Flux745 conclude that the flux through Ais(24.2)From this result, we see that the flux through a surface of fixed area Ahas a maxi-mum value EAwhen the surface is perpendicular to the field (in other words,when the normal to the surface is parallel to the field, that is, in Figure24.2); the flux is zero when the surface is parallel to the field (in other words,when the normal to the surface is perpendicular to the field, that is, We assumed a uniform electric field in the preceding discussion. In more gen-eral situations, the electric field may vary over a surface. Therefore, our definitionof flux given by Equation 24.2 has meaning only over a small element of area.Consider a general surface divided up into a large number of small elements, eachof area /H9004A. The variation in the electric field over one element can be neglected ifthe element is sufficiently small. It is convenient to define a vector /H9004Aiwhose mag-nitude represents the area of the ith element of the surface and whose direction isdefined to be perpendicularto the surface element, as shown in Figure 24.3. The elec-tric flux /H9004/H9021Ethrough this element iswhere we have used the definition of the scalar product of two vectorsBy summing the contributions of all elements, we obtain thetotal flux through the surface.1If we let the area of each element approach zero,then the number of elements approaches infinity and the sum is replaced by an in-tegral. Therefore, the general definition of electric flux is(24.3)Equation 24.3 is a surface integral,which means it must be evaluated over the sur-face in question. In general, the value of /H9021Edepends both on the field pattern andon the surface.We are often interested in evaluating the flux through a closed surface,which isdefined as one that divides space into an inside and an outside region, so that onecannot move from one region to the other without crossing the surface. The sur-face of a sphere, for example, is a closed surface.Consider the closed surface in Figure 24.4. The vectors /H9004Aipoint in differentdirections for the various surface elements, but at each point they are normal todA/H9021E/H11005lim/H9004Ai:0 /H9018Ei/H11554/H9004Ai/H11005/H20885surfaceE/H11554(A/H11554B/H11005AB cos /H9258)./H9004/H9021E/H11005Ei /H9004Ai cos /H9258/H11005Ei/H11080/H9004Ai/H9258/H1100590/H11034)./H9258/H110050/H11034/H9021E/H11005EA/H11032/H11005EA cos /H9258QuickLabShine a desk lamp onto a playingcard and notice how the size of theshadow on your desk depends on theorientation of the card with respectto the beam of light. Could a formulalike Equation 24.2 be used to de-scribe how much light was beingblocked by the card? Definition of electric flux 1It is important to note that drawings with field lines have their inaccuracies because a small area ele-ment (depending on its location) may happen to have too many or too few field lines penetrating it.We stress that the basic definition of electric flux is The use of lines is only an aid for visualiz-ing the concept./H20885 E/H11554dA.AθθA′ = A cos θENormal θFigure 24.2Field lines representing auniform electric field penetrating anarea Athat is at an angle /H9258to the field.Because the number of lines that gothrough the area A/H11032is the same as thenumber that go through A, the fluxthrough A/H11032is equal to the flux throughAand is given by /H9021E/H11005EA cos /H9258. ∆AiEiθFigure 24.3A small element ofsurface area /H9004Ai. The electric fieldmakes an angle /H9258with the vector/H9004Ai, defined as being normal tothe surface element, and the fluxthrough the element is equal toEi /H9004Ai cos /H9258.746CHAPTER 24Gauss's Law the surface and, by convention, always point outward. At the element labeled /L51832,the field lines are crossing the surface from the inside to the outside and hence, the flux ithrough this element is positive. For element /L51833,the field lines graze the surface (perpendicular to the vector /H9004Ai); thus, and the flux is zero. For elements such as /L51834, where the field lines are crossing thesurface from outside to inside, and the flux is negative because cos /H9258is negative. The netflux through the surface is proportional to the net num-ber of lines leaving the surface, where the net number means the number leaving thesurface minus the number entering the surface.If more lines are leaving than entering,the net flux is positive. If more lines are entering than leaving, the net flux is nega-tive. Using the symbol to represent an integral over a closed surface, we can writethe net flux /H9021Ethrough a closed surface as(24.4)where Enrepresents the component of the electric field normal to the surface.Evaluating the net flux through a closed surface can be very cumbersome. How-ever, if the field is normal to the surface at each point and constant in magnitude,the calculation is straightforward, as it was in Example 24.1. The next example alsoillustrates this point./H9021E/H11005/H20886 E/H11554dA/H11005/H20886 En dA/H20886180/H11034/H11022/H9258/H1102290/H11034/H9258/H1100590/H11034/H9004/H9021E/H11005E/H11554/H9004A/H9258/H1102190/ 24.4A closed surfacein an electric field. The area vec-tors /H9004Aiare, by convention, nor-mal to the surface and point out-ward. The flux through an areaelement can be positive (ele-ment /L51832), zero (element /L51833), ornegative (element /L51834). Flux Through a Cube EXAMPLE24.2faces (/L51834, /L51835, and the unnumbered ones) is zero because Eisperpendicular to dAon these faces.The net flux through faces /L51832and /L51833is/H9021E/H11005/H208851 E/H11554dA/H11001/H208852 E/H11554dAConsider a uniform electric field Eoriented in the xdirec-tion. Find the net electric flux through the surface of a cubeof edges /H5129, oriented as shown in Figure 24.5.SolutionThe net flux is the sum of the fluxes through allfaces of the cube. First, note that the flux through four of the Karl Friedrich GaussGermanmathematician and astronomer(1777 – 1855) 24.2Gauss's Law747 GAUSS'S LAWIn this section we describe a general relationship between the net electric fluxthrough a closed surface (often called a gaussian surface) and the charge enclosedby the surface. This relationship, known as Gauss's law,is of fundamental impor-tance in the study of electric fields.Let us again consider a positive point charge qlocated at the center of asphere of radius r, as shown in Figure 24.6. From Equation 23.4 we know that themagnitude of the electric field everywhere on the surface of the sphere isAs noted in Example 24.1, the field lines are directed radially outwardand hence perpendicular to the surface at every point on the surface. That is, ateach surface point, Eis parallel to the vector /H9004Airepresenting a local element ofarea /H9004Aisurrounding the surface point. Therefore,and from Equation 24.4 we find that the net flux through the gaussian surface iswhere we have moved Eoutside of the integral because, by symmetry, Eis constantover the surface and given by Furthermore, because the surface isspherical, Hence, the net flux through the gaussian surface isRecalling from Section 23.3 that we can write this equation in theform(24.5)We can verify that this expression for the net flux gives the same result as Example24.1: / C2/N/H11080m2)/H110051.13/H11003105 N/H11080m2/C. (8.85/H1100310/H1100212/H9021E/H11005(1.00/H1100310/H110026 C)/H9021E/H11005q/H92800ke/H110051/(4/H9266/H92800),/H9021E/H11005keqr2 (4/H9266r2)/H110054/H9266keq/H20886 dA/H11005A/H110054/H9266r2.E/H11005keq/r2./H9021E/H11005/H20886 E/H11554dA/H11005/H20886 E dA/H11005E /H20886 dAE/H11554/H9004Ai/H11005E /H9004AiE/H11005keq/r2.24.2y z/H5129/H5129/H5129xEdA2dA1dA3/L51832/L51833/L51834 /L51835dA4For /L51832, Eis constant and directed inward but dA1is directedoutward thus, the flux through this face isbecause the area of each face is For /L51833, Eis constant and outward and in the same direc-tion as dA2(/H9258/H110050°); hence, the flux through this face isTherefore, the net flux over all six faces is0/H9021E/H11005/H11002E/H51292/H11001E/H51292/H110010/H110010/H110010/H110010/H11005/H208852 E/H11554dA/H11005/H208852 E(cos 0/H11034)dA/H11005E /H208852 dA/H11005/H11001EA/H11005E/H51292A/H11005/H51292./H208851E/H11554dA/H11005/H208851 E(cos 180/H11034)dA/H11005/H11002E /H208851 dA/H11005/H11002EA/H11005/H11002E/H51292(/H9258/H11005180/H11034); Figure 24.5A closed surface in the shape of a cube in a uniformelectric field oriented parallel to the xaxis. The net flux through theclosed surface is zero. Side /L51835is the bottom of the cube, and side /L51832is opposite side /L51833. 11.6 GaussiansurfacerqdAE+iFigure 24.6A spherical gaussiansurface of radius rsurrounding apoint charge q. When the charge isat the center of the sphere, theelectric field is everywhere normalto the surface and constant in mag-nitude.748CHAPTER 24Gauss's Law Note from Equation 24.5 that the net flux through the spherical surface isproportional to the charge inside. The flux is independent of the radius rbecausethe area of the spherical surface is proportional to r2, whereas the electric field isproportional to 1/r2. Thus, in the product of area and electric field, the depen-dence on rcancels.Now consider several closed surfaces surrounding a charge q,as shown in Fig-ure 24.7. Surface S1is spherical, but surfaces S2and S3are not. From Equation24.5, the flux that passes through S1has the value q//H92800. As we discussed in the pre-vious section, flux is proportional to the number of electric field lines passingthrough a surface. The construction shown in Figure 24.7 shows that the numberof lines through S1is equal to the number of lines through the nonspherical sur-faces S2and S3. Therefore, we conclude that the net flux through anyclosed sur-face is independent of the shape of that surface. The net flux through anyclosed surface surrounding a point charge qis given by q//H92800.Now consider a point charge located outside a closed surface of arbitraryshape, as shown in Figure 24.8. As you can see from this construction, any electricfield line that enters the surface leaves the surface at another point. The numberof electric field lines entering the surface equals the number leaving the surface.Therefore, we conclude that the net electric flux through a closed surface thatsurrounds no charge is zero.If we apply this result to Example 24.2, we can eas-ily see that the net flux through the cube is zero because there is no charge insidethe cube.Suppose that the charge in Example 24.1 is just outside the sphere, 1.01 m from its center.What is the total flux through the sphere?Let us extend these arguments to two generalized cases: (1) that of manypoint charges and (2) that of a continuous distribution of charge. We once againuse the superposition principle, which states that the electric field due to manycharges is the vector sum of the electric fields produced by the individualcharges. Therefore, we can express the flux through any closed surface aswhere Eis the total electric field at any point on the surface produced by the vec-tor addition of the electric fields at that point due to the individual charges./H20886 E/H11554dA/H11005/H20886 (E1/H11001E2/H11001/H11080 /H11080 /H11080)/H11554dAQuick Quiz 24.1The net electric flux through aclosed surface is zero if there is nocharge insideS3S2S1q qFigure 24.7Closed surfaces of various shapes surround-ing a charge q. The net electric flux is the same through allsurfaces. Figure 24.8A point charge lo-cated outside a closed surface. Thenumber of lines entering the sur-face equals the number leaving thesurface.24.2Gauss's Law749Consider the system of charges shown in Figure 24.9. The surface Ssurroundsonly one charge, q1; hence, the net flux through Sis q1//H92800. The flux through Sdue to charges q2and q3outside it is zero because each electric field line that en-ters Sat one point leaves it at another. The surface S/H11032surrounds charges q2and q3;hence, the net flux through it is Finally, the net flux through surfaceS/H11033is zero because there is no charge inside this surface. That is, allthe electricfield lines that enter S/H11033at one point leave at another.Gauss's law,which is a generalization of what we have just described, statesthat the net flux through anyclosed surface is(24.6)where qinrepresents the net charge inside the surface and Erepresents the elec-tric field at any point on the surface.A formal proof of Gauss's law is presented in Section 24.6. When using Equa-tion 24.6, you should note that although the charge qinis the net charge inside thegaussian surface, Erepresents the total electric field,which includes contributionsfrom charges both inside and outside the surface.In principle, Gauss's law can be solved for Eto determine the electric fielddue to a system of charges or a continuous distribution of charge. In practice, how-ever, this type of solution is applicable only in a limited number of highly symmet-ric situations. As we shall see in the next section, Gauss's law can be used to evalu-ate the electric field for charge distributions that have spherical, cylindrical, orplanar symmetry. If one chooses the gaussian surface surrounding the charge dis-tribution carefully, the integral in Equation 24.6 can be simplified. You should alsonote that a gaussian surface is a mathematical construction and need not coincidewith any real physical surface.For a gaussian surface through which the net flux is zero, the following four statementscould be true.Which of the statements must be true?(a) There are no charges inside the sur-face. (b) The net charge inside the surface is zero. (c) The electric field is zero everywhereon the surface. (d) The number of electric field lines entering the surface equals the num-ber leaving the surface.Quick Quiz 24.2/H9021E/H11005/H20886 E/H11554dA/H11005qin/H92800(q2/H11001q3)//H92800. Sq1q2q3S'S″Figure 24.9The net electric fluxthrough any closed surface de-pends only on the charge insidethat surface. The net flux throughsurface Sis q1//H92800, the net fluxthrough surface S/H11032is and the net flux through surfaceS/H11033is zero.(q2/H11001q3)//H92800 ,Gauss's lawGauss's law is useful for evaluatingEwhen the charge distribution hashigh symmetry CONCEPTUALEXAMPLE24.3lines from the charge pass through the sphere, regardless ofits radius.(c) The flux does not change when the shape of the gauss-ian surface changes because all electric field lines from thecharge pass through the surface, regardless of its shape.(d) The flux does not change when the charge is movedto another location inside that surface because Gauss's lawrefers to the total charge enclosed, regardless of where thecharge is located inside the surface.A spherical gaussian surface surrounds a point charge q. De-scribe what happens to the total flux through the surface if(a) the charge is tripled, (b) the radius of the sphere is dou-bled, (c) the surface is changed to a cube, and (d) the chargeis moved to another location inside the surface.Solution(a) The flux through the surface is tripled because flux is proportional to the amount of charge insidethe surface.(b) The flux does not change because all electric field750CHAPTER 24Gauss's LawAPPLICATION OF GAUSS'S LAW TOCHARGED INSULATORSAs mentioned earlier, Gauss's law is useful in determining electric fields when thecharge distribution is characterized by a high degree of symmetry. The followingexamples demonstrate ways of choosing the gaussian surface over which the sur-face integral given by Equation 24.6 can be simplified and the electric field deter-mined. In choosing the surface, we should always take advantage of the symmetryof the charge distribution so that we can remove Efrom the integral and solve forit. The goal in this type of calculation is to determine a surface that satisfies one ormore of the following conditions:1. The value of the electric field can be argued by symmetry to be constant overthe surface.2. The dot product in Equation 24.6 can be expressed as a simple algebraic prod-uct E dAbecause Eand dAare parallel.3. The dot product in Equation 24.6 is zero because Eand dAare perpendicular.4. The field can be argued to be zero over the surface.All four of these conditions are used in examples throughout the remainder ofthis chapter.24.3 The Electric Field Due to a Point Charge EXAMPLE24.4Starting with Gauss's law, calculate the electric field due to anisolated point charge q.SolutionA single charge represents the simplest possiblecharge distribution, and we use this familiar case to show howto solve for the electric field with Gauss's law. We choose aspherical gaussian surface of radius rcentered on the pointcharge, as shown in Figure 24.10. The electric field due to apositive point charge is directed radially outward by symmetryand is therefore normal to the surface at every point. Thus, asin condition (2), Eis parallel to dAat each point. Therefore,and Gauss's law givesBy symmetry, Eis constant everywhere on the surface, whichsatisfies condition (1), so it can be removed from the inte-gral. Therefore,/H20886 E dA/H11005E /H20886 dA/H11005E(4/H9266r2)/H11005q/H92800/H9021E/H11005/H20886 E/H11554dA/H11005/H20886 E dA/H11005q/H92800E/H11554dA/H11005E dAGaussiansurfacerqdAE+Figure 24.10The point charge qis at the center of the sphericalgaussian surface, and Eis parallel to dAat every point on thesurface.where we have used the fact that the surface area of a sphereis 4/H9266r2. Now, we solve for the electric field:This is the familiar electric field due to a point charge that wedeveloped from Coulomb's law in Chapter 23.ke qr2E/H11005q4/H9266/H92800r2/H11005 A Spherically Symmetric Charge Distribution EXAMPLE24.5SolutionBecause the charge distribution is sphericallysymmetric, we again select a spherical gaussian surface of ra-dius r, concentric with the sphere, as shown in Figure 24.11a.For this choice, conditions (1) and (2) are satisfied, as theyAn insulating solid sphere of radius ahas a uniform volumecharge density /H9267and carries a total positive charge Q(Fig.24.11). (a) Calculate the magnitude of the electric field at apoint outside the sphere.11.6 24.3Application of Gauss's Law to Charged Insulators751 (a)Gaussiansphere(b)GaussianspherereraraFigure 24.11A uniformly charged insulating sphere of radius aand total charge Q. (a) The magnitude of the electric field at a pointexterior to the sphere is (b) The magnitude of the electricfield inside the insulating sphere is due only to the charge within thegaussian sphere defined by the dashed circle and is keQr/a3.keQ/r2.EaE =keQr2ra Figure 24.12A plot of Eversus rfor a uniformly charged insulat-ing sphere. The electric field inside the sphere varies linearlywith r. The field outside the sphere is the same as that of apoint charge Qlocated at r/H110050.(r/H11022a)(r/H11021a)were for the point charge in Example 24.4. Following the lineof reasoning given in Example 24.4, we find that(for Note that this result is identical to the one we obtained for apoint charge. Therefore, we conclude that, for a uniformlycharged sphere, the field in the region external to the sphereis equivalentto that of a point charge located at the center ofthe sphere.(b) Find the magnitude of the electric field at a point in-side the sphere.SolutionIn this case we select a spherical gaussian surfacehaving radius r/H11021a, concentric with the insulated sphere(Fig. 24.11b). Let us denote the volume of this smallersphere by V/H11032. To apply Gauss's law in this situation, it is im-portant to recognize that the charge qinwithin the gaussiansurface of volume V/H11032is less than Q. To calculate qin, we usethe fact that By symmetry, the magnitude of the electric field is constanteverywhere on the spherical gaussian surface and is normalqin/H11005/H9267V/H11032/H11005/H9267(43/H9266r3)qin/H11005/H9267V/H11032:r/H11022a)ke Qr2E/H11005to the surface at each point—both conditions (1) and (2)are satisfied. Therefore, Gauss's law in the region givesSolving for EgivesBecause by definition and since this expression for Ecan be written as(for r/H11021a)Note that this result for Ediffers from the one we ob-tained in part (a). It shows that E:0 as r:0. Therefore,the result eliminates the problem that would exist at r/H110050 ifEvaried as 1/r2inside the sphere as it does outside thesphere. That is, if for r/H11021a, the field would be infi-nite at r/H110050, which is physically impossible. Note also thatthe expressions for parts (a) and (b) match when r/H11005a.A plot of Eversus ris shown in Figure 24.12. E/H110081/r2keQa3 rE/H11005Qr4/H9266/H92800a3/H11005ke/H110051/(4/H9266/H92800),/H9267/H11005Q/43/H9266a3E/H11005qin4/H9266/H92800r2/H11005/H9 r/H20886 E dA/H11005E /H20886 dA/H11005E(4/H9266r2)/H11005qin/H92800r/H11021a The Electric Field Due to a Thin Spherical Shell EXAMPLE24.6the shell is equivalent to that due to a point charge Qlocatedat the center:(for r/H11022a)(b) The electric field inside the spherical shell is zero.This follows from Gauss's law applied to a spherical surface ofradius r/H11021aconcentric with the shell (Fig. 24.13c). Becauseke Qr2E/H11005A thin spherical shell of radius ahas a total charge Qdistrib-uted uniformly over its surface (Fig. 24.13a). Find the electricfield at points (a) outside and (b) inside the shell.Solution(a) The calculation for the field outside the shellis identical to that for the solid sphere shown in Example24.5a. If we construct a spherical gaussian surface of radius r/H11022aconcentric with the shell (Fig. 24.13b), the charge in-side this surface is Q. Therefore, the field at a point outside752CHAPTER 24Gauss's Law A Cylindrically Symmetric Charge Distribution EXAMPLE24.7Find the electric field a distance rfrom a line of positivecharge of infinite length and constant charge per unit length/H9261(Fig.

24.14a).SolutionThe symmetry of the charge distribution re-quires that Ebe perpendicular to the line charge and di-rected outward, as shown in Figure 24.14a and b. To reflectthe symmetry of the charge distribution, we select a cylindri-cal gaussian surface of radius rand length /H5129that is coaxialwith the line charge. For the curved part of this surface, Eisconstant in magnitude and perpendicular to the surface ateach point—satisfaction of conditions (1) and (2). Further-more, the flux through the ends of the gaussian cylinder iszero because Eis parallel to these surfaces—the first applica-tion we have seen of condition (3).We take the surface integral in Gauss's law over the entiregaussian surface. Because of the zero value of for theends of the cylinder, however, we can restrict our attention toonly the curved surface of the cylinder.The total charge inside our gaussian surface is /H9261/H5129. Apply-ing Gauss's law and conditions (1) and (2), we find that forthe curved surface/H9021E/H11005/H20886 E/H11554dA/H11005E /H20886 dA/H11005EA/H11005qin/H92800/H11005/H9261/H5129/H92800E/H11554dA11.7 +++++++++++++++++++++++++++++++++++EGaussiansurfaceaaraGaussiansurface (a) (c)(b)Ein = 0rFigure 24.13(a) The electric field inside a uniformly charged spherical shell is zero. The fieldoutside is the same as that due to a point charge Qlocated at the center of the shell. (b) Gaussiansurface for r/H11022a. (c) Gaussian surface for r/H11021a. Gaussiansurface+++ +++EdA/H5129r (a)E (b)Figure 24.14(a) An infinite line of charge surrounded by a cylin-drical gaussian surface concentric with the line. (b) An end viewshows that the electric field at the cylindrical surface is constant inmagnitude and perpendicular to the surface.of the spherical symmetry of the charge distribution and be-cause the net charge inside the surface is zero—satisfactionof conditions (1) and (2) again—application of Gauss's lawshows that E/H110050 in the region r/H11021a.We obtain the same results using Equation 23.6 and inte-grating over the charge distribution. This calculation israther complicated. Gauss's law allows us to determine theseresults in a much simpler way.24.3Application of Gauss's Law to Charged Insulators753The area of the curved surface is therefore,(24.7)Thus, we see that the electric field due to a cylindrically sym-metric charge distribution varies as 1/r, whereas the field ex-ternal to a spherically symmetric charge distribution varies as1/r2. Equation 24.7 was also derived in Chapter 23 (see Prob-lem 35[b]), by integration of the field of a point charge.If the line charge in this example were of finite length,the result for Ewould not be that given by Equation 24.7. Afinite line charge does not possess sufficient symmetry for usto make use of Gauss's law. This is because the magnitude of2ke /H9261rE/H11005/H92612/H9266/H92800r/H11005E(2/H9266r/H5129)/H11005/H9261/H5129/H92800A/H110052/H9266r/H5129; A Nonconducting Plane of Charge EXAMPLE24.8Because the distance from each flat end of the cylinder tothe plane does not appear in Equation 24.8, we conclude thatE/H11005/H9268/2/H92800at any distance from the plane. That is, the field isuniform everywhere.An important charge configuration related to this exam-ple consists of two parallel planes, one positively charged andthe other negatively charged, and each with a surface chargedensity /H9268(see Problem 58). In this situation, the electricfields due to the two planes add in the region between theplanes, resulting in a field of magnitude /H9268//H92800, and cancelelsewhere to give a field of zero.Find the electric field due to a nonconducting, infinite planeof positive charge with uniform surface charge density /H9268.SolutionBy symmetry, Emust be perpendicular to theplane and must have the same magnitude at all pointsequidistant from the plane. The fact that the direction of Eisaway from positive charges indicates that the direction of Eon one side of the plane must be opposite its direction on theother side, as shown in Figure 24.15. A gaussian surface thatreflects the symmetry is a small cylinder whose axis is perpen-dicular to the plane and whose ends each have an area Aandare equidistant from the plane. Because Eis parallel to thecurved surface—and, therefore, perpendicular to dAevery-where on the surface—condition (3) is satisfied and there isno contribution to the surface integral from this surface. Forthe flat ends of the cylinder, conditions (1) and (2) are satis-fied. The flux through each end of the cylinder is EA; hence, the total flux through the entire gaussian surface isjust that through the ends, Noting that the total charge inside the surface is qin/H11005/H9268A,we use Gauss's law and find that(24.8)/H92682/H92800E/H11005/H9021E/H110052EA/H11005qin/H92800/H11005/H9268A/H92800/H9021E/H110052EA.E++++++++++++++++++ ++++ ++++++++++++++++AGaussiancylinderEFigure 24.15A cylindrical gaussian surface penetrating an infi-nite plane of charge. The flux is EAthrough each end of the gauss-ian surface and zero through its curved surface.the electric field is no longer constant over the surface ofthe gaussian cylinder—the field near the ends of the linewould be different from that far from the ends. Thus, condi-tion (1) would not be satisfied in this situation. Further-more, Eis not perpendicular to the cylindrical surface at allpoints—the field vectors near the ends would have a compo-nent parallel to the line. Thus, condition (2) would not besatisfied. When there is insufficient symmetry in the chargedistribution, as in this situation, it is necessary to use Equa-tion 23.6 to calculate E.For points close to a finite line charge and far from theends, Equation 24.7 gives a good approximation of the valueof the field.It is left for you to show (see Problem 29) that the electricfield inside a uniformly charged rod of finite radius and infi-nite length is proportional to r. CONCEPTUALEXAMPLE24.9Explain why Gauss's law cannot be used to calculate the electric field near an electric di-pole, a charged disk, or a triangle with a point charge at each corner.754CHAPTER 24Gauss's LawCONDUCTORS IN ELECTROSTATIC EQUILIBRIUMAs we learned in Section 23.2, a good electrical conductor contains charges (elec-trons) that are not bound to any atom and therefore are free to move about withinthe material. When there is no net motion of charge within a conductor, the con-ductor is in electrostatic equilibrium.As we shall see, a conductor in electrosta-tic equilibrium has the following properties:1. The electric field is zero everywhere inside the conductor.2. If an isolated conductor carries a charge, the charge resides on its surface.3. The electric field just outside a charged conductor is perpendicular to the sur-face of the conductor and has a magnitude /H9268//H92800, where /H9268is the surface chargedensity at that point.4. On an irregularly shaped conductor, the surface charge density is greatest at lo-cations where the radius of curvature of the surface is smallest.We verify the first three properties in the discussion that follows. The fourthproperty is presented here without further discussion so that we have a completelist of properties for conductors in electrostatic equilibrium.We can understand the first property by considering a conducting slab placedin an external field E(Fig. 24.16). We can argue that the electric field inside theconductor mustbe zero under the assumption that we have electrostatic equilib-rium. If the field were not zero, free charges in the conductor would accelerateunder the action of the field. This motion of electrons, however, would mean thatthe conductor is not in electrostatic equilibrium. Thus, the existence of electro-static equilibrium is consistent only with a zero field in the conductor.Let us investigate how this zero field is accomplished. Before the external fieldis applied, free electrons are uniformly distributed throughout the conductor.When the external field is applied, the free electrons accelerate to the left in Fig-ure 24.16, causing a plane of negative charge to be present on the left surface. Themovement of electrons to the left results in a plane of positive charge on the rightsurface. These planes of charge create an additional electric field inside the con-ductor that opposes the external field. As the electrons move, the surface chargedensity increases until the magnitude of the internal field equals that of the exter-nal field, and the net result is a net field of zero inside the conductor. The time ittakes a good conductor to reach equilibrium is of the order of 10/H1100216s, which formost purposes can be considered instantaneous.We can use Gauss's law to verify the second property of a conductor in electro-static equilibrium. Figure 24.17 shows an arbitrarily shaped conductor. A gaussiansurface is drawn inside the conductor and can be as close to the conductor's sur-face as we wish. As we have just shown, the electric field everywhere inside the con-ductor is zero when it is in electrostatic equilibrium. Therefore, the electric fieldmust be zero at every point on the gaussian surface, in accordance with condition(4) in Section 24.3. Thus, the net flux through this gaussian surface is zero. Fromthis result and Gauss's law, we conclude that the net charge inside the gaussian sur-24.4 Properties of a conductor inelectrostatic equilibriumSolutionThe charge distributions of all these configurations do not have sufficientsymmetry to make the use of Gauss's law practical. We cannot find a closed surface sur-rounding any of these distributions that satisfies one or more of conditions (1) through(4) listed at the beginning of this section. ++++++++————EE Figure 24.17A conductor of ar-bitrary shape. The broken line rep-resents a gaussian surface just in-side the conductor.Figure 24.16A conducting slabin an external electric field E. Thecharges induced on the two sur-faces of the slab produce an elec-tric field that opposes the externalfield, giving a resultant field of zeroinside the slab.Gaussiansurface24.4Conductors in Electrostatic Equilibrium755 face is zero. Because there can be no net charge inside the gaussian surface (whichis arbitrarily close to the conductor's surface), any net charge on the conductormust reside on its surface. Gauss's law does not indicate how this excess chargeis distributed on the conductor's surface.We can also use Gauss's law to verify the third property. We draw a gaussiansurface in the shape of a small cylinder whose end faces are parallel to the surfaceof the conductor (Fig. 24.18). Part of the cylinder is just outside the conductor,and part is inside. The field is normal to the conductor's surface from the condi-tion of electrostatic equilibrium. (If Ehad a component parallel to the conduc-tor's surface, the free charges would move along the surface; in such a case, theconductor would not be in equilibrium.) Thus, we satisfy condition (3) in Section24.3 for the curved part of the cylindrical gaussian surface—there is no fluxthrough this part of the gaussian surface because Eis parallel to the surface.There is no flux through the flat face of the cylinder inside the conductor becausehere E/H110050—satisfaction of condition (4). Hence, the net flux through the gauss-ian surface is that through only the flat face outside the conductor, where the fieldis perpendicular to the gaussian surface. Using conditions (1) and (2) for thisface, the flux is EA, where Eis the electric field just outside the conductor and Aisthe area of the cylinder's face. Applying Gauss's law to this surface, we obtainwhere we have used the fact that qin/H11005/H9268A. Solving for Egives(24.9)E/H11005/H9268/H92800/H9021E/H11005/H20886 E dA/H11005EA/H11005qin/H92800/H11005/H9268A/H92800Electric field just outside acharged conductorA++++++++++++++++++++En Figure

24.18A gaussian surfacein the shape of a small cylinder isused to calculate the electric fieldjust outside a charged conductor.The flux through the gaussian sur-face is EnA. Remember that Eiszero inside the conductor.Electric field pattern surrounding a charged conductingplate placed near an oppositely charged conducting cylin-der. Small pieces of thread suspended in oil align with theelectric field lines. Note that (1) the field lines are perpen-dicular to both conductors and (2) there are no lines insidethe cylinder (E/H110050). A Sphere Inside a Spherical Shell EXAMPLE24.10SolutionFirst note that the charge distributions on boththe sphere and the shell are characterized by spherical sym-metry around their common center. To determine the elec-tric field at various distances rfrom this center, we construct asphercial gaussian surface for each of the four regions of in-terest. Such a surface for region /L51833is shown in Figure 24.19.To find Einside the solid sphere (region /L51832), consider aA solid conducting sphere of radius acarries a net positivecharge 2Q. A conducting spherical shell of inner radius band outer radius cis concentric with the solid sphere and car-ries a net charge /H11002Q. Using Gauss's law, find the electricfield in the regions labeled /L51832, /L51833, /L51834, and /L51835in Figure 24.19and the charge distribution on the shell when the entire sys-tem is in electrostatic equilibrium.756CHAPTER 24Gauss's Law How would the electric flux through a gaussian surface surrounding the shell in Example24.10 change if the solid sphere were off-center but still inside the shell?Optional SectionEXPERIMENTAL VERIFICATION OFGAUSS'S LAW AND COULOMB'S LAWWhen a net charge is placed on a conductor, the charge distributes itself on thesurface in such a way that the electric field inside the conductor is zero. Gauss'slaw shows that there can be no net charge inside the conductor in this situation. Inthis section, we investigate an experimental verification of the absence of thischarge.We have seen that Gauss's law is equivalent to Equation 23.6, the expressionfor the electric field of a distribution of charge. Because this equation arisesfrom Coulomb's law, we can claim theoretically that Gauss's law and Coulomb'slaw are equivalent. Hence, it is possible to test the validity of both laws by at-tempting to detect a net charge inside a conductor or, equivalently, a nonzeroelectric field inside the conductor. If a nonzero field is detected within the con-ductor, Gauss's law and Coulomb's law are invalid. Many experiments, including24.5Quick Quiz 24.3–Qrabc2Q/L51832/L51834/L51833/L51835 Figure 24.19A solid conducting sphere of radius aand carrying acharge 2Qsurrounded by a conducting spherical shell carrying acharge /H11002Q.gaussian surface of radius r/H11021a. Because there can be nocharge inside a conductor in electrostatic equilibrium, we seethat qin/H110050; thus, on the basis of Gauss's law and symmetry,for r/H11021a.In region /L51833—between the surface of the solid sphere andthe inner surface of the shell—we construct a sphericalgaussian surface of radius rwhere a/H11021r/H11021band note that thecharge inside this surface is /H110012Q(the charge on the solidsphere). Because of the spherical symmetry, the electric fieldE1/H110050lines must be directed radially outward and be constant inmagnitude on the gaussian surface. Following Example 24.4and using Gauss's law, we find that(for a/H11021r/H11021b)In region /L51835, where r/H11022c, the spherical gaussian surfacewe construct surrounds a total charge of Therefore, application of Gauss's law tothis surface gives(for r/H11022c)In region /L51834, the electric field must be zero because thespherical shell is also a conductor in equilibrium. If we con-struct a gaussian surface of radius rwhere b/H11021r/H11021c, we seethat qinmust be zero because From this argument, weconclude that the charge on the inner surface of the spheri-cal shell must be /H110022Qto cancel the charge /H110012Qon the solidsphere. Because the net charge on the shell is /H11002Q, we con-clude that its outer surface must carry a charge /H11001Q.E3/H110050.keQr2E4/H110052Q/H11001(/H11002Q)/H11005Q.qin/H110052keQr2 E2/H110052Q4/H9266/H92800r2/H11005E2A/H11005E2(4/H9266r2)/H11005qin/H92800/H110052Q/H9280024.5Experimental Verification of Gauss's Law and Coulomb's Law757early work by Faraday, Cavendish, and Maxwell, have been performed to detectthe field inside a conductor. In all reported cases, no electric field could be de-tected inside a conductor.Here is one of the experiments that can be performed.2A positively chargedmetal ball at the end of a silk thread is lowered through a small opening into anuncharged hollow conductor that is insulated from ground (Fig. 24.20a). The pos-itively charged ball induces a negative charge on the inner wall of the hollow con-ductor, leaving an equal positive charge on the outer wall (Fig. 24.20b). The pres-ence of positive charge on the outer wall is indicated by the deflection of theneedle of an electrometer (a device used to measure charge and that measurescharge only on the outer surface of the conductor). The ball is then lowered andallowed to touch the inner surface of the hollow conductor (Fig. 24.20c). Chargeis transferred between the ball and the inner surface so that neither is charged af-ter contact is made. The needle deflection remains unchanged while this happens,indicating that the charge on the outer surface is unaffected. When the ball is re-moved, the electrometer reading remains the same (Fig. 24.20d). Furthermore,the ball is found to be uncharged; this verifies that charge was transferred betweenthe ball and the inner surface of the hollow conductor. The overall effect is that the charge that was originally on the ball now appears on the hollow conduc-tor. The fact that the deflection of the needle on the electrometer measuring thecharge on the outer surface remained unchanged regardless of what was happen-ing inside the hollow conductor indicates that the net charge on the system alwaysresided on the outer surface of the conductor.If we now apply another positive charge to the metal ball and place it near theoutside of the conductor, it is repelled by the conductor. This demonstrates thatoutside the conductor, a finding consistent with the fact that the conductorcarries a net charge. If the charged metal ball is now lowered into the interior ofthe charged hollow conductor, it exhibits no evidence of an electric force. Thisshows that E/H110050 inside the hollow conductor.This experiment verifies the predictions of Gauss's law and therefore verifiesCoulomb's law. The equivalence of Gauss's law and Coulomb's law is due to theinverse-square behavior of the electric force. Thus, we can interpret this experi-ment as verifying the exponent of 2 in the 1/r2behavior of the electric force. Ex-periments by Williams, Faller, and Hill in 1971 showed that the exponent of rinCoulomb's law is (2/H11001/H9254), where In the experiment we have described, the charged ball hanging in the hollowconductor would show no deflection even in the case in which an external electricfield is applied to the entire system. The field inside the conductor is still zero.This ability of conductors to "block" external electric fields is utilized in manyplaces, from electromagnetic shielding for computer components to thin metalcoatings on the glass in airport control towers to keep radar originating outsidethe tower from disrupting the electronics inside. Cellular telephone users ridingtrains like the one pictured at the beginning of the chapter have to speak loudly tobe heard above the noise of the train. In response to complaints from other pas-sengers, the train companies are considering coating the windows with a thinmetallic conductor. This coating, combined with the metal frame of the train car,blocks cellular telephone transmissions into and out of the train./H9254/H11005(2.7/H110063.1)/H1100310/H1100216!E/HS110050 2The experiment is often referred to as Faraday's ice-pail experimentbecause Faraday, the first to performit, used an ice pail for the hollow conductor.000(a) (b) (c) (d)++++++ +++++++++++++++++++++++——————————————++++++++++++++++++++++++++++++++++++0Hollow conductor Figure 24.20An experimentshowing that any charge trans-ferred to a conductor resides on itssurface in electrostatic equilibrium.The hollow conductor is insulatedfrom ground, and the small metalball is supported by an insulatingthread.QuickLabWrap a radio or cordless telephone inaluminum foil and see if it still works.Does it matter if the foil touches theantenna? 758CHAPTER 24Gauss's LawOptional SectionFORMAL DERIVATION OF GAUSS'S LAWOne way of deriving Gauss's law involves solid angles.Consider a spherical surfaceof radius rcontaining an area element /H9004A. The solid angle /H9004/H9024(uppercase Greekomega) subtended at the center of the sphere by this element is defined to beFrom this equation, we see that has no dimensions because /H9004Aand r2both havedimensions L2. The dimensionless unit of a solid angle is the steradian.(You maywant to compare this equation to Equation 10.1b, the definition of the radian.) Be-cause the surface area of a sphere is 4/H9266r2, the total solid angle subtended by thesphere isNow consider a point chargeqsurrounded by a closed surface of arbitraryshape (Fig. 24.21). The total electric flux through this surface can be obtained byevaluating for each small area element /H9004A and summing over all elements.The flux through each element iswhere ris the distance from the charge to the area element, /H9258is the angle betweenthe electric field Eand /H9004Afor the element, and for a point charge. InFigure 24.22, we see that the projection of the area element perpendicular to theradius vector is /H9004Acos /H9258. Thus, the quantity /H9004Acos /H9258/r2is equal to the solid angle/H9004/H9024that the surface element /H9004A subtends at the charge q. We also see that /H9004/H9024isequal to the solid angle subtended by the area element of a spherical surface of ra-dius r. Because the total solid angle at a point is 4/H9266steradians, the total fluxE/H11005keq/r2/H9004/H9021E/H11005E/H11554/H9004A/H11005E /H9004A cos /H9258/H11005keq /H9004A cos /H9258r2E/H11554/H9004A/H9024/H110054/H9266r2r2/H110054/H9266 steradians/H9004/H9024/H9004/H9024/H11013/H9004Ar224.6 θΔAΔΩqE ΔΩqrΔAΔAθEΔA cosθθFigure 24.21A closed surface ofarbitrary shape surrounds a pointcharge q. The net electric fluxthrough the surface is independentof the shape of the surface. Figure 24.22The area element /H9004Asubtends a solid angle at the charge q./H9004/H9024 /H11005(/H9004A cos /H9258)/r2Summary759through the closed surface isThus we have derived Gauss's law, Equation 24.6. Note that this result is indepen-dent of the shape of the closed surface and independent of the position of thecharge within the surface.SUMMARYElectric flux is proportional to the number of electric field lines that penetrate asurface. If the electric field is uniform and makes an angle /H9258with the normal to asurface of area A, the electric flux through the surface is(24.2)In general, the electric flux through a surface is(24.3)You need to be able to apply Equations 24.2 and 24.3 in a variety of situations, par-ticularly those in which symmetry simplifies the calculation.Gauss's lawsays that the net electric flux /H9021Ethrough any closed gaussian sur-face is equal to the netcharge inside the surface divided by /H92800: (24.6)Using Gauss's law, you can calculate the electric field due to various symmetriccharge distributions. Table 24.1 lists some typical results./H9021E/H11005/H20886 E/H11554dA/H11005qin/H92800/H9021E/H11005/H20885surfaceE/H11554dA/H9021E/H11005EA cos /H9258/H9021E/H11005keq /H20886 dA cos /H9258r2/H11005keq /H20886d/H9024/H110054/H9266keq/H11005q/H92800 TABLE 24.1Typical Electric Field Calculations Using Gauss's LawCharge Distribution Electric Field LocationInsulating sphere of radiusR,uniform charge density,and total charge QThin spherical shell of radiusRand total charge QLine charge of infinite length Outside theand charge per unit length /H9261lineNonconducting, infinite Everywherecharged plane having outsidesurface charge density /H9268the planeConductor having surface Just outsidecharge density /H9268the conductorInside theconductorke QR3 rke Qr2r/H11021Rr/H11022R0ke Qr2r/H11021Rr/H11022R/H20902 0/H9268/H92800/H92682/H928002ke /H9261r/H20902/H20902760CHAPTER 24Gauss's LawA conductor in electrostatic equilibriumhas the following properties:1. The electric field is zero everywhere inside the conductor.2. Any net charge on the conductor resides entirely on its surface.3. The electric field just outside the conductor is perpendicular to its surface andhas a magnitude /H9268//H92800, where /H9268is the surface charge density at that point.4. On an irregularly shaped conductor, the surface charge density is greatestwhere the radius of curvature of the surface is the smallest.Problem-Solving HintsGauss's law, as we have seen, is very powerful in solving problems involvinghighly symmetric charge distributions. In this chapter, you encountered threekinds of symmetry: planar, cylindrical, and spherical. It is important to reviewExamples 24.4 through 24.10 and to adhere to the following procedure whenusing Gauss's law:•Select a gaussian surface that has a symmetry to match that of the chargedistribution and satisfies one or more of the conditions listed in Section24.3. For point charges or spherically symmetric charge distributions, thegaussian surface should be a sphere centered on the charge as in Examples24.4, 24.5, 24.6, and 24.10. For uniform line charges or uniformly chargedcylinders, your gaussian surface should be a cylindrical surface that is coax-ial with the line charge or cylinder as in Example 24.7. For planes of charge,a useful choice is a cylindrical gaussian surface that straddles the plane, asshown in Example 24.8. These choices enable you to simplify the surface in-tegral that appears in Gauss's law and represents the total electric fluxthrough that surface.•Evaluate the qin//H92800term in Gauss's law, which amounts to calculating the to-tal electric charge qininside the gaussian surface. If the charge density isuniform (that is, if /H9261, /H9268, or /H9267is constant), simply multiply that charge densityby the length, area, or volume enclosed by the gaussian surface. If thecharge distribution is nonuniform,integrate the charge density over the re-gion enclosed by the gaussian surface. For example, if the charge is distrib-uted along a line, integrate the expression where dqis the chargeon an infinitesimal length element dx. For a plane of charge, integratewhere dAis an infinitesimal element of area. For a volume ofcharge, integrate where dVis an infinitesimal element of volume.•Once the terms in Gauss's law have been evaluated, solve for the electricfield on the gaussian surface if the charge distribution is given in the prob-lem. Conversely, if the electric field is known, calculate the charge distribu-tion that produces the field.dq/H11005/H9267 dV,dq/H11005/H9268 dA,dq/H11005/H9261 dx,QUESTIONS3.If more electric field lines are leaving a gaussian surfacethan entering, what can you conclude about the netcharge enclosed by that surface?4.A uniform electric field exists in a region of space inwhich there are no charges. What can you concludeabout the net electric flux through a gaussian surfaceplaced in this region of space?1.The Sun is lower in the sky during the winter than it is inthe summer. How does this change the flux of sunlighthitting a given area on the surface of the Earth? Howdoes this affect the weather?2.If the electric field in a region of space is zero, can youconclude no electric charges are in that region? Explain.Problems7615.If the total charge inside a closed surface is known butthe distribution of the charge is unspecified, can you useGauss's law to find the electric field? Explain.6.Explain why the electric flux through a closed surfacewith a given enclosed charge is independent of the size orshape of the surface.7.Consider the electric field due to a nonconducting infi-nite plane having a uniform charge density. Explain whythe electric field does not depend on the distance fromthe plane in terms of the spacing of the electric fieldlines.8.Use Gauss's law to explain why electric field lines must be-gin or end on electric charges. (Hint:Change the size ofthe gaussian surface.)9.On the basis of the repulsive nature of the force betweenlike charges and the freedom of motion of charge withinthe conductor, explain why excess charge on an isolatedconductor must reside on its surface.10.A person is placed in a large, hollow metallic sphere thatis insulated from ground. If a large charge is placed onthe sphere, will the person be harmed upon touching theinside of the sphere? Explain what will happen if the per-son also has an initial charge whose sign is opposite thatof the charge on the sphere.11.How would the observations described in Figure 24.20differ if the hollow conductor were grounded? Howwould they differ if the small charged ball were an insula-tor rather than a conductor?12.What other experiment might be performed on the ballin Figure 24.20 to show that its charge was transferred tothe hollow conductor?13.What would happen to the electrometer reading if thecharged ball in Figure 24.20 touched the inner wall of theconductor? the outer wall?14.You may have heard that one of the safer places to be dur-ing a lightning storm is inside a car. Why would this bethe case?15.Two solid spheres, both of radius R, carry identical totalcharges Q. One sphere is a good conductor, while theother is an insulator. If the charge on the insulatingsphere is uniformly distributed throughout its interiorvolume, how do the electric fields outside these twospheres compare? Are the fields identical inside the twospheres?PROBLEMS 6.A uniform electric field intersects a surface ofarea A. What is the flux through this area if the surfacelies (a) in the yzplane? (b) in the xzplane? (c) in the xyplane?7.A point charge qis located at the center of a uniformring having linear charge density /H9261and radius a, asshown in Figure P24.7. Determine the total electric fluxai/H11001bjSection 24.1Electric Flux1.An electric field with a magnitude of 3.50 kN/C is ap-plied along the xaxis. Calculate the electric fluxthrough a rectangular plane 0.350 m wide and 0.700 mlong if (a) the plane is parallel to the yzplane; (b) theplane is parallel to the xy plane; and (c) the plane con-tains the yaxis, and its normal makes an angle of 40.0°with the xaxis.2.A vertical electric field of magnitude 2.00/H11003104N/Cexists above the Earth's surface on a day when a thun-derstorm is brewing. A car with a rectangular size of ap-proximately 6.00 m by 3.00 m is traveling along a road-way sloping downward at 10.0°. Determine the electricflux through the bottom of the car.3.A 40.0-cm-diameter loop is rotated in a uniform electricfield until the position of maximum electric flux isfound. The flux in this position is measured to be 5.20/H11003105N/H11080m2/C. What is the magnitude of the elec-tric field?4.A spherical shell is placed in a uniform electric field.Find the total electric flux through the shell.5.Consider a closed triangular box resting within a hori-zontal electric field of magnitude N/C,as shown in Figure P24.5. Calculate the electric fluxthrough (a) the vertical rectangular surface, (b) theslanted surface, and (c) the entire surface of the box.E/H110057.80/H110031041, 2, 3= straightforward, intermediate, challenging = full solution available in the Student Solutions Manual and Study GuideWEB= solution posted at http://www.saunderscollege.com/physics/= Computer useful in solving problem = Interactive Physics= paired numerical/symbolic problems 30.0 cm60.0°10.0 cmEFigure P24.5 Figure P24.7Rqaλ762CHAPTER 24Gauss's Law WEBthrough a sphere centered at the point charge and hav-ing radius R, where 8.A pyramid with a 6.00-m-square base and height of 4.00 m is placed in a vertical electric field of 52.0 N/C.Calculate the total electric flux through the pyramid'sfour slanted surfaces.9.A cone with base radius Rand height his located on ahorizontal table. A horizontal uniform field Epene-trates the cone, as shown in Figure P24.9. Determinethe electric flux that enters the left-hand side of thecone.R/H11021a.located a very small distance from the center of a verylarge square on the line perpendicular to the square andgoing through its center. Determine the approximateelectric flux through the square due to the pointcharge. (c) Explain why the answers to parts (a) and(b) are identical.14.Calculate the total electric flux through the parabo-loidal surface due to a constant electric field of magni-tude E0in the direction shown in Figure P24.14. 16.A point charge of 12.0/H9262C is placed at the center of aspherical shell of radius 22.0 cm. What is the total elec-tric flux through (a) the surface of the shell and (b) any hemispherical surface of the shell? (c) Do theresults depend on the radius? Explain.17.A point charge of 0.046 2/H9262C is inside a pyramid. Deter-mine the total electric flux through the surface of thepyramid.18.An infinitely long line charge having a uniform chargeper unit length /H9261lies a distance dfrom point O, asshown in Figure P24.18. Determine the total electricflux through the surface of a sphere of radius Rcentered at Oresulting from this line charge. (Hint:Consider both cases: when and whenR/H11022d.)R/H11021d,15.A point charge Qis located just above the center of theflat face of a hemisphere of radius R, as shown in FigureP24.15. What is the electric flux (a) through the curvedsurface and (b) through the flat face? 13.(a) A point charge qis located a distance dfrom an infi-nite plane. Determine the electric flux through theplane due to the point charge. (b) A point charge qisSection 24.2Gauss's Law10.The electric field everywhere on the surface of a thinspherical shell of radius 0.750 m is measured to beequal to 890 N/C and points radially toward the centerof the sphere. (a) What is the net charge within thesphere's surface? (b) What can you conclude about thenature and distribution of the charge inside the spheri-cal shell?11.The following charges are located inside a submarine:and (a) Calcu-late the net electric flux through the submarine. (b) Is the number of electric field lines leaving the sub-marine greater than, equal to, or less than the numberentering it?12.Four closed surfaces, S1through S4, together with thecharges /H110022Q, Q, and /H11002Qare sketched in FigureP24.12. Find the electric flux through each surface./H1100284.0 /H9262C.27.0 /H9262C,5.00 /H9262C, /H110029.00 /H9262C,hREFigure P24.9 Figure P24.12Figure P24.14 Figure P24.15 –Q+Q–2QS2S3S1S4drE0 Q0RδProblems763 19.A point charge is located at the center of acube of side In addition, six other identi-cal point charges having are positionedsymmetrically around Q, as shown in Figure P24.19. De-termine the electric flux through one face of the cube.20.A point charge Qis located at the center of a cube ofside L. In addition, six other identical negative pointcharges are positioned symmetrically around Q, asshown in Figure P24.19. Determine the electric fluxthrough one face of the cube.q/H11005/H110021.00 /H9262CL/H110050.100 m.Q/H110055.00 /H9262C23.A charge of 170/H9262C is at the center of a cube of side80.0 cm. (a) Find the total flux through each face of thecube. (b) Find the flux through the whole surface ofthe cube. (c) Would your answers to parts

(a) or (b) change if the charge were not at the center? Ex-plain.24.The total electric flux through a closed surface in theshape of a cylinder is (a) What isthe net charge within the cylinder? (b) From the infor-mation given, what can you say about the charge withinthe cylinder? (c) How would your answers to parts (a) and (b) change if the net flux were25.The line agis a diagonal of a cube (Fig. P24.25). Apoint charge qis located on the extension of line ag,very close to vertex aof the cube. Determine the elec-tric flux through each of the sides of the cube that meetat the point a./H110028.60/H11003104 N/H11080m2/C?8.60/H11003104 N/H11080m2/C. WEBSection 24.3Application of Gauss's Law to Charged Insulators26.Determine the magnitude of the electric field at the sur-face of a lead-208 nucleus, which contains 82 protonsand 126 neutrons. Assume that the lead nucleus has avolume 208 times that of one proton, and consider aproton to be a sphere of radius 1.20/H1100310/H1100215m.27.A solid sphere of radius 40.0 cm has a total positivecharge of 26.0/H9262C uniformly distributed throughout itsvolume. Calculate the magnitude of the electric field(a) 0 cm, (b) 10.0 cm, (c) 40.0 cm, and (d) 60.0 cmfrom the center of the sphere.28.A cylindrical shell of radius 7.00 cm and length 240 cmhas its charge uniformly distributed on its curved surface.The magnitude of the electric field at a point 19.0 cm ra-dially outward from its axis (measured from the midpointof the shell) is 36.0 kN/C. Use approximate relationshipsto find (a) the net charge on the shell and (b) the electricfield at a point 4.00 cm from the axis, measured radiallyoutward from the midpoint of the shell.29.Consider a long cylindrical charge distribution of radiusRwith a uniform charge density /H9267. Find the electricfield at distance rfrom the axis where r/H11021R.21.Consider an infinitely long line charge having uniformcharge per unit length /H9261. Determine the total electricflux through a closed right circular cylinder of length Land radius Rthat is parallel to the line charge, if the dis-tance between the axis of the cylinder and the linecharge is d. (Hint:Consider both cases: when and when 22.A 10.0-/H9262C charge located at the origin of a cartesian co-ordinate system is surrounded by a nonconducting hol-low sphere of radius 10.0 cm. A drill with a radius of1.00 mm is aligned along the zaxis, and a hole is drilledin the sphere. Calculate the electric flux through thehole.R/H11022d.)R/H11021d,Figure P24.18dROλ Figure P24.19Problems 19 and 20.Figure P24.25LLqqqqQqq Ldcab efghq764CHAPTER 24Gauss's Law30.A nonconducting wall carries a uniform charge densityof 8.60 /H9262C/cm2. What is the electric field 7.00 cm infront of the wall? Does your result change as the dis-tance from the wall is varied?31.Consider a thin spherical shell of radius 14.0 cm with atotal charge of 32.0/H9262C distributed uniformly on its sur-face. Find the electric field (a) 10.0 cm and (b) 20.0 cmfrom the center of the charge distribution.32.In nuclear fission, a nucleus of uranium-238, which con-tains 92 protons, divides into two smaller spheres, eachhaving 46 protons and a radius of 5.90/H1100310/H1100215m. Whatis the magnitude of the repulsive electric force pushingthe two spheres apart?33.Fill two rubber balloons with air. Suspend both of themfrom the same point on strings of equal length. Rubeach with wool or your hair, so that they hang apart witha noticeable separation between them. Make order-of-magnitude estimates of (a) the force on each, (b) thecharge on each, (c) the field each creates at the centerof the other, and (d) the total flux of electric field cre-ated by each balloon. In your solution, state the quanti-ties you take as data and the values you measure or esti-mate for them.34.An insulating sphere is 8.00 cm in diameter and carriesa 5.70-/H9262C charge uniformly distributed throughout itsinterior volume. Calculate the charge enclosed by aconcentric spherical surface with radius (a) r/H110052.00 cmand (b) r/H110056.00 cm.35.A uniformly charged, straight filament 7.00 m in lengthhas a total positive charge of 2.00/H9262C. An unchargedcardboard cylinder 2.00 cm in length and 10.0 cm in ra-dius surrounds the filament at its center, with the fila-ment as the axis of the cylinder. Using reasonable ap-proximations, find (a) the electric field at the surface ofthe cylinder and (b) the total electric flux through thecylinder.36.The charge per unit length on a long, straight filamentis /H1100290.0 /H9262C/m. Find the electric field (a) 10.0 cm, (b) 20.0 cm, and (c) 100 cm from the filament, wheredistances are measured perpendicular to the length ofthe filament.37.A large flat sheet of charge has a charge per unit area of9.00 /H9262C/m2. Find the electric field just above the sur-face of the sheet, measured from its midpoint.Section 24.4Conductors in Electrostatic Equilibrium38.On a clear, sunny day, a vertical electrical field of about130 N/C points down over flat ground. What is the sur-face charge density on the ground for these conditions?39.A long, straight metal rod has a radius of 5.00 cm and acharge per unit length of 30.0 nC/m. Find the electricfield (a) 3.00 cm, (b) 10.0 cm, and (c) 100 cm from theaxis of the rod, where distances are measured perpen-dicular to the rod.40.A very large, thin, flat plate of aluminum of area Ahas atotal charge Quniformly distributed over its surfaces. Ifthe same charge is spread uniformly over the uppersurface of an otherwise identical glass plate, comparethe electric fields just above the center of the upper sur-face of each plate.41.A square plate of copper with 50.0-cm sides has no netcharge and is placed in a region of uniform electricfield of 80.0 kN/C directed perpendicularly to theplate. Find (a) the charge density of each face of theplate and (b) the total charge on each face.42.A hollow conducting sphere is surrounded by a largerconcentric, spherical, conducting shell. The innersphere has a charge /H11002Q, and the outer sphere has acharge 3Q. The charges are in electrostatic equilibrium.Using Gauss's law, find the charges and the electricfields everywhere.43.Two identical conducting spheres each having a radiusof 0.500 cm are connected by a light 2.00-m-long con-ducting wire. Determine the tension in the wire if 60.0/H9262C is placed on one of the conductors. (Hint:As-sume that the surface distribution of charge on eachsphere is uniform.)44.The electric field on the surface of an irregularlyshaped conductor varies from 56.0 kN/C to 28.0 kN/C.Calculate the local surface charge density at the pointon the surface where the radius of curvature of the sur-face is (a) greatest and (b) smallest.45.A long, straight wire is surrounded by a hollow metalcylinder whose axis coincides with that of the wire. Thewire has a charge per unit length of /H9261, and the cylinderhas a net charge per unit length of 2/H9261. From this infor-mation, use Gauss's law to find (a) the charge per unitlength on the inner and outer surfaces of the cylinderand (b) the electric field outside the cylinder, a distancerfrom the axis.46.A conducting spherical shell of radius 15.0 cm carries anet charge of /H110026.40/H9262C uniformly distributed on itssurface. Find the electric field at points (a) just outsidethe shell and (b) inside the shell.47.A thin conducting plate 50.0 cm on a side lies in the xyplane. If a total charge of 4.00/H1100310/H110028C is placed onthe plate, find (a) the charge density on the plate, (b) the electric field just above the plate, and (c) theelectric field just below the plate.48.A conducting spherical shell having an inner radius of aand an outer radius of bcarries a net charge Q. If apoint charge qis placed at the center of this shell, determine the surface charge density on (a) the innersurface of the shell and (b) the outer surface of theshell.49.A solid conducting sphere of radius 2.00 cm has acharge 8.00/H9262C. A conducting spherical shell of innerradius 4.00 cm and outer radius 5.00 cm is concentricwith the solid sphere and has a charge /H110024.00/H9262C. Findthe electric field at (a) r/H110051.00 cm, (b) r/H110053.00 cm,(c) r/H110054.50 cm, and (d) r/H110057.00 cm from the center ofthis charge configuration.WEBProblems76550.A positive point charge is at a distance of R/2 from thecenter of an uncharged thin conducting spherical shellof radius R. Sketch the electric field lines set up by thisarrangement both inside and outside the shell.(Optional)Section 24.5Experimental Verification of Gauss's Law and Coulomb's LawSection 24.6Formal Derivation of Gauss's Law51.A sphere of radius Rsurrounds a point charge Q, lo-cated at its center. (a) Show that the electric fluxthrough a circular cap of half-angle /H9258(Fig. P24.51) isWhat is the flux for (b) /H9258/H1100590° and (c) /H9258/H11005180°?/H9021E/H11005Q2/H92800 (1/H11002cos /H9258)net charge enclosed by this surface, as a function of r.Note that the charge inside this surface is less than 3Q.(i) Find the electric field in the region r/H11021a. (j) Deter-mine the charge on the inner surface of the conductingshell. (k) Determine the charge on the outer surface ofthe conducting shell. (l) Make a plot of the magnitudeof the electric field versus r.54.Consider two identical conducting spheres whose sur-faces are separated by a small distance. One sphere isgiven a large net positive charge, while the other isgiven a small net positive charge. It is found that theforce between them is attractive even though bothspheres have net charges of the same sign. Explain howthis is possible.55.A solid, insulating sphere of radius ahas a uniformcharge density /H9267and a total charge Q. Concentric withthis sphere is an uncharged, conducting hollow spherewhose inner and outer radii are band c, as shown in Fig-ure P24.55. (a) Find the magnitude of the electric fieldin the regions and r/H11022c. (b) Determine the induced charge per unit area on theinner and outer surfaces of the hollow sphere.b/H11021r/H11021c,r/H11021a, a/H11021r/H11021b,WEB 56.For the configuration shown in Figure P24.55, supposethat a/H110055.00 cm, b/H1100520.0 cm, and c/H1100525.0 cm.Furthermore, suppose that the electric field at a point10.0 cm from the center is 3.60/H11003103N/C radially in-ward, while the electric field at a point 50.0 cm from thecenter is 2.00/H11003102N/C radially outward. From this in-formation, find (a) the charge on the insulating sphere,ADDITIONAL PROBLEMS52.A nonuniform electric field is given by the expressionwhere a, b, and care constants.Determine the electric flux through a rectangular sur-face in the xyplane, extending from x/H110050 to x/H11005wandfrom y/H110050 to y/H11005h.53.A solid insulating sphere of radius acarries a net positivecharge 3Q, uniformly distributed throughout its vol-ume. Concentric with this sphere is a conducting spheri-cal shell with inner radius band outer radius c, and hav-ing a net charge /H11002Q, as shown in Figure P24.53. (a) Construct a spherical gaussian surface of radius r/H11022cand find the net charge enclosed by this surface. (b) What is the direction of the electric field at r/H11022c?(c) Find the electric field at r/H11022c. (d) Find the electricfield in the region with radius rwhere c/H11022r/H11022b. (e) Construct a spherical gaussian surface of radius r,where c/H11022r/H11022b, and find the net charge enclosed bythis surface. (f) Construct a spherical gaussian surfaceof radius r, where b/H11022r/H11022a, and find the net charge en-closed by this surface. (g) Find the electric field in theregion b/H11022r/H11022a. (h) Construct a spherical gaussiansurface of radius r/H11021a, and find an expression for theE/H11005ayi/H11001bzj/H11001cxk,Figure P24.51Figure P24.53 Figure P24.55Problems 55 and 56.RQθr–Q3Qcba InsulatorConductoracb766CHAPTER 24Gauss's Law(b) the net charge on the hollow conducting sphere,and (c) the total charge on the inner and outer surfacesof the hollow conducting sphere.57.An infinitely long cylindrical insulating shell of inner ra-dius aand outer radius bhas a uniform volume chargedensity /H9267(C/m3). A line of charge density /H9261(C/m) isplaced along the axis of the shell. Determine the elec-tric field intensity everywhere.58.Two infinite, nonconducting sheets of charge are paral-lel to each other, as shown in Figure P24.58. The sheeton the left has a uniform surface charge density /H9268, andthe one on the right has a uniform charge density /H11002/H9268.Calculate the value of the electric field at points (a) tothe left of, (b) in between, and (c) to the right of thetwo sheets. (Hint:See Example 24.8.)the size of the cavity with a uniform negative chargedensity /H11002/H9267.)61.Review Problem.An early (incorrect) model of thehydrogen atom, suggested by J. J. Thomson, proposedthat a positive cloud of charge /H11001ewas uniformly distrib-uted throughout the volume of a sphere of radius R,with the electron an equal-magnitude negative pointcharge /H11002eat the center. (a) Using Gauss's law, showthat the electron would be in equilibrium at the centerand, if displaced from the center a distance would experience a restoring force of the formwhere Kis a constant. (b) Show that(c) Find an expression for the frequency fof simple harmonic oscillations that an electron of massmewould undergo if displaced a short distance (/H11021R)from the center and released. (d) Calculate a numericalvalue for Rthat would result in a frequency of electronvibration of 2.47/H110031015Hz, the frequency of the lightin the most intense line in the hydrogen spectrum.62.A closed surface with dimensions andis located as shown in Figure P24.62. Theelectric field throughout the region is nonuniform andgiven by N/C, where xis in meters.Calculate the net electric flux leaving the closed sur-face. What net charge is enclosed by the surface?E/H11005(3.0/H110012.0x2)ic/H110050.600 ma/H11005b/H110050.400 mK/H11005kee2/R3.F/H11005/H11002Kr,r/H11021R , 59.Repeat the calculations for Problem 58 when bothsheets have positive uniform surface charge densities ofvalue /H9268.60.A sphere of radius 2ais made of a nonconducting mate-rial that has a uniform volume charge density /H9267. (As-sume that the material does not affect the electricfield.) A spherical cavity of radius ais now removedfrom the sphere, as shown in Figure P24.60. Show thatthe electric field within the cavity is uniform and isgiven by and (Hint:The fieldwithin the cavity is the superposition of the field due tothe original uncut sphere, plus the field due to a sphereEy/H11005/H9267a/3/H92800 .Ex/H110050Figure P24.58 Figure P24.60Figure P24.62σ–σ yx2aaaycxzbEa63.A solid insulating sphere of radius Rhas a nonuniformcharge density that varies with raccording to the expres-sion where Ais a constant and is meas-ured from the center of the sphere. (a) Show that theelectric field outside the sphere is(b) Show that the electric field insidethe sphere is (Hint:Note that thetotal charge Qon the sphere is equal to the integral of /H9267dV, where rextends from 0 to R; also note that thecharge qwithin a radius r/H11021Ris less than Q. To evaluatethe integrals, note that the volume element dVfor aspherical shell of radius rand thickness dris equal to64.A point charge Qis located on the axis of a disk of ra-dius Rat a distance bfrom the plane of the disk (Fig.P24.64). Show that if one fourth of the electric fluxfrom the charge passes through the disk, then R/H11005!3b.4/H9266r2 dr.)E/H11005Ar3/5/H92800 .(r/H11021R)E/H11005AR5/5/H92800r2. (r/H11022R)r/H11021R/H9267/H11005Ar2,WEBAnswers to Quick Quizzes767 ANSWERS TOQUICKQUIZZES24.3Any gaussian surface surrounding the system enclosesthe same amount of charge, regardless of how the com-ponents of the system are moved. Thus, the fluxthrough the gaussian surface would be the same as it iswhen the sphere and shell are concentric.24.1Zero, because there is no net charge within the surface.24.2(b) and (d). Statement (a) is not necessarily true be-cause an equal number of positive and negative chargescould be present inside the surface. Statement (c) is notnecessarily true, as can be seen from Figure 24.8: Anonzero electric field exists everywhere on the surface,but the charge is not enclosed within the surface; thus,the net flux is zero.a frequency described by the expressionf/H1100512/H9266 !/H9267eme/H92800 Figure P24.64Figure P24.67Problems 67 and 68.65.A spherically symmetric charge distribution has acharge density given by where ais constant.Find the electric field as a function of r. (Hint:Note thatthe charge within a sphere of radius Ris equal to the in-tegral of /H9267dV, where rextends from 0 to R. To evaluatethe integral, note that the volume element dVfor aspherical shell of radius rand thickness dris equal to66.An infinitely long insulating cylinder of radius Rhas avolume charge density that varies with the radius aswhere /H92670, a, and bare positive constants and ris the dis-tance from the axis of the cylinder. Use Gauss's law todetermine the magnitude of the electric field at radialdistances (a) r/H11021Rand (b) r/H11022R.67.Review Problem.A slab of insulating material (infi-nite in two of its three dimensions) has a uniform posi-tive charge density /H9267. An edge view of the slab is shownin Figure P24.67. (a) Show that the magnitude of theelectric field a distance xfrom its center and inside theslab is (b) Suppose that an electron ofcharge /H11002eand mass meis placed inside the slab. If it isreleased from rest at a distance xfrom the center, showthat the electron exhibits simple harmonic motion withE/H11005/H9267x//H92800 ./H9267/H11005/H92670/H20898a/H11002rb/H208994/H9266r2 dr.)/H9267/H11005a/r,RQbxy Od68.A slab of insulating material has a nonuniform positivecharge density where xis measured from thecenter of the slab, as shown in Figure P24.67, and Cis aconstant. The slab is infinite in the yand zdirections.Derive expressions for the electric field in (a) the exte-rior regions and (b) the interior region of the slab69.(a) Using the mathematical similarity betweenCoulomb's law and Newton's law of universal gravita-tion, show that Gauss's law for gravitation can be writtenaswhere minis the mass inside the gaussian surface andrepresents the gravitational field at any pointon the gaussian surface. (b) Determine the gravita-tional field at a distance rfrom the center of the Earthwhere r/H11021RE, assuming that the Earth's mass density isuniform.g/H11005Fg/m/H20886g/H11554dA/H11005/H110024/H9266Gmin(/H11002d/2/H11021x/H11021d/2)./H9267/H11005Cx2,chapterElectric Potential PUZZLERJennifer is holding on to an electricallycharged sphere that reaches an electricpotential of about 100 000 V. The devicethat generates this high electric potentialis called a Van de Graaff generator.Whatcauses Jennifer's hair to stand on endlike the needles of a porcupine? Why isshe safe in this situation in view of thefact that 110 V from a wall outlet can killyou?(Henry Leap and Jim Lehman) Chapter Outline25.1Potential Difference and ElectricPotential25.2Potential Differences in aUniform Electric Field25.3Electric Potential and PotentialEnergy Due to Point Charges25.4Obtaining the Value of theElectric Field from the ElectricPotential25.5Electric Potential Due toContinuous Charge Distributions25.6Electric Potential Due to aCharged Conductor25.7(Optional)The Millikan Oil-DropExperiment25.8(Optional)Applications ofElectrostaticsPUZZLER 76825.1Potential Difference and Electric Potential769he concept of potential energy was introduced in Chapter 8 in connectionwith such conservative forces as the force of gravity and the elastic force exertedby a spring. By using the law of conservation of energy, we were able to avoidworking directly with forces when solving various problems in mechanics. In thischapter we see that the concept of potential energy is also of great value in thestudy of electricity. Because the electrostatic force given by Coulomb's law is con-servative, electrostatic phenomena can be conveniently described in terms of anelectric potential energy. This idea enables us to define a scalar quantity known aselectric potential.Because the electric potential at any point in an electric field is ascalar function, we can use it to describe electrostatic phenomena more simplythan if we were to rely only on the concepts of the electric field and electric forces.In later chapters we shall see that the concept of electric potential is of great prac-tical value.POTENTIAL DIFFERENCE AND ELECTRIC POTENTIALWhen a test charge q0is placed in an electric field Ecreated by some othercharged object, the electric force acting on the test charge is q0E. (If the field isproduced by more than one charged object, this force acting on the test charge isthe vector sum of the individual forces exerted on it by the various other chargedobjects.) The force q0Eis conservative because the individual forces described byCoulomb's law are conservative. When the test charge is moved in the field bysome external agent, the work done by the field on the charge is equal to the neg-ative of the work done by the external agent causing the displacement. For an in-finitesimal displacement ds, the work done by the electric field on the charge isAs this amount of work is done by the field, the potential energyof the charge–field system is decreased by an amount For a finitedisplacement of the charge from a point Ato a point B, the change in potentialenergy of the system is(25.1)The integration is performed along the path that q0follows as it moves from AtoB, and the integral is called either a path integral or a line integral (the two terms aresynonymous). Because the force q0Eis conservative, this line integral does notdepend on the path taken from Ato B.If the path between Aand Bdoes not make any difference in Equation 25.1, why don't wejust use the expression where dis the straight-line distance between Aand B?The potential energy per unit charge U/q0is independent of the value of q0and has a unique value at every point in an electric field. This quantity U/q0iscalled the electric potential(or simply the potential) V. Thus, the electric poten-tial at any point in an electric field is(25.2)V/H11005Uq0/H9004U/H11005/H11002q0Ed,Quick Quiz 25.1/H9004U/H11005/H11002q0 /H20885BA E/H11554ds/H9004U/H11005UB/H11002UAdU/H11005/H11002q0E/H11554ds.F/H11554ds/H11005q0E/H11554ds.25.1T Change in potential energy11.8 The fact that potential energy is a scalar quantity means that electric potential alsois a scalar quantity.The potential differencebetween any two points Aand Bin anelectric field is defined as the change in potential energy of the system divided bythe test charge q0:(25.3)Potential difference should not be confused with difference in potential energy.The potential difference is proportional to the change in potential energy, and wesee from Equation 25.3 that the two are related by Electric potential is a scalar characteristic of an electric field, indepen-dent of the charges that may be placed in the field. However, when we speakof potential energy, we are referring to the charge–field system.Because weare usually interested in knowing the electric potential at the location of a chargeand the potential energy resulting from the interaction of the charge with thefield, we follow the common convention of speaking of the potential energy as if itbelonged to the charge.Because the change in potential energy of a charge is the negative of the workdone by the electric field on the charge (as noted in Equation 25.1), the potentialdifference /H9004Vbetween points Aand Bequals the work per unit charge that an ex-ternal agent must perform to move a test charge from Ato Bwithout changing thekinetic energy of the test charge.Just as with potential energy, only differences in electric potential are meaning-ful. To avoid having to work with potential differences, however, we often take thevalue of the electric potential to be zero at some convenient point in an electricfield. This is what we do here: arbitrarily establish the electric potential to be zeroat a point that is infinitely remote from the charges producing the field. Havingmade this choice, we can state that the electric potential at an arbitrary pointin an electric field equals the work required per unit charge to bring a posi-tive test charge from infinity to that point.Thus, if we take point Ain Equation25.3 to be at infinity, the electric potential at any point Pis(25.4)In reality, VPrepresents the potential difference /H9004Vbetween the point Pand apoint at infinity. (Eq. 25.4 is a special case of Eq. 25.3.)Because electric potential is a measure of potential energy per unit charge, theSI unit of both electric potential and potential difference is joules per coulomb,which is defined as a volt(V):That is, 1 J of work must be done to move a 1-C charge through a potential differ-ence of 1 V.Equation 25.3 shows that potential difference also has units of electric fieldtimes distance. From this, it follows that the SI unit of electric field (N/C) can alsobe expressed in volts per meter:1 NC/H110051 Vm1 V/H110131 JCVP/H11005/H11002/H20885P/H11009 E/H11554ds/H9004U/H11005q0/H9004V./H9004V/H11005/H9004Uq0/H11005/H11002/H20885BA E/H11554ds/H9004V/H11005VB/H11002VA770CHAPTER 25Electric Potential Potential difference Definition of volt25.2Potential Differences in a Uniform Electric Field771A unit of energy commonly used in atomic and nuclear physics is the electronvolt (eV), which is defined asthe energy an electron (or proton) gains or losesby moving through a potential difference of 1 V .Because 1 V/H110051J / C a n d b e -cause the fundamental charge is approximately the electron volt isrelated to the joule as follows:(25.5)For instance, an electron in the beam of a typical television picture tube may havea speed of 3.5/H11003107m/s. This corresponds to a kinetic energy of 5.6/H1100310/H1100216J,which is equivalent to 3.5/H11003103eV. Such an electron has to be accelerated fromrest through a potential difference of 3.5 kV to reach this speed.POTENTIAL DIFFERENCES IN AUNIFORM ELECTRIC FIELDEquations 25.1 and 25.3 hold in all electric fields, whether uniform or varying, butthey can be simplified for a uniform field. First, consider a uniform electric fielddirected along the negative yaxis, as shown in Figure 25.1a. Let us calculate thepotential difference between two points Aand Bseparated by a distance d, wheredis measured parallel to the field lines. Equation 25.3 givesBecause Eis constant, we can remove it from the integral sign; this gives(25.6)The minus sign indicates that point Bis at a lower electric potential than point A;that is, Electric field lines always point in the direction of decreas-ing electric potential,as shown in Figure 25.1a.Now suppose that a test charge q0moves from Ato B. We can calculate thechange in its potential energy from Equations 25.3 and 25.6: (25.7)/H9004U/H11005q0 /H9004V/H11005/H11002q0EdVB/H11021VA ./H9004V/H11005/H11002E /H20885BA ds/H11005/H11002EdVB/H11002VA/H11005/H9004V/H11005/H11002/H20885BA E/H11554ds/H11005/H11002/H20885BA E cos 0/H11034 ds/H11005/H11002/H20885BA E ds25.21 eV/H110051.60/H1100310/H1100219 C/H11080V/H110051.60/H1100310/H1100219 J1.60/H1100310/H1100219 C, dBAqE(a) (b)gdBAmFigure 25.1(a) When theelectric field Eis directed down-ward, point Bis at a lower elec-tric potential than point A. Apositive test charge that movesfrom point Ato point Bloseselectric potential energy. (b) Amass mmoving downward in thedirection of the gravitationalfield gloses gravitational poten-tial energy.The electron volt Potential difference in a uniformelectric field772CHAPTER 25Electric PotentialFrom this result, we see that if q0is positive, then /H9004Uis negative. We conclude thata positive charge loses electric potential energy when it moves in the direc-tion of the electric field.This means that an electric field does work on a positivecharge when the charge moves in the direction of the electric field. (This is analo-gous to the work done by the gravitational field on a falling mass, as shown in Fig-ure 25.1b.) If a positive test charge is released from rest in this electric field, it ex-periences an electric force q0Ein the direction of E(downward in Fig. 25.1a).Therefore, it accelerates downward, gaining kinetic energy. As the charged parti-cle gains kinetic energy, it loses an equal amount of potential energy.If q0is negative, then /H9004Uis positive and the situation is reversed: A negativecharge gains electric potential energy when it moves in the direction of theelectric field.If a negative charge is released from rest in the field E, it acceler-ates in a direction opposite the direction of the field.Now consider the more general case of a charged particle that is free to movebetween any two points in a uniform electric field directed along the xaxis, asshown in Figure 25.2. (In this situation, the charge is not being moved by an exter-nal agent as before.) If srepresents the displacement vector between points AandB, Equation 25.3 gives(25.8)where again we are able to remove Efrom the integral because it is constant. Thechange in potential energy of the charge is(25.9)Finally, we conclude from Equation 25.8 that all points in a plane perpendicu-lar to a uniform electric field are at the same electric potential. We can see this inFigure 25.2, where the potential difference is equal to the potential differ-ence (Prove this to yourself by working out the dot product forwhere the angle /H9258between Eand sis arbitrary as shown in Figure 25.2, andthe dot product for where /H9258/H110050.) Therefore, The name equipo-tential surface is given to any surface consisting of a continuous distribu-tion of points having the same electric potential.Note that because no work is done in moving a test charge be-tween any two points on an equipotential surface. The equipotential surfaces of auniform electric field consist of a family of planes that are all perpendicular to thefield. Equipotential surfaces for fields with other symmetries are described in latersections.The labeled points in Figure 25.3 are on a series of equipotential surfaces associated with anelectric field. Rank (from greatest to least) the work done by the electric field on a posi-tively charged particle that moves from Ato B; from Bto C; from Cto D; from Dto E.Quick Quiz 25.2/H9004U/H11005q0/H9004V,VB/H11005VC . sA:C ,sA:B ,E/H11554sVC/H11002VA .VB/H11002VA/H9004U/H11005q0 /H9004V/H11005/H11002q0 E/H11554s/H9004V/H11005/H11002 /H20885BA E/H11554ds/H11005/H11002E/H11554/H20885BA ds/H11005/H11002E/H11554s An equipotential surface11.9QuickLabIt takes an electric field of about 30 000 V/cm to cause a spark in dryair. Shuffle across a rug and reach to-ward a doorknob. By estimating thelength of the spark, determine theelectric potential difference betweenyour finger and the doorknob aftershuffling your feet but before touch-ing the knob. (If it is very humid onthe day you attempt this, it may notwork. Why?) EBCAsFigure 25.2A uniform electricfield directed along the positive xaxis. Point Bis at a lower electricpotential than point A. Points Band Care at the sameelectric po-tential. ABCED9 V8 V7 V6 VFigure 25.3Four equipotential surfaces. 25.2Potential Differences in a Uniform Electric Field773The Electric Field Between Two Parallel Plates of Opposite Charge EXAMPLE25.1A battery produces a specified potential difference betweenconductors attached to the battery terminals. A 12-V batteryis connected between two parallel plates, as shown in Figure25.4. The separation between the plates is d/H110050.30 cm, andwe assume the electric field between the plates to be uniform. 1The electric field vanishes within a conductor in electrostatic equilibrium; thus, the path integralbetween any two points in the conductor must be zero. A more complete discussion of thispoint is given in Section 25.6./H20885 E/H11554ds+–12 VABdFigure 25.4A 12-V battery connected to two parallel plates. Theelectric field between the plates has a magnitude given by the poten-tial difference /H9004Vdivided by the plate separation d.(This assumption is reasonable if the plate separation is smallrelative to the plate dimensions and if we do not considerpoints near the plate edges.) Find the magnitude of the elec-tric field between the plates.SolutionThe electric field is directed from the positiveplate (A) to the negative one (B), and the positive plate is ata higher electric potential than the negative plate is. The po-tential difference between the plates must equal the potentialdifference between the battery terminals. We can understandthis by noting that all points on a conductor in equilibriumare at the same electric potential1; no potential difference ex-ists between a terminal and any portion of the plate to whichit is connected. Therefore, the magnitude of the electric fieldbetween the plates is, from Equation 25.6,This configuration, which is shown in Figure 25.4 andcalled a parallel-plate capacitor,is examined in greater detail inChapter 26.4.0/H11003103 V/mE/H11005/H20841VB/H11002VA/H20841d/H1100512 V0.30/H1100310/H110022 m/H11005Motion of a Proton in a Uniform Electric Field EXAMPLE25.2From Equation 25.6, we have(b) Find the change in potential energy of the proton forthis displacement.SolutionThe negative sign means the potential energy of the protondecreases as it moves in the direction of the electric field. Asthe proton accelerates in the direction of the field, it gains ki-netic energy and at the same time loses electric potential en-ergy (because energy is conserved).ExerciseUse the concept of conservation of energy to findthe speed of the proton at point B.Answer2.77/H11003106m/s./H110026.4/H1100310/H1100215 J /H11005 /H11005(1.6/H1100310/H1100219 C)(/H110024.0/H11003104 V)/H9004U/H11005q0 /H9004V/H11005e /H9004V /H110024.0/H11003104 V/H11005/H9004V/H11005/H11002Ed/H11005/H11002(8.0/H11003104 V/m)(0.50 m)A proton is released from rest in a uniform electric field thathas a magnitude of 8.0/H11003104V/m and is directed along thepositive xaxis (Fig. 25.5). The proton undergoes a displace-ment of 0.50 m in the direction of E. (a) Find the change inelectric potential between points Aand B.SolutionBecause the proton (which, as you remember,carries a positive charge) moves in the direction of the field,we expect it to move to a position of lower electric potential. dBA++++++++ ––––––––vBvA = 0E Figure 25.5A proton accelerates from Ato Bin the direction ofthe electric field.774CHAPTER 25Electric PotentialELECTRIC POTENTIAL AND POTENTIAL ENERGYDUE TO POINT CHARGESConsider an isolated positive point charge q. Recall that such a charge produces anelectric field that is directed radially outward from the charge. To find the electricpotential at a point located a distance rfrom the charge, we begin with the gen-eral expression for potential difference:where Aand Bare the two arbitrary points shown in Figure 25.6. At any fieldpoint, the electric field due to the point charge is (Eq. 23.4), where is a unit vector directed from the charge toward the field point. The quantity can be expressed asBecause the magnitude of is 1, the dot product where /H9258is the angle between and ds. Furthermore, dscos /H9258 is the projection of dsonto r; thus, dscos /H9258/H11005dr. That is, any displacement dsalong the path frompoint Ato point Bproduces a change drin the magnitude of r, the radial distance to the charge creating the field. Making these substitutions, we find that hence, the expression for the potential difference becomes(25.10)The integral of is independent of the path between points Aand B—as it mustbe because the electric field of a point charge is conservative. Furthermore, Equa-tion 25.10 expresses the important result that the potential difference between anytwo points Aand Bin a field created by a point charge depends only on the radialcoordinates rAand rB. It is customary to choose the reference of electric potentialto be zero at With this reference, the electric potential created by a pointcharge at any distance rfrom the charge is(25.11)Electric potential is graphed in Figure 25.7 as a function of r, the radial dis-tance from a positive charge in the xyplane. Consider the following analogy togravitational potential: Imagine trying to roll a marble toward the top of a hillshaped like Figure 25.7a. The gravitational force experienced by the marble isanalogous to the repulsive force experienced by a positively charged object as itapproaches another positively charged object. Similarly, the electric potentialgraph of the region surrounding a negative charge is analogous to a "hole" withrespect to any approaching positively charged objects. A charged object must beinfinitely distant from another charge before the surface is "flat" and has an elec-tric potential of zero.V/H11005ke qrrA/H11005/H11009.E/H11554dsVB/H11002VA/H11005keq /H209001rB/H110021rA/H20901 VB/H11002VA/H11005/H11002 /H20885 Er dr/H11005/H11002keq /H20885rBrA drr2/H11005keqr/H20901rBrAE/H11554ds/H11005(keq/r2)dr;rˆrˆ/H11554ds/H11005ds cos /H9258,rˆE/H11554ds/H11005ke qr2 rˆ/H11554dsE/H11554dsrˆE/H11005keqrˆ/r2VB/H11002VA/H11005/H11002/H20885BA E/H11554ds25.3 drdsθrArBB qrrAˆFigure 25.6The potential differ-ence between points Aand Bdueto a point charge qdepends onlyonthe initial and final radial coordi-nates rAand rB. The two dashed cir-cles represent cross-sections ofspherical equipotential surfaces. Electric potential created by apoint charge25.3Electric Potential and Potential Energy Due to Point Charges775 2.52.01.51.00.50xyElectric potential (V) (b)(a) + Figure 25.7(a) The electric potential in the plane around a single positive charge is plottedon the vertical axis. (The electric potential function for a negative charge would look like a holeinstead of a hill.) The red line shows the 1/rnature of the electric potential, as given by Equation25.11. (b) View looking straight down the vertical axis of the graph in part (a), showing concen-tric circles where the electric potential is constant. These circles are cross sections of equipoten-tial spheres having the charge at the center.776CHAPTER 25Electric PotentialA spherical balloon contains a positively charged object at its center. As the balloon is in-flated to a greater volume while the charged object remains at the center, does the electricpotential at the surface of the balloon increase, decrease, or remain the same? How aboutthe magnitude of the electric field? The electric flux?We obtain the electric potential resulting from two or more point charges byapplying the superposition principle. That is, the total electric potential at somepoint Pdue to several point charges is the sum of the potentials due to the individ-ual charges. For a group of point charges, we can write the total electric potentialat Pin the form(25.12)where the potential is again taken to be zero at infinity and riis the distance fromthe point Pto the charge qi. Note that the sum in Equation 25.12 is an algebraicsum of scalars rather than a vector sum (which we use to calculate the electric fieldof a group of charges). Thus, it is often much easier to evaluate Vthan to evaluateE. The electric potential around a dipole is illustrated in Figure 25.8.We now consider the potential energy of a system of two charged particles. IfV1is the electric potential at a point Pdue to charge q1, then the work an externalagent must do to bring a second charge q2from infinity to Pwithout accelerationis q2V1. By definition, this work equals the potential energy Uof the two-particlesystem when the particles are separated by a distance r12(Fig. 25.9). Therefore, wecan express the potential energy as2(25.13)Note that if the charges are of the same sign, Uis positive. This is consistent withthe fact that positive work must be done by an external agent on the system tobring the two charges near one another (because like charges repel). If thecharges are of opposite sign, Uis negative; this means that negative work must bedone against the attractive force between the unlike charges for them to bebrought near each other.If more than two charged particles are in the system, we can obtain the totalpotential energy by calculating Ufor every pair of charges and summing the termsalgebraically. As an example, the total potential energy of the system of threecharges shown in Figure 25.10 is(25.14)Physically, we can interpret this as follows: Imagine that q1is fixed at the positionshown in Figure 25.10 but that q2and q3are at infinity. The work an externalagent must do to bring q2from infinity to its position near q1is whichis the first term in Equation 25.14. The last two terms represent the work requiredto bring q3from infinity to its position near q1and q2. (The result is independentof the order in which the charges are transported.)keq1q2/r12 ,U/H11005ke /H20898q1q2r12/H11001q1q3r13/H11001q2q3r23/H20899U/H11005ke q1q2r12V/H11005ke /H9018iqiriQuick Quiz 25.3 2The expression for the electric potential energy of a system made up of two point charges, Equation25.13, is of the same form as the equation for the gravitational potential energy of a system made up oftwo point masses, Gm1m2/r(see Chapter 14). The similarity is not surprising in view of the fact thatboth expressions are derived from an inverse-square force law.Electric potential due to severalpoint charges Electric potential energy due totwo charges25.3Electric Potential and Potential Energy Due to Point Charges777 2.52.01.51.00.50–0.5–1.0–1.5–2.0–2.5xy (a) (b)Electric potential (V) +–Figure 25.8(a) The electric potential in the plane containing a dipole. (b) Top view of thefunction graphed in part (a).q1q2r12 q2q1q3r13r12r23Figure 25. 9If two point chargesare separated by a distance r12, thepotential energy of the pair ofcharges is given by keq1q2/r12 . Figure 25.10Three pointcharges are fixed at the positionsshown. The potential energy of thissystem of charges is given by Equa-tion 25.14.778CHAPTER 25Electric PotentialThe Electric Potential Due to Two Point Charges EXAMPLE25.3SolutionWhen the charge is at infinity, , and whenthe charge is at P, ; therefore,Therefore, because , positive work would have tobe done by an external agent to remove the charge frompoint Pback to infinity.ExerciseFind the total potential energy of the system illus-trated in Figure 25.11b.Answer/H110025.48/H1100310/H110022J.W/H11005/H11002 /H9004U/H1100218.9/H1100310/H110023 J/H11005/H9004U/H11005q3VP/H110020/H11005(3.00/H1100310/H110026 C)(/H110026.29/H11003103 V)Uf/H11005q3VPUi/H110050A charge q1/H110052.00/H9262C is located at the origin, and a chargeq2/H11005/H110026.00/H9262C is located at (0, 3.00) m, as shown in Figure25.11a. (a) Find the total electric potential due to thesecharges at the point P, whose coordinates are (4.00, 0) m.SolutionFor two charges, the sum in Equation 25.12 gives (b) Find the change in potential energy of a 3.00-/H9262Ccharge as it moves from infinity to point P(Fig. 25.11b)./H110026.29/H11003103 V/H11005VP/H11005ke /H20898q1r1/H11001q2r2/H20899 OBTAINING THE VALUE OF THE ELECTRIC FIELDFROM THE ELECTRIC POTENTIALThe electric field Eand the electric potential Vare related as shown in Equation25.3. We now show how to calculate the value of the electric field if the electric po-tential is known in a certain region.From Equation 25.3 we can express the potential difference dVbetween twopoints a distance dsapart as(25.15)If the electric field has only one component Ex, then Therefore,Equation 25.15 becomes or(25.16)Ex/H11005/H11002dVdxdV/H11005/H11002Ex dx,E/H11554ds/H11005Ex dx.dV/H11005/H11002E/H11554ds25.4(a)3.00 m4.00 mPx–6.00 µCy 2.00 µC(b)3.00 m4.00 mx–6.00 µCy 2.00 µC3.00 µCµ µµµµ Figure 25.11(a) The electric potential at Pdue to the two charges is the algebraic sum of the poten-tials due to the individual charges. (b) What is the potential energy of the three-charge system?/H110058.99/H11003109 N/H11080m2C2 /H208982.00/H1100310/H110026 C4.00 m/H11001/H110026.00/H1100310/H110026 C5.00 m/H2089925.4Obtaining the Value of the Electric Field from the Electric Potential779That is, the magnitude of the electric field in the direction of some coordinate isequal to the negative of the derivative of the electric potential with respect to thatcoordinate. Recall from the discussion following Equation 25.8 that the electricpotential does not change for any displacement perpendicular to an electric field.This is consistent with the notion, developed in Section 25.2, that equipotentialsurfaces are perpendicular to the field, as shown in Figure 25.12. A small positivecharge placed at rest on an electric field line begins to move along the direction ofEbecause that is the direction of the force exerted on the charge by the chargedistribution creating the electric field (and hence is the direction of a). Becausethe charge starts with zero velocity, it moves in the direction of the change in ve-locity—that is, in the direction of a. In Figures 25.12a and 25.12b, a charge placedat rest in the field will move in a straight line because its acceleration vector is al-ways parallel to its velocity vector. The magnitude of vincreases, but its directiondoes not change. The situation is different in Figure 25.12c. A positive chargeplaced at some point near the dipole first moves in a direction parallel to Eat thatpoint. Because the direction of the electric field is different at different locations,however, the force acting on the charge changes direction, and ais no longer par-allel to v. This causes the moving charge to change direction and speed, but itdoes not necessarily follow the electric field lines. Recall that it is not the velocityvector but rather the acceleration vector that is proportional to force.If the charge distribution creating an electric field has spherical symmetrysuch that the volume charge density depends only on the radial distance r, thenthe electric field is radial. In this case, and thus we can express dVin the form dVTherefore,(25.17)For example, the electric potential of a point charge is Because Vis afunction of ronly, the potential function has spherical symmetry. Applying Equa-tion 25.17, we find that the electric field due to the point charge is afamiliar result. Note that the potential changes only in the radial direction, not inEr/H11005keq/r2,V/H11005keq/r.Er/H11005/H11002dVdr/H11005/H11002Er dr.E/H11554ds/H11005Er dr, (a)E(b)q (c)+Figure 25.12Equipotential surfaces (dashed blue lines) and electric field lines (red lines) for(a) a uniform electric field produced by an infinite sheet of charge, (b) a point charge, and (c) an electric dipole. In all cases, the equipotential surfaces are perpendicular to the electric fieldlines at every point. Compare these drawings with Figures 25.2, 25.7b, and 25.8b.780CHAPTER 25Electric Potentialany direction perpendicular to r. Thus, V(like Er) is a function only of r. Again,this is consistent with the idea that equipotential surfaces are perpendicular tofield lines.In this case the equipotential surfaces are a family of spheres concen-tric with the spherically symmetric charge distribution (Fig. 25.12b).The equipotential surfaces for an electric dipole are sketched in Figure25.12c. When a test charge undergoes a displacement dsalong an equipotentialsurface, then because the potential is constant along an equipotential sur-face. From Equation 25.15, then, ; thus, Emust be perpendicu-lar to the displacement along the equipotential surface. This shows that theequipotential surfaces must always be perpendicular to the electric field lines.In general, the electric potential is a function of all three spatial coordinates.If V(r) is given in terms of the cartesian coordinates, the electric field componentsEx, Ey, and Ezcan readily be found from V(x, y, z) as the partial derivatives3For example, if then/H11128V/H11128x/H11005/H11128/H11128x (3x2y/H11001y2/H11001yz)/H11005/H11128/H11128x (3x2y)/H110053y ddx (x2)/H110056xyV/H110053x2y/H11001y2/H11001yz,Ex/H11005/H11002/H11128V/H11128x Ey/H11005/H11002/H11128V/H11128y Ez/H11005/H11002/H11128V/H11128zdV/H11005/H11002E/H11554ds/H110050dV/H110050 The Electric Potential Due to a Dipole EXAMPLE25.4(How would this result change if point Phappened to be lo-cated to the left of the negative charge?)(b) Calculate Vand Exat a point far from the dipole.SolutionIf point Pis far from the dipole, such that then a2can be neglected in the term and Vbecomes Using Equation 25.16 and this result, we can calculate theelectric field at a point far from the dipole:()(c) Calculate Vand Exif point Pis located anywhere be-tween the two charges.SolutionEx/H11005/H11002dVdx/H11005/H11002ddx /H20898/H110022keqxx2/H11002a2/H20899/H110052keq/H20898/H11002x2/H11002a2(x2/H11002a2)2/H20899V/H11005ke /H9018qiri/H11005ke /H20898qa/H11002x/H11002qx/H11001a/H20899/H11005/H110022keqxx2/H11002a2xWa4keqax3Ex/H11005/H11002dVdx/H11005(xWa)2keqax2\ electric dipole consists of two charges of equal magnitudeand opposite sign separated by a distance 2a, as shown in Fig-ure 25.13. The dipole is along the xaxis and is centered atthe origin. (a) Calculate the electric potential at point P.SolutionFor point Pin Figure 25.13,2keqax2/H11002a2/H11005V/H11005ke /H9018qiri/H11005ke /H20898qx/H11002a/H11002qx/H11001a/H20899 3In vector notation, Eis often written where /H11612is called the gradient operator.E/H11005/H11002 /H11612V/H11005/H11002/H20898i /H11128/H11128x/H11001j /H11128/H11128y/H11001k /H11128/H11128z/H20899VEquipotential surfaces areperpendicular to the electric fieldlines aaqPxxy–qFigure 25.13An electric dipole located on the xaxis.25.5Electric Potential Due to Continuous Charge Distributions781ELECTRIC POTENTIAL DUE TO CONTINUOUS CHARGEDISTRIBUTIONSWe can calculate the electric potential due to a continuous charge distribution intwo ways. If the charge distribution is known, we can start with Equation 25.11 forthe electric potential of a point charge. We then consider the potential due to asmall charge element dq, treating this element as a point charge (Fig. 25.14). Theelectric potential dVat some point Pdue to the charge element dqis(25.18)where ris the distance from the charge element to point P. To obtain the total po-tential at point P, we integrate Equation 25.18 to include contributions from all el-ements of the charge distribution. Because each element is, in general, a differentdistance from point Pand because keis constant, we can express Vas(25.19)In effect, we have replaced the sum in Equation 25.12 with an integral. Note thatthis expression for Vuses a particular reference: The electric potential is taken tobe zero when point Pis infinitely far from the charge distribution.If the electric field is already known from other considerations, such as Gauss'slaw, we can calculate the electric potential due to a continuous charge distributionusing Equation 25.3. If the charge distribution is highly symmetric, we first evalu-ate Eat any point using Gauss's law and then substitute the value obtained intoEquation 25.3 to determine the potential difference /H9004Vbetween any two points.We then choose the electric potential Vto be zero at some convenient point.We illustrate both methods with several examples.V/H11005ke /H20885 dqrdV/H11005ke dqr25.5 Electric Potential Due to a Uniformly Charged Ring EXAMPLE25.5we can remove from the integral, and Vreduces to(25.20)The only variable in this expression for Vis x. This is not sur-prising because our calculation is valid only for points alongthe xaxis, where yand zare both zero.(b) Find an expression for the magnitude of the electricfield at point P.SolutionFrom symmetry, we see that along the xaxis Ecan have only an xcomponent. Therefore, we can use Equa-keQ!x2/H11001a2V/H11005ke!x2/H11001a2 /H20885 dq/H11005!x2/H11001a2(a) Find an expression for the electric potential at a point Plocated on the perpendicular central axis of a uniformlycharged ring of radius aand total charge Q.SolutionLet us orient the ring so that its plane is perpen-dicular to an xaxis and its center is at the origin. We can thentake point Pto be at a distance xfrom the center of the ring,as shown in Figure 25.15. The charge element dqis at a dis-tance from point P. Hence, we can express VasBecause each element dqis at the same distance from point P,V/H11005ke /H20885 dqr/H11005ke /H20885 dq!x2/H11001a2!x2/H11001a2We can check these results by considering the situation at the center of the dipole, where x/H110050, V/H110050, and /H110022keq/a2.Ex/H11005ExerciseVerify the electric field result in part (c) by calcu-lating the sum of the individual electric field vectors at theorigin due to the two charges. rPdq Figure 25.14The electric poten-tial at the point Pdue to a continu-ous charge distribution can be cal-culated by dividing the chargedbody into segments of charge dqand summing the electric potentialcontributions over all segments.782CHAPTER 25Electric Potential Px√x2 + a2dqaFigure 25.15A uniformly charged ring of radius alies in a planeperpendicular to the xaxis. All segments dqof the ring are the samedistance from any point Plying on the xaxis.tion 25.16:(25.21)This result agrees with that obtained by direct integration(see Example 23.8). Note that at x/H110050 (the center ofthe ring). Could you have guessed this from Coulomb's law?ExerciseWhat is the electric potential at the center of thering? What does the value of the field at the center tell youabout the value of Vat the center?AnswerBecause at the cen-Ex/H11005/H11002dV/dx/H110050V/H11005keQ /a.Ex/H110050keQx(x2/H11001a2)3/2 /H11005 /H11005/H11002keQ(/H1100212)(x2/H11001a2)/H110023/2(2x) Ex/H11005/H11002dVdx/H11005/H11002keQ ddx (x2/H11001a2)/H110021/2ter, Vhas either a maximum or minimum value; it is, in fact,a maximum. Electric Potential Due to a Uniformly Charged Disk EXAMPLE25.6from the definition of surface charge density (see Section23.5), we know that the charge on the ring is Hence, the potential at the point Pdue tothis ring isTo find the totalelectric potential at P, we sum over all ringsmaking up the disk. That is, we integrate dVfrom r/H110050 to r/H11005a:This integral is of the form unduand has the valuewhere and This gives(25.22)(b) As in Example 25.5, we can find the electric field atany axial point from(25.23)The calculation of Vand Efor an arbitrary point off the axisis more difficult to perform, and we do not treat this situationin this text.2/H9266ke /H9268 /H208981/H11002x!x2/H11001a2/H20899Ex/H11005/H11002dVdx/H110052/H9266ke /H9268[(x2/H11001a2)1/2/H11002x]V/H11005u/H11005r2/H11001x2.n/H11005/H1100212un/H110011/(n/H110011),V/H11005/H9266ke /H9268/H20885a0 2r dr!r2/H11001x2/H11005/H9266ke /H9268/H20885a0 (r2/H11001x2)/H110021/2 2r drdV/H11005ke dq!r2/H11001x2/H11005ke /H92682/H9266r dr!r2/H11001x2/H9268 dA/H11005/H92682/H9266r dr.dq/H11005Find (a) the electric potential and (b) the magnitude of theelectric field along the perpendicular central axis of a uni-formly charged disk of radius aand surface charge density /H9268.Solution(a) Again, we choose the point Pto be at a dis-tance xfrom the center of the disk and take the plane of thedisk to be perpendicular to the xaxis. We can simplify theproblem by dividing the disk into a series of charged rings.The electric potential of each ring is given by Equation 25.20.Consider one such ring of radius rand width dr, as indicatedin Figure 25.16. The surface area of the ring is dA/H110052/H9266r dr; Figure 25.16A uniformly charged disk of radius alies in a planeperpendicular to the xaxis. The calculation of the electric potentialat any point Pon the xaxis is simplified by dividing the disk intomany rings each of area 2/H9266r dr.drdA = 2πrdr√r2 + x2xPraπ25.5Electric Potential Due to Continuous Charge Distributions783Electric Potential Due to a Finite Line of Charge EXAMPLE25.7Evaluating V, we find that(25.24)keQ/H5129 ln/H20898/H5129/H11001!/H51292/H11001a2a/H20899V/H11005A rod of length located along the xaxis has a total chargeQand a uniform linear charge density /H9261/H11005Q/. F i n d t h eelectric potential at a point Plocated on the yaxis a distanceafrom the origin (Fig. 25.17).SolutionThe length element dxhas a charge dq/H11005/H9261dx.Because this element is a distance from point P,we can express the potential at point Pdue to this element asTo obtain the total potential at P, we integrate this expressionover the limits x/H110050 to x/H11005. Noting that keand /H9261are con-stants, we find thatThis integral has the following value (see Appendix B):/H20885 dx!x2/H11001a2/H11005ln(x/H11001!x2/H11001a2)V/H11005ke /H9261 /H20885/H51290 dx!x2/H11001a2/H11005ke Q/H5129 /H20885/H51290 dx!x2/H11001a2/H5129dV/H11005ke dqr/H11005ke /H9261 dx!x2/H11001a2r/H11005!x2/H11001a2/H5129/H5129 Electric Potential Due to a Uniformly Charged Sphere EXAMPLE25.8Because the potential must be continuous at r/H11005R, wecan use this expression to obtain the potential at the surfaceof the sphere. That is, the potential at a point such as Cshown in Figure 25.18 is(for (b) Find the potential at a point inside the sphere, that is,for r/H11021R .r/H11005R)VC/H11005ke QRAn insulating solid sphere of radius Rhas a uniform positivevolume charge density and total charge Q. (a) Find the elec-tric potential at a point outside the sphere, that is, for Take the potential to be zero at SolutionIn Example 24.5, we found that the magnitudeof the electric field outside a uniformly charged sphere of ra-dius Ris(for where the field is directed radially outward when Qis posi-tive. In this case, to obtain the electric potential at an exteriorpoint, such as Bin Figure 25.18, we use Equation 25.4 andthe expression for Ergiven above:(for Note that the result is identical to the expression for the elec-tric potential due to a point charge (Eq. 25.11).r/H11022R)VB/H11005ke QrVB/H11005/H11002/H20885r/H11009 Er dr/H11005/H11002keQ /H20885r/H11009 drr2r/H11022R)Er/H11005ke Qr2r/H11005/H11009.r/H11022R.dx/H5129xx0dqraPy Figure 25.17A uniform line charge of length located along the xaxis. To calculate the electric potential at P, the line charge isdivided into segments each of length dxand each carrying a chargedq/H11005/H9261dx./H5129 RrQDCBFigure 25.18A uniformly charged insulating sphere of radius Rand total charge Q. The electric potentials at points Band Careequivalent to those produced by a point charge Qlocated at the cen-ter of the sphere, but this is not true for point D.ELECTRIC POTENTIAL DUE TO ACHARGED CONDUCTORIn Section 24.4 we found that when a solid conductor in equilibrium carries a netcharge, the charge resides on the outer surface of the conductor. Furthermore, weshowed that the electric field just outside the conductor is perpendicular to thesurface and that the field inside is zero.We now show that every point on the surface of a charged conductor inequilibrium is at the same electric potential.Consider two points Aand Bonthe surface of a charged conductor, as shown in Figure 25.20. Along a surface pathconnecting these points, Eis always perpendicular to the displacement ds; there-25.6784CHAPTER 25Electric PotentialSolutionIn Example 24.5 we found that the electric fieldinside an insulating uniformly charged sphere is(for We can use this result and Equation 25.3 to evaluate the po-tential difference at some interior point D:Substituting into this expression and solving forVD, we obtain(for (25.25)At r/H11005R, this expression gives a result that agrees with thatfor the potential at the surface, that is, VC. A plot of Vversusrfor this charge distribution is given in Figure 25.19.ExerciseWhat are the magnitude of the electric field andthe electric potential at the center of the sphere?r/H11021R)VD/H11005keQ2R /H208983/H11002r2R2/H20899VC/H11005keQ/RVD/H11002VC/H11005/H11002/H20885rR Er dr/H11005/H11002keQR3 /H20885rR r dr/H11005keQ2R3 (R2/H11002r2)VD/H11002VCr/H11021R)Er/H11005keQR3rAnswerV0/H110053keQ/2R .E/H110050;VV0V023RrVB =keQrVD =keQ2R(3 –r2R2)V0 =3keQ2R Figure 25.19A plot of electric potential Vversus distance rfromthe center of a uniformly charged insulating sphere of radius R. Thecurve for VDinside the sphere is parabolic and joins smoothly withthe curve for VBoutside the sphere, which is a hyperbola. The poten-tial has a maximum value V0at the center of the sphere. We couldmake this graph three dimensional (similar to Figures 25.7a and25.8a) by spinning it around the vertical axis. +BAE++++++++++++++++++++++++++++Figure 25.20An arbitrarily shaped conductor carrying a posi-tive charge. When the conductor is in electrostatic equilibrium,all of the charge resides at the surface, E/H110050 inside the conduc-tor, and the direction of Ejust outside the conductor is perpen-dicular to the surface. The electric potential is constant insidethe conductor and is equal to the potential at the surface. Notefrom the spacing of the plus signs that the surface charge densityis nonuniform.25.6Electric Potential Due to a Charged Conductor785 the surface of any charged conductor in electrostatic equilibrium is an equipo-tential surface. Furthermore, because the electric field is zero inside the con-ductor, we conclude from the relationship that the electric poten-tial is constant everywhere inside the conductor and equal to its value at thesurface.Er/H11005/H11002dV/drfore Using this result and Equation 25.3, we conclude that the potentialdifference between Aand Bis necessarily zero:This result applies to any two points on the surface. Therefore, Vis constant every-where on the surface of a charged conductor in equilibrium. That is,VB/H11002VA/H11005/H11002/H20885BA E/H11554ds/H110050E/H11554ds/H110050. (a)++++++++++++++++RVkeQRkeQr(b)rEkeQr2rR(c)Figure 25.21(a) The excesscharge on a conducting sphere ofradius Ris uniformly distributed onits surface. (b) Electric potentialversus distance rfrom the center ofthe charged conducting sphere. (c) Electric field magnitude versusdistance rfrom the center of thecharged conducting sphere. Electric field pattern of a charged conducting plateplaced near an oppositely charged pointed conductor.Small pieces of thread suspended in oil align with theelectric field lines. The field surrounding the pointedconductor is most intense near the pointed end and atother places where the radius of curvature is small.Because this is true about the electric potential, no work is required to move a testcharge from the interior of a charged conductor to its surface.Consider a solid metal conducting sphere of radius Rand total positive chargeQ, as shown in Figure 25.21a. The electric field outside the sphere is keQ/r2andpoints radially outward. From Example 25.8, we know that the electric potential atthe interior and surface of the sphere must be keQ/Rrelative to infinity. The po-tential outside the sphere is keQ/r. Figure 25.21b is a plot of the electric potentialas a function of r, and Figure 25.21c shows how the electric field varies with r.When a net charge is placed on a spherical conductor, the surface charge den-sity is uniform, as indicated in Figure 25.21a. However, if the conductor is non-spherical, as in Figure 25.20, the surface charge density is high where the radius ofcurvature is small and the surface is convex (as noted in Section 24.4), and it is lowwhere the radius of curvature is small and the surface is concave. Because the elec-tric field just outside the conductor is proportional to the surface charge density,we see that the electric field is large near convex points having small radii ofcurvature and reaches very high values at sharp points.Figure 25.22 shows the electric field lines around two spherical conductors:one carrying a net charge Q, and a larger one carrying zero net charge. In thiscase, the surface charge density is not uniform on either conductor. The spherehaving zero net charge has negative charges induced on its side that faces theThe surface of a chargedconductor is an equipotentialsurface786CHAPTER 25Electric Potential charged sphere and positive charges induced on its side opposite the chargedsphere. The blue curves in the figure represent the cross-sections of the equipo-tential surfaces for this charge configuration. As usual, the field lines are perpen-dicular to the conducting surfaces at all points, and the equipotential surfaces areperpendicular to the field lines everywhere. Trying to move a positive charge inthe region of these conductors would be like moving a marble on a hill that is flaton top (representing the conductor on the left) and has another flat area partwaydown the side of the hill (representing the conductor on the right).Two Connected Charged Spheres EXAMPLE25.9Two spherical conductors of radii r1and r2are separated by adistance much greater than the radius of either sphere. Thespheres are connected by a conducting wire, as shown in Fig-ure 25.23. The charges on the spheres in equilibrium are q1and q2, respectively, and they are uniformly charged. Findthe ratio of the magnitudes of the electric fields at the sur-faces of the spheres.SolutionBecause the spheres are connected by a conduct-ing wire, they must both be at the same electric potential:Therefore, the ratio of charges isV/H11005ke q1r1/H11005ke q2r2QQ = 0—————+++Figure 25.22The electric field lines (in red) around two spherical conductors. The smallersphere has a net charge Q, and the larger one has zero net charge. The blue curves are cross-sections of equipotential surfaces. r1 r2q1 q2Figure 25.23Two charged spherical conductors connected by aconducting wire. The spheres are at the sameelectric potential V.25.6Electric Potential Due to a Charged Conductor787 A Cavity Within a ConductorNow consider a conductor of arbitrary shape containing a cavity as shown in Fig-ure 25.24. Let us assume that no charges are inside the cavity. In this case, theelectric field inside the cavity must be zeroregardless of the charge distribu-tion on the outside surface of the conductor. Furthermore, the field in the cavity iszero even if an electric field exists outside the conductor.To prove this point, we use the fact that every point on the conductor is at thesame electric potential, and therefore any two points Aand Bon the surface of thecavity must be at the same potential. Now imagine that a field Eexists in the cavityand evaluate the potential difference defined by Equation 25.3:If Eis nonzero, we can always find a path between Aand Bfor which is apositive number; thus, the integral must be positive. However, becausethe integral of must be zero for all paths between any twopoints on the conductor, which implies that Eis zero everywhere. This contradic-tion can be reconciled only if Eis zero inside the cavity. Thus, we conclude that acavity surrounded by conducting walls is a field-free region as long as no chargesare inside the cavity.Corona DischargeA phenomenon known as corona discharge is often observed near a conductorsuch as a high-voltage power line. When the electric field in the vicinity of the con-ductor is sufficiently strong, electrons are stripped from air molecules. This causesthe molecules to be ionized, thereby increasing the air’s ability to conduct. Theobserved glow (or corona discharge) results from the recombination of free elec-trons with the ionized air molecules. If a conductor has an irregular shape, theelectric field can be very high near sharp points or edges of the conductor; conse-quently, the ionization process and corona discharge are most likely to occuraround such points.(a) Is it possible for the magnitude of the electric field to be zero at a location where theelectric potential is not zero? (b) Can the electric potential be zero where the electric fieldis nonzero?Quick Quiz 25.4E/H11554dsVB/H11002VA/H110050,E/H11554dsVB/H11002VA/H11005/H11002/H20885BA E/H11554dsVB/H11002VA(1)Because the spheres are very far apart and their surfaces uni-formly charged, we can express the magnitude of the electricfields at their surfaces asandE2/H11005ke q2r2 2E1/H11005ke q1r1 2q1q2/H11005r1r2Taking the ratio of these two fields and making use of Equa-tion (1), we find thatHence, the field is more intense in the vicinity of the smallersphere even though the electric potentials of both spheresare the same.E1E2/H11005r2r1 ABFigure 25.24A conductor inelectrostatic equilibrium contain-ing a cavity. The electric field in thecavity is zero, regardless of thecharge on the conductor.788CHAPTER 25Electric PotentialOptional SectionTHE MILLIKAN OIL-DROP EXPERIMENTDuring the period from 1909 to 1913, Robert Millikan performed a brilliant set ofexperiments in which he measured e, the elementary charge on an electron, anddemonstrated the quantized nature of this charge. His apparatus, diagrammed inFigure 25.25, contains two parallel metallic plates. Charged oil droplets from an at-omizer are allowed to pass through a small hole in the upper plate. A horizontallydirected light beam (not shown in the diagram) is used to illuminate the oildroplets, which are viewed through a telescope whose long axis is at right angles tothe light beam. When the droplets are viewed in this manner, they appear as shin-ing stars against a dark background, and the rate at which individual drops fall canbe determined.4Let us assume that a single drop having a mass mand carrying a charge qis be-ing viewed and that its charge is negative. If no electric field is present between theplates, the two forces acting on the charge are the force of gravity mgacting down-ward and a viscous drag force FDacting upward as indicated in Figure 25.26a. Thedrag force is proportional to the drop’s speed. When the drop reaches its terminalspeed v, the two forces balance each other (mg/H11005FD).Now suppose that a battery connected to the plates sets up an electric field be-tween the plates such that the upper plate is at the higher electric potential. In thiscase, a third force qEacts on the charged drop. Because qis negative and Eis di-rected downward, this electric force is directed upward, as shown in Figure 25.26b.If this force is sufficiently great, the drop moves upward and the drag force actsdownward. When the upward electric force qEbalances the sum of the gravita-tional force and the downward drag force the drop reaches a new terminalspeed v/H11032in the upward direction.With the field turned on, a drop moves slowly upward, typically at rates of hun-dredths of a centimeter per second. The rate of fall in the absence of a field iscomparable. Hence, one can follow a single droplet for hours, alternately risingand falling, by simply turning the electric field on and off.F/H11032D ,F/H11032D25.7 4At one time, the oil droplets were termed

"Millikan's Shining Stars." Perhaps this description has lostits popularity because of the generations of physics students who have experienced hallucinations, nearblindness, migraine headaches, and so forth, while repeating Millikan's experiment! qv–+Battery SwitchCharged plateCharged plateTelescopeAtomizerOil dropletsPin hole FDFD qE mgEv′ (b) Field onvmgq(a) Field off ′Figure 25.25Schematic drawing of the Millikan oil-drop apparatus.Figure 25.26The forces actingon a negatively charged oil dropletin the Millikan experiment.25.8Applications of Electrostatics789After recording measurements on thousands of droplets, Millikan and his co-workers found that all droplets, to within about 1% precision, had a charge equalto some integer multiple of the elementary charge e:/H110021, /H110022, /H110023, . . .where Millikan's experiment yields conclusive evidence thatcharge is quantized. For this work, he was awarded the Nobel Prize in Physics in1923.Optional SectionAPPLICATIONS OF ELECTROSTATICSThe practical application of electrostatics is represented by such devices as light-ning rods and electrostatic precipitators and by such processes as xerography andthe painting of automobiles. Scientific devices based on the principles of electro-statics include electrostatic generators, the field-ion microscope, and ion-driverocket engines.The Van de Graaff GeneratorIn Section 24.5 we described an experiment that demonstrates a method for trans-ferring charge to a hollow conductor (the Faraday ice-pail experiment). When acharged conductor is placed in contact with the inside of a hollow conductor, allof the charge of the charged conductor is transferred to the hollow conductor. Inprinciple, the charge on the hollow conductor and its electric potential can be in-creased without limit by repetition of the process.In 1929 Robert J. Van de Graaff (1901–1967) used this principle to design andbuild an electrostatic generator. This type of generator is used extensively in nu-clear physics research. A schematic representation of the generator is given in Fig-ure 25.27. Charge is delivered continuously to a high-potential electrode by meansof a moving belt of insulating material. The high-voltage electrode is a hollow con-ductor mounted on an insulating column. The belt is charged at point Aby meansof a corona discharge between comb-like metallic needles and a grounded grid.The needles are maintained at a positive electric potential of typically 104V. Thepositive charge on the moving belt is transferred to the hollow conductor by a sec-ond comb of needles at point B. Because the electric field inside the hollow con-ductor is negligible, the positive charge on the belt is easily transferred to the con-ductor regardless of its potential. In practice, it is possible to increase the electricpotential of the hollow conductor until electrical discharge occurs through the air.Because the "breakdown" electric field in air is about 3/H11003106V/m, a sphere 1 min radius can be raised to a maximum potential of 3/H11003106V. The potential can beincreased further by increasing the radius of the hollow conductor and by placingthe entire system in a container filled with high-pressure gas.Van de Graaff generators can produce potential differences as large as 20 mil-lion volts. Protons accelerated through such large potential differences receiveenough energy to initiate nuclear reactions between themselves and various targetnuclei. Smaller generators are often seen in science classrooms and museums. If aperson insulated from the ground touches the sphere of a Van de Graaff genera-tor, his or her body can be brought to a high electric potential. The hair acquires anet positive charge, and each strand is repelled by all the others. The result is a25.8e/H110051.60/H1100310/H1100219 C.q/H11005ne n/H110050, 11.10Hollow conductor Belt GroundB A+ Insulator+++++++++++++++++ ++++++++ ————————Groundedgrid Figure 25.27Schematic diagramof a Van de Graaff generator.Charge is transferred to the hollowconductor at the top by means of amoving belt. The charge is de-posited on the belt at point Aandtransferred to the hollow conduc-tor at point B. 790CHAPTER 25Electric Potentialscene such as that depicted in the photograph at the beginning of this chapter. Inaddition to being insulated from ground, the person holding the sphere is safe inthis demonstration because the total charge on the sphere is very small (on the or-der of 1/H9262C). If this amount of charge accidentally passed from the spherethrough the person to ground, the corresponding current would do no harm.The Electrostatic PrecipitatorOne important application of electrical discharge in gases is the electrostatic precipi-tator.This device removes particulate matter from combustion gases, thereby re-ducing air pollution. Precipitators are especially useful in coal-burning powerplants and in industrial operations that generate large quantities of smoke. Cur-rent systems are able to eliminate more than 99% of the ash from smoke.Figure 25.28a shows a schematic diagram of an electrostatic precipitator. Ahigh potential difference (typically 40 to 100 kV) is maintained between a wirerunning down the center of a duct and the walls of the duct, which are grounded.The wire is maintained at a negative electric potential with respect to the walls, sothe electric field is directed toward the wire. The values of the field near the wirebecome high enough to cause a corona discharge around the wire; the dischargeionizes some air molecules to form positive ions, electrons, and such negative ionsas O2/H11002. The air to be cleaned enters the duct and moves near the wire. As the elec-trons and negative ions created by the discharge are accelerated toward the outerwall by the electric field, the dirt particles in the air become charged by collisionsand ion capture. Because most of the charged dirt particles are negative, they tooare drawn to the duct walls by the electric field. When the duct is periodicallyshaken, the particles break loose and are collected at the bottom.InsulatorClean airoutWeightDirtyair inDirt out(a)(c)(b) Figure 25.28(a) Schematic diagram of an electrostatic precipitator. The high negative electricpotential maintained on the central coiled wire creates an electrical discharge in the vicinity ofthe wire. Compare the air pollution when the electrostatic precipitator is (b) operating and (c) turned off. QuickLabSprinkle some salt and pepper on anopen dish and mix the two together.Now pull a comb through your hairseveral times and bring the comb towithin 1 cm of the salt and pepper.What happens? How is what happenshere related to the operation of anelectrostatic precipitator? 25.8Applications of Electrostatics791In addition to reducing the level of particulate matter in the atmosphere(compare Figs. 25.28b and c), the electrostatic precipitator recovers valuable mate-rials in the form of metal oxides.Xerography and Laser PrintersThe basic idea of xerography5was developed by Chester Carlson, who was granteda patent for the xerographic process in 1940. The one feature of this process thatmakes it unique is the use of a photoconductive material to form an image. (A pho-toconductor is a material that is a poor electrical conductor in the dark but that be-comes a good electrical conductor when exposed to light.)The xerographic process is illustrated in Figure 25.29a to d. First, the surface of a plate or drum that has been coated with a thin film of photoconductive mater-ial (usually selenium or some compound of selenium) is given a positive electrosta-tic charge in the dark. An image of the page to be copied is then focused by a lensonto the charged surface. The photoconducting surface becomes conducting onlyin areas where light strikes it. In these areas, the light produces charge carriers inthe photoconductor that move the positive charge off the drum. However, positive 5The prefix xero- is from the Greek word meaning "dry." Note that no liquid ink is used anywhere in xerography.Selenium-coateddrum(a) Charging the drum (b) Imaging the document (d) Transferring the toner to the paperLaserbeamInterlaced patternof laser lines(e) Laser printer drumNegativelychargedtoner(c) Applying the tonerLensLight causes some areasof drum to becomeelectrically conducting,removing positive charge Figure 25.29The xerographic process: (a) The photoconductive surface of the drum is posi-tively charged. (b) Through the use of a light source and lens, an image is formed on the surfacein the form of positive charges. (c) The surface containing the image is covered with a negativelycharged powder, which adheres only to the image area. (d) A piece of paper is placed over thesurface and given a positive charge. This transfers the image to the paper as the negativelycharged powder particles migrate to the paper. The paper is then heat-treated to "fix" the pow-der. (e) A laser printer operates similarly except the image is produced by turning a laser beamon and off as it sweeps across the selenium-coated drum.792CHAPTER 25Electric Potentialcharges remain on those areas of the photoconductor not exposed to light, leaving a latent image of the object in the form of a positive surface charge dis-tribution.Next, a negatively charged powder called a toner is dusted onto the photocon-ducting surface. The charged powder adheres only to those areas of the surfacethat contain the positively charged image. At this point, the image becomes visible.The toner (and hence the image) are then transferred to the surface of a sheet ofpositively charged paper.Finally, the toner is "fixed" to the surface of the paper as the toner melts whilepassing through high-temperature rollers. This results in a permanent copy of theoriginal.A laser printer (Fig. 25.29e) operates by the same principle, with the excep-tion that a computer-directed laser beam is used to illuminate the photoconductorinstead of a lens.SUMMARYWhen a positive test charge q0is moved between points Aand Bin an electric fieldE, the change in the potential energyis(25.1)The electric potentialis a scalar quantity and has units of joules percoulomb (J/C), where The potential difference/H9004Vbetween points Aand Bin an electric field Eisdefined as(25.3)The potential difference between two points Aand Bin a uniform electricfield Eis(25.6)where dis the magnitude of the displacement in the direction parallel to E.An equipotential surface is one on which all points are at the same electricpotential. Equipotential surfaces are perpendicular to electric field lines. If we define at the electric potential due to a point charge atany distance rfrom the charge is(25.11)We can obtain the electric potential associated with a group of point charges bysumming the potentials due to the individual charges.The potential energy associated with a pair of point chargesseparated bya distance r12is(25.13)This energy represents the work required to bring the charges from an infiniteseparation to the separation r12. We obtain the potential energy of a distributionof point charges by summing terms like Equation 25.13 over all pairs of particles.U/H11005ke q1q2r12V/H11005ke qrrA/H11005/H11009,V/H110050/H9004V/H11005/H11002Ed/H9004V/H11005/H9004Uq0/H11005/H11002/H20885BA E/H11554ds1 J/C/H110131 V.V/H11005U/q0/H9004U/H11005/H11002q0 /H20885BA E/H11554dsSummary793 If we know the electric potential as a function of coordinates x, y, z, we can ob-tain the components of the electric field by taking the negative derivative of theelectric potential with respect to the coordinates. For example, the xcomponentof the electric field is(25.16)The electric potential due to a continuous charge distributionis(25.19)Every point on the surface of a charged conductor in electrostatic equilibriumis at the same electric potential. The potential is constant everywhere inside theconductor and equal to its value at the surface.Table 25.1 lists electric potentials due to several charge distributions.V/H11005ke /H20885 dqrEx/H11005/H11002dVdx Problem-Solving HintsCalculating Electric Potential•Remember that electric potential is a scalar quantity, so components neednot be considered. Therefore, when using the superposition principle toevaluate the electric potential at a point due to a system of point charges,simply take the algebraic sum of the potentials due to the various charges.However, you must keep track of signs. The potential is positive for positivecharges, and it is negative for negative charges.•Just as with gravitational potential energy in mechanics, only changesin elec-tric potential are significant; hence, the point where you choose the poten-TABLE 25.1Electric Potential Due to Various Charge DistributionsCharge Distribution Electric Potential LocationUniformly charged Along perpendicular centralring of radius aaxis of ring, distance xfrom ring centerUniformly charged Along perpendicular centraldisk of radius aaxis of disk, distance xfrom disk centerUniformly charged,insulatingsolidsphere of radius Rand total charge QIsolated conductingsphere of radius Rand total charge QV/H11005ke QRV/H11005ke Qrr/H11349Rr/H11022RV/H11005ke Q!x2/H11001a2 V/H11005keQ2R /H208983/H11002r2R2/H20899V/H11005ke Qr/H20902/H20902V/H110052/H9266ke /H9268[(x2/H11001a2)1/2/H11002x]r/H11021Rr/H11350R794CHAPTER 25Electric Potentialtial to be zero is arbitrary. When dealing with point charges or a charge dis-tribution of finite size, we usually define V/H110050 to be at a point infinitely farfrom the charges.•You can evaluate the electric potential at some point Pdue to a continuousdistribution of charge by dividing the charge distribution into infinitesimalelements of charge dqlocated at a distance rfrom P. Then, treat one chargeelement as a point charge, such that the potential at Pdue to the element isObtain the total potential at Pby integrating dVover the en-tire charge distribution. In performing the integration for most problems,you must express dqand rin terms of a single variable. To simplify the inte-gration, consider the geometry involved in the problem carefully. Review Ex-amples 25.5 through 25.7 for guidance.•Another method that you can use to obtain the electric potential due to a fi-nite continuous charge distribution is to start with the definition of poten-tial difference given by Equation 25.3. If you know or can easily obtain E(from Gauss's law), then you can evaluate the line integral of An ex-ample of this method is given in Example 25.8.•Once you know the electric potential at a point, you can obtain the electricfield at that point by remembering that the electric field component in a specified direction is equal to the negative of the derivative of the electricpotential in that direction. Example 25.4 illustrates this procedure.E/H11554ds.dV/H11005kedq/r. QUESTIONSsphere is zero. Does this imply that the potential is zeroinside the sphere? Explain.9.The potential of a point charge is defined to be zero at aninfinite distance. Why can we not define the potential ofan infinite line of charge to be zero at 10.Two charged conducting spheres of different radii areconnected by a conducting wire, as shown in Figure25.23. Which sphere has the greater charge density?11.What determines the maximum potential to which thedome of a Van de Graaff generator can be raised?12.Explain the origin of the glow sometimes observedaround the cables of a high-voltage power line.13.Why is it important to avoid sharp edges or points on con-ductors used in high-voltage equipment?14.How would you shield an electronic circuit or laboratoryfrom stray electric fields? Why does this work?15.Why is it relatively safe to stay in an automobile with ametal body during a severe thunderstorm?16.Walking across a carpet and then touching someone canresult in a shock. Explain why this occurs.r/H11005/H11009 ?1.Distinguish between electric potential and electric poten-tial energy.2.A negative charge moves in the direction of a uniformelectric field. Does the potential energy of the charge in-crease or decrease? Does it move to a position of higheror lower potential?3.Give a physical explanation of the fact that the poten-tial energy of a pair of like charges is positive whereas the potential energy of a pair of unlike charges is nega-tive.4.A uniform electric field is parallel to the xaxis. In whatdirection can a charge be displaced in this field withoutany external work being done on the charge?5.Explain why equipotential surfaces are always perpendic-ular to electric field lines.6.Describe the equipotential surfaces for (a) an infinite lineof charge and (b) a uniformly charged sphere.7.Explain why, under static conditions, all points in a con-ductor must be at the same electric potential.8.The electric field inside a hollow, uniformly chargedProblems795PROBLEMS 11.A 4.00-kg block carrying a charge Q/H1100550.0/H9262C is con-nected to a spring for which k/H11005100 N/m. The blocklies on a frictionless horizontal track, and the system isimmersed in a uniform electric field of magnitude E/H110055.00/H11003105V/m, directed as shown in Figure P25.11. Ifthe block is released from rest when the spring is un-stretched (at x/H110050), (a) by what maximum amountdoes the spring expand? (b) What is the equilibriumposition of the block? (c) Show that the block's motionis simple harmonic, and determine its period. (d) Repeat part (a) if the coefficient of kinetic frictionbetween block and surface is 0.200.12.A block having mass mand charge Qis connected to aspring having constant k. The block lies on a frictionlesshorizontal track, and the system is immersed in a uni-form electric field of magnitude E,directed as shown inFigure P25.11. If the block is released from rest whenthe spring is unstretched (at x/H110050), (a) by what maxi-mum amount does the spring expand? (b) What is theequilibrium position of the block? (c) Show that theblock's motion is simple harmonic, and determine itsperiod.(d) Repeat part (a) if the coefficier kineticfriction between block and surface is /H9262k.Section 25.1Potential Difference and Electric Potential1.How much work is done (by a battery, generator, orsome other source of electrical energy) in moving Avo-gadro's number of electrons from an initial point wherethe electric potential is 9.00 V to a point where the po-tential is /H110025.00 V ? (The potential in each case is mea-sured relative to a common reference point.)2.An ion accelerated through a potential difference of115 V experiences an increase in kinetic energy of 7.37/H1100310/H1100217J. Calculate the charge on the ion.3.(a) Calculate the speed of a proton that is acceleratedfrom rest through a potential difference of 120 V. (b) Calculate the speed of an electron that is acceler-ated through the same potential difference.4. Review Problem.Through what potential differencewould an electron need to be accelerated for it toachieve a speed of 40.0% of the speed of light, startingfrom rest? The speed of light is c/H110053.00/H11003108m/s; review Section 7.7.5.What potential difference is needed to stop an electronhaving an initial speed of 4.20/H11003105m/s?Section 25.2Potential Differences in a Uniform Electric Field6.A uniform electric field of magnitude 250 V/m is directed in the positive xdirection. A /H1100112.0-/H9262C charge moves from the origin to the point (x, y)/H11005(20.0 cm, 50.0 cm). (a) What was the change in the potential energy of this charge? (b) Through what po-tential difference did the charge move?7.The difference in potential between the acceleratingplates of a TV set is about 25 000 V. If the distance be-tween these plates is 1.50 cm, find the magnitude of theuniform electric field in this region.8.Suppose an electron is released from rest in a uniformelectric field whose magnitude is 5.90/H11003103V/m. (a) Through what potential difference will it havepassed after moving 1.00 cm? (b) How fast will the elec-tron be moving after it has traveled 1.00 cm?9.An electron moving parallel to the xaxis has an initialspeed of 3.70/H11003106m/s at the origin. Its speed is re-duced to 1.40/H11003105m/s at the point x/H110052.00 cm. Cal-culate the potential difference between the origin andthat point. Which point is at the higher potential?10.A uniform electric field of magnitude 325 V/m is directed in the negative ydirection as shown in Figure P25.10. The coordinates of point Aare (/H110020.200, /H110020.300) m, and those of point Bare (0.400, 0.500) m. Calculate the potential differenceusing the blue path.VB/H11002VA,1, 2, 3= straightforward, intermediate, challenging = full solution available in the Student Solutions Manual and Study GuideWEB= solution posted at http://www.saunderscollege.com/physics/= Computer useful in solving problem = Interactive Physics= paired numerical/symbolic problems yBxEAFigure P25.10 Figure P25.11Problems 11 and 12.km, QEx = 0WEB796CHAPTER 25Electric Potential13.On planet Tehar, the acceleration due to gravity is thesame as that on Earth but there is also a strong down-ward electric field with the field being uniform close tothe planet's surface. A 2.00-kg ball having a charge of 5.00/H9262C is thrown upward at a speed of 20.1 m/s and ithits the ground after an interval of 4.10 s. What is thepotential difference between the starting point and thetop point of the trajectory?14.An insulating rod having linear charge density /H9261/H1100540.0 /H9262C/m and linear mass density /H9262/H110050.100 kg/m isreleased from rest in a uniform electric field E/H11005100 V/m directed perpendicular to the rod (Fig.P25.14). (a) Determine the speed of the rod after it hastraveled 2.00 m. (b) How does your answer to part (a)change if the electric field is not perpendicular to therod? Explain. 18.A charge /H11001qis at the origin. A charge /H110022qis at x/H110052.00 m on the xaxis. For what finite value(s) of xis (a) the electric field zero? (b) the electric potential zero?19.The Bohr model of the hydrogen atom states that thesingle electron can exist only in certain allowed orbitsaround the proton. The radius of each Bohr orbit is r/H11005n2(0.052 9 nm) where n/H110051, 2, 3,....Calculatethe electric potential energy of a hydrogen atom whenthe electron is in the (a) first allowed orbit, n/H110051; (b) second allowed orbit, n/H110052; and (c) when the elec-tron has escaped from the atom Express youranswers in electron volts.20.Two point charges nC and are separated by 35.0 cm. (a) What is the potential en-ergy of the pair? What is the significance of the alge-braic sign of your answer? (b) What is the electric po-tential at a point midway between the charges?21.The three charges in Figure P25.21 are at the vertices ofan isosceles triangle. Calculate the electric potential atthe midpoint of the base, taking q/H110057.00/H9262C.22.Compare this problem with Problem 55 in Chapter 23.Fouridentical point charges (q/H11005/H1100110.0/H9262C) are located onthe corners of a rectangle, as shown in Figure P23.55.The dimensions of the rectangle are L/H1100560.0 cm andW/H1100515.0 cm. Calculate the electric potential energy ofthe charge at the lower left corner due to the otherthree charges.Q2/H11005/H110023.00 nCQ1/H11005/H110015.00(r/H11005/H11009).string makes an angle /H9258/H1100560.0° with a uniform electricfield of magnitude E/H11005300 V/m. Determine the speedof the particle when the string is parallel to the electricfield (point ain Fig. P25.15).Section 25.3Electric Potential and Potential Energy Due to Point ChargesNote:Unless stated otherwise, assume a reference level of po-tential at 16.(a) Find the potential at a distance of 1.00 cm from aproton. (b) What is the potential difference betweentwo points that are 1.00 cm and 2.00 cm from a proton?(c) Repeat parts (a) and (b) for an electron.17.Given two 2.00-/H9262C charges, as shown in Figure P25.17,and a positive test charge

/H1100310/H1100218C at the ori-gin, (a) what is the net force exerted on qby the two2.00-/H9262C charges? (b) What is the electric field at the ori-gin due to the two 2.00-/H9262C charges? (c) What is theelectric potential at the origin due to the two 2.00-/H9262Ccharges? q/H110051.28r/H11005/H11009.V/H110050 15.A particle having charge /H9262C and mass m/H110050.010 0 kg is connected to a string that is L/H110051.50 mlong and is tied to the pivot point Pin Figure P25.15.The particle, string, and pivot point all lie on a horizon-tal table. The particle is released from rest when theq/H11005/H110012.002.00yq0x = 0.800 mx = –0.800 mxCCμ2.00μ θTop ViewEPamqLλ, μEE,Figure P25.14 Figure P25.15Figure P25.17Problems797collide? (Hint:Consider conservation of energy andconservation of linear momentum.) (b) If the sphereswere conductors, would the speeds be greater or lessthan those calculated in part (a)?29.A small spherical object carries a charge of 8.00 nC. Atwhat distance from the center of the object is the poten-tial equal to 100 V ? 50.0 V ? 25.0 V ? Is the spacing of theequipotentials proportional to the change in potential?30.Two point charges of equal magnitude are locatedalong the yaxis equal distances above and below the xaxis, as shown in Figure P25.30. (a) Plot a graph ofthe potential at points along the xaxis over the intervalYou should plot the potential in units ofkeQ/a. (b) Let the charge located at /H11002abe negativeand plot the potential along the yaxis over the interval/H110024a/H11021y/H110214a./H110023a/H11021x/H110213a. 31.In Rutherford's famous scattering experiments that ledto the planetary model of the atom, alpha particles(charge /H110012e, mass/H110056.64/H1100310/H1100227kg) were fired at a gold nucleus (charge /H1100179e). An alpha particle, ini-tially very far from the gold nucleus, is fired with a ve-locity of 2.00/H11003107m/s directly toward the center ofthe nucleus. How close does the alpha particle get tothis center before turning around? Assume the gold nu-cleus remains stationary.32.An electron starts from rest 3.00 cm from the center ofa uniformly charged insulating sphere of radius 2.00 cmand total charge 1.00 nC. What is the speed of the elec-tron when it reaches the surface of the sphere?33.Calculate the energy required to assemble the array ofcharges shown in Figure P25.33, where a/H110050.200 m, b/H110050.400 m, and q/H110056.00/H9262C.34.Four identical particles each have charge qand mass m.They are released from rest at the vertices of a square ofside L. How fast is each charge moving when their dis-tance from the center of the square doubles?23.Show that the amount of work required to assemblefour identical point charges of magnitude Qat the cor-ners of a square of side sis 5.41keQ2/s.24.Compare this problem with Problem 18 in Chapter 23.Twopoint charges each of magnitude 2.00/H9262C are locatedon the xaxis. One is at x/H110051.00 m, and the other is at x/H11005/H110021.00 m. (a) Determine the electric potential onthe yaxis at y/H110050.500 m. (b) Calculate the electric po-tential energy of a third charge, of /H110023.00/H9262C, placed onthe yaxis at y/H110050.500 m.25.Compare this problem with Problem 22 in Chapter 23.Fiveequal negative point charges /H11002qare placed symmetri-cally around a circle of radius R. Calculate the electricpotential at the center of the circle.26.Compare this problem with Problem 17 in Chapter 23.Three equal positive charges qare at the corners of anequilateral triangle of side a, as shown in Figure P23.17.(a) At what point, if any, in the plane of the charges isthe electric potential zero? (b) What is the electric po-tential at the point Pdue to the two charges at the baseof the triangle?27.Review Problem.Two insulating spheres having radii0.300 cm and 0.500 cm, masses 0.100 kg and 0.700 kg,and charges /H110022.00/H9262C and 3.00/H9262C are released fromrest when their centers are separated by 1.00 m. (a) How fast will each be moving when they collide? (Hint:Consider conservation of energy and linear mo-mentum.) (b) If the spheres were conductors would thespeeds be larger or smaller than those calculated in part(a)? Explain.28.Review Problem.Two insulating spheres having radiir1and r2, masses m1and m2, and charges /H11002q1and q2are released from rest when their centers are separatedby a distance d. (a) How fast is each moving when they2.00 cm4.00 cmq –q –qFigure P25.21aaxyQ >O QFigure P25.30WEB798CHAPTER 25Electric Potential 35.How much work is required to assemble eight identicalpoint charges, each of magnitude q, at the corners of acube of side s?Section 25.4Obtaining the Value of the Electric Fieldfrom the Electric Potential36.The potential in a region between x/H110050 and x/H110056.00 m is where a/H1100510.0 V and b/H11005/H110027.00 V/m. Determine (a) the potential at x/H110050, 3.00 m, and 6.00 m and (b) the magnitude and direction of the electric field at x/H110050, 3.00 m, and 6.00 m.37.Over a certain region of space, the electric potential isFind the expressions for the x, y,and zcomponents of the electric field over this region.What is the magnitude of the field at the point P, whichhas coordinates (1, 0, /H110022) m?38.The electric potential inside a charged sphericalconductor of radius Ris given by V/H11005keQ/Randoutside the conductor is given by V/H11005keQ/r. Usingderive the electric field (a) inside and(b) outside this charge distribution.39.It is shown in Example 25.7 that the potential at a pointPa distance aabove one end of a uniformly chargedrod of length lying along the xaxis isUse this result to derive an expression for the ycompo-nent of the electric field at P. (Hint:Replace awith y.)40.When an uncharged conducting sphere of radius aisplaced at the origin of an xyzcoordinate system that liesin an initially uniform electric field the result-ing electric potential is for points outside the sphere, where V0is the (constant)electric potential on the conductor. Use this equation todetermine the x, y, and zcomponents of the resultingelectric field.V(x, y, z)/H11005V0/H11002E0z/H11001E0a3z(x2/H11001y2/H11001z2)3/2E/H11005E0k,V/H11005keQ/H5129 ln/H20898/H5129/H11001!/H51292/H11001a2a/H20899/H5129Er/H11005/H11002dV/dr,V/H110055x/H110023x2y/H110012yz2.V/H11005a/H11001 25.5Electric Potential Due to Continuous Charge Distributions41.Consider a ring of radius Rwith the total charge Qspread uniformly over its perimeter. What is the poten-tial difference between the point at the center of thering and a point on its axis a distance 2Rfrom the center?42.Compare this problem with Problem 33 in Chapter 23.Auniformly charged insulating rod of length 14.0 cm isbent into the shape of a semicircle, as shown in FigureP23.33. If the rod has a total charge of /H110027.50/H9262C, findthe electric potential at O, the center of the semicircle.43.A rod of length L(Fig. P25.43) lies along the xaxis withits left end at the origin and has a nonuniform chargedensity /H9261/H11005/H9251x(where /H9251is a positive constant). (a) What are the units of /H9251? (b) Calculate the electricpotential at A. 46.A wire of finite length that has a uniform linear chargedensity /H9261is bent into the shape shown in Figure P25.46.Find the electric potential at point O.44.For the arrangement described in the previous prob-lem, calculate the electric potential at point Bthat lieson the perpendicular bisector of the rod a distance babove the xaxis.45.Calculate the electric potential at point Pon the axis ofthe annulus shown in Figure P25.45, which has a uni-form charge density /H9268.abxPbByxLdAq–2q 2q3qbaFigure P25.33 Figure P25.43Problems 43 and 44. Figure P25.45WEBProblems799 Section 25.6Electric Potential Due to a Charged Conductor47.How many electrons should be removed from an ini-tially uncharged spherical conductor of radius 0.300 mto produce a potential of 7.50 kV at the surface?48.Two charged spherical conductors are connected by along conducting wire, and a charge of 20.0/H9262C is placedon the combination. (a) If one sphere has a radius of4.00 cm and the other has a radius of 6.00 cm, what isthe electric field near the surface of each sphere? (b) What is the electric potential of each sphere?49.A spherical conductor has a radius of 14.0 cm andcharge of 26.0/H9262C. Calculate the electric field and theelectric potential at (a) r/H1100510.0 cm, (b) r/H1100520.0 cm,and (c) r/H1100514.0 cm from the center.50.Two concentric spherical conducting shells of radii a/H110050.400 m and b/H110050.500 m are connected by a thin wire,as shown in Figure P25.50. If a total charge Q/H1100510.0/H9262C is placed on the system, how much chargesettles on each sphere?ADDITIONAL PROBLEMS53.The liquid-drop model of the nucleus suggests thathigh-energy oscillations of certain nuclei can split thenucleus into two unequal fragments plus a few neu-trons. The fragments acquire kinetic energy from theirmutual Coulomb repulsion. Calculate the electric po-tential energy (in electron volts) of two spherical frag-ments from a uranium nucleus having the followingcharges and radii: 38eand 5.50/H1100310/H1100215m; 54eand 6.20/H1100310/H1100215m. Assume that the charge is distributeduniformly throughout the volume of each sphericalfragment and that their surfaces are initially in contactat rest. (The electrons surrounding the nucleus can beneglected.)54.On a dry winter day you scuff your leather-soled shoesacross a carpet and get a shock when you extend the tipof one finger toward a metal doorknob. In a dark roomyou see a spark perhaps 5 mm long. Make order-of-magnitude estimates of (a) your electric potential and(b) the charge on your body before you touch the door-knob. Explain your reasoning.55.The charge distribution shown in Figure P25.55 is re-ferred to as a linear quadrupole. (a) Show that the po-tential at a point on the xaxis where x/H11022ais(b) Show that the expression obtained in part (a) whenreduces toV/H110052keQa2x3xWaV/H110052keQa2x3/H11002xa2WEB 56.(a) Use the exact result from Problem 55 to find theelectric field at any point along the axis of the linearquadrupole for (b) Evaluate Eat x/H110053aif a/H110052.00 mm and Q/H110053.00/H9262C.57.At a certain distance from a point charge, the magni-tude of the electric field is 500 V/m and the electric po-tential is /H110023.00 kV. (a) What is the distance to thecharge? (b) What is the magnitude of the charge?58.An electron is released from rest on the axis of a uni-form positively charged ring, 0.100 m from the ring'sx/H11022a.(Optional)Section 25.7The Millikan Oil-Drop Experiment(Optional)Section 25.8Applications of Electrostatics51.Consider a Van de Graaff generator with a 30.0-cm-diameter dome operating in dry air. (a) What is themaximum potential of the dome? (b) What is the maxi-mum charge on the dome?52.The spherical dome of a Van de Graaff generator canbe raised to a maximum potential of 600 kV; then addi-tional charge leaks off in sparks, by producing break-down of the surrounding dry air. Determine (a) thecharge on the dome and (b) the radius of the dome.+Q–2Q+Qxy(a,0)(–a,0)Quadrupoleabq1q2Wire2R2RORFigure P25.46 Figure P25.50Figure P25.55800CHAPTER 25Electric Potentialcenter. If the linear charge density of the ring is /H110010.100 /H9262C/m and the radius of the ring is 0.200 m,how fast will the electron be moving when it reaches thecenter of the ring?59.(a) Consider a uniformly charged cylindrical shell hav-ing total charge Q, radius R, and height h. Determinethe electrostatic potential at a point a distance dfromthe right side of the cylinder, as shown in Figure P25.59.(Hint:Use the result of Example 25.5 by treating thecylinder as a collection of ring charges.) (b) Use the result of Example 25.6 to solve the same problem for asolid cylinder.63.From Gauss's law, the electric field set up by a uniformline of charge iswhere is a unit vector pointing radially away from theline and /H9261is the charge per unit length along the line.Derive an expression for the potential difference be-tween and 64.A point charge qis located at x/H11005/H11002R, and a pointcharge /H110022qis located at the origin. Prove that theequipotential surface that has zero potential is a spherecentered at (/H110024R/3, 0, 0) and having a radius r/H110052R/3.65.Consider two thin, conducting, spherical shells asshown in cross-section in Figure P25.65. The inner shellhas a radius r1/H1100515.0 cm and a charge of 10.0 nC. Theouter shell has a radius r2/H1100530.0 cm and a charge of/H1100215.0 nC. Find (a) the electric field Eand (b) theelectric potential Vin regions A, B, and C, with V/H110050 atr/H11005/H11009.r/H11005r2 .r/H11005r1rˆE/H11005/H20898/H92612/H9266/H92800r/H20899rˆWEB 66.The xaxis is the symmetry axis of a uniformly chargedring of radius Rand charge Q(Fig. P25.66). A pointcharge Qof mass Mis located at the center of the ring.When it is displaced slightly, the point charge acceler-60.Two parallel plates having charges of equal magnitudebut opposite sign are separated by 12.0 cm. Each platehas a surface charge density of 36.0 nC/m2. A proton isreleased from rest at the positive plate. Determine (a) the potential difference between the plates, (b) theenergy of the proton when it reaches the negative plate,(c) the speed of the proton just before it strikes the neg-ative plate, (d) the acceleration of the proton, and (e) the force on the proton. (f) From the force, findthe magnitude of the electric field and show that it isequal to that found from the charge densities on theplates.61.Calculate the work that must be done to charge a spher-ical shell of radius Rto a total charge Q.62.A Geiger–Müller counter is a radiation detector that es-sentially consists of a hollow cylinder (the cathode) ofinner radius raand a coaxial cylindrical wire (the an-ode) of radius rb(Fig. P25.62). The charge per unitlength on the anode is /H9261, while the charge per unitlength on the cathode is /H11002/H9261. (a) Show that the magni-tude of the potential difference between the wire andthe cylinder in the sensitive region of the detector is(b) Show that the magnitude of the electric field overthat region is given bywhere ris the distance from the center of the anode tothe point where the field is to be calculated.E/H11005/H9004Vln(ra/rb) /H208981r/H20899/H9004V/H110052ke /H9261 ln/H20898rarb/H20899dRhFigure P25.59rbλra–λCathodeAnodeλ CBAr1r2Figure P25.62 Figure P25.65Problems801ates along the xaxis to infinity. Show that the ultimatespeed of the point charge isv/H11005/H208982keQ2MR/H208991/2RQvxUniformlycharged ringQFigure P25.66 Figure P25.68Figure P25.69 Figure P25.70(b) For the dipole arrangement shown, express Vinterms of cartesian coordinates using r/H11005(x2+y2)1/2andUsing these results and taking calculate the fieldcomponents Exand Ey.70.Figure P25.70 shows several equipotential lines each la-beled by its potential in volts. The distance between thelines of the square grid represents 1.00 cm. (a) Is themagnitude of the field bigger at Aor at B? Why? (b) What is Eat B? (c) Represent what the field lookslike by drawing at least eight field lines.rWa,cos /H9258/H11005y(x2/H11001y2)1/2 69.A dipole is located along the yaxis as shown in FigureP25.69. (a) At a point P, which is far from the dipolethe electric potential iswhere p/H110052qa. Calculate the radial component Erandthe perpendicular component E/H9258of the associated elec-tric field. Note that Do these re-sults seem reasonable for /H9258/H1100590° and 0°? for r/H110050? E/H9258/H11005/H11002(1/r)(/H11128V//H11128/H9258).V/H11005ke p cos /H9258r2(rWa),67.An infinite sheet of charge that has a surface chargedensity of 25.0 nC/m2lies in the yzplane, passesthrough the origin, and is at a potential of 1.00 kV atthe point . A long wire having a linearcharge density of 80.0 nC/m lies parallel to the yaxisand intersects the xaxis at x/H110053.00 m. (a) Determine,as a function of x, the potential along the xaxis betweenwire and sheet. (b) What is the potential energy of a2.00-nC charge placed at x/H110050.800 m?68.The thin, uniformly charged rod shown in FigureP25.68 has a linear charge density /H9261. Find an expressionfor the electric potential at P.y/H110050, z/H110050a–qa+qr1r2rθxyPErEθθ baLxPy 71.A disk of radius Rhas a nonuniform surface chargedensity /H9268/H11005Cr, where Cis a constant and ris measuredfrom the center of the disk (Fig. P25.71). Find (by di-rect integration) the potential at P.× B×02468A802CHAPTER 25Electric Potential ANSWERS TOQUICKQUIZZESelectric potential is zero at the center of the dipole, butthe magnitude of the field at that point is not zero. (Thetwo charges in a dipole are by definition of oppositesign; thus, the electric field lines created by the twocharges extend from the positive to the negative chargeand do not cancel anywhere.) This is the situation wepresented in Example 25.4c, in which the equations weobtained give and .Ex/HS110050V/H11005025.1We do if the electric field is uniform. (This is preciselywhat we do in the next section.) In general, however, anelectric field changes from one place to another.25.2B:C, C:D, A:B, D:E. Moving from Bto Cde-creases the electric potential by 2 V, so the electric fieldperforms 2 J of work on each coulomb of charge thatmoves. Moving from Cto Ddecreases the electric poten-tial by 1 V, so 1 J of work is done by the field. It takes nowork to move the charge from Ato Bbecause the elec-tric potential does not change. Moving from Dto Ein-creases the electric potential by 1 V, and thus the fielddoes /H110021J o f w o r k , j u s t a s r a i s i n g a m a s s t o a h i g h e r e l e -vation causes the gravitational field to do negative workon the mass.25.3The electric potential decreases in inverse proportion tothe radius (see Eq. 25.11). The electric field magnitudedecreases as the reciprocal of the radius squared (seeEq. 23.4). Because the surface area increases as r2whilethe electric field magnitude decreases as 1/r2, the elec-tric flux through the surface remains constant (see Eq. 24.1).25.4(a) Yes. Consider four equal charges placed at the cor-ners of a square. The electric potential graph for this sit-uation is shown in the figure. At the center of thesquare, the electric field is zero because the individualfields from the four charges cancel, but the potential isnot zero. This is also the situation inside a charged con-ductor. (b) Yes again. In Figure 25.8, for instance, theFigure P25.71electric potential energy. (Hint:Imagine that the sphereis constructed by adding successive layers of concentricshells of charge and use 73.The results of Problem 62 apply also to an electrostaticprecipitator (see Figs. 25.28a and P25.62). An appliedvoltage is to produce an elec-tric field of magnitude 5.50 MV/m at the surface of thecentral wire. The outer cylindrical wall has uniform ra-dius ra/H110050.850 m. (a) What should be the radius rbofthe central wire? You will need to solve a transcendentalequation. (b) What is the magnitude of the electricfield at the outer wall?/H9004V/H11005Va/H11002Vb/H1100550.0 kVdU/H11005V dq.)dq/H11005(4/H9266r2 dr)/H9267RPx xyElectric potential (V)6543210 72.A solid sphere of radius Rhas a uniform charge density/H9267and total charge Q. Derive an expression for its total 2.2This is the Nearest One Head803 chapterCapacitance and Dielectrics PUZZLERMany electronic components carry awarning label like this one. What is thereinside these devices that makes them sodangerous? Why wouldn't you be safe ifyou unplugged the equipment beforeopening the case?(George Semple) Chapter Outline26.1Definition of Capacitance26.2Calculating Capacitance26.3Combinations of Capacitors26.4Energy Stored in a ChargedCapacitor26.5Capacitors with Dielectrics26.6(Optional) Electric Dipole in anElectric Field26.7(Optional) An Atomic Descriptionof DielectricsPUZZLER 803804CHAPTER 26Capacitance and Dielectricsn this chapter, we discuss capacitors—devices that store electric charge. Capaci-tors are commonly used in a variety of electric circuits. For instance, they areused to tune the frequency of radio receivers, as filters in power supplies, toeliminate sparking in automobile ignition systems, and as energy-storing devices inelectronic flash units.A capacitor consists of two conductors separated by an insulator. We shall seethat the capacitance of a given capacitor depends on its geometry and on the ma-terial—called a dielectric—that separates the conductors.DEFINITION OF CAPACITANCEConsider two conductors carrying charges of equal magnitude but of oppositesign, as shown in Figure 26.1. Such a combination of two conductors is called a ca-pacitor.The conductors are called plates.A potential difference /H9004Vexists betweenthe conductors due to the presence of the charges. Because the unit of potentialdifference is the volt, a potential difference is often called a voltage.We shall usethis term to describe the potential difference across a circuit element or betweentwo points in space.What determines how much charge is on the plates of a capacitor for a givenvoltage? In other words, what is the capacity of the device for storing charge at aparticular value of /H9004V? Experiments show that the quantity of charge Qon a ca-pacitor1is linearly proportional to the potential difference between the conduc-tors; that is, The proportionality constant depends on the shape and sepa-ration of the conductors.2We can write this relationship as if we definecapacitance as follows:Q/H11005C /H9004VQ/H11008/H9004V.26.1 The capacitance Cof a capacitor is the ratio of the magnitude of the charge oneither conductor to the magnitude of the potential difference between them:(26.1)C/H11013Q/H9004VI Note that by definition capacitance is always a positive quantity.Furthermore, the po-tential difference /H9004Vis always expressed in Equation 26.1 as a positive quantity. Be-cause the potential difference increases linearly with the stored charge, the ratioQ//H9004Vis constant for a given capacitor. Therefore, capacitance is a measure of acapacitor's ability to store charge and electric potential energy.From Equation 26.1, we see that capacitance has SI units of coulombs per volt.The SI unit of capacitance is the farad (F), which was named in honor of MichaelFaraday:The farad is a very large unit of capacitance. In practice, typical devices have ca-pacitances ranging from microfarads (10/H110026F) to picofarads (10/H1100212F). For practi-cal purposes, capacitors often are labeled "mF" for microfarads and "mmF" for mi-cromicrofarads or, equivalently, "pF" for picofarads.1 F/H110051 C/VDefinition of capacitance 1Although the total charge on the capacitor is zero (because there is as much excess positive chargeon one conductor as there is excess negative charge on the other), it is common practice to refer to themagnitude of the charge on either conductor as "the charge on the capacitor."2The proportionality between /H9004Vand Qcan be proved from Coulomb's law or by experiment.13.5 –Q+QFigure 26.1A capacitor consistsof two conductors carrying chargesof equal magnitude but oppositesign.26.2Calculating Capacitance805 Let us consider a capacitor formed from a pair of parallel plates, as shown inFigure 26.2. Each plate is connected to one terminal of a battery (not shown inFig. 26.2), which acts as a source of potential difference. If the capacitor is initiallyuncharged, the battery establishes an electric field in the connecting wires whenthe connections are made. Let us focus on the plate connected to the negative ter-minal of the battery. The electric field applies a force on electrons in the wire justoutside this plate; this force causes the electrons to move onto the plate. Thismovement continues until the plate, the wire, and the terminal are all at the sameelectric potential. Once this equilibrium point is attained, a potential differenceno longer exists between the terminal and the plate, and as a result no electricfield is present in the wire, and the movement of electrons stops. The plate nowcarries a negative charge. A similar process occurs at the other capacitor plate,with electrons moving from the plate to the wire, leaving the plate positivelycharged. In this final configuration, the potential difference across the capacitorplates is the same as that between the terminals of the battery.Suppose that we have a capacitor rated at 4 pF. This rating means that the ca-pacitor can store 4 pC of charge for each volt of potential difference between thetwo conductors. If a 9-V battery is connected across this capacitor, one of the con-ductors ends up with a net charge of /H1100236 pC and the other ends up with a netcharge of /H1100136 pC.CALCULATING CAPACITANCEWe can calculate the capacitance of a pair of oppositely charged conductors in thefollowing manner: We assume a charge of magnitude Q, and we calculate the po-tential difference using the techniques described in the preceding chapter. Wethen use the expression to evaluate the capacitance. As we might ex-pect, we can perform this calculation relatively easily if the geometry of the capaci-tor is simple.We can calculate the capacitance of an isolated spherical conductor of radiusRand charge Qif we assume that the second conductor making up the capacitor isa concentric hollow sphere of infinite radius. The electric potential of the sphereof radius Ris simply keQ/R, and setting at infinity as usual, we have(26.2)This expression shows that the capacitance of an isolated charged sphere is pro-portional to its radius and is independent of both the charge on the sphere andthe potential difference.C/H11005Q/H9004V/H11005QkeQ/R/H11005Rke/H110054/H9266/H92800RV/H110050C/H11005Q//H9004V26.2QuickLabRoll some socks into balls and stuffthem into a shoebox. What deter-mines how many socks fit in the box?Relate how hard you push on thesocks to /H9004Vfor a capacitor. How doesthe size of the box influence its “sockcapacity”? A collection of capacitors used in a variety of applica-tions. d–Q+QArea = AFigure 26.2A parallel-plate ca-pacitor consists of two parallel con-ducting plates, each of area A, sepa-rated by a distance d. When thecapacitor is charged, the platescarry equal amounts of charge.One plate carries positive charge,and the other carries negativecharge.806CHAPTER 26Capacitance and DielectricsThe capacitance of a pair of conductors depends on the geometry of the con-ductors. Let us illustrate this with three familiar geometries, namely, parallelplates, concentric cylinders, and concentric spheres. In these examples, we assumethat the charged conductors are separated by a vacuum. The effect of a dielectricmaterial placed between the conductors is treated in Section 26.5.Parallel-Plate CapacitorsTwo parallel metallic plates of equal area Aare separated by a distance d, as shownin Figure 26.2. One plate carries a charge Q,and the other carries a charge /H11002Q.Let us consider how the geometry of these conductors influences the capacity ofthe combination to store charge. Recall that charges of like sign repel one an-other. As a capacitor is being charged by a battery, electrons flow into the negativeplate and out of the positive plate. If the capacitor plates are large, the accumu-lated charges are able to distribute themselves over a substantial area, and theamount of charge that can be stored on a plate for a given potential difference in-creases as the plate area is increased. Thus, we expect the capacitance to be pro-portional to the plate area A.Now let us consider the region that separates the plates. If the battery has aconstant potential difference between its terminals, then the electric field betweenthe plates must increase as dis decreased. Let us imagine that we move the platescloser together and consider the situation before any charges have had a chanceto move in response to this change. Because no charges have moved, the electricfield between the plates has the same value but extends over a shorter distance.Thus, the magnitude of the potential difference between the plates (Eq.25.6) is now smaller. The difference between this new capacitor voltage and theterminal voltage of the battery now exists as a potential difference across the wiresconnecting the battery to the capacitor. This potential difference results in an elec-tric field in the wires that drives more charge onto the plates, increasing the po-tential difference between the plates. When the potential difference between theplates again matches that of the battery, the potential difference across the wiresfalls back to zero, and the flow of charge stops. Thus, moving the plates closer to-gether causes the charge on the capacitor to increase. If dis increased, the chargedecreases. As a result, we expect the device’s capacitance to be inversely propor-tional to d./H9004V/H11005Ed Figure 26.3(a) The electric field between the plates of a parallel-plate capacitor is uniformnear the center but nonuniform near the edges. (b) Electric field pattern of two oppositelycharged conducting parallel plates. Small pieces of thread on an oil surface align with the elec-tric field.+Q–Q(a) (b)26.2Calculating Capacitance807We can verify these physical arguments with the following derivation. The sur-face charge density on either plate is If the plates are very close to-gether (in comparison with their length and width), we can assume that the elec-tric field is uniform between the plates and is zero elsewhere. According to the lastparagraph of Example 24.8, the value of the electric field between the plates isBecause the field between the plates is uniform, the magnitude of the potentialdifference between the plates equals Ed(see Eq. 25.6); therefore,Substituting this result into Equation 26.1, we find that the capacitance is(26.3)That is, the capacitance of a parallel-plate capacitor is proportional to thearea of its plates and inversely proportional to the plate separation,just aswe expect from our conceptual argument.A careful inspection of the electric field lines for a parallel-plate capacitor re-veals that the field is uniform in the central region between the plates, as shown inFigure 26.3a. However, the field is nonuniform at the edges of the plates. Figure26.3b is a photograph of the electric field pattern of a parallel-plate capacitor.Note the nonuniform nature of the electric field at the ends of the plates. Suchend effects can be neglected if the plate separation is small compared with thelength of the plates.Many computer keyboard buttons are constructed of capacitors, as shown in Figure 26.4.When a key is pushed down, the soft insulator between the movable plate and the fixedplate is compressed. When the key is pressed, the capacitance (a) increases, (b) decreases,or (c) changes in a way that we cannot determine because the complicated electric circuitconnected to the keyboard button may cause a change in /H9004V.Quick Quiz 26.1C/H11005/H92800AdC/H11005Q/H9004V/H11005QQd//H92800A/H9004V/H11005Ed/H11005Qd/H92800AE/H11005/H9268/H92800/H11005Q/H92800AKeyMovableplateSoftinsulatorFixedplateB Parallel-Plate Capacitor EXAMPLE26.1ExerciseWhat is the capacitance for a plate separation of3.00 mm?Answer0.590 pF.1.77 pF /H110051.77/H1100310/H1100212 F/H11005A parallel-plate capacitor has an area and a plate separation mm. Find its capacitance.SolutionFrom Equation 26.3, we find thatC/H11005/H92800 Ad/H11005(8.85/H1100310/H1100212 C2/N/H11080m2)/H208982.00/H1100310/H110024 m21.00/H1100310/H110023 m/H20899d/H110051.00A/H110052.00/H1100310/H110024 m2Figure 26.4One type of com-puter keyboard button.808CHAPTER 26Capacitance and Dielectrics The Cylindrical Capacitor EXAMPLE26.2by (b/a), a positive quantity. Aspredicted, the capacitance is proportional to the length ofthe cylinders. As we might expect, the capacitance also de-pends on the radii of the two cylindrical conductors. FromEquation 26.4, we see that the capacitance per unit length ofa combination of concentric cylindrical conductors is(26.5)An example of this type of geometric arrangement is a coaxialcable,which consists of two concentric cylindrical conductorsseparated by an insulator. The cable carries electrical signalsin the inner and outer conductors. Such a geometry is espe-cially useful for shielding the signals from any possible exter-nal influences.C/H5129/H1100512ke ln/H20898ba/H20899/H9004V/H11005/H20841Vb/H11002Va/H20841/H110052ke/H9261 lnA solid cylindrical conductor of radius aand charge Qiscoaxial with a cylindrical shell of negligible thickness, radiusand charge /H11002Q(Fig. 26.5a). Find the capacitance ofthis cylindrical capacitor if its length is /H5129.SolutionIt is difficult to apply physical arguments to thisconfiguration, although we can reasonably expect the capaci-tance to be proportional to the cylinder length /H5129for the samereason that parallel-plate capacitance is proportional to platearea: Stored charges have more room in which to be distrib-uted. If we assume that /H5129is much greater than aand b, we canneglect end effects. In this case, the electric field is perpen-dicular to the long axis of the cylinders and is confined to theregion between them (Fig. 26.5b). We must first calculate thepotential difference between the two cylinders, which is givenin general bywhere Eis the electric field in the region In Chap-ter 24, we showed using Gauss’s law that the magnitude of theelectric field of a cylindrical charge distribution having linearcharge density /H9261is (Eq. 24.7). The same resultapplies here because, according to Gauss’s law, the charge onthe outer cylinder does not contribute to the electric field in-side it. Using this result and noting from Figure 26.5b that Eis along r, we find thatSubstituting this result into Equation 26.1 and using the factthat we obtain(26.4)where /H9004Vis the magnitude of the potential difference, given/H51292ke ln/H20898ba/H20899C/H11005Q/H9004V/H11005Q2keQ/H5129 ln/H20898ba/H20899/H11005/H9261/H11005Q//H5129,Vb/H11002Va/H11005/H11002/H20885ba Er dr/H11005/H110022ke/H9261 /H20885ba drr/H11005/H110022ke/H9261 ln/H20898ba/H20899Er/H110052ke/H9261/ra/H11021r/H11021b.Vb/H11002Va/H11005/H11002/H20885ba E/H11554dsb/H11022a, The Spherical Capacitor EXAMPLE26.3SolutionAs we showed in Chapter 24, the field outsidea spherically symmetric charge distribution is radial andgiven by the expression In this case, this result ap-plies to the field between the spheres From(a/H11021r/H11021b).keQ/r2.A spherical capacitor consists of a spherical conducting shellof radius band charge /H11002Qconcentric with a smaller conduct-ing sphere of radius aand charge Q(Fig. 26.6). Find the ca-pacitance of this device.ba/H5129 (a)(b)Gaussiansurface–QaQbrFigure 26.5(a) A cylindrical capacitor consists of a solid cylindri-cal conductor of radius aand length /H5129surrounded by a coaxial cylin-drical shell of radius b. (b) End view. The dashed line represents theend of the cylindrical gaussian surface of radius rand length /H5129.Cylindrical and Spherical CapacitorsFrom the definition of capacitance, we can, in principle, find the capacitance ofany geometric arrangement of conductors. The following examples demonstratethe use of this definition to calculate the capacitance of the other familiar geome-tries that we mentioned: cylinders and spheres.26.3Combinations of Capacitors809 What is the magnitude of the electric field in the region outside the spherical capacitor de-scribed in Example 26.3?COMBINATIONS OF CAPACITORSTwo or more capacitors often are combined in electric circuits. We can calculatethe equivalent capacitance of certain combinations using methods described inthis section. The circuit symbols for capacitors and batteries, as well as the colorcodes used for them in this text, are given in Figure 26.7. The symbol for the ca-pacitor reflects the geometry of the most common model for a capacitor—a pairof parallel plates. The positive terminal of the battery is at the higher potentialand is represented in the circuit symbol by the longer vertical line.Parallel CombinationTwo capacitors connected as shown in Figure 26.8a are known as a parallel combina-tion of capacitors. Figure 26.8b shows a circuit diagram for this combination of ca-pacitors. The left plates of the capacitors are connected by a conducting wire tothe positive terminal of the battery and are therefore both at the same electric po-tential as the positive terminal. Likewise, the right plates are connected to the neg-ative terminal and are therefore both at the same potential as the negative termi-nal. Thus, the individual potential differences across capacitors connected inparallel are all the same and are equal to the potential difference appliedacross the combination.In a circuit such as that shown in Figure 26.8, the voltage applied across thecombination is the terminal voltage of the battery. Situations can occur in which26.3Quick Quiz 26.2Figure 26.6A spherical capacitor consists of an inner sphere ofradius asurrounded by a concentric spherical shell of radius b. Theelectric field between the spheres is directed radially outward whenthe inner sphere is positively charged.ab– Q+Q ExerciseShow that as the radius bof the outer sphere ap-proaches infinity, the capacitance approaches the valuea/ke/H110054/H9266/H92800a. Figure 26.7Circuit symbols forcapacitors, batteries, and switches.Note that capacitors are in blueand batteries and switches are inred.CapacitorsymbolBatterysymbol+–Switchsymbol13.5 Gauss's law we see that only the inner sphere contributes to this field. Thus, the potential difference between thespheres isThe magnitude of the potential difference isSubstituting this value for /H9004Vinto Equation 26.1, we obtain(26.6)abke(b/H11002a)C/H11005Q/H9004V/H11005/H9004V/H11005/H20841Vb/H11002Va/H20841/H11005keQ (b/H11002a)ab /H11005keQ /H208981b/H110021a/H20899Vb/H11002Va/H11005/H11002/H20885ba Er dr/H11005/H11002keQ /H20885ba drr2/H11005keQ /H209001r/H20901ba810CHAPTER 26Capacitance and Dielectrics the parallel combination is in a circuit with other circuit elements; in such situa-tions, we must determine the potential difference across the combination by ana-lyzing the entire circuit.When the capacitors are first connected in the circuit shown in Figure 26.8,electrons are transferred between the wires and the plates; this transfer leaves theleft plates positively charged and the right plates negatively charged. The energysource for this charge transfer is the internal chemical energy stored in the bat-tery, which is converted to electric potential energy associated with the charge sep-aration. The flow of charge ceases when the voltage across the capacitors is equalto that across the battery terminals. The capacitors reach their maximum chargewhen the flow of charge ceases. Let us call the maximum charges on the two ca-pacitors Q1and Q2. The total charge Qstored by the two capacitors is(26.7)That is, the total charge on capacitors connected in parallel is the sum of thecharges on the individual capacitors. Because the voltages across the capacitorsare the same, the charges that they carry areSuppose that we wish to replace these two capacitors by one equivalent capacitorhaving a capacitance Ceq, as shown in Figure 26.8c. The effect this equivalent ca-pacitor has on the circuit must be exactly the same as the effect of the combina-tion of the two individual capacitors. That is, the equivalent capacitor must store Qunits of charge when connected to the battery. We can see from Figure 26.8c thatthe voltage across the equivalent capacitor also is /H9004Vbecause the equivalent capac-Q1/H11005C1 /H9004V Q2/H11005C2 /H9004VQ/H11005Q1/H11001Q2(a)+–C2+–C1+– (b)∆V+–Q2C2Q1C1∆V1 = ∆V2 = ∆V ∆V+–Ceq = C1 + C2 (c)∆VFigure 26.8(a) A parallel combination of two capacitors in an electric circuit in which the po-tential difference across the battery terminals is /H9004V. (b) The circuit diagram for the parallel com-bination. (c) The equivalent capacitance is Ceq/H11005C1/H11001C2 .26.3Combinations of Capacitors811itor is connected directly across the battery terminals. Thus, for the equivalent ca-pacitor,Substituting these three relationships for charge into Equation 26.7, we haveIf we extend this treatment to three or more capacitors connected in parallel,we find the equivalent capacitance to be(26.8)Thus, the equivalent capacitance of a parallel combination of capacitors isgreater than any of the individual capacitances.This makes sense because weare essentially combining the areas of all the capacitor plates when we connectthem with conducting wire.Series CombinationTwo capacitors connected as shown in Figure 26.9a are known as a series combina-tion of capacitors. The left plate of capacitor 1 and the right plate of capacitor 2are connected to the terminals of a battery. The other two plates are connected toeach other and to nothing else; hence, they form an isolated conductor that is ini-tially uncharged and must continue to have zero net charge. To analyze this com-bination, let us begin by considering the uncharged capacitors and follow whathappens just after a battery is connected to the circuit. When the battery is con-Ceq/H11005C1/H11001C2/H11001C3/H11001/H11080 /H11080 /H11080 (parallel combination)Ceq/H11005C1/H11001C2 /H20898parallelcombination/H20899Ceq /H9004V/H11005C1 /H9004V/H11001C2 /H9004V Q/H11005Ceq /H9004V (a)+–C2 ∆VC1∆V1∆V2+Q –Q +Q –Q (b)+–∆VCeq Figure 26.9(a) A series combination of two capacitors. The charges on the two capacitors arethe same. (b) The capacitors replaced by a single equivalent capacitor. The equivalent capaci-tance can be calculated from the relationship1Ceq/H110051C1/H110011C2812CHAPTER 26Capacitance and Dielectricsnected, electrons are transferred out of the left plate of C1and into the right plateof C2. As this negative charge accumulates on the right plate of C2, an equivalentamount of negative charge is forced off the left plate of C2, and this left platetherefore has an excess positive charge. The negative charge leaving the left plateof C2travels through the connecting wire and accumulates on the right plate ofC1. As a result, all the right plates end up with a charge /H11002Q, and all the left platesend up with a charge /H11001Q. Thus, the charges on capacitors connected in seriesare the same.From Figure 26.9a, we see that the voltage /H9004Vacross the battery terminals issplit between the two capacitors:(26.9)where /H9004V1and /H9004V2are the potential differences across capacitors C1and C2, re-spectively. In general, the total potential difference across any number of ca-pacitors connected in series is the sum of the potential differences acrossthe individual capacitors.Suppose that an equivalent capacitor has the same effect on the circuit as theseries combination. After it is fully charged, the equivalent capacitor must have acharge of /H11002Qon its right plate and a charge of /H11001Qon its left plate. Applying thedefinition of capacitance to the circuit in Figure 26.9b, we haveBecause we can apply the expression to each capacitor shown in Figure26.9a, the potential difference across each isSubstituting these expressions into Equation 26.9 and noting that we haveCanceling Q, we arrive at the relationshipWhen this analysis is applied to three or more capacitors connected in series, therelationship for the equivalent capacitance is(26.10)This demonstrates that the equivalent capacitance of a series combination isalways less than any individual capacitance in the combination.1Ceq/H110051C1/H110011C2/H110011C3/H11001/H11080 /H11080 /H11080 /H20898seriescombination/H208991Ceq/H110051C1/H110011C2 /H20898seriescombination/H20899QCeq/H11005QC1/H11001QC2/H9004V/H11005Q/Ceq ,/H9004V1/H11005QC1 /H9004V2/H11005QC2Q/H11005C /H9004V/H9004V/H11005QCeq/H9004V/H11005/H9004V1/H11001/H9004V2 Equivalent Capacitance EXAMPLE26.4SolutionUsing Equations 26.8 and 26.10, we reduce thecombination step by step as indicated in the figure. The 1.0-/H9262F and 3.0-/H9262F capacitors are in parallel and combine ac-Find the equivalent capacitance between aand bfor the com-bination of capacitors shown in Figure 26.10a. All capaci-tances are in microfarads.26.4Energy Stored in a Charged Capacitor813 ENERGY STORED IN A CHARGED CAPACITORAlmost everyone who works with electronic equipment has at some time verifiedthat a capacitor can store energy. If the plates of a charged capacitor are con-nected by a conductor, such as a wire, charge moves between the plates and theconnecting wire until the capacitor is uncharged. The discharge can often be ob-served as a visible spark. If you should accidentally touch the opposite plates of acharged capacitor, your fingers act as a pathway for discharge, and the result is anelectric shock. The degree of shock you receive depends on the capacitance andon the voltage applied to the capacitor. Such a shock could be fatal if high voltagesare present, such as in the power supply of a television set. Because the chargescan be stored in a capacitor even when the set is turned off, unplugging the televi-sion does not make it safe to open the case and touch the components inside.Consider a parallel-plate capacitor that is initially uncharged, such that the ini-tial potential difference across the plates is zero. Now imagine that the capacitor isconnected to a battery and develops a maximum charge Q. (We assume that thecapacitor is charged slowly so that the problem can be considered as an electrosta-tic system.) When the capacitor is connected to the battery, electrons in the wirejust outside the plate connected to the negative terminal move into the plate togive it a negative charge. Electrons in the plate connected to the positive terminalmove out of the plate into the wire to give the plate a positive charge. Thus,charges move only a small distance in the wires. To calculate the energy of the capacitor, we shall assume a different process —one that does not actually occur but gives the same final result. We can make this26.4cording to the expression /H9262F. The 2.0-/H9262F and 6.0-/H9262F capacitors also are in parallel and have anequivalent capacitance of 8.0/H9262F. Thus, the upper branch inFigure 26.10b consists of two 4.0-/H9262F capacitors in series,which combine as follows:Ceq/H1100511/2.0 /H9262F/H110052.0 /H9262F 1Ceq/H110051C1/H110011C2/H1100514.0 /H9262F/H1100114.0 /H9262F/H1100512.0 /H9262FCeq/H11005C1/H11001C2/H110054.0 The lower branch in Figure 26.10b consists of two 8.0-/H9262F ca-pacitors in series, which combine to yield an equivalent ca-pacitance of 4.0/H9262F. Finally, the 2.0-/H9262F and 4.0-/H9262F capacitorsin Figure 26.10c are in parallel and thus have an equivalentcapacitance of 6.0/H9262F.ExerciseConsider three capacitors having capacitances of3.0/H9262F, 6.0/H9262F, and 12/H9262F. Find their equivalent capacitancewhen they are connected (a) in parallel and (b) in series.Answer(a) 21/H9262F; (b) 1.7/H9262F.4.04.08.08.0ba(b)4.0ba(c)2.06.0ba(d)4.08.0ba(a)2.06.03.01.0 Figure 26.10To find the equivalent capacitance of the capacitors in part (a), wereduce the various combinations in steps as indicated in parts (b), (c), and (d), usingthe series and parallel rules described in the text. 13.5 814CHAPTER 26Capacitance and Dielectricsassumption because the energy in the final configuration does not depend on theactual charge-transfer process. We imagine that we reach in and grab a smallamount of positive charge on the plate connected to the negative terminal and ap-ply a force that causes this positive charge to move over to the plate connected tothe positive terminal. Thus, we do work on the charge as we transfer it from oneplate to the other. At first, no work is required to transfer a small amount ofcharge dqfrom one plate to the other.3However, once this charge has been trans-ferred, a small potential difference exists between the plates. Therefore, workmust be done to move additional charge through this potential difference. Asmore and more charge is transferred from one plate to the other, the potential dif-ference increases in proportion, and more work is required.Suppose that qis the charge on the capacitor at some instant during thecharging process. At the same instant, the potential difference across the capacitoris /H9004V/H11005q/C. From Section 25.2, we know that the work necessary to transfer an in-crement of charge dqfrom the plate carrying charge /H11002qto the plate carryingcharge q(which is at the higher electric potential) isThis is illustrated in Figure 26.11. The total work required to charge the capacitorfrom to some final charge isThe work done in charging the capacitor appears as electric potential energy Ustored in the capacitor. Therefore, we can express the potential energy stored in acharged capacitor in the following forms:(26.11)This result applies to any capacitor, regardless of its geometry. We see that for agiven capacitance, the stored energy increases as the charge increases and as thepotential difference increases. In practice, there is a limit to the maximum energyU/H11005Q22C/H1100512Q /H9004V/H1100512C(/H9004V)2W/H11005/H20885Q0 qC dq/H110051C /H20885Q0 q dq/H11005Q22Cq/H11005Qq/H110050dW/H11005/H9004V dq/H11005qC dq Energy stored in a chargedcapacitorQuickLabHere's how to find out whether yourcalculator has a capacitor to protectvalues or programs during batterychanges: Store a number in your cal-culator's memory, remove the calcu-lator battery for a moment, and thenquickly replace it. Was the numberthat you stored preserved while thebattery was out of the calculator?(You may want to write down any crit-ical numbers or programs that arestored in the calculator before tryingthis!) 3We shall use lowercase qfor the varying charge on the capacitor while it is charging, to distinguish itfrom uppercase Q, which is the total charge on the capacitor after it is completely charged.V dqq∆Figure 26.11A plot of potential difference versus charge fora capacitor is a straight line having a slope 1/C. The work re-quired to move charge dqthrough the potential difference /H9004Vacross the capacitor plates is given by the area of the shadedrectangle. The total work required to charge the capacitor to afinal charge Qis the triangular area under the straight line,. (Don't forget that J/C; hence, the unitfor the area is the joule.)1 V/H110051W/H1100512Q /H9004V26.4Energy Stored in a Charged Capacitor815(or charge) that can be stored because, at a sufficiently great value of /H9004V, dis-charge ultimately occurs between the plates. For this reason, capacitors are usuallylabeled with a maximum operating voltage.You have three capacitors and a battery. How should you combine the capacitors and thebattery in one circuit so that the capacitors will store the maximum possible energy?We can consider the energy stored in a capacitor as being stored in the elec-tric field created between the plates as the capacitor is charged. This description isreasonable in view of the fact that the electric field is proportional to the chargeon the capacitor. For a parallel-plate capacitor, the potential difference is relatedto the electric field through the relationship /H9004V/H11005Ed. Furthermore, its capaci-tance is (Eq. 26.3). Substituting these expressions into Equation 26.11,we obtain(26.12)Because the volume V(volume, not voltage!) occupied by the electric field is Ad,the energy per unit volumeknown as the energy density,is(26.13)Although Equation 26.13 was derived for a parallel-plate capacitor, the expressionis generally valid. That is, the energy density in any electric field is propor-tional to the square of the magnitude of the electric field at a given point.uE/H1100512/H92800E2uE/H11005U/V/H11005U/Ad,U/H1100512 /H92800Ad (E2d2)/H1100512 (/H92800Ad)E2C/H11005/H92800A/dQuick Quiz 26.3 Energy stored in a parallel-platecapacitorEnergy density in an electric field This bank of capacitors stores electrical en-ergy for use in the particle accelerator atFermiLab, located outside Chicago. Be-cause the electric utility company cannotprovide a large enough burst of energy tooperate the equipment, these capacitorsare slowly charged up, and then the energyis rapidly "dumped" into the accelerator. Inthis sense, the setup is much like a fire-protection water tank on top of a building.The tank collects water and stores it for sit-uations in which a lot of water is needed ina short time. 816CHAPTER 26Capacitance and DielectricsRewiring Two Charged Capacitors EXAMPLE26.5 As noted eargler, To express /H9004Vfin terms of the given quantities andwe substitute the value of Qfrom Equation (1) to obtain(b) Find the total energy stored in the capacitors beforeand after the switches are closed and the ratio of the final en-ergy to the initial energy.SolutionBefore the switches are closed, the total energystored in the capacitors isAfter the switches are closed, the total energy stored in thecapacitors isUsing Equation (1), we can express this asTherefore, the ratio of the final energy stored to the initialenergy stored is/H20898C1/H11002C2C1/H11001C2/H208992UfUi/H1100512 (C1/H11002C2)2(/H9004Vi)2(C1/H11001C2)12 (C1/H11001C2)(/H9004Vi)2/H1100512 (C1/H11002C2)2(/H9004Vi)2(C1/H11001C2)Uf/H1100512 Q2(C1/H11001C2)/H11005 /H1100512 (C1/H11001C2)/H20898QC1/H11001C2/H208992/H1100512 Q2C1/H11001C2 Uf/H1100512C1(/H9004Vf)2/H1100112C2(/H9004Vf)2/H1100512 (C1/H11001C2)(/H9004Vf)212 (C1/H11001C2)(/H9004Vi)2Ui/H1100512C1(/H9004Vi)2/H1100112C2(/H9004Vi)2/H11005/H9004Vf/H11005/H20898C1/H11002C2C1/H11001C2/H20899/H9004Vi/H9004Vi ,C1, C2,/H9004V1f/H11005/H9004V2f/H11005/H9004Vf./H9004V2f/H11005Q2fC2/H11005Q/H20898C2C1/H11001C2/H20899C2/H11005QC1/H11001C2Two capacitors C1and C2(where are charged tothe same initial potential difference /H9004Vi, but with oppositepolarity. The charged capacitors are removed from the bat-tery, and their plates are connected as shown in Figure26.12a. The switches S1and S2are then closed, as shown inFigure 26.12b. (a) Find the final potential difference /H9004Vfbe-tween aand bafter the switches are closed.SolutionLet us identify the left-hand plates of the capaci-tors as an isolated system because they are not connected tothe right-hand plates by conductors. The charges on the left-hand plates before the switches are closed areThe negative sign for Q2iis necessary because the charge onthe left plate of capacitor C2is negative. The total charge Qin the system is(1)After the switches are closed, the total charge in the systemremains the same:(2)The charges redistribute until the entire system is at the samepotential /H9004Vf. Thus, the final potential difference across C1must be the same as the final potential difference across C2.To satisfy this requirement, the charges on the capacitors af-ter the switches are closed areDividing the first equation by the second, we have(3)Combining Equations (2) and (3), we obtainUsing Equation (3) to find Q1fin terms of Q, we haveFinally, using Equation 26.1 to find the voltage across each ca-pacitor, we find that/H9004V1f/H11005Q1fC1/H11005Q/H20898C1C1/H11001C2/H20899C1/H11005QC1/H11001C2Q1f/H11005C1C2 Q2f/H11005C1C2 Q/H20898C2C1/H11001C2/H20899/H11005Q/H20898C1C1/H11001C2/H20899 Q2f/H11005Q/H20898C2C1/H11001C2/H20899Q/H11005Q1f/H11001Q2f/H11005C1C2 Q2f/H11001Q2f/H11005Q2f/H208981/H11001C1C2/H20899Q1f/H11005C1C2 Q2fQ1fQ2f/H11005C1 /H9004VfC2 /H9004Vf/H11005C1C2 Q1f/H11005C1 /H9004Vf and Q2f/H11005C2 /H9004VfQ/H11005Q1f/H11001Q2fQ/H11005Q1i/H11001Q2i/H11005(C1/H11002C2)/H9004ViQ1i/H11005C1 /H9004Vi and Q2i/H11005/H11002C2 /H9004ViC1/H11022C2) +–Q1i+ba(a)–C1 Q2i–+C2S1S2+ba(b)–S1S2Q1fC1 Q2fC2Figure 26.1226.4Energy Stored in a Charged Capacitor817 You charge a parallel-plate capacitor, remove it from the battery, and prevent the wires con-nected to the plates from touching each other. When you pull the plates apart, do the fol-lowing quantities increase, decrease, or stay the same? (a) C; (b) Q; (c) Ebetween theplates; (d) /H9004V; (e) energy stored in the capacitor.Repeat Quick Quiz 26.4, but this time answer the questions for the situation in which thebattery remains connected to the capacitor while you pull the plates apart.One device in which capacitors have an important role is the defibrillator (Fig.26.13). Up to 360 J is stored in the electric field of a large capacitor in a defibrilla-tor when it is fully charged. The defibrillator can deliver all this energy to a patientin about 2 ms. (This is roughly equivalent to 3 000 times the power output of a 60-W lightbulb!) The sudden electric shock stops the fibrillation (random contrac-tions) of the heart that often accompanies heart attacks and helps to restore thecorrect rhythm.A camera's flash unit also uses a capacitor, although the total amount of en-ergy stored is much less than that stored in a defibrillator. After the flash unit's ca-pacitor is charged, tripping the camera's shutter causes the stored energy to besent through a special lightbulb that briefly illuminates the subject being pho-tographed.Quick Quiz 26.5Quick Quiz 26.4 webTo learn more about defibrillators, visitwww.physiocontrol.com This ratio is less than unity, indicating that the final energy is less than the initial energy. At first, you might think thatthe law of energy conservation has been violated, but this is not the case. The "missing" energy is radiated away in the form of electromagnetic waves, as we shall see in Chap-ter 34. Figure 26.13In a hospitalor at an emergency scene, youmight see a patient being re-vived with a defibrillator. Thedefibrillator's paddles are ap-plied to the patient's chest,and an electric shock is sentthrough the chest cavity. Theaim of this technique is to re-store the heart's normalrhythm pattern.818CHAPTER 26Capacitance and DielectricsCAPACITORS WITH DIELECTRICSA dielectric is a nonconducting material, such as rubber, glass, or waxed paper.When a dielectric is inserted between the plates of a capacitor, the capacitance in-creases. If the dielectric completely fills the space between the plates, the capaci-tance increases by a dimensionless factor /H9260, which is called the dielectric con-stant.The dielectric constant is a property of a material and varies from onematerial to another. In this section, we analyze this change in capacitance in termsof electrical parameters such as electric charge, electric field, and potential differ-ence; in Section 26.7, we shall discuss the microscopic origin of these changes.We can perform the following experiment to illustrate the effect of a dielectricin a capacitor: Consider a parallel-plate capacitor that without a dielectric has acharge Q0and a capacitance C0. The potential difference across the capacitor isFigure 26.14a illustrates this situation. The potential difference ismeasured by a voltmeter, which we shall study in greater detail in Chapter 28. Notethat no battery is shown in the figure; also, we must assume that no charge canflow through an ideal voltmeter, as we shall learn in Section 28.5. Hence, there isno path by which charge can flow and alter the charge on the capacitor. If a dielec-tric is now inserted between the plates, as shown in Figure 26.14b, the voltmeterindicates that the voltage between the plates decreases to a value /H9004V. The voltageswith and without the dielectric are related by the factor /H9260as follows:Because /H9004V/H11021/H9004V0, we see that Because the charge Q0on the capacitor does not change, we conclude thatthe capacitance must change to the value(26.14)That is, the capacitance increases by the factor /H9260when the dielectric completely fillsthe region between the plates.4For a parallel-plate capacitor, where (Eq. 26.3), we can express the capacitance when the capacitor is filled with a di-electric as(26.15)From Equations 26.3 and 26.15, it would appear that we could make the ca-pacitance very large by decreasing d, the distance between the plates. In practice,the lowest value of dis limited by the electric discharge that could occur throughthe dielectric medium separating the plates. For any given separation d, the maxi-mum voltage that can be applied to a capacitor without causing a discharge de-pends on the dielectric strength (maximum electric field) of the dielectric. If themagnitude of the electric field in the dielectric exceeds the dielectric strength,then the insulating properties break down and the dielectric begins to conduct.Insulating materials have values of /H9260greater than unity and dielectric strengthsC/H11005/H9260 /H92800AdC0/H11005/H92800A/dC/H11005/H9260C0C/H11005Q0/H9004V/H11005Q0/H9004V0//H9260/H11005/H9260 Q0/H9004V0/H9260/H110221./H9004V/H11005/H9004V0/H9260/H9004V0/H11005Q0/C0 .26.5 The capacitance of a filledcapacitor is greater than that of anempty one by a factor /H9260. 4If the dielectric is introduced while the potential difference is being maintained constant by a battery,the charge increases to a value Q/H11005/H9260Q0. The additional charge is supplied by the battery, and the ca-pacitance again increases by the factor /H9260.26.5Capacitors with Dielectrics819 greater than that of air, as Table 26.1 indicates. Thus, we see that a dielectric pro-vides the following advantages:•Increase in capacitance•Increase in maximum operating voltage•Possible mechanical support between the plates, which allows the plates to beclose together without touching, thereby decreasing dand increasing CC0Q0+–CQ0Dielectric ∆V∆V0+– (a)(b)Figure 26.14A charged capacitor (a) before and (b) after insertion of a dielectric between theplates. The charge on the plates remains unchanged, but the potential difference decreases from/H9004V0to /H9004V/H11005/H9004V0//H9260. Thus, the capacitance increases from C0to /H9260C0. TABLE 26.1Dielectric Constants and Dielectric Strengthsof Various Materials at Room TemperatureDielectric DielectricMaterial Constant /H9260Strengtha(V/m)Air (dry) 1.000 59 3/H11003106Bakelite 4.9 24/H11003106Fused quartz 3.78 8/H11003106Neoprene rubber 6.7 12/H11003106Nylon 3.4 14/H11003106Paper 3.7 16/H11003106Polystyrene 2.56 24/H11003106Polyvinyl chloride 3.4 40/H11003106Porcelain 6 12/H11003106Pyrex glass 5.6 14/H11003106Silicone oil 2.5 15/H11003106Strontium titanate 233 8/H11003106Teflon 2.1 60/H11003106Vacuum 1.000 00 —Water 80 —aThe dielectric strength equals the maximum electric field that can exist in adielectric without electrical breakdown. Note that these values dependstrongly on the presence of impurities and flaws in the materials.820CHAPTER 26Capacitance and Dielectrics Types of CapacitorsCommercial capacitors are often made from metallic foil interlaced with thinsheets of either paraffin-impregnated paper or Mylar as the dielectric material.These alternate layers of metallic foil and dielectric are rolled into a cylinder toform a small package (Fig. 26.15a). High-voltage capacitors commonly consist of anumber of interwoven metallic plates immersed in silicone oil (Fig. 26.15b). Smallcapacitors are often constructed from ceramic materials. Variable capacitors (typi-cally 10 to 500 pF) usually consist of two interwoven sets of metallic plates, onefixed and the other movable, and contain air as the dielectric.Often, an electrolytic capacitoris used to store large amounts of charge at rela-tively low voltages. This device, shown in Figure 26.15c, consists of a metallic foil incontact with an electrolyte—a solution that conducts electricity by virtue of the mo-tion of ions contained in the solution. When a voltage is applied between the foiland the electrolyte, a thin layer of metal oxide (an insulator) is formed on the foil, (a) Kirlian photograph created by dropping a steel ball into a high-energy electric field. Kirlianphotography is also known as electrophotography.(b) Sparks from static electricity discharge be-tween a fork and four electrodes. Many sparks were used to create this image because only onespark forms for a given discharge. Note that the bottom prong discharges to both electrodes atthe bottom right. The light of each spark is created by the excitation of gas atoms along its path.(a) (b) Metal foil PaperPlates OilElectrolyteCase Metallic foil + oxide layerContacts(a) (b) (c)Figure 26.15Three commercial capacitor designs. (a) A tubular capacitor, whose plates areseparated by paper and then rolled into a cylinder. (b) A high-voltage capacitor consisting ofmany parallel plates separated by insulating oil. (c) An electrolytic capacitor.26.5Capacitors with Dielectrics821 A Paper-Filled Capacitor EXAMPLE26.6the paper is 1.0 mm, the maximum voltage that can be ap-plied before breakdown isHence, the maximum charge isExerciseWhat is the maximum energy that can be storedin the capacitor?Answer2.6/H1100310/H110023J.0.32 /H9262CQmax/H11005C /H9004Vmax/H11005(20/H1100310/H1100212 F)(16/H11003103 V)/H11005 /H1100516/H11003103 V/H9004Vmax/H11005Emaxd/H11005(16/H11003106 V/m)(1.0/H1100310/H110023 m)A parallel-plate capacitor has plates of dimensions 2.0 cm by3.0 cm separated by a 1.0-mm thickness of paper. (a) Find itscapacitance.SolutionBecause /H9260/H110053.7 for paper (see Table 26.1), wehave(b) What is the maximum charge that can be placed onthe capacitor?SolutionFrom Table 26.1 we see that the dielectricstrength of paper is 16/H11003106V/m. Because the thickness of20 pF /H1100520/H1100310/H1100212 F/H11005C/H11005/H9260 /H92800Ad/H110053.7(8.85/H1100310/H1100212 C2/N/H11080m2)/H208986.0/H1100310/H110024 m21.0/H1100310/H110023 m/H20899and this layer serves as the dielectric. Very large values of capacitance can be ob-tained in an electrolytic capacitor because the dielectric layer is very thin, and thusthe plate separation is very small.Electrolytic capacitors are not reversible as are many other capacitors—theyhave a polarity, which is indicated by positive and negative signs marked on the de-vice. When electrolytic capacitors are used in circuits, the polarity must be alignedproperly. If the polarity of the applied voltage is opposite that which is intended,the oxide layer is removed and the capacitor conducts electricity instead of storingcharge.If you have ever tried to hang a picture, you know it can be difficult to locate a wooden studin which to anchor your nail or screw. A carpenter’s stud-finder is basically a capacitor withits plates arranged side by side instead of facing one another, as shown in Figure 26.16.When the device is moved over a stud, does the capacitance increase or decrease?Quick Quiz 26.6 CapacitorplatesStud-finderWall boardStud (b)(a)Figure 26.16A stud-finder. (a)The materials between the plates of the capacitor are the wall-board and air. (b) When the capacitor moves across a stud in the wall, the materials between theplates are the wallboard and the wood. The change in the dielectric constant causes a signal lightto illuminate.822CHAPTER 26Capacitance and DielectricsEnergy Stored Before and After EXAMPLE26.7ExerciseSuppose that the capacitance in the absence of adielectric is 8.50 pF and that the capacitor is charged to a po-tential difference of 12.0 V. If the battery is disconnected anda slab of polystyrene is inserted between the plates, what isAnswer373 pJ.U0/H11002U ?A parallel-plate capacitor is charged with a battery to a chargeQ0, as shown in Figure 26.17a. The battery is then removed,and a slab of material that has a dielectric constant /H9260is in-serted between the plates, as shown in Figure 26.17b. Findthe energy stored in the capacitor before and after the dielec-tric is inserted.SolutionThe energy stored in the absence of the dielec-tric is (see Eq. 26.11):After the battery is removed and the dielectric inserted, thecharge on the capacitor remains the same.Hence, the energystored in the presence of the dielectric isBut the capacitance in the presence of the dielectric isso UbecomesBecause /H9260/H110221, the final energy is less than the initial energy.We can account for the “missing” energy by noting that thedielectric, when inserted, gets pulled into the device (see thefollowing discussion and Figure 26.18). An external agentmust do negative work to keep the dielectric from accelerat-ing. This work is simply the difference (Alternatively,the positive work done by the system on the external agent isU0/H11002U.)U/H11002U0 .U/H11005Q0 22/H9260C0/H11005U0/H9260C/H11005/H9260C0 ,U/H11005Q0 22CU0/H11005Q0 22C0 As we have seen, the energy of a capacitor not connected to a battery is low-ered when a dielectric is inserted between the plates; this means that negativework is done on the dielectric by the external agent inserting the dielectric intothe capacitor. This, in turn, implies that a force that draws it into the capacitormust be acting on the dielectric. This force originates from the nonuniform na-ture of the electric field of the capacitor near its edges, as indicated in Figure26.18. The horizontal component of this fringe field acts on the induced charges onthe surface of the dielectric, producing a net horizontal force directed into thespace between the capacitor plates.A fully charged parallel-plate capacitor remains connected to a battery while you slide a di-electric between the plates. Do the following quantities increase, decrease, or stay the same?(a) C; (b) Q; (c) Ebetween the plates; (d) /H9004V; (e) energy stored in the capacitor.Quick Quiz 26.7Figure 26.17–+Q0C0 ∆V0(a)Dielectric–+Q0(b)26.6Electric Dipole in an Electric Field823 Optional SectionELECTRIC DIPOLE IN AN ELECTRIC FIELDWe have discussed the effect on the capacitance of placing a dielectric between theplates of a capacitor. In Section 26.7, we shall describe the microscopic origin ofthis effect. Before we can do so, however, we need to expand upon the discussionof the electric dipole that we began in Section 23.4 (see Example 23.6). The elec-tric dipole consists of two charges of equal magnitude but opposite sign separatedby a distance 2a, as shown in Figure 26.19. The electric dipole momentof thisconfiguration is defined as the vector pdirected from /H11002qto /H11001qalong the linejoining the charges and having magnitude 2aq:(26.16)Now suppose that an electric dipole is placed in a uniform electric field E, asshown in Figure 26.20. We identify Eas the field external to the dipole, distin-guishing it from the field due to the dipole, which we discussed in Section 23.4.The field Eis established by some other charge distribution, and we place the di-pole into this field. Let us imagine that the dipole moment makes an angle /H9258with the field. The electric forces acting on the two charges are equal in magnitude but op-posite in direction as shown in Figure 26.20 (each has a magnitude Thus,the net force on the dipole is zero. However, the two forces produce a net torqueon the dipole; as a result, the dipole rotates in the direction that brings the dipolemoment vector into greater alignment with the field. The torque due to the forceon the positive charge about an axis through Oin Figure 26.20 is Fasin /H9258, where asin /H9258is the moment arm of Fabout O. This force tends to produce a clockwiserotation. The torque about Oon the negative charge also is Fasin /H9258; here again,the force tends to produce a clockwise rotation. Thus, the net torque about OisBecause and we can express /H9270as(26.17)/H9270/H110052aqE sin /H9258/H11005pE sin /H9258p/H110052aq,F/H11005qE/H9270/H110052Fa sin /H9258F/H11005qE).p/H110132aq26.6+Q–Q+–+–+–+–+–+–+–Figure 26.18The nonuniform electric field near the edges of a parallel-plate capacitor causesa dielectric to be pulled into the capacitor. Note that the field acts on the induced surfacecharges on the dielectric, which are nonuniformly distributed. + qθ– qFE– FO+–Figure 26.20An electric dipolein a uniform external electric field.The dipole moment pis at an an-gle /H9258to the field, causing the di-pole to experience a torque.+ q– q2ap–+Figure 26.19An electric dipoleconsists of two charges of equalmagnitude but opposite sign sepa-rated by a distance of 2a. The elec-tric dipole moment pis directedfrom /H11002qto /H11001q.824CHAPTER 26Capacitance and DielectricsIt is convenient to express the torque in vector form as the cross product of thevectors pand E:(26.18)We can determine the potential energy of the system of an electric dipole inan external electric field as a function of the orientation of the dipole with respectto the field. To do this, we recognize that work must be done by an external agentto rotate the dipole through an angle so as to cause the dipole moment vector tobecome less aligned with the field. The work done is then stored as potential en-ergy in the system of the dipole and the external field. The work dWrequired torotate the dipole through an angle d/H9258is (Eq. 10.22). Becauseand because the work is transformed into potential energy U, we findthat, for a rotation from /H9258ito /H9258f, the change in potential energy isThe term that contains cos /H9258iis a constant that depends on the initial orienta-tion of the dipole. It is convenient for us to choose so that cos 90°/H110050. Furthermore, let us choose at as our reference of poten-tial energy. Hence, we can express a general value of as(26.19)We can write this expression for the potential energy of a dipole in an electric fieldas the dot product of the vectors pand E:(26.20)To develop a conceptual understanding of Equation 26.19, let us compare thisexpression with the expression for the potential energy of an object in the gravita-tional field of the Earth, (see Chapter 8). The gravitational expression in-cludes a parameter associated with the object we place in the field—its mass m.Likewise, Equation 26.19 includes a parameter of the object in the electric field—its dipole moment p. The gravitational expression includes the magnitude of thegravitational field g. Similarly, Equation 26.19 includes the magnitude of the elec-tric field E. So far, these two contributions to the potential energy expressions ap-pear analogous. However, the final contribution is somewhat different in the twocases. In the gravitational expression, the potential energy depends on how highwe lift the object, measured by h. In Equation 26.19, the potential energy dependson the angle /H9258through which we rotate the dipole. In both cases, we are making achange in the system. In the gravitational case, the change involves moving an ob-ject in a translational sense, whereas in the electrical case, the change involves mov-ing an object in a rotational sense. In both cases, however, once the change ismade, the system tends to return to the original configuration when the object isreleased: the object of mass mfalls back to the ground, and the dipole begins torotate back toward the configuration in which it was aligned with the field. Thus,apart from the type of motion, the expressions for potential energy in these twocases are similar.U/H11005mghU/H11005/H11002p/H11554EU/H11005/H11002pE cos /H9258U/H11005Uf/H9258i/H1100590/H11034Ui/H110050/H9258i/H11005cos/H9258i/H1100590/H11034, /H11005pE /H20900/H11002cos /H9258/H20901/H9258f/H9258i/H11005pE(cos /H9258i/H11002cos /H9258f)Uf/H11002Ui/H11005/H20885/H9258f/H9258i /H9270 d/H9258/H11005/H20885/H9258f/H9258i p/H9005 sin /H9258 d/H9258/H11005pE /H20885/H9258f/H9258i sin /H9258 d/H9258/H9270/H11005pE sin /H9258dW/H11005/H9270 d/H9258/H9270/H11005p/H11547ETorque on an electric dipole in anexternal electric field Potential energy of a dipole in anelectric field26.6Electric Dipole in an Electric Field825Molecules are said to be polarized when a separation exists between the averageposition of the negative charges and the average position of the positive charges inthe molecule. In some molecules, such as water, this condition is always present—such molecules are called polar molecules.Molecules that do not possess a per-manent polarization are called nonpolar molecules.We can understand the permanent polarization of water by inspecting thegeometry of the water molecule. In the water molecule, the oxygen atom isbonded to the hydrogen atoms such that an angle of 105° is formed between thetwo bonds (Fig. 26.21). The center of the negative charge distribution is near theoxygen atom, and the center of the positive charge distribution lies at a point mid-way along the line joining the hydrogen atoms (the point labeled/H11003in Fig. 26.21).We can model the water molecule and other polar molecules as dipoles becausethe average positions of the positive and negative charges act as point charges. As aresult, we can apply our discussion of dipoles to the behavior of polar molecules.Microwave ovens take advantage of the polar nature of the water molecule.When in operation, microwave ovens generate a rapidly changing electric fieldthat causes the polar molecules to swing back and forth, absorbing energy fromthe field in the process. Because the jostling molecules collide with each other, theenergy they absorb from the field is converted to internal energy, which corre-sponds to an increase in temperature of the food.Another household scenario in which the dipole structure of water is ex-ploited is washing with soap and water. Grease and oil are made up of nonpolarmolecules, which are generally not attracted to water. Plain water is not very usefulfor removing this type of grime. Soap contains long molecules called surfactants.Ina long molecule, the polarity characteristics of one end of the molecule can be dif-ferent from those at the other end. In a surfactant molecule, one end acts like anonpolar molecule and the other acts like a polar molecule. The nonpolar endcan attach to a grease or oil molecule, and the polar end can attach to a water mol-ecule. Thus, the soap serves as a chain, linking the dirt and water molecules to-gether. When the water is rinsed away, the grease and oil go with it.A symmetric molecule (Fig. 26.22a) has no permanent polarization, but polar-ization can be induced by placing the molecule in an electric field. A field directedto the left, as shown in Figure 26.22b, would cause the center of the positivecharge distribution to shift to the left from its initial position and the center of thenegative charge distribution to shift to the right. This induced polarization is the ef-fect that predominates in most materials used as dielectrics in capacitors.The H2O Molecule EXAMPLE26.8obtainBecause there are 1021molecules in the sample, the totalwork required is1.6/H1100310/H110023 JWtotal/H11005(1021)(1.6/H1100310/H1100224 J)/H11005 /H110051.6/H1100310/H1100224 J /H11005pE/H11005(6.3/H1100310/H1100230 C/H11080m)(2.5/H11003105 N/C)W/H11005U90/H11002U0/H11005(/H11002pE cos 90/H11034)/H11002(/H11002pE cos 0/H11034) The water (H2O) molecule has an electric dipole moment of6.3/H1100310/H1100230C/H11080m. A sample contains 1021water molecules,with the dipole moments all oriented in the direction of anelectric field of magnitude 2.5/H11003105N/C. How much workis required to rotate the dipoles from this orientationto one in which all the dipole moments are perpen-dicular to the field SolutionThe work required to rotate one molecule 90° isequal to the difference in potential energy between the 90°orientation and the 0° orientation. Using Equation 26.19, we(/H9258/H1100590/H11034)?(/H9258/H110050/H11034)OHH105°−−++/H11003 E(a)(b)++−−+−+Figure 26.21The water mole-cule, H2O, has a permanent polar-ization resulting from its bentgeometry. The center of the posi-tive charge distribution is at thepoint /H11003. Figure 26.22(a) A symmetricmolecule has no permanent polar-ization. (b) An external electricfield induces a polarization in themolecule.826CHAPTER 26Capacitance and DielectricsOptional SectionAN ATOMIC DESCRIPTION OF DIELECTRICSIn Section 26.5 we found that the potential difference /H9004V0between the plates of acapacitor is reduced to V0//H9260when a dielectric is introduced. Because the poten-tial difference between the plates equals the product of the electric field and theseparation d, the electric field is also reduced. Thus, if E0is the electric field with-out the dielectric, the field in the presence of a dielectric is(26.21)Let us first consider a dielectric made up of polar molecules placed in theelectric field between the plates of a capacitor. The dipoles (that is, the polar mol-ecules making up the dielectric) are randomly oriented in the absence of an elec-tric field, as shown in Figure 26.23a. When an external field E0due to charges onthe capacitor plates is applied, a torque is exerted on the dipoles, causing them topartially align with the field, as shown in Figure 26.23b. We can now describe thedielectric as being polarized. The degree of alignment of the molecules with theelectric field depends on temperature and on the magnitude of the field. In gen-eral, the alignment increases with decreasing temperature and with increasingelectric field.If the molecules of the dielectric are nonpolar, then the electric field due tothe plates produces some charge separation and an induced dipole moment.Theseinduced dipole moments tend to align with the external field, and the dielectric ispolarized. Thus, we can polarize a dielectric with an external field regardless ofwhether the molecules are polar or nonpolar.With these ideas in mind, consider a slab of dielectric material placed betweenthe plates of a capacitor so that it is in a uniform electric field E0, as shown in Fig-ure 26.24a. The electric field due to the plates is directed to the right and polar-izes the dielectric. The net effect on the dielectric is the formation of an inducedpositive surface charge density /H9268indon the right face and an equal negative surfacecharge density /H11002/H9268indon the left face, as shown in Figure 26.24b. These inducedsurface charges on the dielectric give rise to an induced electric field Eindin thedirection opposite the external field E0. Therefore, the net electric field Ein theE/H11005E0/H9260/H900426.7

–++–+–+–+–+–+–(a) (b)E0Figure 26.23(a) Polar mole-cules are randomly oriented in theabsence of an external electricfield. (b) When an external field isapplied, the molecules partiallyalign with the field. Figure 26.24(a) When a dielectric is polarized, the dipole moments of the molecules in thedielectric are partially aligned with the external field E0. (b) This polarization causes an inducednegative surface charge on one side of the dielectric and an equal induced positive surfacecharge on the opposite side. This separation of charge results in a reduction in the net electricfield within the dielectric.E0–+–+–+–+–+–+–+–+–+–+–+–+(a)E0 (b)–+–+–+–+–+–σindσindEindσσ26.7An Atomic Description of Dielectrics827dielectric has a magnitude(26.22)In the parallel-plate capacitor shown in Figure 26.25, the external field E0isrelated to the charge density /H9268on the plates through the relationship The induced electric field in the dielectric is related to the induced charge density/H9268indthrough the relationship Because substitu-tion into Equation 26.22 gives(26.23)Because this expression shows that the charge density /H9268indinduced on thedielectric is less than the charge density /H9268on the plates. For instance, if wesee that the induced charge density is two-thirds the charge density on the plates.If no dielectric is present, then and as expected. However, if the di-electric is replaced by an electrical conductor, for which then Equation26.22 indicates that this corresponds to That is, the surfacecharge induced on the conductor is equal in magnitude but opposite in sign tothat on the plates, resulting in a net electric field of zero in the conductor./H9268ind/H11005/H9268.E0/H11005Eind ;E/H110050,/H9268ind/H110050/H9260/H110051/H9260/H110053,/H9260/H110221,/H9268ind/H11005/H20898/H9260/H110021/H9260/H20899/H9268/H9268/H9260/H92800/H11005/H9268/H92800/H11002/H9268ind/H92800E/H11005E0//H9260/H11005/H9268//H9260/H92800 ,Eind/H11005/H9268ind//H92800 .E0/H11005/H9268//H92800 .E/H11005E0/H11002Eind Effect of a Metallic Slab EXAMPLE26.9SolutionIn the result for part (a), we let a:0:which is the original capacitance.C/H11005lima:0 /H92800Ad/H11002a/H11005/H92800AdA parallel-plate capacitor has a plate separation dand platearea A. An uncharged metallic slab of thickness ais insertedmidway between the plates. (a) Find the capacitance of thedevice.SolutionWe can solve this problem by noting that anycharge that appears on one plate of the capacitor must in-duce a charge of equal magnitude but opposite sign on thenear side of the slab, as shown in Figure 26.26a. Conse-quently, the net charge on the slab remains zero, and theelectric field inside the slab is zero. Hence, the capacitor isequivalent to two capacitors in series, each having a plate sep-aration as shown in Figure 26.26b.Using the rule for adding two capacitors in series (Eq.26.10), we obtain Note that Capproaches infinity as aapproaches d. Why?(b) Show that the capacitance is unaffected if the metallicslab is infinitesimally thin.C/H11005/H92800Ad/H11002a1C/H110051C1/H110011C2/H110051/H92800A(d/H11002a)/2/H110011/H92800A(d/H11002a)/2(d/H11002a)/2,++++++++++++++– ———+ ++++++σ– σ– ——————σindσσindσσ Figure 26.25Induced charge ona dielectric placed between theplates of a charged capacitor. Notethat the induced charge density onthe dielectric is lessthan the chargedensity on the plates. (b)(d – a)/2(d – a)/2 (a)da(d – a)/2(d – a)/2σ–σ——————++++++++++σσFigure 26.26(a) A parallel-plate capacitor of plate separation dpartially filled with a metallic slab of thickness a. (b) The equivalentcircuit of the device in part (a) consists of two capacitors in series,each having a plate separation (d/H11002a)/2.828CHAPTER 26Capacitance and Dielectrics A Partially Filled Capacitor EXAMPLE26.10SolutionIn Example 26.9, we found that we could insert ametallic slab between the plates of a capacitor and considerthe combination as two capacitors in series. The resulting ca-pacitance was independent of the location of the slab. Fur-thermore, if the thickness of the slab approaches zero, thenthe capacitance of the system approaches the capacitancewhen the slab is absent. From this, we conclude that we caninsert an infinitesimally thin metallic slab anywhere betweenthe plates of a capacitor without affecting the capacitance.Thus, let us imagine sliding an infinitesimally thin metallicslab along the bottom face of the dielectric shown in Figure26.27a. We can then consider this system to be the series com-bination of the two capacitors shown in Figure 26.27b: onehaving a plate separation d/3 and filled with a dielectric, andthe other having a plate separation 2d/3 and air between itsplates.From Equations 26.15 and 26.3, the two capacitances areUsing Equation 26.10 for two capacitors combined in series,we have Because the capacitance without the dielectric is we see thatC/H11005/H208983/H92602/H9260/H110011/H20899 C0C0/H11005/H92800A/d,C/H11005/H208983/H92602/H9260/H110011/H20899 /H92800Ad /H11005d3/H92800A /H208981/H9260/H110012/H20899/H11005d3/H92800A /H208981/H110012/H9260/H9260/H208991C/H110051C1/H110011C2/H11005d/3/H9260/H92800A/H110012d/3/H92800AC1/H11005/H9260/H92800Ad/3 and C2/H11005/H92800A2d/3A parallel-plate capacitor with a plate separation dhas a ca-pacitance C0in the absence of a dielectric. What is the capac-itance when a slab of dielectric material of dielectric constant/H9260and thickness is inserted between the plates (Fig.26.27a)?13d(c) Show that the answer to part (a) does not depend onwhere the slab is inserted.SolutionLet us imagine that the slab in Figure 26.26a ismoved upward so that the distance between the upper edgeof the slab and the upper plate is b. Then, the distance be-tween the lower edge of the slab and the lower plate isAs in part (a), we find the total capacitance of theseries combination:d/H11002b/H11002a.This is the same result as in part (a). It is independent of thevalue of b, so it does not matter where the slab is located.C/H11005/H92800Ad/H11002a /H11005b/H92800A/H11001d/H11002b/H11002a/H92800A/H11005d/H11002a/H92800A1C/H110051C1/H110011C2/H110051/H92800Ab/H110011/H92800A(d/H11002b/H11002a)13– d23– dd(a)κ (b)C1C213– d23– dκ Figure 26.27(a) A parallel-plate capacitor of plate separation dpartially filled with a dielectric of thickness d/3. (b) The equivalentcircuit of the capacitor consists of two capacitors connected in series.Summary829SUMMARYA capacitorconsists of two conductors carrying charges of equal magnitude butopposite sign. The capacitance Cof any capacitor is the ratio of the charge Qoneither conductor to the potential difference /H9004Vbetween them:(26.1)This relationship can be used in situations in which any two of the three variablesare known. It is important to remember that this ratio is constant for a given con-figuration of conductors because the capacitance depends only on the geometryof the conductors and not on an external source of charge or potential difference.The SI unit of capacitance is coulombs per volt, or the farad (F), andCapacitance expressions for various geometries are summarized in Table 26.2. If two or more capacitors are connected in parallel, then the potential differ-ence is the same across all of them. The equivalent capacitance of a parallel com-bination of capacitors is(26.8)If two or more capacitors are connected in series, the charge is the same on allof them, and the equivalent capacitance of the series combination is given by(26.10)These two equations enable you to simplify many electric circuits by replacing mul-tiple capacitors with a single equivalent capacitance.Work is required to charge a capacitor because the charging process is equiva-lent to the transfer of charges from one conductor at a lower electric potential toanother conductor at a higher potential. The work done in charging the capacitorto a charge Qequals the electric potential energy Ustored in the capacitor, where(26.11)U/H11005Q22C/H1100512Q /H9004V/H1100512C(/H9004V)21Ceq/H110051C1/H110011C2/H110011C3/H11001/H11080 /H11080 /H11080Ceq/H11005C1/H11001C2/H11001C3/H11001/H11080 /H11080 /H110801 F/H110051 C/V.C/H11013Q/H9004V TABLE 26.2Capacitance and GeometryGeometry Capacitance EquationIsolated charged sphere of radiusR (second charged conductor 26.2assumed at infinity)Parallel-plate capacitor of platearea A and plate separation d26.3Cylindrical capacitor of length/H5129and inner and outer radii 26.4aand b, respectivelySpherical capacitor with inner and outer radii aand b,2 6 . 6respectivelyC/H11005abke (b/H11002a)C/H110054/H9266/H92800RC/H11005/H51292ke ln/H20898ba/H20899C/H11005/H92800 Ad830CHAPTER 26Capacitance and DielectricsWhen a dielectric material is inserted between the plates of a capacitor, the ca-pacitance increases by a dimensionless factor /H9260, called the dielectric constant:(26.14)where C0is the capacitance in the absence of the dielectric. The increase in capac-itance is due to a decrease in the magnitude of the electric field in the presence ofthe dielectric and to a corresponding decrease in the potential difference betweenthe plates—if we assume that the charging battery is removed from the circuit be-fore the dielectric is inserted. The decrease in the magnitude of Earises from aninternal electric field produced by aligned dipoles in the dielectric. This internalfield produced by the dipoles opposes the applied field due to the capacitorplates, and the result is a reduction in the net electric field.The electric dipole moment pof an electric dipole has a magnitude(26.16)The direction of the electric dipole moment vector is from the negative charge to-ward the positive charge.The torque acting on an electric dipole in a uniform electric field Eis(26.18)The potential energy of an electric dipole in a uniform external electric fieldEis(26.20)U/H11005/H11002p/H11554E/H9270/H11005p/H11547Ep/H110132aqC/H11005/H9260C0 Problem-Solving HintsCapacitors•Be careful with units. When you calculate capacitance in farads, make surethat distances are expressed in meters and that you use the SI value of /H92800.When checking consistency of units, remember that the unit for electricfields can be either N/C or V/m.•When two or more capacitors are connected in parallel, the potential differ-ence across each is the same. The charge on each capacitor is proportionalto its capacitance; hence, the capacitances can be added directly to give theequivalent capacitance of the parallel combination. The equivalent capaci-tance is always larger than the individual capacitances.•When two or more capacitors are connected in series, they carry the samecharge, and the sum of the potential differences equals the total potentialdifference applied to the combination. The sum of the reciprocals of the ca-pacitances equals the reciprocal of the equivalent capacitance, which is al-ways less than the capacitance of the smallest individual capacitor.•A dielectric increases the capacitance of a capacitor by a factor /H9260(the dielec-tric constant) over its capacitance when air is between the plates.•For problems in which a battery is being connected or disconnected, notewhether modifications to the capacitor are made while it is connected to thebattery or after it has been disconnected. If the capacitor remains con-nected to the battery, the voltage across the capacitor remains unchanged(equal to the battery voltage), and the charge is proportional to the capaci-Problems831tance, although it may be modified (for instance, by the insertion of a di-electric). If you disconnect the capacitor from the battery before makingany modifications to the capacitor, then its charge remains fixed. In thiscase, as you vary the capacitance, the voltage across the plates changes ac-cording to the expression /H9004V/H11005Q/C.QUESTIONS10.Explain why the work needed to move a charge Qthrough a potential difference /H9004Vis whereas the energy stored in a charged capacitor is Where does the factor come from?11.If the potential difference across a capacitor is doubled,by what factor does the stored energy change?12.Why is it dangerous to touch the terminals of a high-voltage capacitor even after the applied voltage has beenturned off? What can be done to make the capacitor safeto handle after the voltage source has been removed?13.Describe how you can increase the maximum operatingvoltage of a parallel-plate capacitor for a fixed plate sepa-ration.14.An air-filled capacitor is charged, disconnected from thepower supply, and, finally, connected to a voltmeter. Ex-plain how and why the voltage reading changes when adielectric is inserted between the plates of the capacitor.15.Using the polar molecule description of a dielectric, ex-plain how a dielectric affects the electric field inside a ca-pacitor.16.Explain why a dielectric increases the maximum operat-ing voltage of a capacitor even though the physical size ofthe capacitor does not change.17.What is the difference between dielectric strength andthe dielectric constant?18.Explain why a water molecule is permanently polarized.What type of molecule has no permanent polarization?19.If a dielectric-filled capacitor is heated, how does its ca-pacitance change? (Neglect thermal expansion and as-sume that the dipole orientations are temperature depen-dent.)12U/H1100512Q /H9004V.W/H11005Q /H9004V,1.If you were asked to design a capacitor in a situation forwhich small size and large capacitance were required,what factors would be important in your design?2.The plates of a capacitor are connected to a battery. Whathappens to the charge on the plates if the connectingwires are removed from the battery? What happens to thecharge if the wires are removed from the battery and con-nected to each other?3.A farad is a very large unit of capacitance. Calculate thelength of one side of a square, air-filled capacitor that hasa plate separation of 1 m. Assume that it has a capaci-tance of 1 F.4.A pair of capacitors are connected in parallel, while anidentical pair are connected in series. Which pair wouldbe more dangerous to handle after being connected tothe same voltage source? Explain.5.If you are given three different capacitors C1, C2, C3,how many different combinations of capacitance can youproduce?6.What advantage might there be in using two identical ca-pacitors in parallel connected in series with another iden-tical parallel pair rather than a single capacitor?7.Is it always possible to reduce a combination of capacitorsto one equivalent capacitor with the rules we have devel-oped? Explain.8.Because the net charge in a capacitor is always zero, whatdoes a capacitor store?9.Because the charges on the plates of a parallel-plate ca-pacitor are of opposite sign, they attract each other.Hence, it would take positive work to increase the plateseparation. What happens to the external work done inthis process? PROBLEMS2.Two conductors having net charges of /H1100110.0/H9262C and/H1100210.0/H9262C have a potential difference of 10.0 V. Deter-mine (a) the capacitance of the system and (b) the poten-tial difference between the two conductors if the chargeson each are increased to /H11001100/H9262C and /H11002100/H9262C.Section 26.1Definition of Capacitance1.(a) How much charge is on each plate of a 4.00-/H9262F ca-pacitor when it is connected to a 12.0-V battery? (b) If this same capacitor is connected to a 1.50-V bat-tery, what charge is stored?1, 2, 3= straightforward, intermediate, challenging = full solution available in the Student Solutions Manual and Study GuideWEB= solution posted at http://www.saunderscollege.com/physics/= Computer useful in solving problem = Interactive Physics= paired numerical/symbolic problems 832CHAPTER 26Capacitance and Dielectrics WEB WEBSection 26.2Calculating Capacitance3.An isolated charged conducting sphere of radius 12.0 cm creates an electric field of 4.90/H11003104N/C at adistance 21.0 cm from its center. (a) What is its surfacecharge density? (b) What is its capacitance?4.(a) If a drop of liquid has capacitance 1.00 pF, what isits radius? (b) If another drop has radius 2.00 mm, whatis its capacitance? (c) What is the charge on the smallerdrop if its potential is 100 V?5.Two conducting spheres with diameters of 0.400 m and1.00 m are separated by a distance that is large com-pared with the diameters. The spheres are connected bya thin wire and are charged to 7.00/H9262C. (a) How is thistotal charge shared between the spheres? (Neglect anycharge on the wire.) (b) What is the potential of the sys-tem of spheres when the reference potential is taken tobe at 6.Regarding the Earth and a cloud layer 800 m above theEarth as the “plates” of a capacitor, calculate the capaci-tance if the cloud layer has an area of 1.00 km2. Assumethat the air between the cloud and the ground is pureand dry. Assume that charge builds up on the cloud and on the ground until a uniform electric field with a mag-nitude of 3.00/H11003106N/C throughout the space be-tween them makes the air break down and conductelectricity as a lightning bolt. What is the maximumcharge the cloud can hold?7.An air-filled capacitor consists of two parallel plates,each with an area of 7.60 cm2, separated by a distanceof 1.80 mm. If a 20.0-V potential difference is applied tothese plates, calculate (a) the electric field between theplates, (b) the surface charge density, (c) the capaci-tance, and (d) the charge on each plate.8.A 1-megabit computer memory chip contains many60.0-fF capacitors. Each capacitor has a plate area of21.0/H1100310/H1100212m2. Determine the plate separation ofsuch a capacitor (assume a parallel-plate configura-tion). The characteristic atomic diameter is 10/H1100210m/H110050.100 nm. Express the plate separation in nanometers.9.When a potential difference of 150 V is applied to theplates of a parallel-plate capacitor, the plates carry a sur-face charge density of 30.0 nC/cm2. What is the spacingbetween the plates?10.A variable air capacitor used in tuning circuits is madeof Nsemicircular plates each of radius Rand positioneda distancedfrom each other. As shown in FigureP26.10, a second identical set of plates is enmeshed withits plates halfway between those of the first set. The sec-ond set can rotate as a unit. Determine the capacitanceas a function of the angle of rotation /H9258, where corresponds to the maximum capacitance.11.A 50.0-m length of coaxial cable has an inner conductorthat has a diameter of 2.58 mm and carries a charge of8.10/H9262C. The surrounding conductor has an inner di-ameter of 7.27 mm and a charge of /H110028.10/H9262C. (a) What is the capacitance of this cable? (b) What is/H9258/H110050r/H11005/H11009 ?V/H110050the potential difference between the two conductors?Assume the region between the conductors is air.12.A 20.0-/H9262F spherical capacitor is composed of two metal-lic spheres, one having a radius twice as large as theother. If the region between the spheres is a vacuum,determine the volume of this region.13.A small object with a mass of 350 mg carries a charge of30.0 nC and is suspended by a thread between the verti-cal plates of a parallel-plate capacitor. The plates areseparated by 4.00 cm. If the thread makes an angle of15.0° with the vertical, what is the potential differencebetween the plates?14.A small object of mass mcarries a charge qand is sus-pended by a thread between the vertical plates of aparallel-plate capacitor. The plate separation is d. If thethread makes an angle /H9258with the vertical, what is thepotential difference between the plates?15.An air-filled spherical capacitor is constructed with in-ner and outer shell radii of 7.00 and 14.0 cm, respec-tively. (a) Calculate the capacitance of the device. (b) What potential difference between the spheres re-sults in a charge of 4.00/H9262C on the capacitor?16.Find the capacitance of the Earth. (Hint:The outerconductor of the “spherical capacitor” may be consid-ered as a conducting sphere at infinity where Vap-proaches zero.)Section 26.3Combinations of Capacitors17.Two capacitors and C2/H1100512.0/H9262F are con-nected in parallel, and the resulting combination is con-nected to a 9.00-V battery. (a) What is the value of theequivalent capacitance of the combination? What are(b) the potential difference across each capacitor and(c) the charge stored on each capacitor?18.The two capacitors of Problem 17 are now connected inseries and to a 9.00-V battery. Find (a) the value of theequivalent capacitance of the combination, (b) the volt-age across each capacitor, and (c) the charge on eachcapacitor.19.Two capacitors when connected in parallel give anequivalent capacitance of 9.00 pF and an equivalent ca-C1/H110055.00 /H9262FRd/H9258Figure P26.10Problems833pacitance of 2.00 pF when connected in series. What isthe capacitance of each capacitor?20.Two capacitors when connected in parallel give anequivalent capacitance of Cpand an equivalent capaci-tance of Cswhen connected in series. What is the capac-itance of each capacitor?21.Four capacitors are connected as shown in FigureP26.21. (a) Find the equivalent capacitance betweenpoints aand b. (b) Calculate the charge on each capaci-tor if /H9004Vab/H1100515.0 V.24.According to its design specification, the timer circuitdelaying the closing of an elevator door is to have a ca-pacitance of 32.0/H9262F between two points Aand B. (a) When one circuit is being constructed, the inexpen-sive capacitor installed between these two points isfound to have capacitance 34.8/H9262F. To meet the specifi-cation, one additional capacitor can be placed betweenthe two points. Should it be in series or in parallel withthe 34.8-/H9262F capacitor? What should be its capacitance?(b) The next circuit comes down the assembly line withcapacitance 29.8/H9262F between Aand B. What additionalcapacitor should be installed in series or in parallel inthat circuit, to meet the specification?25.The circuit in Figure P26.25 consists of two identicalparallel metallic plates connected by identical metallicsprings to a 100-V battery. With the switch open, theplates are uncharged, are separated by a distanceand have a capacitance When the switch is closed, the distance between theplates decreases by a factor of 0.500. (a) How muchcharge collects on each plate and (b) what is the springconstant for each spring? (Hint:Use the result of Prob-lem 35.)C/H110052.00 /H9262F.d/H110058.00 mm,WEB 26.Figure P26.26 shows six concentric conducting spheres,A, B, C, D, E, and F having radii R, 2R, 3R, 4R, 5R, and6R, respectively. Spheres B and C are connected by aconducting wire, as are spheres D and E. Determine theequivalent capacitance of this system.27.A group of identical capacitors is connected first in se-ries and then in parallel. The combined capacitance inparallel is 100 times larger than for the series connec-tion. How many capacitors are in the group?28.Find the equivalent capacitance between points aand bfor the group of capacitors connected as shown in Fig-ure P26.28 if and29.For the network described in the previous problem ifthe potential difference between points aand bis 60.0 V, what charge is stored on C3?C3/H110052.00 /H9262F.C2/H1100510.0 /H9262F,C1/H110055.00 /H9262F,23.Consider the circuit shown in Figure P26.23, whereand Capaci-tor C1is first charged by the closing of switch S1. SwitchS1is then opened, and the charged capacitor is con-nected to the uncharged capacitor by the closing of S2.Calculate the initial charge acquired by C1and the finalcharge on each./H9004V/H1100520.0 V.C2/H110053.00 /H9262F,C1/H110056.00 /H9262F,22.Evaluate the equivalent capacitance of the configura-tion shown in Figure P26.22. All the capacitors are iden-tical, and each has capacitance C.6.00 μF20.0 μF3.00 μF15.0 μFabμμμμ +–kkd ∆VS C1C2S2S1∆VCCCCCCFigure P26.21 Figure P26.22 Figure P26.23Figure P26.25834CHAPTER 26Capacitance and Dielectrics 30.Find the equivalent capacitance between points aand bin the combination of capacitors shown in FigureP26.30.energy stored in the two capacitors. (b) What potentialdifference would be required across the same two ca-pacitors connected in series so that the combinationstores the same energy as in part (a)? Draw a circuit dia-gram of this circuit.33.A parallel-plate capacitor is charged and then discon-nected from a battery. By what fraction does the storedenergy change (increase or decrease) when the plateseparation is doubled?34.A uniform electric field exists within acertain region. What volume of space contains an en-ergy equal to 1.00/H1100310/H110027J? Express your answer in cu-bic meters and in liters.35.A parallel-plate capacitor has a charge Qand plates ofarea A. Show that the force exerted on each plate by theother is (Hint:Let for an arbi-trary plate separation x; then require that the workdone in separating the two charged plates be36.Plate aof a parallel-plate, air-filled capacitor is con-nected to a spring having force constant k, and plate bisfixed. They rest on a table top as shown (top view) inFigure P26.36. If a charge /H11001Qis placed on plate aand acharge /H11002Qis placed on plate b, by how much does thespring expand?W/H11005/H20885 F dx.)C/H11005/H92800A/xF/H11005Q2/2/H92800A.E/H110053 000 V/mWEB 37.Review Problem.A certain storm cloud has a potentialdifference of 1.00/H11003108V relative to a tree. If, during alightning storm, 50.0 C of charge is transferred throughthis potential difference and 1.00% of the energy is ab-sorbed by the tree, how much water (sap in the tree)initially at 30.0°C can be boiled away? Water has a spe-cific heat of 4 186 J/kg/H11080°C, a boiling point of 100°C,and a heat of vaporization of 2.26/H11003106J/kg.38.Show that the energy associated with a conductingsphere of radius Rand charge Qsurrounded by a vac-uum is 39.Einstein said that energy is associated with mass accord-ing to the famous relationship Estimate the ra-dius of an electron, assuming that its charge is distrib-uted uniformly over the surface of a sphere of radius Rand that the mass–energy of the electron is equal to thetotal energy stored in the resulting nonzero electricfield between Rand infinity. (See Problem 38. Experi-mentally, an electron nevertheless appears to be a pointparticle. The electric field close to the electron must bedescribed by quantum electrodynamics, rather than theclassical electrodynamics that we study.)E/H11005mc2.U/H11005keQ2/2R.Section 26.4Energy Stored in a Charged Capacitor31.(a) A 3.00-/H9262F capacitor is connected to a 12.0-V battery.How much energy is stored in the capacitor? (b) If thecapacitor had been connected to a 6.00-V battery, howmuch energy would have been stored?32.Two capacitors and are con-nected in parallel and charged with a 100-V power sup-ply. (a) Draw a circuit diagram and calculate the totalC2/H110055.00 /H9262FC1/H1100525.0 /H9262Fba6.0 μF5.0 μF7.0 μF4.0 μFμμμμC2C2C1C1 C2C2C3 ba kabABCDEFFigure P26.26 Figure P26.28Problems 28 and 29. Figure P26.30Figure P26.36Problems835Section 26.5Capacitors with Dielectrics40.Find the capacitance of a parallel-plate capacitor thatuses Bakelite as a dielectric, if each of the plates has anarea of 5.00 cm2and the plate separation is 2.00 mm.41.Determine (a) the capacitance and (b) the maximumvoltage that can be applied to a Teflon-filled parallel-plate capacitor having a plate area of 1.75 cm2and plateseparation of 0.040 0 mm.42.(a) How much charge can be placed on a capacitor withair between the plates before it breaks down, if the areaof each of the plates is 5.00 cm2? (b) Find the maxi-mum charge if polystyrene is used between the platesinstead of air.43.A commercial capacitor is constructed as shown in Fig-ure 26.15a. This particular capacitor is rolled from twostrips of aluminum separated by two strips of paraffin-coated paper. Each strip of foil and paper is 7.00 cmwide. The foil is 0.004 00 mm thick, and the paper is0.025 0 mm thick and has a dielectric constant of 3.70. What length should the strips be if a capacitanceof 9.50/H1100310/H110028F is desired? (Use the parallel-plateformula.)44.The supermarket sells rolls of aluminum foil, plasticwrap, and waxed paper. Describe a capacitor made fromsupermarket materials. Compute order-of-magnitude es-timates for its capacitance and its breakdown voltage.45.A capacitor that has air between its plates is connectedacross a potential difference of 12.0 V and stores 48.0/H9262C of charge. It is then disconnected from thesource while still charged. (a) Find the capacitance ofthe capacitor. (b) A piece of Teflon is inserted betweenthe plates. Find its new capacitance. (c) Find the voltageand charge now on the capacitor.46.A parallel-plate capacitor in air has a plate separation of1.50 cm and a plate area of 25.0 cm2. The plates arecharged to a potential difference of 250 V and discon-nected from the source. The capacitor is then im-mersed in distilled water. Determine (a) the charge onthe plates before and after immersion, (b) the capaci-tance and voltage after immersion, and (c) the changein energy of the capacitor. Neglect the conductance ofthe liquid.47.A conducting spherical shell has inner radius aandouter radius c. The space between these two surfaces isfilled with a dielectric for which the dielectric constantis /H92601between aand b, and /H92602between band c(Fig.P26.47). Determine the capacitance of this system.48.A wafer of titanium dioxide has an area of1.00 cm2and a thickness of 0.100 mm. Aluminum isevaporated on the parallel faces to form a parallel-platecapacitor. (a) Calculate the capacitance. (b) When thecapacitor is charged with a 12.0-V battery, what is themagnitude of charge delivered to each plate? (c) Forthe situation in part (b), what are the free and inducedsurface charge densities? (d) What is the magnitude Eof the electric field?(/H9260/H11005173)49.Each capacitor in the combination shown in FigureP26.49 has a breakdown voltage of 15.0 V. What is thebreakdown voltage of the combination? (Optional)Section 26.6Electric Dipole in an Electric Field50.A small rigid object carries positive and negative 3.50-nCcharges. It is oriented so that the positive charge is at thepoint (/H110021.20 mm, 1.10 mm) and the negative charge isat the point (1.40 mm, /H110021.30 mm). (a) Find the electricdipole moment of the object. The object is placed in anelectric field E/H11005(7 800i/H1100249 0 0j) N/C. (b) Find thetorque acting on the object. (c) Find the potential en-ergy of the object in this orientation. (d) If the orienta-tion of the object can change, find the difference be-tween its maximum and its minimum potential energies.51.A small object with electric dipole moment pis placedin a nonuniform electric field That is, thefield is in the xdirection, and its magnitude depends onthe coordinate x. Let /H9258represent the angle between thedipole moment and the xdirection. (a) Prove that thedipole experiences a net force cos /H9258inthe direction toward which the field increases. (b) Con-sider the field created by a spherical balloon centeredat the origin. The balloon has a radius of 15.0 cm andcarries a charge of 2.00/H9262C. Evaluate dE/dxat the point(16 cm, 0, 0). Assume that a water droplet at this pointhas an induced dipole moment of (6.30i)n C/H11080m. Findthe force on it.(Optional)Section 26.7An Atomic Description of Dielectrics52.A detector of radiation called a Geiger–Muller counterconsists of a closed, hollow, conducting cylinder with aF/H11005p(dE/dx)E/H11005E(x)i.20.0 μF10.0 μF20.0 μF20.0 μF20.0 μFμμμμμabc–Q+Qκ2κ1Figure P26.47 Figure P26.49836CHAPTER 26Capacitance and Dielectricsfine wire along its axis. Suppose the internal diame-ter of the cylinder is 2.50 cm and that the wire along theaxis has a diameter of 0.200 mm. If the dielectricstrength of the gas between the central wire and thecylinder is 1.20/H11003106V/m, calculate the maximumvoltage that can be applied between the wire and thecylinder before breakdown occurs in the gas.53.The general form of Gauss's law describes how a chargecreates an electric field in a material, as well as in a vac-uum. It iswhere is the permittivity of the material. (a) A sheet with charge Quniformly distributed over its area Ais surrounded by a dielectric. Show that thesheet creates a uniform electric field with magnitudeat nearby points. (b) Two large sheets ofarea Acarrying opposite charges of equal magnitude Qare a small distance dapart. Show that they create a uni-form electric field of magnitude betweenthem. (c) Assume that the negative plate is at zero po-tential. Show that the positive plate is at a potentialQd/A/H9280. (d) Show that the capacitance of the pair ofplates is ADDITIONAL PROBLEMS54.For the system of capacitors shown in Figure P26.54,find (a) the equivalent capacitance of the system, (b) the potential difference across each capacitor , (c) the charge on each capacitor, and (d) the total energy stored by the group.A/H9280/d/H11005/H9260A/H92800/d.E/H11005Q/A/H9280E/H11005Q/2A/H9280/H9280/H11005/H9260/H92800/H20886 E/H11554dA/H11005q/H928056.A 2.00-nF parallel-plate capacitor is charged to an initialpotential difference and then isolated. Thedielectric material between the plates is mica (/H9260/H110055.00). (a) How much work is required to withdraw themica sheet? (b) What is the potential difference of thecapacitor after the mica is withdrawn?57.A parallel-plate capacitor is constructed using a dielec-tric material whose dielectric constant is 3.00 and whosedielectric strength is 2.00/H11003108V/m. The desired ca-pacitance is 0.250/H9262F, and the capacitor must withstanda maximum potential difference of 4 000 V. Find theminimum area of the capacitor plates.58.A parallel-plate capacitor is constructed using threedielectric materials, as shown in Figure P26.58. You mayassume that /H5129d.(a) Find an expression for the ca-pacitance of the device in terms of the plate area Aandd, /H92601, /H92602, and /H92603. (b) Calculate the capacitance usingthe values cm2, mm, /H92601/H110054.90, /H92602/H110055.60, and /H92603/H110052.10.d/H110052.00A/H110051.00W/H9004Vi/H11005100 V 60.(a) Two spheres have radii aand band their centers area distance dapart. Show that the capacitance of this sys-tem isprovided that dis large compared with aand b. (Hint:Because the spheres are far apart, assume that theC/H110154/H9266/H928001a/H110011b/H110022d59.A conducting slab of thickness dand area Ais insertedinto the space between the plates of a parallel-plate ca-pacitor with spacing sand surface area A, as shown inFigure P26.59. The slab is not necessarily halfway be-tween the capacitor plates. What is the capacitance ofthe system? 55.Consider two long,parallel, and oppositely chargedwires of radius dwith their centers separated by a dis-tance D. Assuming the charge is distributed uniformlyon the surface of each wire, show that the capacitanceper unit length of this pair of wires isC/H5129/H11005/H9266/H92800ln/H20898D/H11002dd/H20899AAdsdd/2/H5129/2/H5129κ2κ3κ1κκκ 4.00 μF2.00 μF6.00 μF3.00 μF 90.0

VµµµµFigure P26.54Figure P26.58 Figure P26.59WEBProblems837 68.It is possible to obtain large potential differences by firstcharging a group of capacitors connected in paralleland then activating a switch arrangement that in effectdisconnects the capacitors from the charging sourceand from each other and reconnects them in a seriesarrangement. The group of charged capacitors is thendischarged in series. What is the maximum potentialdifference that can be obtained in this manner by usingten capacitors each of 500/H9262F and a charging source of800 V? 69.A parallel-plate capacitor of plate separation discharged to a potential difference /H9004V0. A dielectric slab64.When considering the energy supply for an automobile,the energy per unit mass of the energy source is an im-portant parameter. Using the following data, comparethe energy per unit mass (J/kg) for gasoline, lead–acidbatteries, and capacitors. (The ampere A will be intro-duced in Chapter 27 and is the SI unit of electric cur-rent. 1 A/H110051 C/s.)Gasoline:126 000 Btu/gal; density/H11005670 kg/m3Lead–acid battery:12.0 V; 100 A/H11080h; mass/H1100516.0 kgCapacitor:potential difference at full charge/H1100512.0 V; capacitance/H110050.100 F; mass/H110050.100 kg65.An isolated capacitor of unknown capacitance has beencharged to a potential difference of 100 V. When thecharged capacitor is then connected in parallel to anuncharged 10.0-/H9262F capacitor, the voltage across thecombination is 30.0 V. Calculate the unknown capaci-tance.66.A certain electronic circuit calls for a capacitor having acapacitance of 1.20 pF and a breakdown potential of 10 0 0V . I f y o u h a v e a s u p p l y o f 6 . 0 0 - p F c a p a c i t o r s , e a c hhaving a breakdown potential of 200 V, how could youmeet this circuit requirement?67.In the arrangement shown in Figure P26.67, a potentialdifference /H9004Vis applied, and C1is adjusted so that thevoltmeter between points band dreads zero. This “bal-ance” occurs when If andcalculate the value of C2.C4/H1100512.0 /H9262F,C3/H110059.00 /H9262FC1/H110054.00 /H9262F. 63.A capacitor is constructed from two square plates ofsides /H5129and separation d, as suggested in Figure P26.62.You may assume that dis much less than /H5129. The platescarry charges /H11001Q0and /H11002Q0. A block of metal has awidth /H5129, a length /H5129, and a thickness slightly less than d. Itis inserted a distance xinto the capacitor. The chargeson the plates are not disturbed as the block slides in. In a static situation, a metal prevents an electric fieldfrom penetrating it. The metal can be thought of as aperfect dielectric, with /H9260:/H11009. (a) Calculate the storedenergy as a function of x. (b) Find the direction andmagnitude of the force that acts on the metallic block. (c) The area of the advancing front face of the block isessentially equal to /H5129d. Considering the force on theblock as acting on this face, find the stress (force perarea) on it. (d) For comparison, express the energydensity in the electric field between the capacitor platesin terms of Q0, /H5129, d, and /H92800.charge on one sphere does not perturb the charge dis-tribution on the other sphere. Thus, the potential ofeach sphere is expressed as that of a symmetric chargedistribution, , and the total potential at eachsphere is the sum of the potentials due to each sphere.(b) Show that as dapproaches infinity the above resultreduces to that of two isolated spheres in series.61.When a certain air-filled parallel-plate capacitor is con-nected across a battery, it acquires a charge (on eachplate) of q0. While the battery connection is main-tained, a dielectric slab is inserted into and fills the re-gion between the plates. This results in the accumula-tion of an additional charge qon each plate. What is thedielectric constant of the slab?62.A capacitor is constructed from two square plates ofsides /H5129and separation d. A material of dielectric con-stant /H9260is inserted a distance xinto the capacitor, asshown in Figure P26.62. (a) Find the equivalent capaci-tance of the device. (b) Calculate the energy stored inthe capacitor if the potential difference is /H9004V. (c) Findthe direction and magnitude of the force exerted onthe dielectric, assuming a constant potential difference/H9004V. Neglect friction. (d) Obtain a numerical value forthe force assuming that /H9004V/H1100520 0 0V ,and the dielectric is glass (/H9260/H110054.50).(Hint:The system can be considered as two capacitorsconnected in parallel.)d/H110052.00 mm,/H5129/H110055.00 cm,V/H11005keQ/r C1C2C4C3abcd∆VVxd/H5129κFigure P26.62Problems 62 and 63.Figure P26.67838CHAPTER 26Capacitance and Dielectrics 76.Determine the effective capacitance of the combinationshown in Figure P26.76. (Hint:Consider the symmetryinvolved!)pacitors are disconnected from the battery and fromeach other. They are then connected positive plate tonegative plate and negative plate to positive plate. Cal-culate the resulting charge on each capacitor.73.The inner conductor of a coaxial cable has a radius of0.800 mm, and the outer conductor’s inside radius is3.00 mm. The space between the conductors is filledwith polyethylene, which has a dielectric constant of2.30 and a dielectric strength of 18.0/H11003106V/m. Whatis the maximum potential difference that this cable canwithstand?74.You are optimizing coaxial cable design for a majormanufacturer. Show that for a given outer conductor ra-dius b, maximum potential difference capability is at-tained when the radius of the inner conductor iswhere eis the base of natural logarithms.75.Calculate the equivalent capacitance between the pointsaand bin Figure P26.75. Note that this is not a simpleseries or parallel combination. (Hint:Assume a poten-tial difference /H9004Vbetween points aand b. Write expres-sions for /H9004Vabin terms of the charges and capacitancesfor the various possible pathways from ato b, and re-quire conservation of charge for those capacitor platesthat are connected to each other.)a/H11005b/e 72.Capacitors and are chargedas a parallel combination across a 250-V battery. The ca-C2/H110052.00 /H9262FC1/H110056.00 /H9262F71.A vertical parallel-plate capacitor is half filled with a di-electric for which the dielectric constant is 2.00 (Fig.P26.71a). When this capacitor is positioned horizon-tally, what fraction of it should be filled with the samedielectric (Fig. P26.71b) so that the two capacitors haveequal capacitance?of thickness dand dielectric constant /H9260is introducedbetween the plates while the battery remains connected to theplates.(a) Show that the ratio of energy stored after thedielectric is introduced to the energy stored in theempty capacitor is Give a physical explana-tion for this increase in stored energy. (b) What hap-pens to the charge on the capacitor? (Note that this sit-uation is not the same as Example 26.7, in which thebattery was removed from the circuit before the dielec-tric was introduced.)70.A parallel-plate capacitor with plates of area Aand plateseparation dhas the region between the plates filledwith two dielectric materials as in Figure P26.70. As-sume that and that (a) Determine thecapacitance and (b) show that when /H92601/H11005/H92602/H11005/H9260yourresult becomes the same as that for a capacitor contain-ing a single dielectric, C/H11005/H9260/H92800A/d.dVW.dVLU/U0/H11005/H9260. CC3C2C2Cab2.00 µF4.00 µF2.00 µF 4.00 µF8.00 µFµµµµµ (b)(a)dκ1κ2LWFigure P26.70 Figure P26.71Figure P26.76Figure P26.75Answers to Quick Quizzes839ANSWERS TOQUICKQUIZZEScrease. (c) Edecreases because the charge density onthe plates decreases. (d) /H9004Vremains constant becauseof the presence of the battery. (e) The energy stored inthe capacitor decreases (Eq. 26.11).26.6It increases. The dielectric constant of wood (and of allother insulating materials, for that matter) is greaterthan 1; therefore, the capacitance increases (Eq. 26.14).This increase is sensed by the stud-finder’s special cir-cuitry, which causes an indicator on the device to lightup.26.7(a) Cincreases (Eq. 26.14). (b) Qincreases. Becausethe battery maintains a constant /H9004V, Qmust increase ifCincreases. (c) Ebetween the plates remainsconstant because /H9004V/H11005Edand neither /H9004Vnor dchanges. The electric field due to the charges on theplates increases because more charge has flowed ontothe plates. The induced surface charges on the dielec-tric create a field that opposes the increase in the fieldcaused by the greater number of charges on the plates.(d) The battery maintains a constant /H9004V. (e) The energystored in the capacitor increases (Eq. 26.11). You wouldhave to push the dielectric into the capacitor, just as youwould have to do positive work to raise a mass and in-crease its gravitational potential energy.(/H11005Q//H9004V)26.1(a) because the plate separation is decreased. Capaci-tance depends only on how a capacitor is constructedand not on the external circuit.26.2Zero. If you construct a spherical gaussian surface out-side and concentric with the capacitor, the net chargeinside the surface is zero. Applying Gauss’s law to thisconfiguration, we find that at points outside thecapacitor.26.3For a given voltage, the energy stored in a capacitor isproportional to C:. T h u s , y o u w a n t t omaximize the equivalent capacitance. You do this byconnecting the three capacitors in parallel, so that thecapacitances add.26.4(a) Cdecreases (Eq. 26.3). (b) Qstays the same becausethere is no place for the charge to flow. (c) Eremainsconstant (see Eq. 24.8 and the paragraph following it). (d) /H9004Vincreases because /H9004V/H11005Q/C, Qis constant(part b), and Cdecreases (part a). (e) The energystored in the capacitor is proportional to both Qand /H9004V(Eq. 26.11) and thus increases. The additional en-ergy comes from the work you do in pulling the twoplates apart.26.5(a) Cdecreases (Eq. 26.3). (b) Qdecreases. The batterysupplies a constant potential difference /H9004V; thus, chargemust flow out of the capacitor if is to de-C/H11005Q//H9004VU/H11005C(/H9004V)2/2E/H110050