Chittagong Independent University (CIU)

School of Science & Engineering (SSE)

Final Project Report

Project title   : A new method for rooting nonlinear equations based on the Bisection method

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INDEX

Chapter 1
Introduction

Why are nonlinear issues of importance to us? The fact that all of these issues are nonlinear from the outset when a phenomenon is being studied is arguably the most significant factor. Though we admit that we are unable to meet the challenge posed by nonlinear issues as such linearization, the generally used linearization is an approximate tool.

Nonlinear equations are widely used in basic sciences and engineering to model physical issues. It is therefore simpler to employ numerical approaches because it can be difficult to solve them in numerous sciences. An essential component of applied mathematics is the study of solving nonlinear equations. Because the majority of phenomena that occur in the actual world may be represented using a nonlinear equation or set of nonlinear equations.Since it can be highly challenging and occasionally impossible to identify the root(s) of a nonlinear equation, we frequently employ repeating numerical approaches to find the root (s). These days, many complex and challenging issues, as well as nonlinear equations, can be solved thanks to computer breakthroughs and the expansion of applied software.

To discover the roots of nonlinear equations, techniques like artificial neural networks, Monte Carlo, and fuzzy logic are utilized. These techniques can address some of the shortcomings of earlier techniques like Newton Raphson, Bisection Method, False Position, and Secant Method.

The following are some of the proposed approaches' shortcomings: Depending on the starting point (S1), it might not converge (S2), rely on variations in sign (A lower guess (a) such that $f(a)*f(b)0$ cannot be found if a function $f(x)$ is such that it simply touches the x-axis, for instance, $f(x)=x2$). (S3), expensive calculations (S4), unable to identify several roots (S5).

The Monte Carlo and Bisection approach, which largely overcomes the shortcomings of well-known existing methods, is utilized in this study to discover the roots of nonlinear equations. The structure of this study is as follows: A review of earlier techniques is given in section 2. We discuss a Monte Carlo-based method for locating multiple roots of nonlinear equations in Section 3. The Result and Discussion are presented in Section 4. The conclusions are presented in Section 5.

Shortcoming:

Methods	s1	s2	s3	s4	s5
Bisection	☒	☒	☑	☒	☑
Newton Rapson	☑	☑	☒	☑	☑
False Position	☒	☒	☑	☒	☑
Secant	☒	☑	☒	☑	☑
New Method(IBMS)	☒	☒	☒	☒	☒

Chapter 2

Literature View

Summary of other methods:

Bisection technique

The bisection method is one of the most straightforward algorithms for locating algebraic equations. Any continuous function f on in the domain [a,b] can be solved using this method. The Intermediate Value Theorem states that there must be at least one root in this range if f(a).f(b) have opposing signs. The range [a,b] is split into two halves to identify the root. c=(a+b)/2 , The domain will be confined by [a,c] if f(a).f(c) is negative; otherwise, it will be replaced by [a,c]. The distance's length that contains root is half after each repetition.

Several drawbacks include (S3) and (S5)

Method of Newton-Raphson

Newton's method is one of the numerical techniques that derivation uses and is applied to in a variety of domains. Equations can be solved numerically and roughly using this approach, which can converge more quickly than the bisection method [5].

Several drawbacks are (S1), (S2), (S4), and (S5).

Method of false position

The bisection method and the false position method, often known as the regula false method, are comparable. It only differs from the bisection approach in that it converges more quickly. In this approach, a chord connecting the points and is used

to shorten the distance rather than halving it. The chord's intersection with the x-axis is known as the point. If is negative, the domain will be limited by, otherwise [9] will be used in its place.

Several drawbacks include (S3) and (S5).

Secant approach

This approach is distinct from Newton's approach in that it does not call for a function's derivative. Additionally, it is not necessary for our two beginning points to be located on either side of the function's root. Convergence is not guaranteed by this procedure. However, if it is convergent, it will go toward the root quickly [5].

Several drawbacks include (S2) and (S5).

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Chapter 3

Methology

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Algorithm for finding multiple roots of nonlinear equations

Using Monte Carlo method, we divide the desired distance into smaller parts. In each section that specifies the change function, we use the Bisection method to find the root. Extreme points close to the y-axis are used to find the tangent root to the x-axis. Algorithm codes in R software are provided in the appendix.

Step 1:

Generating random sample numbers x from a uniform distribution in the interval [a,b]

Step 2:

∃ xi s.t f(xi) f(xi+1)<0 (Like the green dots in figure).
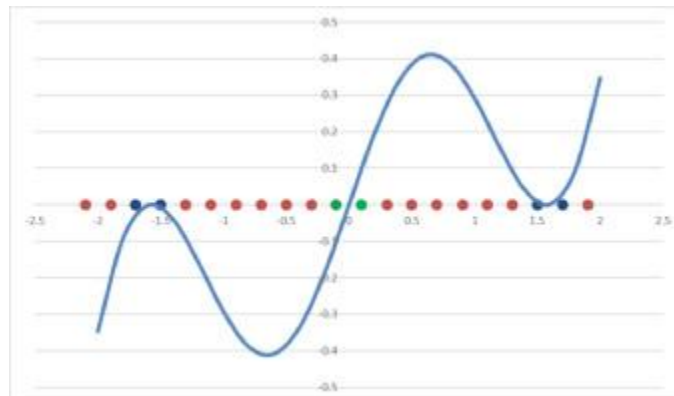

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Fig. 1. Diagram x cos²(x).

</div>

Finding xi.

Step 3:

We use the Bisection method to find the root at distance $[x_i, x_{i+1}]$.

Step 4:

$(f(x_i+\varepsilon)-f(x_i))(f(x_{i+1}+\varepsilon)-f(x_{i+1}))<0$

To find the next root, we repeat the first to third steps with x_{i+1}. Until all the roots are found in the distance $[a,b]$.

Step 5:

For roots that are tangent to the x-axis:

We consider the absolute value of the function, for roots:

$\exists x$ is.t:

$(f(x_i + \varepsilon)-f(x_i))(f(x_{i+1} + \varepsilon)- f(x_{i+1}))<0$

 (Like the blue dots in Fig 1).

Finding x_i.

Step 6:

We use the Bisection method to find the root at distance $[x_i, x_{i+1}]$.

Step 7:

To find the next root, Repeat steps five through seven with x_{i+1}. Until all the roots are found in the distance $[a, b]$.

Step 8:

Finally, we examine the beginning and end points of the interval $[a,b]$

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Chapter 4
Implemantation

The benefit of the suggested approach is that it can locate many nonlinear equations' roots without depending on the initial value or sign changes. The answers before and after must be calculated separately in the prior numerical methods for solving equations of degree n, and as a result, the answers before and after are interdependent. However, the suggested method can find all the equation's roots simultaneously. As a result, this dependence vanishes and this method's benefits virtually outweigh its drawbacks. The root(s) tangent to the x-axis and the root(s) intersecting the x-axis are separated using this method after the root(s) of the equation are discovered.

Chapter 5

Conclusion

In this paper, we used the integration of Monte Carlo method and Bisection method to find the root(s) of nonlinear equations. The approximate position of the root(s) is done by examining a random sample of numbers generated in places where the change function signals and where there are extreme points near the y-axis and convergence to the root using the Bisection method. Hence, the dependence on the initial value disappears and all the roots(s) of the equation can be found at one time. Given the advantages of the proposed method over previous methods, it can be used as a new method in solving algebraic equations.