Hypothesis Testing.

(1) During the 1980s, the general consensus is that about 5% of the nation's children had autism. Some claimed that increases certain chemicals int environment has led to an increase in autism.

(A) Write an appropriate hypothesis test for this situation.

b) Give an appropriate test for this hypothesis, stating what are the neccessary conditions for performing the test.

c) A recent study examined 384 children & found out that 46 showed signs of autism. Perform a test of the hypothesis & state the p-value.

d) What are your conclusions? State how you use the p-value.

→ Solution :-

Step 1 :-

(a) · $\boxed{\text{Null hypothesis}}$:- General consencus state that about 5% of nation's children had autism.

· $\boxed{\text{Alternate hypothesis}}$:- Their was increase of in ~~ce~~ certain chemicals in ~~ev~~ environment which lead to increase the rate of autism.

~~$H_0 = 0.05$~~

$$\boxed{\begin{array}{l} H_0 : \mu = 0.05 \\ H_1 : \mu > 0.05 \end{array}}$$

Step 2 :-

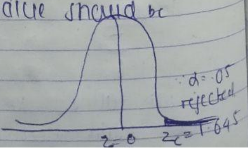
(b) $\boxed{\text{Z test}}$ can be performed as some porportion is been given ~~as~~ related to the children.

Step 3 :- $\boxed{\text{significance value}}$

$\boxed{\alpha = .05}$

decision rule

Step 4 :- This test is one-tailed test, To reject the null hypothesis at z score value should be greater $z_c = 1.645$



(c) $\hat{p} = \dfrac{46}{384}$

Z value

Step 5 :- The data is been showed.

Step 6 :- 'z' test so we need calculate the value of z by given formula.

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{\cancel{0.16}\ 0.1197 - 0.05}{\sqrt{\frac{0.05 \times 0.95}{384}}}$$

$\therefore n = 384, x = 46, p = 0.05$

$\hat{p} = 0.1197$

$q = 1 - p$

$= 1 - 0.05$

$q = 0.95$

$$= \frac{0.0697}{\sqrt{\frac{0.0475}{384}}}$$

$$= \frac{0.0697}{\sqrt{0.0001}}$$

$$= \frac{0.0697}{0.01}$$

$$\boxed{z_{score} = 6.97}.$$

Step 7 :- since

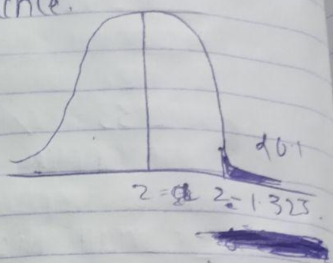
$$\underset{6.97}{Z_{score}} > \underset{1.645}{Z_{critical}}$$

Step 8 :-

(d) We conclude that rejection of null hypothesis should be done. So according general consencus ~~sta~~ 5% of nation's children ~~we~~ were affected by autism this is false because their were more than 5% of nation's children were affected autism. The general consencus should have tested correctly to share nearly proper result of children having autism.

(2) (a) Null hypothesis: 20% of the entire fleet might be out of compliance.

Step 1: Alternate hypothesis: more than 20% of the entire fleet might be out of compliance.



Step 2: Test to be used is z test

Step 3: 10%, 5%, 1%.

$\alpha = 0.1, 0.05, 0.01$.

significance value $\alpha = 0.1$.

By $z_c = 1.32$

Step 4: $Z_{calc} > Z_{critical}$

We will reject the null hypothesis.

In P value:

~~P value~~ significance value > p value.

∴ We will reject our null hypothesis

Step 5: Data gathering:

A company with fleet of 150 cars found that the emission type system of 7 out of the 22 cars tested failed to meet pollution guidelines.

Step 6:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p - q}{n}}}$$

We have, $\boxed{p = 0.20}$, $\boxed{n = 22}$, $\boxed{q = 0.80}$, $\boxed{\hat{p} = 0.31}$

$\hat{q} = 1 - p = 1 - 0.20 = 0.80$

$\hat{p} = \frac{7}{22} = 0.31$

substituting, $z = \frac{0.31 - 0.20}{\sqrt{\frac{0.20 - 0.80}{22}}}$

$= \frac{0.11}{\sqrt{\frac{\cancel{\ }}{22}}}$ ~~0.016~~ $= \frac{0.11}{0.410}$

$Z_s = 0.26$

By $Z_c = 1.32$

P value = 1.003

Step 7:- As,

$$\underset{1.32}{Z_c} > \underset{0.26}{Z_s}$$

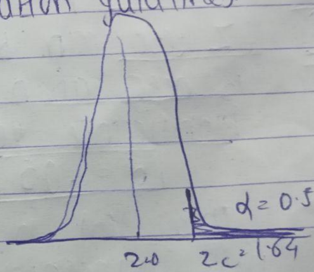
We failed to reject null hypothesis

$$\underset{1.003}{\text{Pvalues}} > \underset{0.10}{\text{significant value.}}$$

∴ 20% of the entire fleet of the cars are failed to meet the population guidlines

b) $\alpha = 5\%$, $\alpha = 0.5$.

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p-q}{n}}}$$



$z = 0.26$

again,

$$\underset{1.6}{Z_c} > \underset{0.26}{Z_s}$$

We failed to reject null hypotheis

ie, Entire fleet of the cars are failed to meet population guidlines.

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Step 5: Data gathering.
A national random sample of ~~1987~~ 891 adults were interviewed and 463 stated that they had never smoked.

Step 6:

$P = 0.44$; $N = 891$

We have,

$P = 0.44$, $N = 891$.

$\hat{q} = 1 - P$

$= 1 - 0.44 = 0.56$.

$\hat{P} = \frac{463}{891} = 0.51$

$$Z_s = \frac{\hat{P} - P}{\sqrt{\frac{P \cdot q}{n}}}$$

$$= \frac{0.51 - 0.44}{\sqrt{\frac{0.44 - 0.56}{891}}}$$

$$= \frac{0.07}{\sqrt{0.66}}$$

$$= 0.106$$

By using Z-table,

$Z_c = 2.23$.

Hence reject null hypothesis for both.

Step 8: Business Implimentation.

Business came to an decision by accepting alternate hypothesis. 44 had increase the no. of years with never smoked.

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c) $Z_s = 0.26$

$\alpha = 0.01$

By table,

$Z_c = 2.33$.

Again

$$\underset{2.33}{Z_c} > \underset{0.26}{Z_s}$$

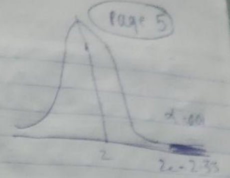
We have to reject null hypothesis

Business implication:

~~a) We shall make cars accordingly to meet up the guide~~

b) "

c) We shall make cars accordingly to meet up the guidlines of following



③ Step 1:

$\boxed{\text{Null hypothesis}}$: There is 44% adult popular had never smoked.

$\boxed{\text{Alternate hypothesis}}$: more than 44% adult pop^n had never smoked.

$H_0 : \mu = 0.44$

$H_A : \mu > 0.44$

Step 2:

We are going to perform the Z-test

Step 3:

significance value = 2%.

$\alpha = 0.2$.

Step 4: Decision Rule,

Z-critical < Z-score ; Reject the null hypothesis

For P-value

P-value < significance value, Reject the null hypothesis

4) Step 1:

$\boxed{\text{Null hypothesis}}$: The distance from the lens to the object and distance from the lens to real image is same.

$\boxed{\text{Alternative Hypothesis :-}}$ The distance from the lens to the object and distance from the lens to real image is not same.

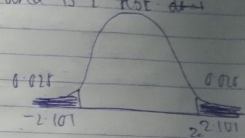
$H_0 : \mu a = \mu b.$

$H_1 : \mu a \neq \mu b$

Step 2: Statistical test to be used is t test. ~~at~~

Step 3: $\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$



Step 4: ~~st~~ decision rule is two tailed test critical value is 2.101

$df = n_1 + n_2 - 2$

$= 25 + 25 - 2$

$\boxed{df = 48}$

tscore > tcritical

and

For p value:

$P\text{value} \leq \alpha$

Step 5: gathered data as per the question:

Step 6: $n = 25$, $\bar{X}_1 = 26.6$, $\bar{X}_2 = 13.8$, $S_1 = 0.1$, $S_2 = 0.5$

~~Step 6~~ $t = \dfrac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$

$= \dfrac{(26.6 - 13.08) - 0}{\sqrt{\dfrac{(0.1)^2}{25} + \dfrac{(0.5)^2}{25}}}$

$= \dfrac{12.8}{\sqrt{0.0104}}$

$= 125.49.$

P value is 0.000.

Step 7:

tscore > tcritical - (we reject null hypothesis)

125.49 > 2.101

For p value:

p value < significance value.

0.000 < 0.05 --- (We reject null hypo.)

significance level (0.05) is greater the p value because it does not exceed.

Step 8:

The distance from the lens to the object & distance from the lens to the real image are not same

focal length $= \frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$

$= \frac{1}{0.1} + \frac{1}{0.5} = \frac{6}{0.5}$

$= 12.$

5) Step 1:

$\boxed{\text{Null Hypothesis}}$: body temperature will be at 98.6.

$\boxed{\text{Alternate hypothesis}}$: body temperature will not be at 98.6.

Step 2: Since the proportion is given, z-test is to be used with 2 tails.

Step 3: significance value:

$\alpha = 0.02$, $\frac{\alpha}{2} = 0.01.$

Step 4: z-test at two tailed

zscore > zcritical

For P value:

~~Pvalue~~ ⇒ $P\text{value} \leq \alpha$

-- (reject null hypothesis)

(6) ~~Step~~

~~Step 1:~~ Null hypothesis : There is no difference between the regular gas tank in term of mileage.

Alternate hypothesis : There is difference between the regular gas tank in term of mileage

$H_0: \mu_a = \mu_b$

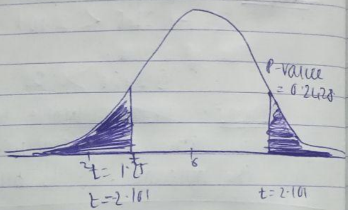
$H_1: \mu_a \neq \mu_b$

Step 2: We are going to perform t test.

Step 3:

$\alpha = .05$

Step 4: decision rule



$t_{score} > t_{critical}$

For pvalue.

$p_{value} \leq \alpha$

-- (Reject null hypothesis)

$df = n_1 + n_2 - 2$

$= 10 + 10 - 2$

$= 18$

Step 5: data gathered as shown in the question

Step 6: $t = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$\bar{X}_1 = \dfrac{\sum X_1}{n_1}$

$= \dfrac{231}{10}$

$\boxed{\bar{X}_1 = 23.1}$

$S_1^2 = \dfrac{\sum x_1^2}{n_1} - (\bar{X}_1)^2$

$= \dfrac{5461}{10} - (23.1)$

$\boxed{S_1^2 = 12.49}$

$\bar{X}_2 = \dfrac{\sum x_2}{n_2}$

$= \dfrac{251}{10}$

$\boxed{X_2 = 25.1}$

$S_2^2 = \dfrac{\sum x_2^2}{n_2} - (\bar{X}_2)^2$

$= \dfrac{6407}{10} - (25.1)^2$

$\boxed{S_2^2 = 10.69}$

Step 5: gathered data as per question.

Step 6: (1) $z = \dfrac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$

$= \dfrac{98.2846 - 98.6}{\frac{0.6824}{\sqrt{52}}}$

$= \dfrac{-0.3154}{0.095} = -3.32$

For Pvalue: $= 0.001$

$H_0: \mu = 98.2$

$H_1: \mu \neq 98.2$

(2) $z = \dfrac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$

$= \dfrac{98.2846 - 98.2}{\frac{0.6824}{\sqrt{52}}}$

$= \dfrac{0.0846}{0.095}$

$= 0.89$

For P value:

$= 2 \times P(z > 0.89) = 2 \times [1 - P(z < 0.89)$

$= 2 \times (1 - 0.8133) = 2 \times 0.1867 = 0.37$

Step 7:

- $98.6 - c = -3.32 > -2.33$

For Pvalue:

$0.001 > 0.02$

--- (Reject null hypothesis).

- $98.2 - c = 0.89 < 2.33$

For Pvalue:

$0.37 > 0.02$

--- (~~Re~~ Accept null hypothesis)

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$\therefore t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$= \dfrac{23.1 - 25.1}{\sqrt{12.88\left(\frac{1}{10} + \frac{1}{10}\right)}}$

$= \dfrac{-2}{1.606}$

$= -1.245$

$s^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$

$= \dfrac{10(12.49) + 10(10.69)}{10 + 10 - 2}$

$= \dfrac{124.9 + 106.9}{18}$

$\boxed{s^2 = 12.88}$

Step 7: Conclusion.

$t = -1.245 < 2.101$ ∴ [Fail to reject null hypothesis].

For p value.

$0.2428 > 0.05$

Step 8: ~~Testing~~ While testing cars with regular gas is same as premium gas because it depends upon the mileage. It is budget friendly car drivers also.

(7) **Step 1:**

$\boxed{\text{Null hypothesis}}$: sugar content of several national brands of cereals of children and adult are in equal percentage of weight.

$\boxed{\text{Alternate hypothesis}}$: sugar content of several national brands of cereals of children and adult are not in equal percentage of weight.

$H_0 : \mu_1 = \mu_2$

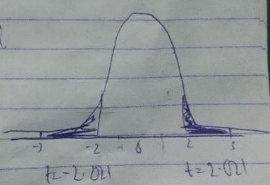
$H_1 : \mu_1 \neq \mu_2$

Step 2: Test t is to be used.

Step 3: The value of $\alpha = .05$

Step 4: two-tailed test

$t_{critical} = \pm 2.021$



$df = n_1 + n_2 - 2$

$= 19 + 29 - 2 = 46$

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Decision rule is reject the null H_0 if $>$ t critical value.

For p value:

p value $<$ significance value

$0.000 < 0.05$

∴ ~~pvalue~~ reject null H_0 at 0.05 because it does not exceed 0.05.

Step 5: data gathered as shown in question.

Step 6: $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$= 8.47$

$s^2 = \dfrac{(19)(39.03)^2 + (29)(53.96)^2}{19 + 289 - 2}$

$= 2464.831$

Step 7:

$t_{calc} > t_{critical}$

$8.47 > 2.021$

∴ We reject null H_0

For p value:

pvalue $<$ significance value

$0.000 < 0.05$

We reject the null hypothesis

Step 8: Business has a difference in sugar content of several national brands of cereals between the children and adult products.