

Assignment 4

2022-10-30

Module 8 - DEA Hope Valley Health Care Association

The Hope Valley Health Care Association owns and operates six nursing homes in adjoining states. An evaluation of their efficiency has been undertaken using two inputs and two outputs. The inputs are staffing labor (measured in average hours per day) and the cost of supplies (in thousands of dollars per day). The outputs are the number of patient-days reimbursed by third- party sources and the number of patient-days reimbursed privately. A summary of performance data is shown in the table below.

```
library(Benchmarking)
```

```
## Warning: package 'Benchmarking' was built under R version 4.1.2
## Loading required package: lpSolveAPI
## Warning: package 'lpSolveAPI' was built under R version 4.1.2
## Loading required package: ucminf
## Warning: package 'ucminf' was built under R version 4.1.2
## Loading required package: quadprog
##
## Loading Benchmarking version 0.30h, (Revision 244, 2022/05/05 16:31:31) ...
## Build 2022/05/05 16:31:40
```

```
library(lpSolveAPI)
```

```
## We used vectors to represent our inputs and outputs. we have 2 inputs (Staff hours, Supplies) and 2
outputs ("Reimbursed Patient_Days", "Privately Paid Patient_Day") .
```

```
x <- matrix(c(150, 400, 320, 520, 350, 320, 0.2, 0.7, 1.2, 2.0, 1.2, 0.7), ncol = 2)
y <- matrix(c(14000, 14000, 42000, 28000, 19000, 14000, 3500, 21000, 10500, 42000, 25000, 15000), ncol = 2)
colnames(y) <- c("Reimbursed Patient_Days", "Privately Paid Patient_Days")
colnames(x) <- c("Staff_Hours", "Supplies")
Table <- cbind(x, y)
row.names(Table) = c("F1", "F2", "F3", "F4", "F5", "F6")
Table
```

	Staff_Hours	Supplies	Reimbursed Patient_Days	Privately Paid Patient_Days
## F1	150	0.2	14000	3500
## F2	400	0.7	14000	21000
## F3	320	1.2	42000	10500
## F4	520	2.0	28000	42000
## F5	350	1.2	19000	25000
## F6	320	0.7	14000	15000

DEA Analysis using all of the DEA assumptions (FDH, CRS, VRS, IRS, DRS, and FRH)

```
CRS <- dea(x,y, RTS = "crs")
print(CRS)
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675
```

CRS - The efficiency rates for facilities 1, 2, 3, and 4 are high, whereas those for facilities 5 and 6 are 98% and 87%, respectively.

```
peers(CRS)
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     1     2     4
## [6,]     1     2     4
```

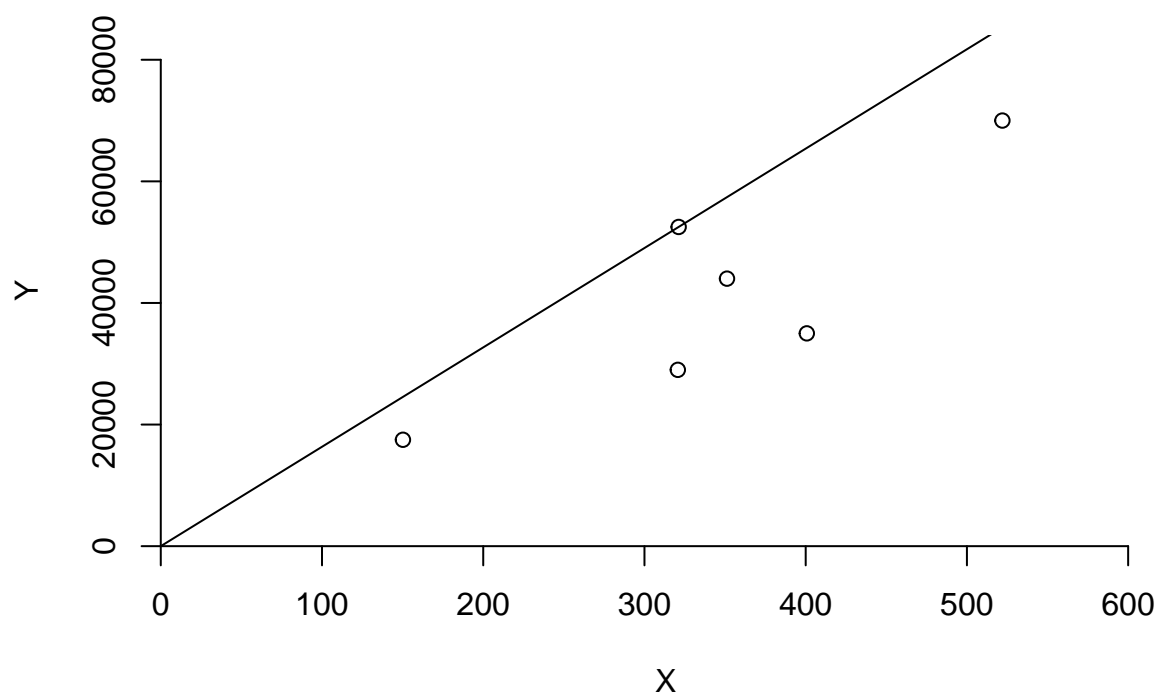
```
CRS_Weights <- lambda(CRS)
CRS_Weights
```

```
##           L1           L2 L3           L4
## [1,] 1.0000000 0.0000000  0 0.0000000
## [2,] 0.0000000 1.0000000  0 0.0000000
## [3,] 0.0000000 0.0000000  1 0.0000000
## [4,] 0.0000000 0.0000000  0 1.0000000
## [5,] 0.2000000 0.08048142  0 0.5383307
## [6,] 0.3428571 0.39499264  0 0.1310751
```

The weights for facility 5 are 0.20, 0.08, 0.54. The weights for facility 6 are 0.34, 0.39, 0.13

```
#Plotting the results
dea.plot(x,y,RTS="crs", main="Constant Returns to Scale (CRS) Graph")
```

Constant Returns to Scale (CRS) Graph



```
FDH <- dea(x,y, RTS= "fdh")
```

```
FDH #All of the facilities are effective.
```

```
## [1] 1 1 1 1 1 1
```

```
peers(FDH) #Each facility's peer is its own.
```

```
##      peer1
```

```
## [1,]      1
```

```
## [2,]      2
```

```
## [3,]      3
```

```
## [4,]      4
```

```
## [5,]      5
```

```
## [6,]      6
```

```
FDH_Weights <- lambda(FDH)
```

```
FDH_Weights
```

```
##      L1 L2 L3 L4 L5 L6
```

```
## [1,]  1  0  0  0  0  0
```

```
## [2,]  0  1  0  0  0  0
```

```
## [3,]  0  0  1  0  0  0
```

```
## [4,]  0  0  0  1  0  0
```

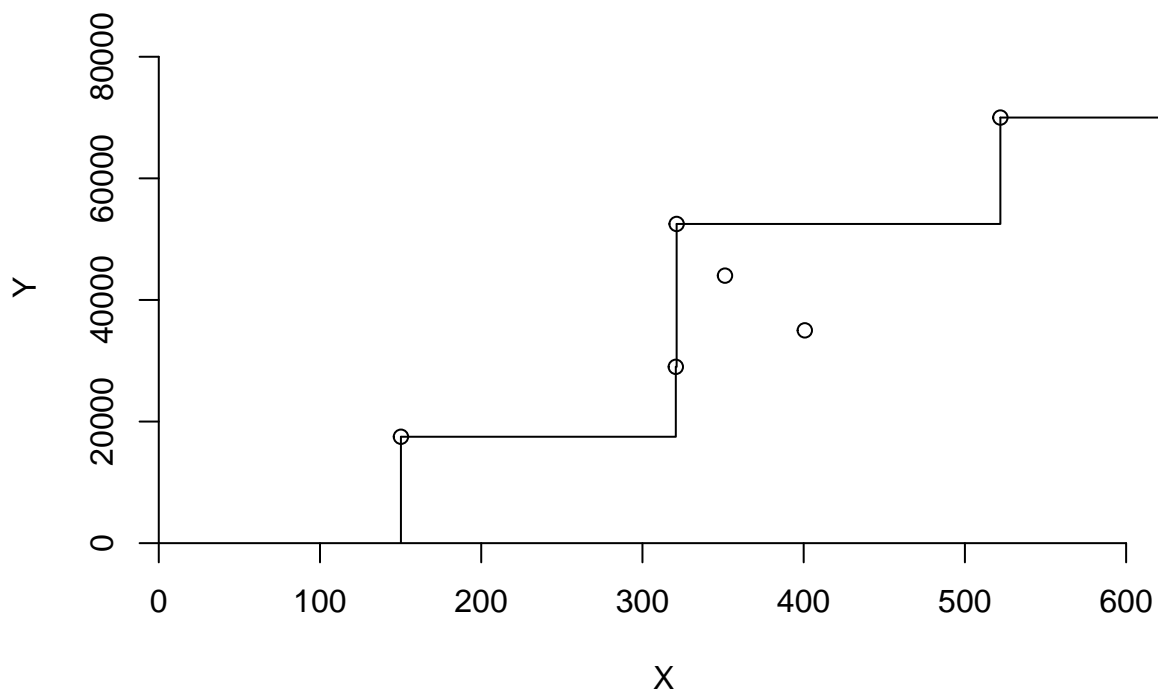
```
## [5,]  0  0  0  0  1  0
```

```
## [6,]  0  0  0  0  0  1
```

```
# Plotting the results
```

```
dea.plot(x,y,RTS="fdh", main="Free disposability hull (FDH) Graph")
```

Free disposability hull (FDH) Graph



```
VRS <- dea(x,y, RTS = "vrs")
VRS #Except for facility 6, all facilities are effective.
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963
```

```
peers(VRS) #peers for facility 6 are 1,2,5
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     5    NA    NA
## [6,]     1     2     5
```

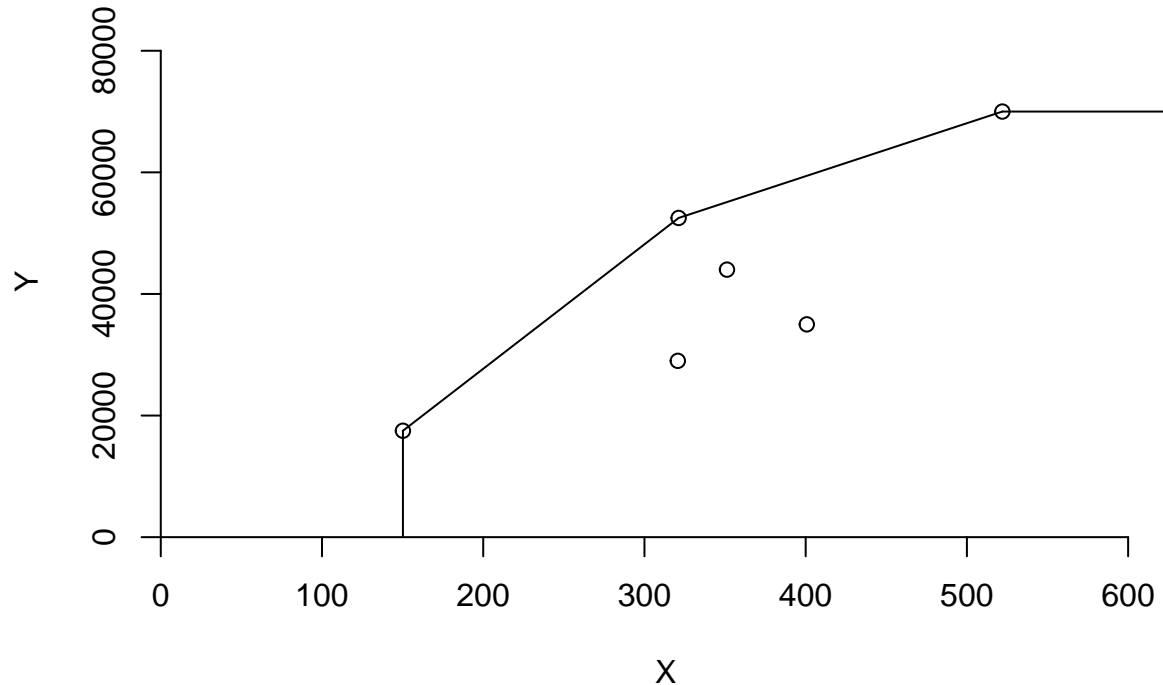
```
VRS_Weights <- lambda(VRS)
VRS_Weights
```

```
##           L1          L2 L3 L4          L5
## [1,] 1.0000000 0.0000000  0  0 0.0000000
## [2,] 0.0000000 1.0000000  0  0 0.0000000
## [3,] 0.0000000 0.0000000  1  0 0.0000000
## [4,] 0.0000000 0.0000000  0  1 0.0000000
## [5,] 0.0000000 0.0000000  0  0 1.0000000
## [6,] 0.4014399 0.3422606  0  0 0.2562995
```

```
# Plotting the results
```

```
dea.plot(x,y,RTS="vrs", main="Variable Returns to Scale (VRS) Graph")
```

Variable Returns to Scale (VRS) Graph



```
IRS <- dea(x,y, RTS= "irs")
IRS #Except for facility, all facilities are efficient.
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963
```

```
peers(IRS)
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     5    NA    NA
## [6,]     1     2     5
```

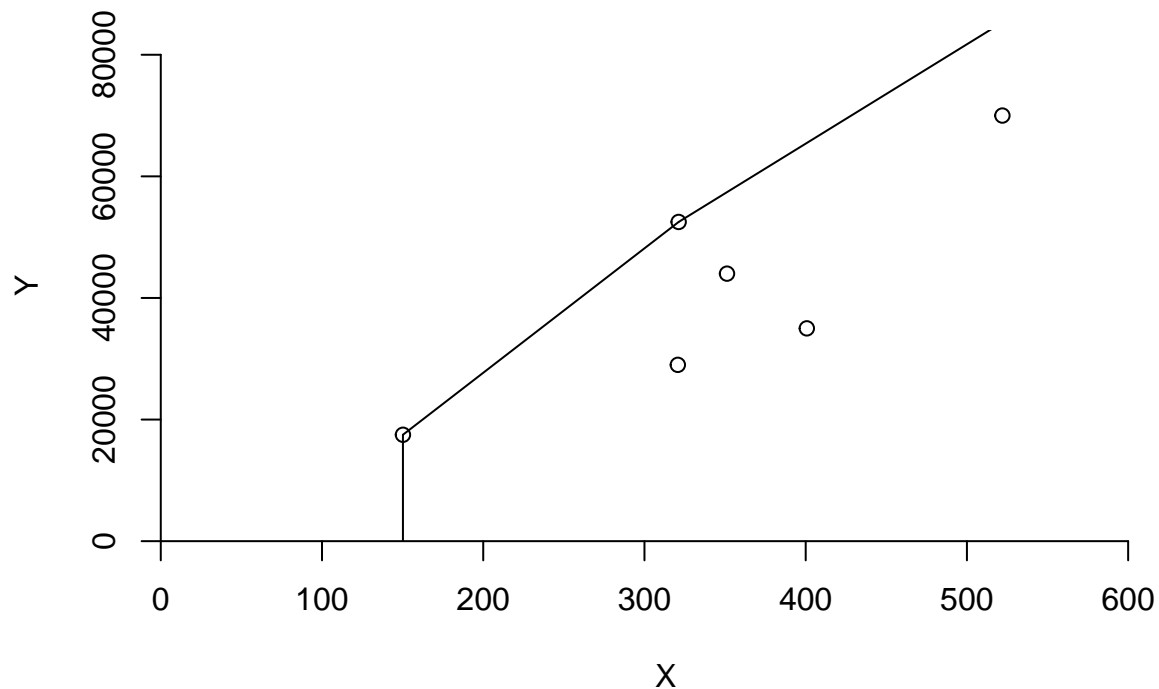
```
IRS_Weights <- lambda(IRS)
IRS_Weights
```

```
##          L1          L2 L3 L4          L5
## [1,] 1.0000000 0.0000000 0 0 0.0000000
## [2,] 0.0000000 1.0000000 0 0 0.0000000
## [3,] 0.0000000 0.0000000 1 0 0.0000000
## [4,] 0.0000000 0.0000000 0 1 0.0000000
## [5,] 0.0000000 0.0000000 0 0 1.0000000
## [6,] 0.4014399 0.3422606 0 0 0.2562995
```

```
# Plotting the results
```

```
dea.plot(x,y,RTS="irs", main="Increasing Returns to Scale (IRS) Graph")
```

Increasing Returns to Scale (IRS) Graph



```
DRS <- dea(x,y, RTS= "drs")
DRS #Except for facility 5,6, all facilities are efficient.
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675
```

```
peers(DRS) # The peers units for for facilities 5,6 are 1,2,4
```

```
##      peer1 peer2 peer3
## [1,]    1    NA    NA
## [2,]    2    NA    NA
## [3,]    3    NA    NA
## [4,]    4    NA    NA
## [5,]    1     2     4
## [6,]    1     2     4
```

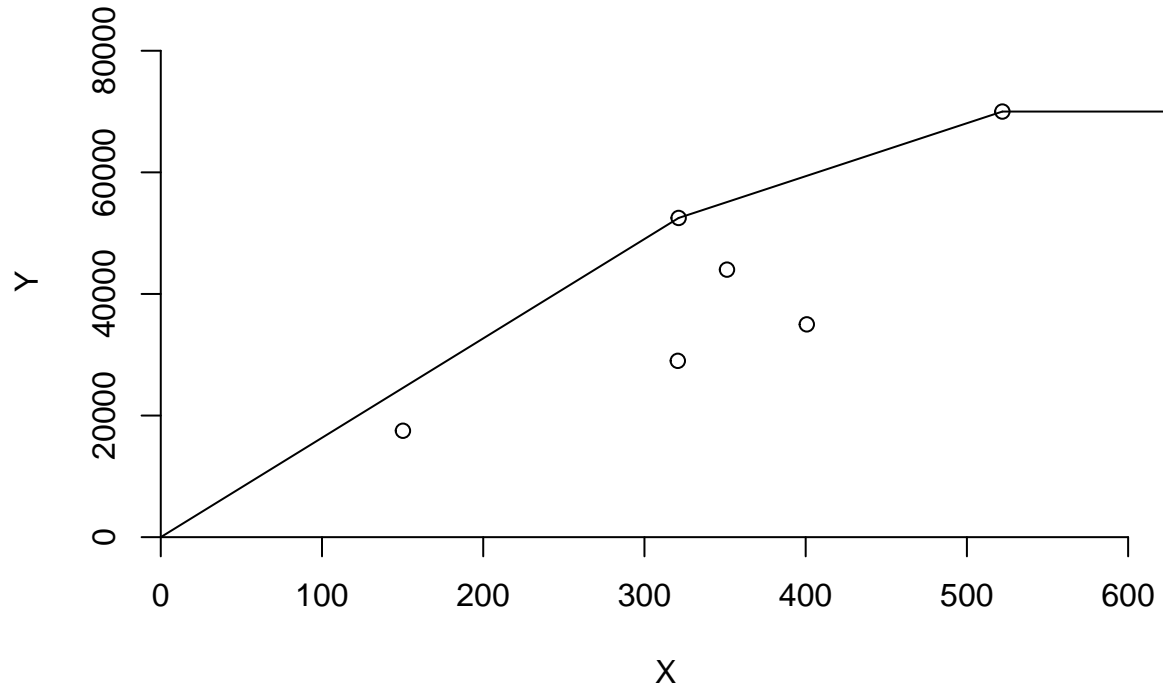
```
DRS_Weights <- lambda(DRS)
DRS_Weights
```

```
##           L1           L2 L3           L4
## [1,] 1.0000000 0.0000000  0 0.0000000
## [2,] 0.0000000 1.0000000  0 0.0000000
## [3,] 0.0000000 0.0000000  1 0.0000000
## [4,] 0.0000000 0.0000000  0 1.0000000
## [5,] 0.2000000 0.08048142  0 0.5383307
## [6,] 0.3428571 0.39499264  0 0.1310751
```

```
# Plotting the results
```

```
dea.plot(x,y,RTS="drs", main="Decreasing Returns to Scale (DRS) Graph")
```

Decreasing Returns to Scale (DRS) Graph



```
FRH <- dea(x,y, RTS= "add")
```

```
FRH #Every facility is effective.
```

```
## [1] 1 1 1 1 1 1
```

```
peers(FRH) #Each facility's peer unit is itself.
```

```
##      peer1
```

```
## [1,]      1
```

```
## [2,]      2
```

```
## [3,]      3
```

```
## [4,]      4
```

```
## [5,]      5
```

```
## [6,]      6
```

```
FRH_Weights <- lambda(FRH)
```

```
FRH_Weights
```

```
##      L1 L2 L3 L4 L5 L6
```

```
## [1,]  1  0  0  0  0  0
```

```
## [2,]  0  1  0  0  0  0
```

```
## [3,]  0  0  1  0  0  0
```

```
## [4,]  0  0  0  1  0  0
```

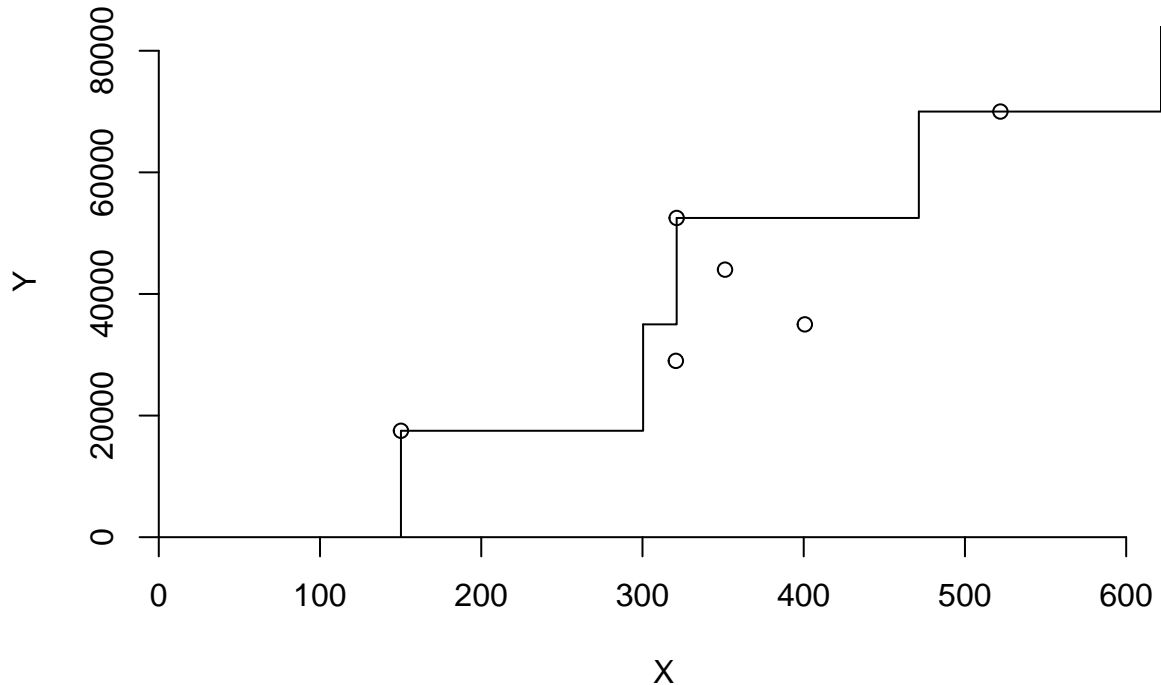
```
## [5,]  0  0  0  0  1  0
```

```
## [6,]  0  0  0  0  0  1
```

```
# Plot the results
```

```
dea.plot(x,y,RTS="add", main="Free Replicability Hull (FRH) Graph")
```

Free Replicability Hull (FRH) Graph

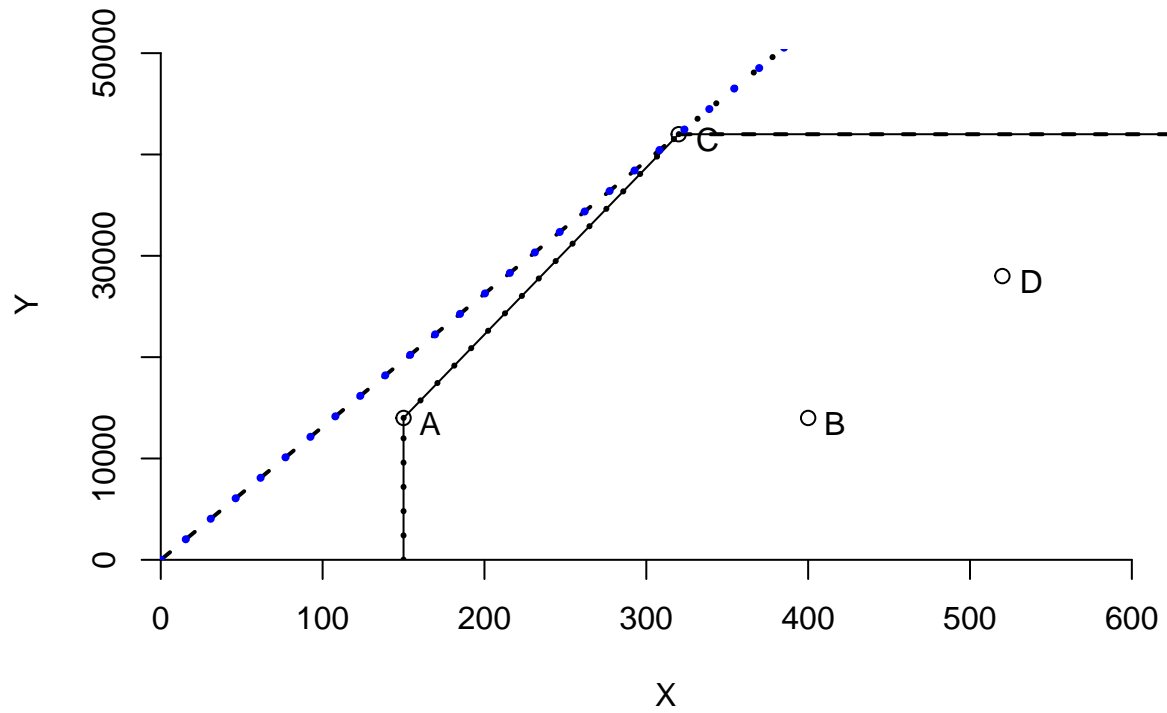


```
as.data.frame(Table)
```

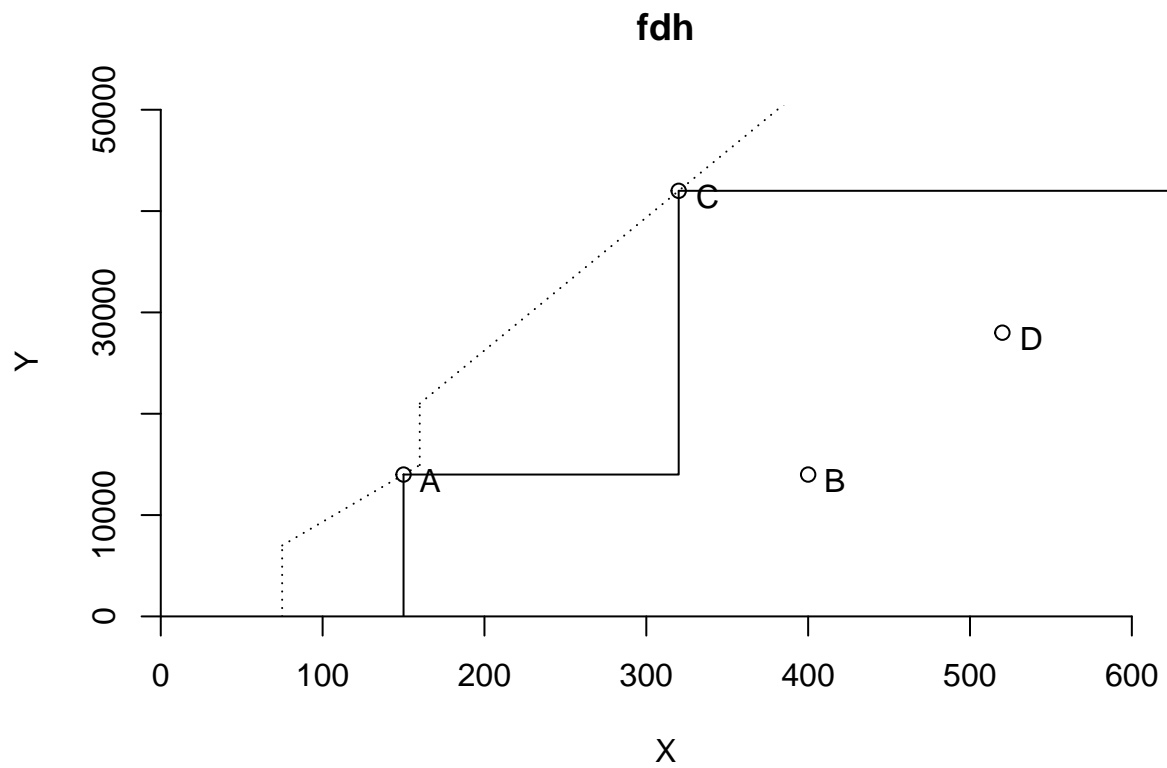
##	Staff_Hours	Supplies	Reimbursed Patient_Days	Privately Paid Patient_Days
## F1	150	0.2	14000	3500
## F2	400	0.7	14000	21000
## F3	320	1.2	42000	10500
## F4	520	2.0	28000	42000
## F5	350	1.2	19000	25000
## F6	320	0.7	14000	15000

Plot of different technologies

```
library(Benchmarking)
x <- matrix(c(150, 400, 320, 520),ncol=1,dimnames=list(LETTERS[1:4],"x"))
y <- matrix(c(14000,14000,42000,28000),ncol=1,dimnames=list(LETTERS[1:4],"y"))
dea.plot(x,y,RTS="vrs",ORIENTATION="in-out",txt=rownames(x))
dea.plot(x,y,RTS="drs",ORIENTATION="in-out",add=TRUE,lty="dashed",lwd=2)
dea.plot(x,y,RTS="irs",ORIENTATION="in-out",add=TRUE,lty="dotted",lwd=3)
dea.plot(x,y,RTS="crs",ORIENTATION="in-out",add=TRUE,lty="dotted",lwd=4, col="blue")
```

```
dea.plot(x,y,RTS="fdh",ORIENTATION="in-out",txt=rownames(x),main="fdh")
dea.plot(x,y,RTS="fdh+",ORIENTATION="in-out",add=TRUE,lty="dotted",param=.5)
```



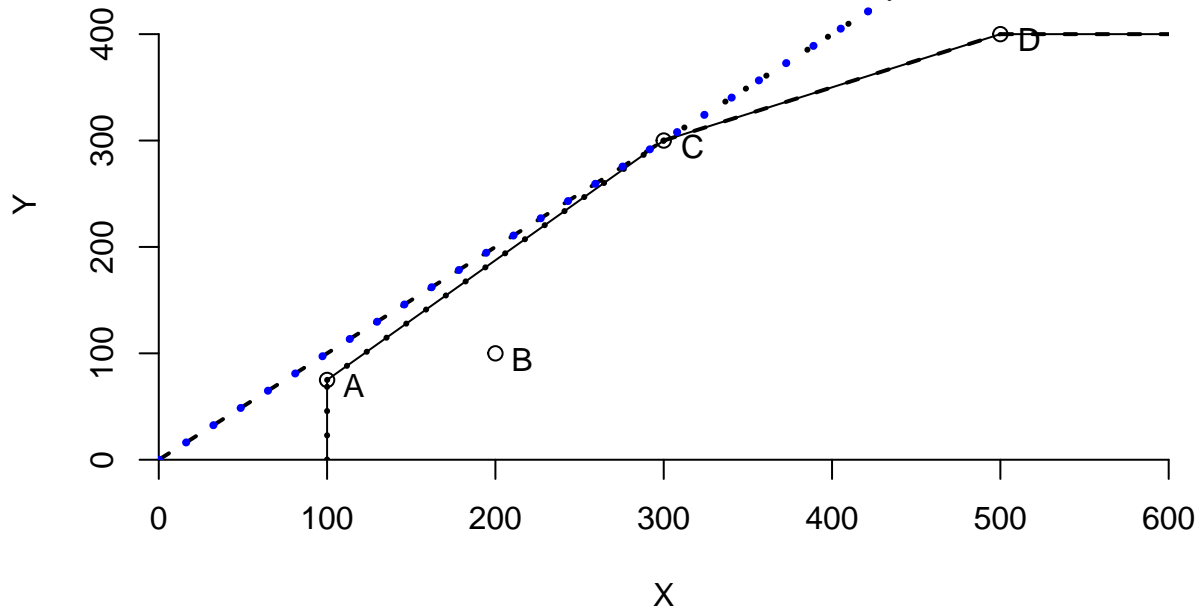
```
# Plot of different technologies
```

```
library(Benchmarking)
```

```

x <- matrix(c(100,200,300,500),ncol=1,dimnames=list(LETTERS[1:4],"x"))
y <- matrix(c(75,100,300,400),ncol=1,dimnames=list(LETTERS[1:4],"y"))
dea.plot(x,y,RTS="vrs",ORIENTATION="in-out",txt=rownames(x))
dea.plot(x,y,RTS="drs",ORIENTATION="in-out",add=TRUE,lty="dashed",lwd=2)
dea.plot(x,y,RTS="irs",ORIENTATION="in-out",add=TRUE,lty="dotted",lwd=3)
dea.plot(x,y,RTS="crs",ORIENTATION="in-out",add=TRUE,lty="dotted",lwd=4,col="blue")

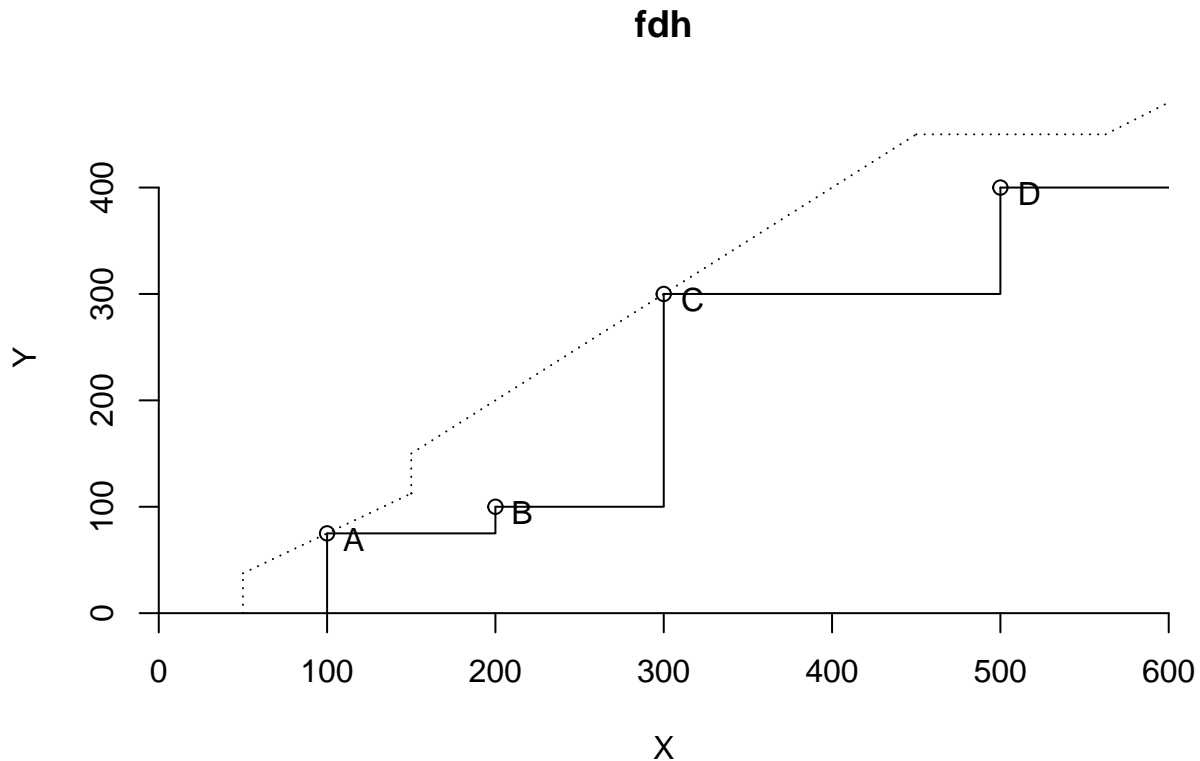
```



```

dea.plot(x,y,RTS="fdh",ORIENTATION="in-out",txt=rownames(x),main="fdh")
dea.plot(x,y,RTS="fdh+",ORIENTATION="in-out",add=TRUE,lty="dotted",param=.5)

```



#3. Summarize your results in a tabular format

```
df<-data.frame (CRS = c(1.0000, 1.0000, 1.0000, 1.0000, 0.9775, 0.8675),
FDH= c(1,1,1,1,1,1), VRS= c(1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 0.8963),IRS =c( 1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 0.8963),
DRS =c( 1.0000, 1.0000, 1.0000, 1.0000, 0.9775, 0.8675),FRH =c( 1.0000, 1.0000, 1.0000, 1.0000, 0.9775, 0.8675))
df
```

	CRS	FDH	VRS	IRS	DRS	FRH
## 1	1.0000	1	1.0000	1.0000	1.0000	1
## 2	1.0000	1	1.0000	1.0000	1.0000	1
## 3	1.0000	1	1.0000	1.0000	1.0000	1
## 4	1.0000	1	1.0000	1.0000	1.0000	1
## 5	0.9775	1	1.0000	1.0000	0.9775	1
## 6	0.8675	1	0.8963	0.8963	0.8675	1

#In each DEA assumption, the efficiency results at each facility

#Observation - CRS and DRS give same results, FDH and FRH gave same results, and finally both VRS and IRS gave same results as well.

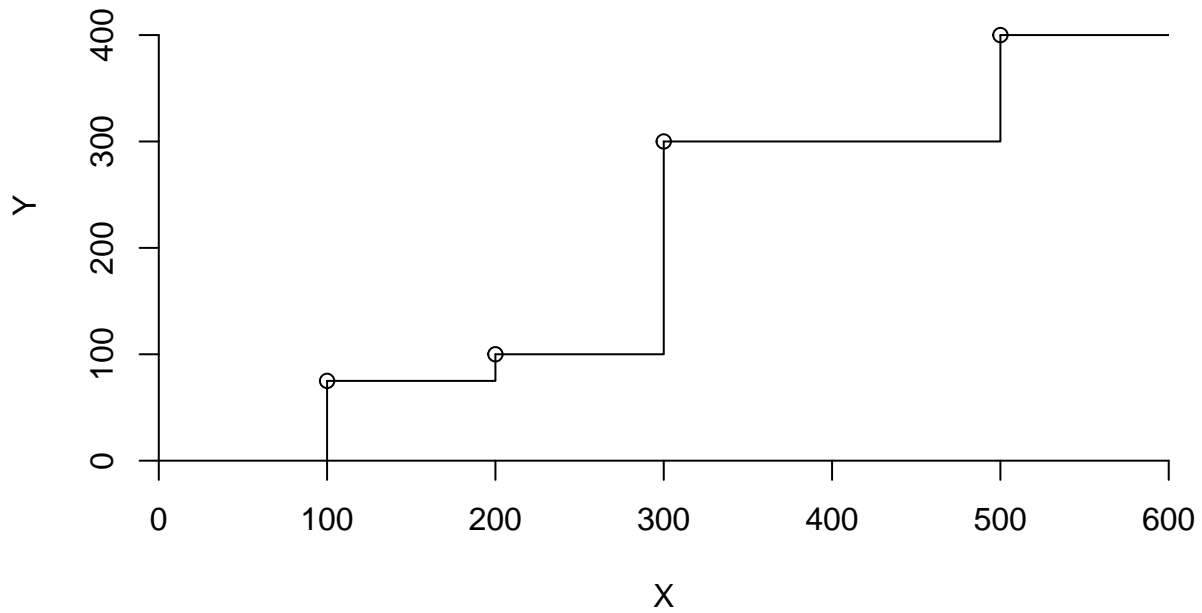
```
results <- cbind(Table, df)
results[, -c(1:4)]
```

	CRS	FDH	VRS	IRS	DRS	FRH
## F1	1.0000	1	1.0000	1.0000	1.0000	1
## F2	1.0000	1	1.0000	1.0000	1.0000	1
## F3	1.0000	1	1.0000	1.0000	1.0000	1
## F4	1.0000	1	1.0000	1.0000	1.0000	1
## F5	0.9775	1	1.0000	1.0000	0.9775	1
## F6	0.8675	1	0.8963	0.8963	0.8675	1

#4. Compare and contrast the above results #Compare between different assumptions

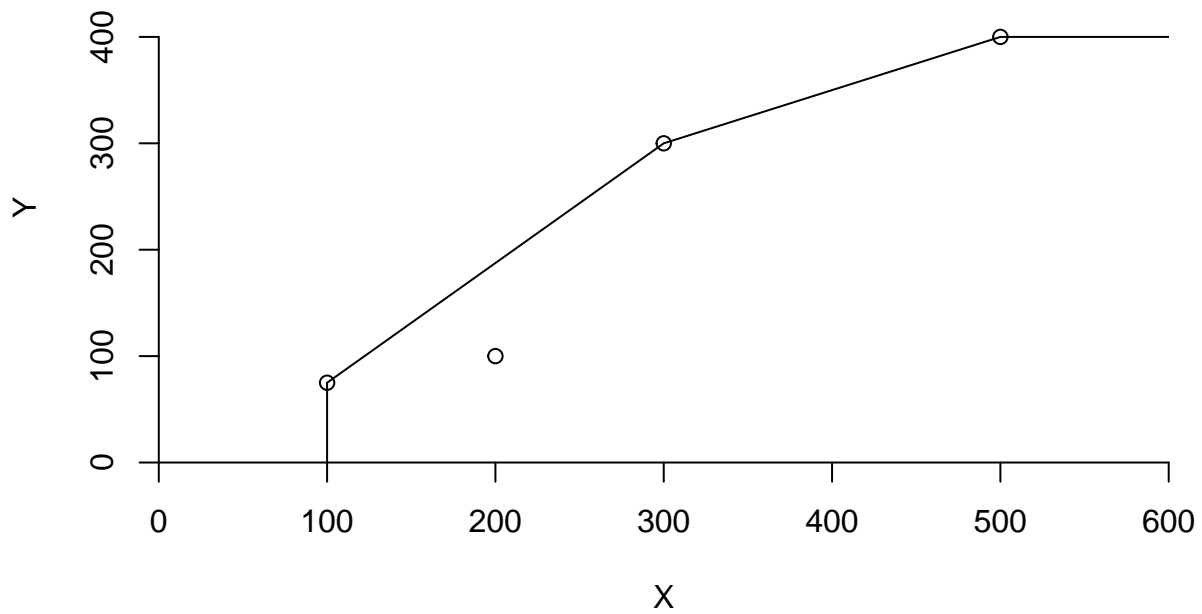
```
dea.plot(x,y,RTS="fdh", main="Free disposability hull (FDH) Graph")
```

Free disposability hull (FDH) Graph



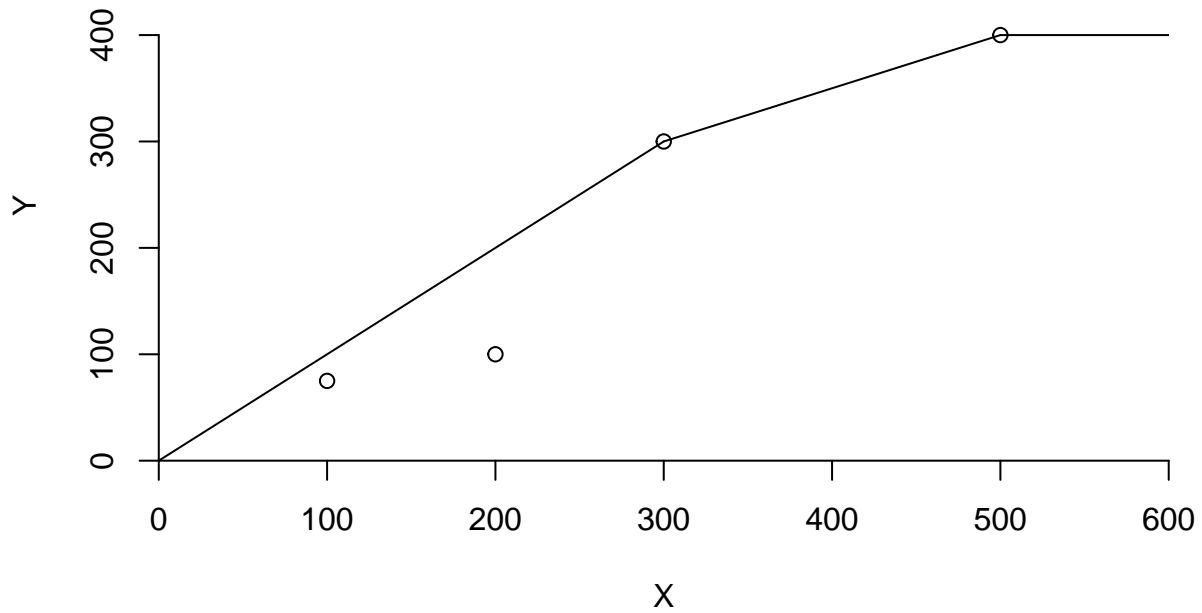
```
dea.plot(x,y,RTS="vrs", main="Variable Returns to Scale (VRS) Graph")
```

Variable Returns to Scale (VRS) Graph



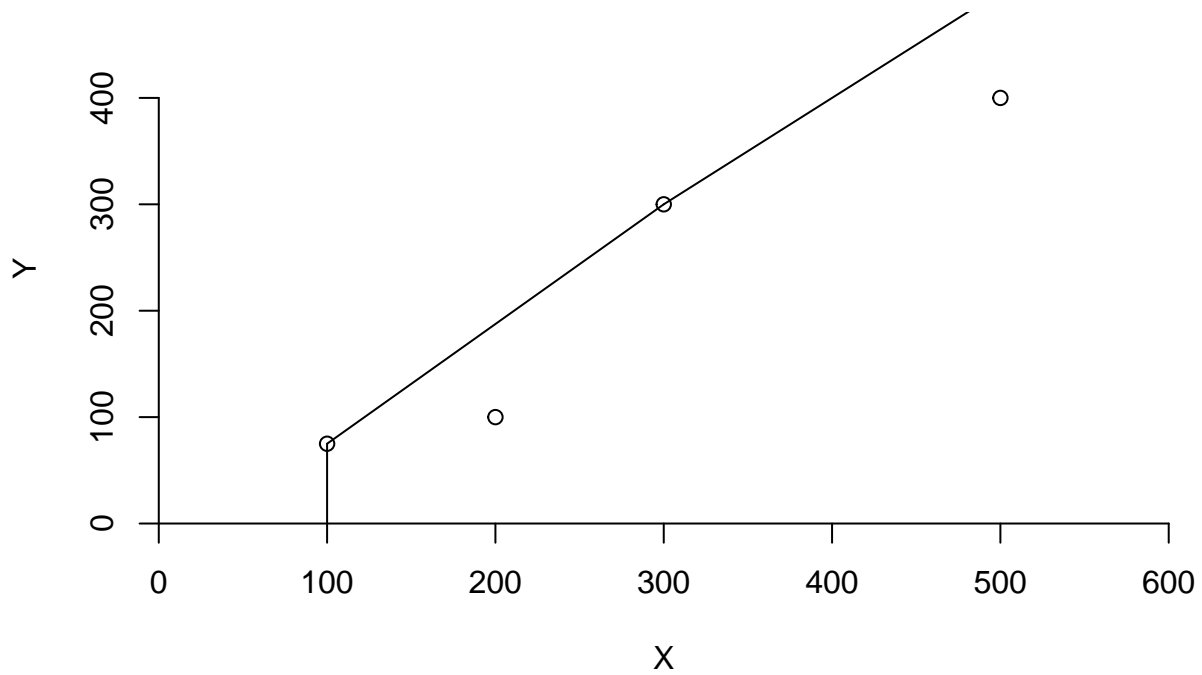
```
dea.plot(x,y,RTS="drs", main="Decreasing Returns to Scale (DRS) Graph")
```

Decreasing Returns to Scale (DRS) Graph



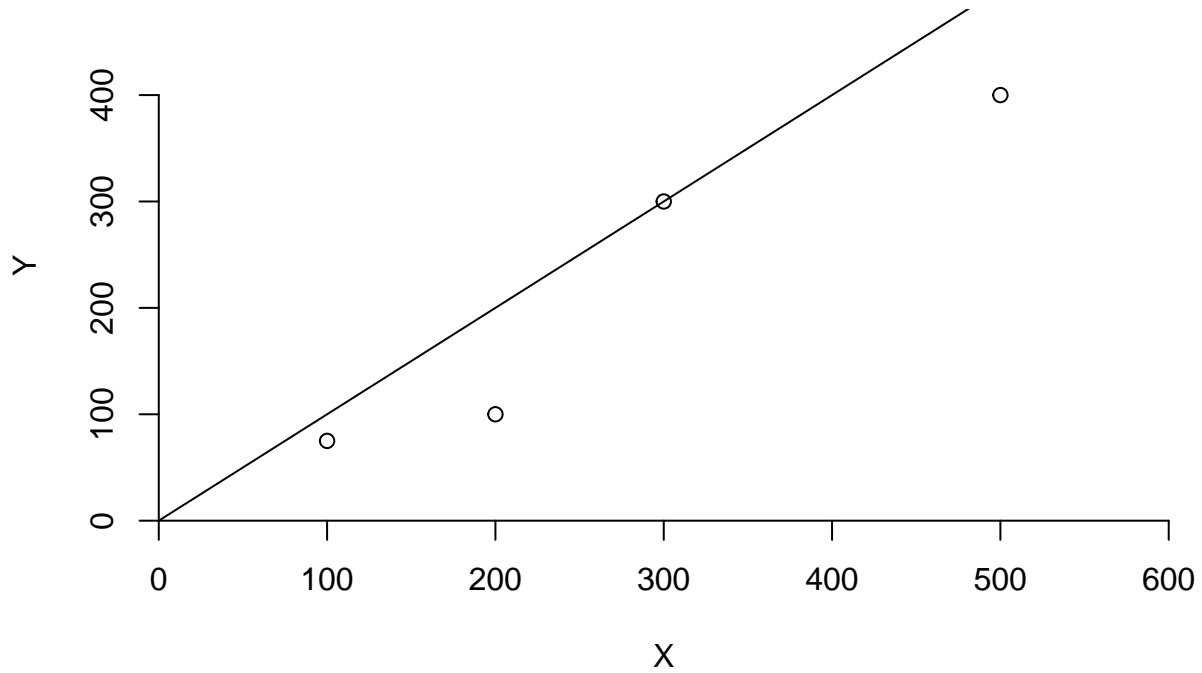
```
dea.plot(x,y,RTS="irs", main="Increasing Returns to Scale (IRS) Graph")
```

Increasing Returns to Scale (IRS) Graph



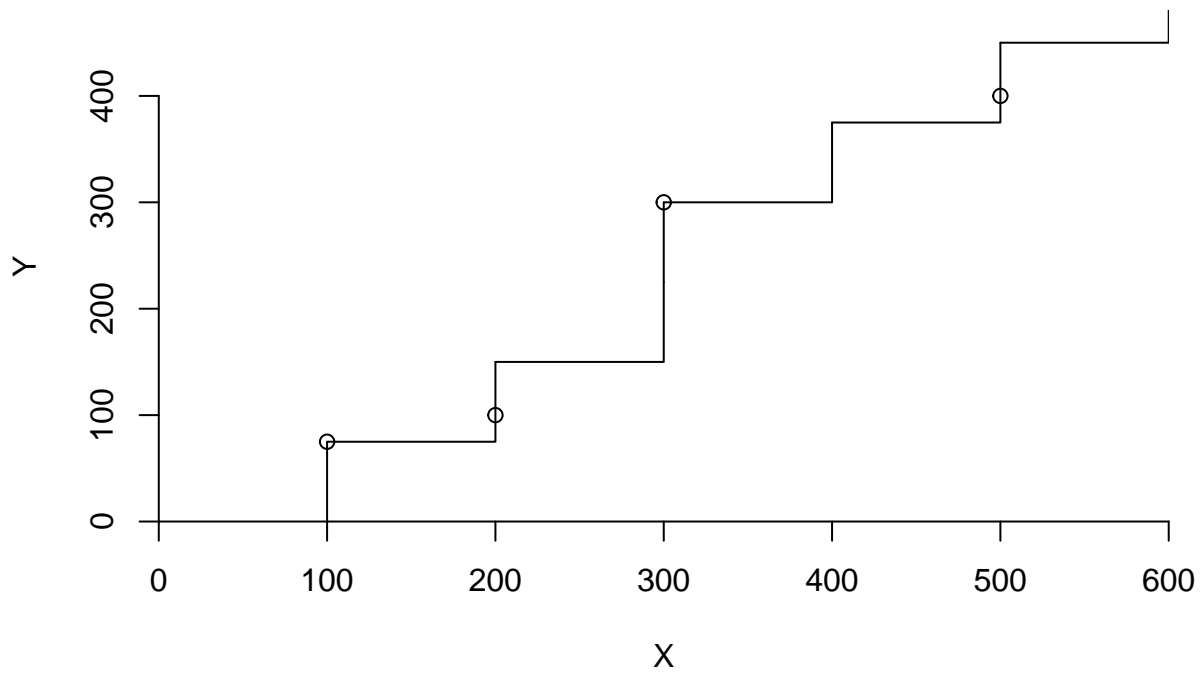
```
dea.plot(x,y,RTS="crs", main="Constant Returns to Scale (CRS) Graph")
```

Constant Returns to Scale (CRS) Graph



```
dea.plot(x,y,RTS="add", main="Free Replicability Hull (FRH) Graph")
```

Free Replicability Hull (FRH) Graph



#A summary of the weights assigned to each Facility in each DEA assumption

```
Weights_tbl <- cbind(FDH_Weights, CRS_Weights, VRS_Weights, IRS_Weights, DRS_Weights, FRH_Weights)

row.names(Weights_tbl) <- c("F1", "F2", "F3", "F4", "F5", "F6")

colnames(Weights_tbl) <- c("FDH", "FDH", "FDH", "FDH", "FDH", "FDH", "CRS", "CRS", "CRS", "CRS", "VRS", "VRS", "VRS", "VRS", "VRS", "VRS", "IRS", "IRS", "IRS", "IRS", "DRS", "DRS", "DRS", "DRS", "DRS", "DRS", "FRH", "FRH", "FRH", "FRH", "FRH", "FRH", "FRH", "FRH")

as.data.frame(Weights_tbl)
```

##	FDH	FDH	FDH	FDH	FDH	FDH	CRS	CRS	CRS	CRS	CRS	VRS
## F1	1	0	0	0	0	0	1.0000000	0.0000000	0	0.0000000	1.0000000	
## F2	0	1	0	0	0	0	0.0000000	1.0000000	0	0.0000000	0.0000000	
## F3	0	0	1	0	0	0	0.0000000	0.0000000	1	0.0000000	0.0000000	
## F4	0	0	0	1	0	0	0.0000000	0.0000000	0	1.0000000	0.0000000	
## F5	0	0	0	0	1	0	0.2000000	0.08048142	0	0.5383307	0.0000000	
## F6	0	0	0	0	0	1	0.3428571	0.39499264	0	0.1310751	0.4014399	

##	VRS	VRS	VRS	VRS	IRS	IRS	IRS	IRS	IRS	IRS	DRS
## F1	0.0000000	0	0	0.0000000	1.0000000	0.0000000	0	0	0.0000000	1.0000000	
## F2	1.0000000	0	0	0.0000000	0.0000000	1.0000000	0	0	0.0000000	0.0000000	
## F3	0.0000000	1	0	0.0000000	0.0000000	0.0000000	1	0	0.0000000	0.0000000	
## F4	0.0000000	0	1	0.0000000	0.0000000	0.0000000	0	1	0.0000000	0.0000000	
## F5	0.0000000	0	0	1.0000000	0.0000000	0.0000000	0	0	1.0000000	0.2000000	
## F6	0.3422606	0	0	0.2562995	0.4014399	0.3422606	0	0	0.2562995	0.3428571	

##	DRS	DRS	DRS	FRH	FRH	FRH	FRH	FRH	FRH
## F1	0.0000000	0	0.0000000	1	0	0	0	0	0
## F2	1.0000000	0	0.0000000	0	1	0	0	0	0
## F3	0.0000000	1	0.0000000	0	0	1	0	0	0
## F4	0.0000000	0	1.0000000	0	0	0	1	0	0
## F5	0.08048142	0	0.5383307	0	0	0	0	1	0
## F6	0.39499264	0	0.1310751	0	0	0	0	0	1

The above table summarizes the weights for each facility under each DEA assumption

Summary

All facilities are efficient under FDH and FRH.

Except for Facility 5,6, all facilities were efficient under CRS and DRS.

Except for facility 6, all were efficient under VRS and IRS assumptions.

They are peer units for efficient facilities.

IN the VRS and IRS assumptions for inefficient facilities were 1, 2, and 5.

The peer units under CRS and DRS were 1, 2, and 4.