📚 Fluid Mechanics & Hydraulics Overview

Brief Overview

This note covers **Fluid Mechanics and Hydraulics** and was created from a PDF document of 147 pages (source: https://api.turbolearn.ai/storage/v1/object/public/pdf/54b3424b-31bb-4e83-a4d1-8e31f4abf605/f62e5ba1-1b73-491d-a5ba-29bf39c1dbf3.pdf). It introduces fluid statics, flow regimes, pressure measurement, and channel optimisation with equations and worked examples.

Key Points

- Fundamentals of fluid properties (density, viscosity, surface tension)
- Pressure measurement techniques (manometers, Bourdon gauge)
- Flow regimes, Bernoulli's equation, and pump analysis
- Open-channel flow, channel optimisation, and hydraulics engineering

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🌊 Unit-I: Fluid Mechanics & Hydraulics

> **Fluid mechanics and hydraulics** is the branch of science dealing with the behavior of fluids (liquids or gases) at rest and in motion, covering statics, kinematics, and dynamics.

📚 Classification of Fluid Studies

Aspect	Focus	Pressure Forces	Typical Quantities
Fluid Statics	Fluids at rest	✓	Pressure distribution, hydrostatic forces
Fluid Kinematics	Fluids in motion	❌	Velocity, acceleration, streamlines
Fluid Dynamics	Fluids in motion	✓	Shear stress, momentum, energy

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⚖️ Physical Properties of Fluids

1. Density (ρ)

> Mass per unit volume
> $\boxed{\rho = \frac{\text{mass}}{\text{volume}}} \quad \text{units: kg/m}^3$

- Liquids ≈ constant
- Gases vary with p, T

2. Specific Weight (w)

> Weight per unit volume
> $w = \rho, g \quad \text{units: N/m}^3$
> Standard water: $w = 9810, \text{N/m}^3$

3. Specific Volume

> Volume per unit mass
> $\text{sp. vol} = \frac{1}{\rho} \quad \text{m}^3/\text{kg}$

4. Specific Gravity (S)

> Ratio: fluid density to reference

- Liquids → vs. water @4 °C
- Gases → vs. air
  $\rho_{\text{fluid}} = S \times 1000, \text{kg/m}^3$

Quick reference:
$\rho_{\text{Hg}} = 13600, \text{kg/m}^3 \quad (S = 13.6)$

✏️ Solved Density Problems

Ex-1 – 1 L liquid weighing 7 N

- $w = 7000, \mathrm{N/m^3}$
- $\rho = \frac{w}{g} = 713.5, \mathrm{kg/m^3}$
- $S = 0.7135$

Ex-2 – Petrol $S = 0.7$, 1 L

- $\rho = 700, \mathrm{kg/m^3}; w = 6867, \mathrm{N/m^3}; W = 6.867, \mathrm{N}$

🍯 Viscosity (μ)

> Property resisting relative motion between layers.

Newton's relation:

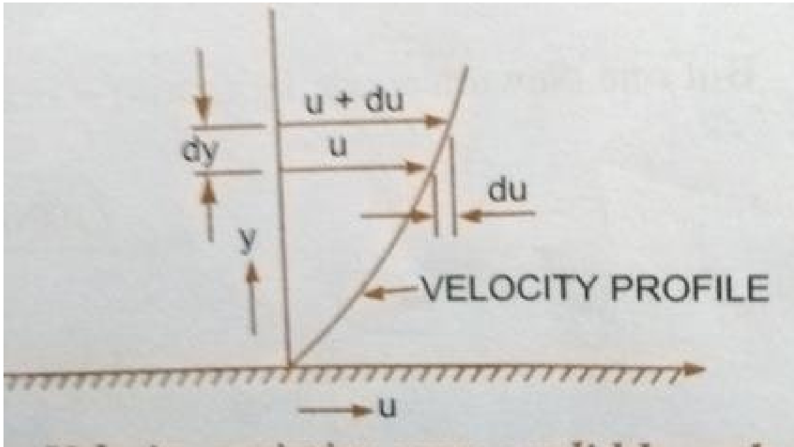
$$\boxed{\tau = \mu, \frac{du}{dy}}$$

Quantity	SI Unit	Common Conversion
μ	$\mathrm{N\cdotps/m^2}$	$1, \mathrm{poise} = 0.1; \mathrm{N\cdotps/m^2}$
ν	$\mathrm{m^2/s}$	$1, \mathrm{stoke} = 10^{-4}; \mathrm{m^2/s}$

Temperature effect:

- Liquids → $\mu \downarrow$ with $T \uparrow$
- Gases → $\mu \uparrow$ with $T \uparrow$

Illustration of velocity gradient near a wall:



Velocity variation near a solid boundar

Shear is largest where $\frac{du}{dy}$ is steepest.

Ex-3 – Plate separated 0.025 mm, speed 0.6 m/s, $\tau = 2; \mathrm{N/m}^2$

$\mu = \frac{\tau}{du/dy} = \frac{2 \times 0.025 \times 10^{-3}}{0.6} = 8.33 \times 10^{-5}; \mathrm{N}\backslash\mathrm{cdotps/m}^2$

🔲 Types of Fluids

Type	Viscous	Newtonian $\tau \propto \frac{du}{dy}$	Example
Ideal	❌	—	Theory only
Real	✔️	May/may not	Water, air
Newtonian	✔️	✔️	Water, air, petrol
Non-Newtonian	✔️	❌	Blood, paint
Ideal plastic	✔️* (after yield)	✔️	–

💧 Surface Tension (σ)

> Tensile force per unit length acting along interface.

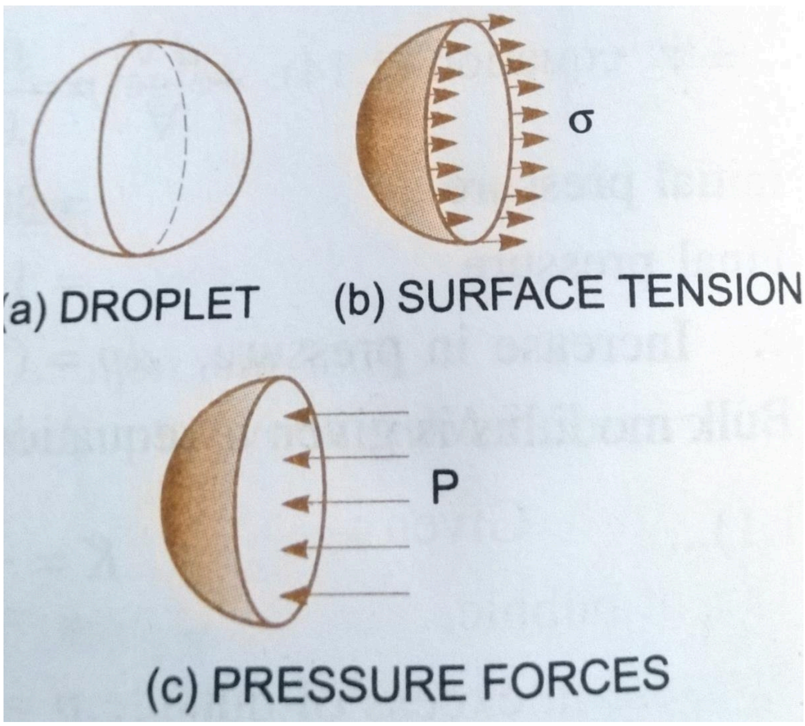
Units: $\mathrm{N/m}$ (S.I.)

Curved surface excess pressures

1. Droplet: $p = \frac{4\sigma}{d}$
2. Soap bubble (two films): $p = \frac{8\sigma}{d}$
3. Cyl. jet: $p = \frac{2\sigma}{d}$

Example: 0.04 mm water droplet, $\sigma = 0.0725; \mathrm{N/m}$
$p = \frac{4 \times 0.0725}{0.04 \times 10^{-3}} = 7.25; \mathrm{kPa\ (in\ excess)}$

Realistic diagram of pressure balance on a droplet:



🧪 Capillarity

> Rise/fall of liquid in narrow tube relative to free surface.

General rise/fall formula (vertical tube)

$$\boxed{h = \frac{4, \sigma, \cos\theta}{\rho, g, d}}$$

Contact angle θ	Liquid-solid	Observed h
$\approx 0^\circ$	Water-clean glass	Rise

$\approx 128^\circ$	Hg-glass	Fall

Design limit: Capillary rise ≤ 0.2 mm water
→ Requires $d \geq 14.8; \mathrm{cm}$

🔩 Compressibility & Bulk Modulus

> **K**: ratio of pressure increment to volumetric strain
> $K = \frac{\Delta p}{-\Delta V/V}$; Compressibility $= 1/K$

Units: $\mathrm{N/m^2}$ (Pa)
Typical water $K \approx 2 \times 10^9; \mathrm{Pa}$

💨 Vapour Pressure

- Molecules leave liquid → exert partial pressure
- ↑ with temperature; high for volatile fluids
- **Practical note:** Mercury's p_{vap} extremely low → used in barometers & thermometers

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📏 Fluid Pressure & Pascal's Law

> Pressure at a point: $p = \frac{dF}{dA}$
> For uniform force: $p = \frac{F}{A}$

Pascal's principle: Pressure at any point in static fluid is **isotropic**
→ Basis of hydraulic jack, lift, brake, crane, press.

🎚️ Absolute vs Gauge vs Vacuum

Relations:

$p_{\mathrm{abs}} = p_{\mathrm{atm}} + p_{\mathrm{gauge}}$
$p_{\mathrm{vac}} = p_{\mathrm{atm}} - p_{\mathrm{abs}}$

Reference values

- Sea-level standard $p_{\text{atm}} = 101.3; \text{kPa} = 10.33; \text{m H}_2\text{O} = 760; \text{mm Hg}$

🌡️ Pressure Measurement & Manometers

Simple Manometers

> **Simple manometer**: A glass tube with one end connected to the measurement point and the other open to atmosphere.

Three main types:

1. **Piezometer** - simplest form for gauge pressure
2. **U-tube Manometer** - U-shaped tube with heavy liquid (typically mercury)
3. **Single Column Manometer** - modified U-tube with large reservoir

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📊 Piezometer

> Measures **gauge pressure** directly through liquid column height.

Working principle:

- Pressure at point A: $p = w \times h$ where h is the liquid rise
- Limited to moderate pressures (cannot measure vacuum)

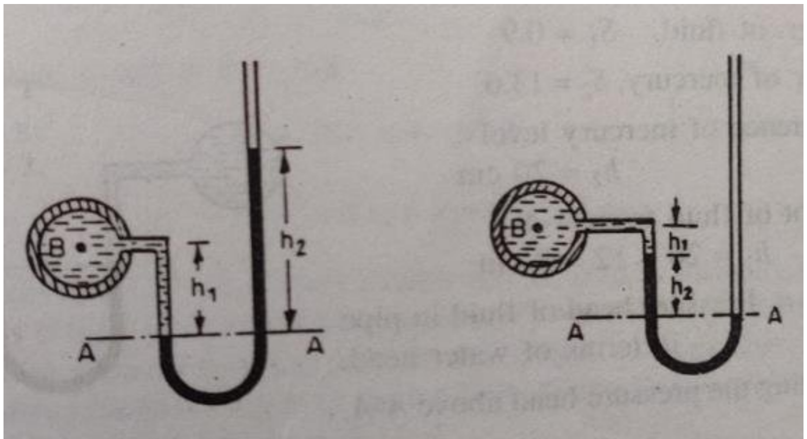
🔧 U-tube Manometer

Configuration:

- Contains **heavy liquid** (usually mercury, $S = 13.6$)
- Heavy liquid specific gravity must exceed fluid being measured

Two applications:

(a) Gauge Pressure Measurement



(a) For Gauge pressure. (b) For Vacuum pressure.

Pressure balance equation: $h + h_1 S_1 = h_2 S_2$

Solving for pressure head: $h = h_2 S_2 - h_1 S_1$

In terms of pressure: $p = \rho_2 g h_2 - \rho_1 g h_1$

(b) Vacuum Pressure Measurement

Resulting equation: $h = -(h_2 S_2 + h_1 S_1)$

In terms of pressure: $p = -(\rho_2 g h_2 + \rho_1 g h_1)$

🔍 Worked Examples

Problem 1: U-tube with mercury measuring fluid ($S = 0.9$), pipe center 12 cm below mercury level, mercury difference = 20 cm.

Solution:

- $h_1 = 20 - 12 = 8$ cm
- $h + (8)(0.9) = (20)(13.6)$
- $h = 264.8$ cm of water
- $p = 2648$ kg/m² = **2.597 N/cm²**

Problem 2: Vacuum measurement with fluid ($S = 0.8$), mercury difference = 40 cm, fluid height = 15 cm.

Solution:

- $h = -[(40)(13.6) + (15)(0.8)]$
- $p = -54543.6$ N/m² = **-0.5454 N/cm²**

📐 Single Column Manometers

> Modified U-tube with large reservoir (≈100× tube area) for enhanced sensitivity.

Key advantage: Reservoir level change is negligible, pressure determined solely by height in measuring limb.

Types:

1. **Vertical Single Column**
2. **Inclined Single Column** (more sensitive due to longer measurement scale)

Mathematical Analysis

Basic principle: $A \times \Delta h = a \times h_2$ therefore $\Delta h = \frac{ah_2}{A}$

Final pressure equation (simplified): $h = h_2 S_2 - h_1 S_1$

Inclined version: $h = L \sin\theta \times S_2 - h_1 S_1$

📐 Worked Example

Problem: Single column manometer with reservoir area = 100× tube area, measuring fluid ($S = 0.9$), mercury ($S = 13.6$), $h_2 = 40$ cm, $h_1 = 20$ cm.

Solution:

- Using full equation: $h = \frac{1}{100}(0.4)(13.6 - 0.9) + (0.4)(13.6) - (0.2)(0.9)$
- $p = 52134$ N/m² = **5.21 N/cm²**

🔍 Differential Manometers

> Measures **pressure difference** between two points using U-tube with heavy liquid.

1. **U-tube Differential Manometer**

Two cases:

(a) Different Levels & Different Fluids

Pressure difference: $p_A - p_B = hg(\rho_g - \rho_1) + \rho_2 gy - x\rho_1 g$

(b) Same Level & Same Fluid

Simplified equation: $p_A - p_B = hg(\rho_g - \rho_1)$

Worked Example: Two pipes with different fluids, given pressures. Mercury level difference calculated as **18.1 cm**.

2. Inverted U-tube Differential Manometer

> Used for **low pressure differences** with light liquid.

Pressure equation: $p_A - p_B = h_1\rho_1 g - h_2\rho_2 g - h\rho_S g$

Worked Example: Water pipes with oil manometer ($S = 0.8$), height difference = 20 cm.

- Result: $p_A - p_B = 1569.6$ N/m²

🌡️ Bourdon Gauge

> **Mechanical pressure gauge** using elliptical tube that straightens under pressure.

Key features:

- Metallic tube of elliptical section
- Closed at one end, connected to measurement point at other
- Tube straightens → pointer movement via mechanical linkage
- **Calibrated** against known pressures

Applications:

- High pressures (steam boilers, water mains)
- **Vacuum pressures** (compound gauge)

💪 Hydrostatic Forces on Submerged Surfaces

Fundamental Concepts

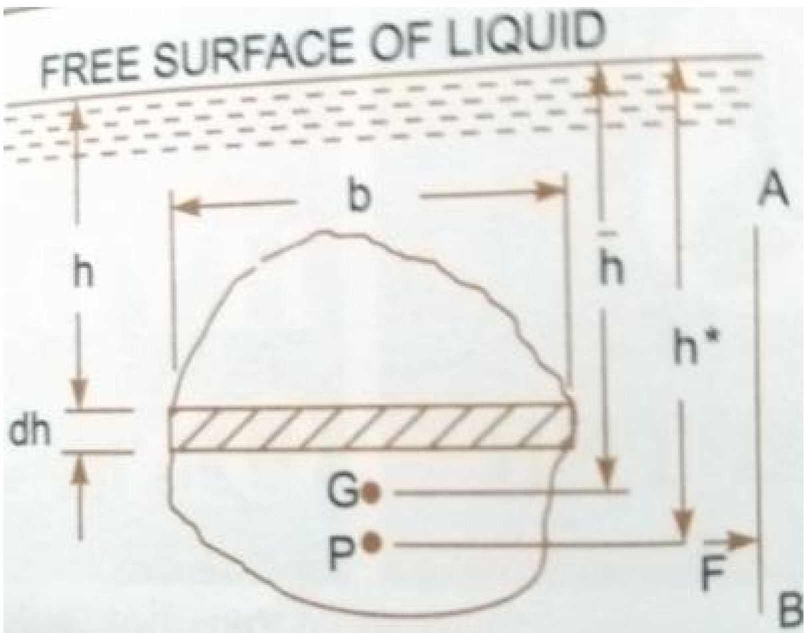
> **Total Pressure (F)**: Force exerted by static fluid on submerged surface, always acts **normal** to surface.

> **Centre of Pressure**: Point of application of total pressure force.

Four cases:

1. Vertical plane surface
2. Horizontal plane surface
3. Inclined plane surface
4. Curved surface

📐 Vertical Plane Surface



(a) Total Pressure

$F = wA\bar{h}$

Where:

- $w = \rho g$ = specific weight of liquid
- A = total area of surface
- $\bar{h}$ = distance of centroid from free surface

(b) Centre of Pressure

Using **principle of moments**:

$h^* = \frac{I_O}{A\bar{h}}$

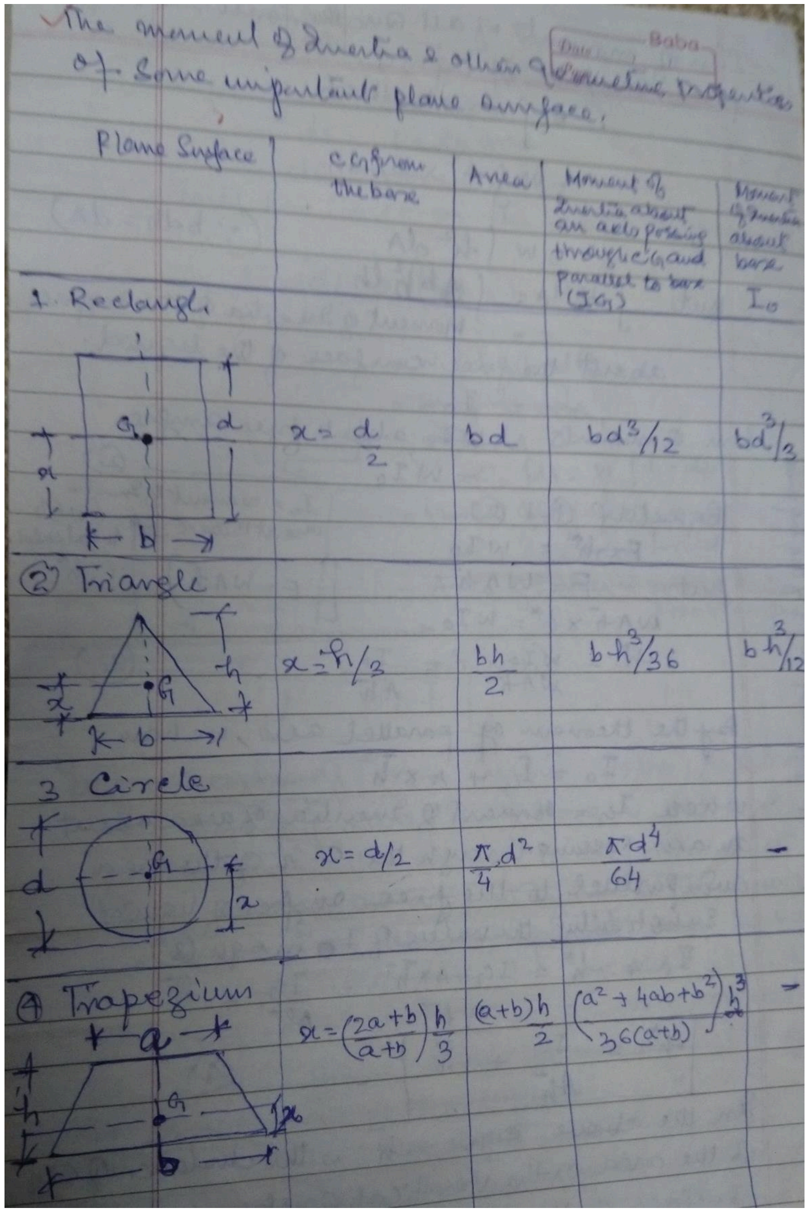
Parallel axis theorem: $I_O = I_G + A\bar{h}^2$

Final equation: $\boxed{h^* = \frac{I_G}{A\bar{h}} + \bar{h}}$

Key insights:

- Centre of pressure **always lies below centroid**
- Position is **independent** of liquid specific weight

📏 Geometric Properties Table



The moment of Inertia & other geometric properties of some important plane surface.

Plane Surface	C.G from the base	Area	Moment of Inertia about an axis passing through C.G and parallel to base (I_G)	Moment of Inertia about base I_0
1. Rectangle	$x = \frac{d}{2}$	bd	$bd^3/12$	$bd^3/3$
2. Triangle	$x = h/3$	$\frac{bh}{2}$	$bh^3/36$	$bh^3/12$
3. Circle	$x = d/2$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	-
4. Trapezium	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)h}{2}$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right)h^3$	-

Common shapes:

- **Rectangle**: $I_G = \frac{bd^3}{12}$
- **Triangle**: $I_G = \frac{bh^3}{36}$
- **Circle**: $I_G = \frac{\pi d^4}{64}$

📋 Worked Examples

Rectangular Plate (2m × 3m):

(a) **Upper edge at water surface:**

- $F = 88290$ N
- $h^* = 2.0$ m

(b) **Upper edge 2.5m below surface:**

- $F = 235440$ N
- $h^* = 4.1875$ m

Circular Plate (Diameter 1.5m, center 3m deep):

- $F = 52000$ N (approx.)
- $h^* = 3.07$ m

Trapezoidal Channel:

- Area = 3 m²
- $\bar{h} = 0.444$ m
- $F = 13079.87$ N
- $h^* = 0.625$ m

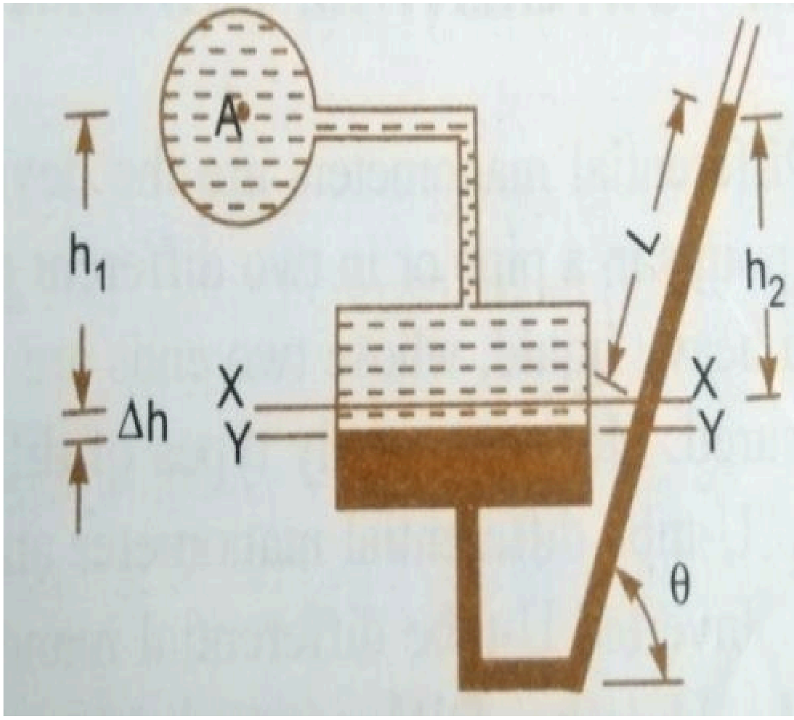
↔️ Horizontal Plane Surface

Special case: All points at same depth

- Pressure intensity: $p = wh$
- Total force: $F = whA = wA\bar{h}$
- **Centre of pressure = Centroid** ($h^* = \bar{h} = h$)

📐 Inclined Plane Surface

> Surface making angle θ with free surface.



Same formulas apply as vertical case, but:

- $\bar{h}$ = vertical distance from free surface to centroid
- All distances measured perpendicular to free surface

📐 Inclined Plane Surface Submerged in Liquid

> **Inclined surface**: A plane surface making angle θ with the free liquid surface, where the depth varies along the surface.

🧮 Key Parameters

Symbol	Meaning
A	**Total area** of inclined surface
$\bar{h}$	**Depth of C.G.** from free liquid surface
h^*	**Depth of centre of pressure** from free surface
θ	**Angle** made by surface plane with free liquid surface

$\bar{y}$	**Distance of C.G.** from reference axis O-O
y^*	**Distance of centre of pressure** from reference axis O-O

📊 Pressure Distribution Analysis

For a **small strip** of area dA at depth h and distance y from axis O-O:

- **Pressure intensity**: $p = wh = wy \sin\theta$
- **Force on strip**: $dF = p \times dA = wh \times dA = wy \sin\theta \times dA$

🔢 Total Pressure Force

Building on the vertical surface approach from previous sections, the **total force** on inclined surface:

$F = \int dF = \int wy \sin\theta \times dA = w \sin\theta \int y \times dA$

Since $\int y \times dA = A\bar{y}$ (moment of area about O-O):

$F = wA\bar{y} \sin\theta$

Using $\bar{h} = \bar{y} \sin\theta$:

$F = wA\bar{h}$

Or equivalently:

$F = \rho g A\bar{h}$

📍 Centre of Pressure Location

Applying the **principle of moments** about axis O-O:

Moment of total force: $F \times y^*$

Sum of moments of all forces: $\int w \sin\theta y^2 dA = w \sin\theta I_O$

Where $I_O = \int y^2 dA$ is the **moment of inertia** about axis O-O.

Equating moments:

$F \times y^* = w \sin\theta I_O$

Substituting $F = wA\bar{h}$ and using $y^* = \frac{h^*}{\sin\theta}$:

$h^* = \frac{\sin^2\theta I_O}{A\bar{h}}$

Applying **parallel axis theorem**: $I_O = I_G + A\bar{y}^2 = I_G + A\frac{\bar{h}^2}{\sin^2\theta}$

Final formula for **centre of pressure depth**:

$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$

🧮 Worked Example: Rectangular Inclined Surface

Given: 2 m wide × 3 m deep rectangular plate, $\theta = 30^\circ$, upper edge 1.5 m below water surface.

📏 Step-by-Step Solution

Distance of C.G. from surface:

$\bar{h} = 1.5 + \frac{3}{2}\sin 30^\circ = 1.5 + 1.5 \times 0.5 = 2.25 \text{ m}$

Total pressure force:

$F = \rho g A\bar{h} = 1000 \times 9.81 \times (2 \times 3) \times 2.25 = 132435 \text{ N}$

Centre of pressure:

$I_G = \frac{bd^3}{12} = \frac{2\times 3^3}{12} = 4.5 \text{ m}^4$

$h^* = \frac{4.5\times\sin^2 30^\circ}{(6\times 2.25)} + 2.25 = 2.333 \text{ m}$

🌊 Unit-II: Types of Fluid Flow

(i) Steady vs Unsteady Flow

Flow Type	Time Dependence	Mathematical Condition
Steady	Properties constant at fixed point	$\frac{\partial v}{\partial t} = 0, \frac{\partial p}{\partial t} = 0, \frac{\partial \rho}{\partial t} = 0$
Unsteady	Properties vary with time	$\frac{\partial v}{\partial t} \neq 0, \frac{\partial p}{\partial t} \neq 0$

(ii) Uniform vs Non-Uniform Flow

Flow Type	Space Dependence	Mathematical Condition
Uniform	Velocity constant along flow direction	$\frac{\partial v}{\partial s} = 0$
Non-uniform	Velocity varies along flow direction	$\frac{\partial v}{\partial s} \neq 0$

(iii) Laminar vs Turbulent Flow

> **Laminar flow**: Fluid particles move along well-defined parallel paths in smooth layers without mixing.

> **Turbulent flow**: Fluid particles move in irregular zig-zag patterns creating eddies and high energy loss.

Key distinction: Determined by **Reynolds number**:

$Re = \frac{VD}{\nu}$

Where:

- V = mean velocity
- D = pipe diameter
- ν = kinematic viscosity

(iv) Compressible vs Incompressible Flow

- **Compressible**: Density varies significantly ($\rho \neq \text{constant}$)
- **Incompressible**: Density essentially constant ($\rho \approx \text{constant}$)

(v) Rotational vs Irrotational Flow

- **Rotational**: Fluid particles rotate about their own axis
- **Irrotational**: Fluid particles don't rotate (zero vorticity)

(vi) Flow Dimensionality

Type	Variables	Description
One-dimensional	1 spatial variable	Flow varies only along flow direction

Two-dimensional	2 spatial variables	Flow varies in plane
Three-dimensional	3 spatial variables	Flow varies in all directions

🌊 Reynolds Number & Flow Regimes

Reynolds Number Definition

> **Reynolds number** determines the flow regime and is calculated as: $Re = \frac{VD}{\nu}$

where:

- V = mean velocity
- D = pipe diameter
- ν = kinematic viscosity

Flow Classification by Reynolds Number

Flow Type	Reynolds Number Range	Characteristics
Laminar	$Re < 2000$	Smooth, parallel layers
Transitional	$2000 < Re < 4000$	May be laminar or turbulent
Turbulent	$Re > 4000$	Irregular, mixing motion

💧 Rate of Flow (Discharge)

> **Discharge (Q)** is the quantity of fluid flowing per second through a section

For incompressible fluids: Volume per second $Q = A \times V$ (units: m^3/sec or litres/sec)

For compressible fluids: Weight per second (units: kgf/sec or N/sec)

📐 Continuity Equation

> **Continuity equation** states that for steady flow, mass flow rate is constant throughout the pipe

General form (compressible/incompressible): $\boxed{V_1\rho_1A_1 = V_2\rho_2A_2}$

For incompressible fluids ($\rho_1 = \rho_2$): $\boxed{V_1A_1 = V_2A_2}$

Worked Example 1: Pipe Contraction

Given: $D_1 = 10$ cm, $D_2 = 15$ cm, $V_1 = 5$ m/s

Solution:

- $A_1 = \frac{\pi}{4}(0.1)^2 = 0.007854$ m²
- $Q = A_1V_1 = 0.03927$ m³/s
- $A_2 = \frac{\pi}{4}(0.15)^2 = 0.01767$ m²
- $V_2 = \frac{A_1V_1}{A_2} = 2.22$ m/s

Worked Example 2: Pipe Branching

Given: 30 cm pipe branches to 20 cm and 15 cm pipes, $V_1 = 2.5$ m/s, $V_2 = 2$ m/s

Solution:

- $Q_1 = 0.1767$ m³/s
- $Q_2 = 0.0628$ m³/s
- $Q_3 = Q_1 - Q_2 = 0.1139$ m³/s
- $V_3 = 6.44$ m/s

⚡ Energy Forms in Fluid Flow

> A fluid body possesses three types of energy:

1. **Pressure Energy**: Energy due to pressure
2. **Kinetic Energy**: Energy due to motion
3. **Potential Energy**: Energy due to position/elevation

🌟 Bernoulli's Theorem

> **Bernoulli's theorem** states that for steady, ideal, incompressible, irrotational flow, total energy is constant

Per unit weight:

- Pressure head: $\frac{p}{\rho g}$
- Kinetic head: $\frac{v^2}{2g}$
- Potential head: z

Bernoulli's equation: $\boxed{\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 = \text{constant}}$

Assumptions:

1. Ideal fluid (zero viscosity)
2. Steady flow
3. Incompressible flow
4. Irrotational flow

Worked Example: Total Head Calculation

Given: $D = 5$ cm, $p = 29.43$ N/cm², $v = 2.0$ m/s, $z = 5$ m

Solution:

- Pressure head = $\frac{29.43 \times 10^4}{1000 \times 9.81} = 30$ m
- Kinetic head = $\frac{2^2}{2 \times 9.81} = 0.204$ m
- **Total head = 30 + 0.204 + 5 = 35.204 m**

Worked Example: Velocity Change in Constriction

Given: $D_1 = 20$ cm, $D_2 = 10$ cm, $V_1 = 4.0$ m/s

Solution:

- $V_1^2/2g = 0.815$ m
- $V_2 = 16$ m/s
- $V_2^2/2g = 13.047$ m
- $Q = 0.1256$ m³/s

📏 Practical Applications of Bernoulli's Equation

Bernoulli's equation is applied to measuring devices:

1. **Venturi meter**
2. **Orifice meter**
3. **Pitot tube**

1. Venturi Meter

> Measures flow rate using a converging-diverging section

Components:

- Converging part (21° angle)
- Throat ($\frac{1}{4}$ to $\frac{1}{2}$ of pipe diameter)
- Diverging part

Flow rate equation: $\boxed{Q_{act} = C_d \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}}$

2. Orifice Meter

> Cheaper alternative using flat plate with sharp-edged hole

Discharge equation: $\boxed{Q_{act} = C_d \frac{a_1 a_0 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}}$

3. Pitot Tube

> Measures local velocity by converting kinetic to pressure energy

Velocity equation: $\boxed{v = C_v \sqrt{2gh}}$

🔢 Hydraulic Coefficients

Coefficient	Definition	Typical Range
C_v	$\frac{\text{actual velocity}}{\text{theoretical velocity}}$	0.95-0.99

C_c	$\frac{\text{area at vena contracta}}{\text{orifice area}}$	0.61-0.69
C_d	$C_v \times C_c$	0.61-0.65

📊 Notches and Weirs

Definitions

> **Notch**: Metallic opening in tank side for measuring small discharge
> **Weir**: Concrete/masonry structure in open channels for large discharge

Key Terms

- **Nappe/Vein**: Sheet of water flowing through
- **Crest/Sill**: Bottom edge of opening

Discharge Equations

Rectangular notch/weir: $Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$

Triangular notch/weir: $Q = \frac{8}{15} C_d \tan\left(\frac{\theta}{2}\right) \sqrt{2g} H^{5/2}$

🌊 Velocity of Approach

> **Velocity of approach** is defined as the velocity with which water approaches or reaches the weir or notch before it flows over it.

When velocity of approach V_a is considered, an additional head $h_a = \frac{V_a^2}{2g}$ acts on water flowing over the weir.

Modified rectangular weir equation with velocity of approach:

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} [(H + h_a)^{3/2} - h_a^{3/2}]$$

📊 Worked Example

Given: Rectangular channel 1 m wide × 0.75 m deep, weir length = 60 cm, head H = 20 cm, $C_d = 0.62$

Solution:

- $A = 1.0 \times 0.75 = 0.75$ m²
- Initial discharge (neglecting velocity of approach): $Q = 0.0982$ m³/sec
- $V_a = \frac{0.0982}{0.75} = 0.1309$ m/s
- $h_a = \frac{0.1309^2}{2 \times 9.81} = 0.00087$ m
- **Final discharge**: $Q = 0.09881$ m³/sec

🔺 Cipolletti Weir

> **Cipolletti weir** is a trapezoidal weir with side slopes of 1 horizontal to 4 vertical.

Key feature: No end contraction factor required due to designed side slopes.

Discharge equation: $Q = \frac{2}{3} C_d \times L \times \sqrt{2g} H^{3/2}$

With velocity of approach: $Q = \frac{2}{3} C_d \times L \times \sqrt{2g}[(H + h_a)^{3/2} - h_a^{3/2}]$

📋 Worked Example

Given: Cipolletti weir L = 60 cm, H = 360 mm, channel = 80 cm × 50 cm, $C_d = 0.6$

Solution:

- Initial discharge: $Q = 0.2296$ m³/sec
- $V_a = \frac{0.2296}{0.4} = 0.574$ m/s
- $h_a = \frac{0.574^2}{2 \times 9.81} = 0.0168$ m
- **Final discharge**: $Q = 0.2435$ m³/sec

📏 Broad-Crested Weir

> **Broad-crested weir**: Weir with wide crest where $2L > H$

Maximum discharge equation: $Q_{max} = 1.705 \times C_d \times L \times H^{3/2}$

🧮 Worked Example

Given: Broad-crested weir L = 50 m, H = 50 cm = 0.5 m, $C_d = 0.6$

Solution:

- **Without velocity of approach**: $Q_{max} = 1.705 \times 0.6 \times 50 \times (0.5)^{3/2} = 18.084$ m³/sec
- **With velocity of consideration** (A = 50 m²): $Q_{max} = 18.412$ m³/sec

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🌊 Submerged or Drowned Weir

> **Submerged weir**: When downstream water level is above the crest of the weir

Total discharge: $Q = Q_1 + Q_2$

Where:

- $Q_1 = \frac{2}{3} C_d \times L \times \sqrt{2g}(H - h)^{3/2}$ (free portion)
- $Q_2 = C_d \times L \times h \times \sqrt{2g(H - h)}$ (drowned portion)

📏 Current Meters

> **Current meter**: Device for measuring velocity of flow at a point in flowing stream

Working principle: Rotating element speed ∝ flow velocity

Types:

1. **Price Current Meter**: Cup-shaped vanes on vertical spindle
2. **Propeller Type**: 3-blade propeller rotating about horizontal axis

Application: Propeller type preferred for slow-moving water (higher RPM at same velocity)

🛶 Floats

> **Floats**: Simple instruments for measuring flow velocity

Types:

1. **Surface Floats**: Single float (weighted bottle/wood) - measures surface velocity
   - *Conversion*: Average velocity ≈ 0.7-0.95 × surface velocity
2. **Double Floats**: Two connected spherical floats - gives direct average velocity
3. **Rod Floats**: Weighted rod/tube - gives direct average velocity when properly sized

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📐 Area-Velocity Method for Discharge Measurement

> **Discharge = Area of flow × Average velocity of flow**

Area Determination Methods:

(i) Simple Segment Method

Area calculation: $A = x_1d_1 + x_2d_2 + x_3d_3 + ... = \sum(x \times d)$

Where:

- x_1, x_2, x_3 = lengths of segments
- d_1, d_2, d_3 = mean depths of segments

🔄 Centrifugal Pumps

> **Centrifugal pump**: Hydraulic machine converting mechanical energy to hydraulic energy using centrifugal force

Working principle: Based on forced vortex flow - pressure rise ∝ square of tangential velocity

Main parts:

1. **Impeller**: Rotating part with backward-curved vanes
2. **Casing**: Airtight passage surrounding impeller (3 types: volute, vortex, guide blade)
3. **Suction pipe**: With foot valve (non-return) and strainer
4. **Delivery pipe**: Delivers water to required height

Energy conversion: Kinetic energy → pressure energy in casing and discharge nozzle

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⚡ Reciprocating Pumps

> **Reciprocating pump**: Mechanical energy converted to hydraulic energy by piston reciprocating in cylinder

Main parts:

1. Cylinder with piston, piston rod, connecting rod, crank
2. Suction pipe with valve
3. Delivery pipe with valve

Working cycle:

- **Suction stroke** (0°-180°): Creates vacuum, liquid enters through suction valve
- **Delivery stroke** (180°-360°): Pressure increases, liquid forced through delivery valve

Single-Acting Pump Calculations:

Discharge: $Q = \frac{ALN}{60}$

Power required: $P = \frac{\rho g ALN(h_s + h_d)}{60000}$ kW

Double-Acting Pump:

Discharge (twice single-acting): $Q = \frac{2ALN}{60}$

Power: $P = \frac{2\rho g ALN(h_s + h_d)}{60000}$ kW

📊 Slip Definition:

> **Slip** = Theoretical discharge - Actual discharge

💧 Slip in Reciprocating Pumps

Definition

> **Slip** is the difference between theoretical discharge and actual discharge of the pump.

Mathematical expression: $\text{Slip} = Q_{th} - Q_{act}$

Percentage Slip

> **Percentage slip** expresses the slip as a percentage of theoretical discharge.

Formula: $\text{Percentage slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = (1 - C_d) \times 100$ where $C_d = \frac{Q_{act}}{Q_{th}}$ is the coefficient of discharge.

Negative Slip

> **Negative slip** occurs when actual discharge exceeds theoretical discharge.

Conditions for negative slip:

- Delivery pipe is **short**
- Suction pipe is **long**
- Pump running at **high speed**

🔧 Classification of Reciprocating Pumps

Based on Piston Contact

1. **Single-acting pump**: Water contacts **one side** of piston
2. **Double-acting pump**: Water contacts **both sides** of piston

Based on Number of Cylinders

1. **Single cylinder pump**
2. **Double cylinder pump**
3. **Triple cylinder pump**

🧮 Worked Examples

Problem 1: Single-Acting Pump Analysis

Given: $N = 50$ rpm, $Q_{act} = 0.01$ m³/s, $D = 200$ mm, $L = 400$ mm

Solution:

- **Area:** $A = \frac{\pi}{4} \times 0.2^2 = 0.031416$ m²

- **Theoretical discharge:** $Q_{th} = \frac{ALN}{60} = \frac{0.031416 \times 0.4 \times 50}{60} = 0.01047$ m³/s
- **Coefficient of discharge:** $C_d = \frac{0.01}{0.01047} = 0.955$
- **Slip:** $0.01047 - 0.01 = 0.00047$ m³/s
- **Percentage slip:** $(1 - 0.955) \times 100 = 4.5$

Problem 2: Double-Acting Pump

Given: $N = 40$ rpm, $Q_{act} = 1.0$ m³/min, $D = 200$ mm, $L = 400$ mm, $h_s = 5$ m, $h_d = 20$ m

Solution:

- **Theoretical discharge:** $Q_{th} = \frac{2ALN}{60} = \frac{2 \times 0.031416 \times 0.4 \times 40}{60} = 0.01675$ m³/s
- **Slip:** $0.01675 - 0.01666 = 0.00009$ m³/s
- **Power required:** $P = \frac{2\rho g ALN(h_s + h_d)}{60000} = \frac{2 \times 1000 \times 9.81 \times 0.031416 \times 0.4 \times 40 \times 25}{60000} = 4.109$ kW

⚡ Turbines: Basic Concepts

> **Turbine**: Hydraulic machine that converts **hydraulic energy** (water energy) into **mechanical energy**

Classification by Energy Type

1. **Impulse turbine**: Only **kinetic energy** at inlet (e.g., Pelton wheel)
2. **Reaction turbine**: Both **kinetic and pressure energy** at inlet

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🎯 Pelton Wheel Turbine

> **Pelton wheel**: **Tangential flow impulse turbine** for **high heads**

Key Features

- Water strikes buckets **tangentially**
- **Atmospheric pressure** throughout
- Named after L.A. Pelton, American engineer

Main Components

1. **Nozzle and flow regulating arrangement** (spear)
2. **Runner and buckets**
3. **Casing**
4. **Breaking jet**

Working Principle

- Nozzle converts pressure to **kinetic energy**
- Jet strikes **splitter** dividing flow equally
- Deflection angle: **160°-170°**
- Generator converts mechanical to electrical energy

🔄 Radial Flow Reaction Turbines

> **Radial flow turbines**: Water flows in **radial direction**

Types

1. **Inward radial flow**: Outer → Inner diameter
2. **Outward radial flow**: Inner → Outer diameter

Main Components

1. **Spiral casing** (decreasing cross-section)
2. **Guide mechanism** (stationary vanes)
3. **Runner** (moving vanes)
4. **Draft tube** (pressure recovery)

🌊 Francis Turbine

> **Francis turbine**: **Inward radial flow reaction turbine** with **mixed flow** (radial inlet, axial outlet)

Main Components

1. **Spiral casing** - maintains uniform pressure
2. **Stay vanes** - reduces swirling
3. **Guide vanes** (wicket gates) - controls flow angle
4. **Runner blades** - converts energy
5. **Draft tube** - recovers pressure

Working Process

1. Water enters **spiral casing**
2. Passes through **stay vanes** and **guide vanes**
3. Strikes **runner blades** at controlled angle
4. Exits through **draft tube** to tailrace

🌀 Axial Flow Reaction Turbines

> **Axial flow turbines**: Water flows **parallel to shaft axis**

Types

1. **Propeller turbine**: **Fixed vanes** on hub
2. **Kaplan turbine**: **Adjustable vanes** on hub

Kaplan Turbine Components

1. **Scroll casing**
2. **Guide vanes mechanism**
3. **Hub with adjustable vanes** (runner)
4. **Draft tube**

Key Equations

- **Peripheral velocity:** $u = \frac{\pi D_0 N}{60}$
- **Flow area:** $A = \frac{\pi}{4}(D_0^2 - D_b^2)$
- **Velocity of flow:** $V_{f1} = V_{f2}$ (inlet = outlet)

🔬 Reynolds Number & Flow Regimes

> **Reynolds number**: Dimensionless quantity determining flow regime

Formula: $Re = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$

Flow Classification

Flow Type	Reynolds Number	Characteristics

Laminar	$Re < 2000$	Smooth, parallel layers
Transitional	$2000 < Re < 4000$	Mixed behavior
Turbulent	$Re > 4000$	Irregular, mixing

Critical Velocities

- **Lower critical velocity**: Transition from laminar to mixed flow
- **Upper critical velocity**: Transition from mixed to turbulent flow

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📊 Velocity Distribution

Laminar Flow

- **Parabolic profile**
- **Maximum velocity** = 2 × mean velocity
- **Zero velocity** at pipe walls

Turbulent Flow

- **Flatter profile**
- **Maximum velocity** at center
- **Non-zero velocity** at walls

⚡ Energy Losses in Pipes

Major Losses (Friction)

1. **Darcy-Weisbach Formula:** $h_f = \frac{4fLV^2}{2gD}$ where $f = \frac{16}{Re}$ for $Re < 2000$ or $f = \frac{0.079}{Re^{1/4}}$ for $4000 < Re < 10^6$
2. **Chezy's Formula:** $V = C\sqrt{mi}$ where $m = \frac{D}{4}$ and $i = \frac{h_f}{L}$

Minor Losses

- **Sudden expansion**
- **Sudden contraction**
- **Pipe entrance/exit**
- **Bends and fittings**

- **Obstructions**

🧮 Practice Problems

Problem 1: Head Loss Calculation

Given: $D = 300$ mm, $L = 50$ m, $V = 3$ m/s, $\nu = 0.01$ stoke, $C = 60$

Solution:

- $Re = \frac{3 \times 0.3}{0.01 \times 10^{-4}} = 900,000$
- $f = \frac{0.079}{(900,000)^{1/4}} = 0.00256$
- **Darcy:** $h_f = \frac{4 \times 0.00256 \times 50 \times 3^2}{2 \times 9.81 \times 0.3} = 0.7828$ m
- **Chezy:** $h_f = 1.665$ m

Problem 2: Power Requirement

Given: $S = 0.7$, $D = 300$ mm, $Q = 500$ L/s, $L = 1000$ m, $\nu = 0.29$ stokes

Solution:

- $V = 7.073$ m/s, $Re = 7.316 \times 10^4$
- $f = 0.0048$, $h_f = 163.18$ m
- **Power required:** $P = \frac{700 \times 9.81 \times 0.5 \times 163.18}{1000} = 560.28$ kW

🔍 Minor Losses in Pipes

> **Minor losses** are head losses occurring in pipelines due to various fittings, changes in cross-section, and other local disturbances beyond friction losses.

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a. Loss of Head Due to Sudden Enlargement

> When a pipe abruptly increases in diameter, eddies form causing energy dissipation.

Formula: $h_e = \frac{(V_1 - V_2)^2}{2g}$

Where:

- V_1 = velocity in smaller pipe
- V_2 = velocity in larger pipe
- g = acceleration due to gravity

b. Loss of Head Due to Sudden Contraction

> Loss occurs when fluid flows from larger pipe into smaller pipe with flow separation at vena contracta.

Formula: $h_c = k\frac{V_2^2}{2g}$

Where:

- $k = \left[\frac{1}{C_c} - 1\right]^2$
- If $C_c = 0.62$, then $k = 0.375$
- If C_c unknown, use $h_c = 0.5\frac{V_2^2}{2g}$

📊 Worked Examples

Problem 1: Sudden Enlargement

- $D_1 = 200$ mm, $D_2 = 400$ mm, $Q = 250$ L/s
- $A_1 = 0.0314$ m², $A_2 = 0.12564$ m²
- $V_1 = 7.96$ m/s, $V_2 = 1.99$ m/s
- **Head loss:** $h_e = \frac{(7.96-1.99)^2}{2\times9.81} = 1.816$ m

Problem 2: Sudden Contraction

- $D_1 = 500$ mm, $D_2 = 250$ mm
- $p_1 = 13.734$ N/cm², $p_2 = 11.772$ N/cm²
- Using Bernoulli's equation with contraction loss
- **Result:** $V_2 = 5.467$ m/s, $h_c = 0.571$ m, $Q = 0.2683$ m³/s

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c. Loss at Pipe Entrance

> Energy loss occurs when liquid enters pipe from large tank/reservoir.

Formula: $h_i = 0.5\frac{V^2}{2g}$ (where V = pipe velocity)

d. Loss at Pipe Exit

> Kinetic energy of exiting jet is dissipated.

Formula: $h_o = \frac{V^2}{2g}$

e. Loss Due to Obstruction

> Head loss from area reduction and subsequent enlargement beyond obstruction.

Formula: $h_{obs} = \frac{V^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]^2$

Where:

- A = pipe area
- a = maximum obstruction area
- C_c = coefficient of contraction

f. Loss Due to Pipe Bends

> Separation and eddy formation in bends cause energy loss.

Formula: $h_b = k\frac{V^2}{2g}$

Where k depends on:

- Bend angle
- Radius of curvature
- Pipe diameter

g. Loss in Pipe Fittings (Valves, Couplings)

> Expressed as coefficient times velocity head.

General Formula: $h_{fittings} = k\frac{V^2}{2g}$

Where k varies by fitting type and size.

📋 Comprehensive Problem

Given: Horizontal pipe $D = 200$ mm, $L = 50$ m, connects tank at $H = 4$ m, $f = 0.009$

Solution: Using Bernoulli's equation including all minor losses: $4 = \frac{V^2}{2g}[1 + 0.5 + \frac{4 \times 0.009 \times 50}{0.2}]$ $V = 2.734\ \text{m/s}, \quad Q = 0.08589\ \text{m³/s}$

📈 Hydraulic and Energy Lines

Hydraulic Gradient Line (HGL)

> Line showing sum of pressure head and datum head at any section.

- HGL = $\frac{p}{w} + z$
- Coincides with free surface in open channels

Total Energy Line (TEL)

> Line showing sum of pressure, datum, and kinetic heads.

- TEL = $\frac{p}{w} + z + \frac{V^2}{2g}$
- Always lies above HGL by velocity head amount

🔗 Pipes in Series (Compound Pipes)

> Pipes of different lengths/diameters connected end-to-end.

Key Principles:

1. **Same discharge** through each pipe: $Q = A_1V_1 = A_2V_2 = A_3V_3$
2. **Total head loss** = sum of all losses (friction + minor)

With minor losses: $H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2gD_1} + \frac{0.5V_2^2}{2g} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2gD_3} + \frac{V_3^2}{2g}$

Neglecting minor losses: $H = \frac{4f_1L_1V_1^2}{2gD_1} + \frac{4f_2L_2V_2^2}{2gD_2} + \frac{4f_3L_3V_3^2}{2gD_3}$

📊 Parallel Pipes Problem

Given: Main divides into two branches (L_1=2000 m, d_1=1.0 m; L_2=2000 m, d_2=0.8 m) with total Q = 3 m³/s, f = 0.005

Solution: Using equal head loss principle: $\frac{V_1^2}{d_1} = \frac{V_2^2}{d_2} \Rightarrow V_1 = \frac{V_2}{0.894}$

With continuity: $Q_1 + Q_2 = 3$ m³/s

- **Results:** $Q_1 = 1.906$ m³/s, $Q_2 = 1.094$ m³/s

💥 Water Hammer in Pipes

> Pressure surge caused when flowing fluid is suddenly stopped (valve closure).

Mechanism:

- Momentum destruction creates high-pressure wave
- Wave travels at sound velocity
- Creates "knocking" noise and hammering action

Factors affecting pressure rise:

1. Flow velocity (V)
2. Pipe length (L)
3. Valve closure time
4. Pipe material elasticity

Cases studied:

1. **Gradual valve closure**
2. **Sudden closure (rigid pipe)**
3. **Sudden closure (elastic pipe)**

Effects:

- Knocking/water hammer sounds
- Enormous hoop stresses
- Potential pipe burst

🌊 Unit-V: Flow Through Open Channels

> **Open channel flow** is liquid flow with **free surface** at **atmospheric pressure**.

Characteristics:

- Flow driven by **gravity** (bed slope)
- **HGL coincides with free surface**
- Examples: rivers, canals, flumes

📐 Classification of Open Channel Flow

Flow Type	Definition	Mathematical Condition
Uniform	Constant depth, velocity, slope, cross-section	$\frac{\partial y}{\partial s} = 0, \frac{\partial v}{\partial s} = 0$
Non-uniform	Varying flow properties	$\frac{\partial y}{\partial s} \neq 0, \frac{\partial v}{\partial s} \neq 0$

Non-uniform flow subdivisions:

1. **Rapidly Varied Flow (R.V.F.)**: Abrupt depth changes over short length
2. **Gradually Varied Flow (G.V.F.)**: Gradual depth changes over long length

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🧮 Chezy's Formula for Velocity

> Empirical formula relating velocity to channel characteristics.

Formula: $V = C\sqrt{mi}$

Where:

- V = velocity (m/s)
- C = Chezy's constant
- m = hydraulic mean depth = $\frac{A}{P}$
- i = bed slope

Discharge: $Q = AV = AC\sqrt{mi}$

📊 Worked Examples

Rectangular Channel (6 m × 3 m):

- $A = 18$ m², $P = 12$ m, $m = 1.5$ m

- $C = 55$, $i = \frac{1}{2000}$
- **Results:** $V = 1.506$ m/s, $Q = 27.108$ m³/s

Trapezoidal Channel:

- Width = 8 m, depth = 2.4 m, side slope = 1:3
- $A = 21.12$ m², $P = 13.058$ m, $m = 1.617$ m
- **Result:** $Q = 21.23$ m³/s

V-Shaped Channel:

- Depth = 4 m, 30° slope each side
- $A = 9.2376$ m², $P = 9.2375$ m, $m = 1.0$ m
- **Result:** $Q = 16.066$ m³/s

📏 Empirical Formulas for Chezy's Constant

Formula	Expression	Parameters
Bazin	$C = \frac{157.6}{1.81 + \frac{k}{\sqrt{m}}}$	k = Bazin's constant (surface roughness)
Ganguillet-Kutter	$C = \frac{23 + \frac{0.00155}{i} + \frac{1}{N}}{1 + \left(23 + \frac{0.00155}{i}\right)\frac{N}{\sqrt{m}}}$	N = Kutter's roughness coefficient
Manning	$C = \frac{1}{N} m^{1/6}$	N = Manning's roughness coefficient

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🔢 Practical Problem Solutions

Using Bazin's Formula (k = 2.36):

- Rectangular channel: 2.5 m × 1.5 m, i = 1/2000
- $m = 0.682$ m, $C = 33.76$
- **Result:** $Q = 2.337$ m³/s

Using Manning's Formula (N = 0.012):

- Rectangular channel: 2 m × 1.5 m, i = 4/8000
- $m = 0.6$ m, $C = 55.12$
- **Result:** $Q = 4.74$ m³/s

🎯 Most Economical Channel Sections

> **Most economical section**: A channel section with **minimum construction cost** achieved by minimizing wetted perimeter for a given discharge.

Key principle: For constant area A, bed slope i, and resistance coefficient C, discharge Q is maximum when wetted perimeter P is minimum.

📊 General Condition

$Q = AC\sqrt{mi} = AC\sqrt{\frac{A}{P}i}$

Since A, C, and i are constant: $Q \propto \frac{1}{\sqrt{P}}$

Therefore: **Maximum discharge occurs when wetted perimeter is minimum**

📐 Most Economical Rectangular Channel

For a rectangular channel with width b and depth d:

Area: $A = b \times d$

Wetted perimeter: $P = b + 2d$

Derivation for minimum perimeter: $\frac{dP}{dd} = 0$

$\frac{d}{dd}\left(\frac{A}{d} + 2d\right) = -\frac{A}{d^2} + 2 = 0$

Resulting conditions:

1. $b = 2d$ (width equals twice the depth)
2. $m = \frac{d}{2}$ (hydraulic mean depth equals half the depth)

📊 Example Problems - Rectangular Channels

Problem 1: Maximum Discharge

Given: $b = 4$ m, $i = \frac{1}{1500}$, $C = 50$

Solution:

- For most economical: $d = \frac{b}{2} = 2$ m
- $A = 4 \times 2 = 8$ m²

- $m = \frac{d}{2} = 1$ m
- $Q = 8 \times 50 \times \sqrt{1 \times \frac{1}{1500}} = 10.328$ m³/s

Problem 2: Find Dimensions for Given Discharge

Given: $Q = 400$ L/s = 0.4 m³/s, $i = \frac{1}{2000}$, $C = 50$

Solution:

- For most economical: $b = 2d$, $m = \frac{d}{2}$
- $A = 2d^2$
- $0.4 = 2d^2 \times 50 \times \sqrt{\frac{d}{2} \times \frac{1}{2000}}$
- $d^{5/2} = 0.253$
- $d = 0.577$ **m**, $b = 1.154$ **m**

📐 Most Economical Trapezoidal Section

For trapezoidal channel with side slope 1 vertical to n horizontal:

Area: $A = (b + nd)d$

Wetted perimeter: $P = b + 2d\sqrt{n^2 + 1}$

Optimisation condition: $\frac{b+2nd}{2} = d\sqrt{n^2 + 1}$

Key result: **Half of top width equals sloping side length**

Hydraulic mean depth: $m = \frac{d}{2}$

📊 Example Problems - Trapezoidal Channels

Problem 1: Most Economical Dimensions

Given: Side slopes $1 : 2$, $A = 40$ m², $i = \frac{1}{1500}$, $C = 50$

Solution:

- $n = \frac{1}{2}$
- From condition: $b = 1.236d$
- $A = 1.736d^2 = 40$

- $d = 4.8$ **m**, $b = 5.933$ **m**
- $m = 2.4$ m
- $Q = 40 \times 50 \times \sqrt{2.4 \times \frac{1}{1500}} = 80$ m³/s

Problem 2: Optimum Dimensions for Given Flow

Given: Side slopes $3:4$, $Q = 0.5$ m³/s, $i = \frac{1}{2000}$, $C = 80$

Approach: Apply trapezoidal optimization condition and solve for dimensions (student exercise)

🔄 Channel Optimization Summary

Channel Type	Width Condition	Depth Condition	Hydraulic Depth
Rectangular	$b = 2d$	$d = \frac{b}{2}$	$m = \frac{d}{2}$
Trapezoidal	$\frac{b+2nd}{2} = d\sqrt{n^2+1}$	Variable	$m = \frac{d}{2}$

Universal principle: For any channel shape, the **most economical section** occurs when the **hydraulic mean depth equals half the flow depth** for the given cross-sectional area.