Homework for Module 2

Quiz, 6 questions

1 point

1. (Difficulty: ★) Write the value for the inner product $\langle \mathbf{v}^{(0)}, \mathbf{v}^{(1)} \rangle$ where

$$\mathbf{v}^{(0)} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \text{ and } \mathbf{v}^{(1)} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix},$$

0

1 point

2. (Difficulty: ★★) Consider the following vectors in $\mathbb{R}^4$

$$\mathbf{v}^{(0)} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}, \mathbf{v}^{(1)} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix} \text{ and } \mathbf{v}^{(2)} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}.$$

You can verify that the vectors are mutually orthogonal and have unit norm.

How many different vectors $\mathbf{v}^{(3)}$ could we find such that $\{\mathbf{v}^{(0)}, \mathbf{v}^{(1)}, \mathbf{v}^{(2)}, \mathbf{v}^{(3)}\}$ is a full **orthogonal** basis in $\mathbb{R}^4$?

- 0
- 1
- 2
- 3
- >3

1 point

3. Let $\mathbf{y} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ -1 \end{bmatrix}$,

what are the expansion coefficients of $\mathbf{y}$ in the basis $\{\mathbf{v}_0, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ you found in the previous question?

Important: *Enter your answer as space separated floating point decimal numbers, e.g. the vector* $\mathbf{y}$ *would be entered as :*

2.0 1.0 0.0 -1.0

0 1 0 -1

1 point

4. Which of the following sets form a basis of $\mathbb{R}^4$?

- $\{\mathbf{y}, \mathbf{v}_0, \mathbf{v}_1, \mathbf{v}_3\}$
- $\{\mathbf{y}, \mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_2, \mathbf{v}_3\}$
- $\{\mathbf{y}, \mathbf{v}_0, \mathbf{v}_2, \mathbf{v}_3\}$
- $\{\mathbf{y}, \mathbf{v}_0, \mathbf{v}_1, \mathbf{v}_2\}$

1 point

5. (Difficulty: ★) If we represent finite-length signals as vectors in Euclidean space, many operations on signals can be encoded as a matrix-vector multiplication. Consider for example a circular shift in $\mathbb{C}^3$: a delay by one (i.e. a right shift) transforms the signal $\mathbf{x} = [x_0\ x_1\ x_2]^T$ into $\mathbf{x}' = [x_2\ x_0\ x_1]^T$ and it can be described by the matrix

$$D = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

so that $\mathbf{x}' = D\mathbf{x}$.

Determine the matrix F that implements the one-step-difference operator in $\mathbb{C}^3$ i.e. the operator that transforms a signal $\mathbf{x}$ into $[(x_0 - x_2)\ (x_1 - x_0)\ (x_2 - x_1)]^T$.

Write the 9 integer matrix coefficients one after the other, row by row and separated by spaces.

1 0 -1 -1 1 0 -1 1

1 point

6. (Difficulty ★) Given the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

compute the matrix A^4 (i.e. the fourth power of A).

(Hint: there is a simple way to do that and, if you've solved the previous question, it should be obvious).

Write the 16 integer matrix coefficients one after the other, row by row and separated by spaces.

1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1

Sayre Blades

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