

Assignment 1 - The LP Model

1. Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of \$32. Each Mini requires 40 minutes of labor and generates a unit profit of \$24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

In the first stage of the solution the various decision variables are identified and formulated

a. Defining the decision variables

Let X_1 and X_2 be the two decision variables two different types of models of backpacks Collegiate and the Mini respectively.

X_1 = Number of Collegiate Backpacks manufactured,

X_2 = Number of Mini type Backpacks manufactured

b. Defining the objective function

The goal is to produce backpacks so as to achieve the highest total profit. Thus, the objective function should calculate the total profit, where the objective will be to maximize the profit.

In this case, the total profit will be presented by

Total Profit = \$32 * Number of Collegiates type backpacks + \$24 *
Number of Mini type backpacks

Hence, $Z=32*X_1+24*X_2$

c. Constraints

There are various constraints in relation with the current problem.

1. Sold limits
2. Time Availability
3. Shipment material related constraints

There is given that there can be maximum 1000 units sold for
Collegiate Backpacks and 1200 for Mini type Backpacks

Hence

$$X_1 \leq 1000$$

$$X_2 \leq 1200$$

Each Collegiate requires 45 minutes of labour to produce and Mini
requires 40 minutes of labour.

There are maximum of 35 labours available and hence time constraints
will be given by

$$45X_1+40X_2 \leq 84000$$

Shipment material related constraints

$$3X_1+2X_2 \leq 5000$$

d. Formulating the problem for LPP

Hence the final Linear programming problem can be formulated as follows

$$\text{Max } Z = 32X_1 + 24X_2$$

Subjected to constraints

$$45X_1 + 40X_2 \leq 84000$$

$$3X_1 + 2X_2 \leq 5000$$

$$X_1 \leq 1000$$

$$X_2 \leq 1200$$

Where $X_1, X_2 \geq 0$

Solution of the problem using Solver in Excel

No of Collegiate, X_1	Number of Mini Backpacks X_2	Objective Function, Z
1000	975	55400

Final Answer

Thus, they should produce 1000 Collegiates and 975 Minis to achieve the maximum total profit of \$55,400.

2. The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way.

This product can be made in three sizes--large, medium, and small--that yield a net unit profit of \$420, \$360, and \$300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

In the first phase the decision variable are studied and the objective function are build for the problem

a: *Formulating the decision variables:*

X_{1L} = number of large units produced per day at Plant 1,

X_{1M} = number of medium units produced per day at Plant 1,

X_{1S} = number of small units produced per day at Plant 1,

X_{2L} = number of large units produced per day at Plant 2,

X_{2M} = number of medium units produced per day at Plant 2,

X_{2S} = number of small units produced per day at Plant 2,

X_{3L} = number of large units produced per day at Plant 3,

X_{3M} = number of medium units produced per day at Plant 3,

X_{3S} = number of small units produced per day at Plant 3.

b: Formulating the Objective function:

Also let the maximization of the profits denoted by Z which is the total net profit per day, which is given by

$$\text{Maximize } Z = 420X_{1L} + 360X_{1M} + 300X_{1S} + 420X_{2L} + 360X_{2M} + 300X_{2S} + 420X_{3L} + 360X_{3M} + 300X_{3S}$$

Which is Subjected to constraints

Capacity constraints

$$X_{1L} + X_{1M} + X_{1S} \leq 750$$

$$X_{2L} + X_{2M} + X_{2S} \leq 900$$

$$X_{3L} + X_{3M} + X_{3S} \leq 450$$

Size availability constraints

$$20X_{1L} + 15X_{1M} + 12X_{1S} \leq 13000$$

$$20X_{2L} + 15X_{2M} + 12X_{2S} \leq 12000$$

$$20X_{3L} + 15X_{3M} + 12X_{3S} \leq 5000$$

Demand constraints

$$X_{1L} + X_{1M} + X_{1S} \leq 900$$

$$X_{2L} + X_{2M} + X_{2S} \leq 1200$$

$$X_{3L} + X_{3M} + X_{3S} \leq 750$$

Also, $X_{1L}, X_{1M}, X_{1S}, X_{2L}, X_{2M}, X_{2S}, X_{3L}, X_{3M}, X_{3S} \geq 0$

Solution of the problem in Excel Solver

The optimized decision variable values are:

X_{1L}	X_{1M}	X_{1S}	X_{2L}	X_{2M}	X_{2S}	X_{3L}	X_{3M}	X_{3S}	Objective Function
350	400	0	0	400	500	1	0	415	709920

Final Answer

The maximized optimized value for the objective function, $Z=709920$