# Testing the Imaging Properties of Synchrotron Radiation and Bremsstrahlung with a Self-Designed X-Ray Camera

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## INTRODUCTION

When charged particles, such as electrons, are accelerated by a magnetic field perpendicularly to their instantaneous direction of motion, they emit electromagnetic radiation, called synchrotron radiation. [1] For relativistic particles such as the ones found in particle accelerators, this radiation occupies the higher frequency end of the electromagnetic spectrum, namely x-rays. Thus, synchrotron radiation has been used as a light source for almost 70 years, given its emission of intense and uniform light [2]. One application of the x-rays produced is in medical radiography as bones absorb x-rays, creating contrast in images. It is also used in radiation therapy for tumours. [3] Furthermore, measurements of synchrotron radiation are widely used in astrophysics to study violent objects in the universe such as pulsars and supernovae remnants [4] e.g. the Crab Nebula. [5]

Bremsstrahlung, German for “braking radiation”, is electromagnetic radiation emitted by a charged particle when it is decelerated by the electric field of an atom in a medium. [6] Specifically, high-speed charged particles (e.g. electrons) collide with atomic electrons via the Coulomb Effect, causing excitations and ionizations [7]. This results in a loss of kinetic energy in such particles, which, as a result of the first law of thermodynamics, leads to the emission of Bremsstrahlung radiation. Similarly to synchrotron radiation, relativistic particles that experience this deflection emit Bremsstrahlung in the x-ray region of the electromagnetic spectrum. However, unlike synchrotron radiation, Bremsstrahlung is seldom used as a light source. We propose to explore if Bremsstrahlung radiation may have further applications in the field of imaging.

In our experiment, we want to use the x-rays generated by synchrotron radiation for spectroscopy to analyse the properties (e.g. density and thickness) of sample of metals that absorb different wavelengths of x-rays, as well as comparing the quality and resolution of the image created to one made with Bremsstrahlung. It will be interesting to see major changes in the shape of the images produced, which will indicate fundamental differences in the two types of radiation. To do this, we will be using an x-ray camera that we designed ourselves – a simplified version of professional devices used in hospitals or in facilities specifically designated to the study of radiation. We believe our proposal is inventive and original as we combine a more investigative aspect of physics, the research of radiation, with engineering in the creation of our own detection device.

## WHY WE WANT TO GO TO CERN

To most of us, the immersive world of particle physics was undiscovered land until recently, but as aspiring physicists and engineers driven by curiosity, we are eager to develop exciting ideas and glean new knowledge. The opportunity to carry out our experiment proposal at CERN and work alongside scientists would not only fulfil our ambitions as a team, but also encourage and embolden future students to expose themselves to groundbreaking science with a proactive, hands-on approach.

## AIM OF THE EXPERIMENT

Our experiment aims to reach two main objectives. Firstly, it will test the effectiveness of our camera by using different anisotropic samples of metals as fixed targets. In places where the samples are thicker/denser, more x-rays from synchrotron radiation and Bremsstrahlung will be absorbed, which will be reflected in the image produced. The more successful our camera, the higher its resolution and the more details we will see.

Secondly, with this experiment we seek to investigate properties of both types of radiation by analysing differences in the images produced. The main characteristics of synchrotron radiation and Bremsstrahlung that will be distinguishable in the images are distribution, intensity, collimation, polarisation and energy. Comparing the two will lead to a valuable conclusion on the uses and applications of the two types of x-rays depending on the features required.

## CAMERA DESIGN

The x-ray camera we designed consists of the following components:

1. Polycrystalline Copper with a thin layer of Magnesium Oxide photocathode
2. Scintillator (provided by CERN)
3. UV CCD camera

The photocathode[1] is necessary to convert the x-rays into high-energy photoelectrons, via the photoelectric effect. The scintillator then converts the photoelectrons back into photons by absorbing them and remitting their energy, resulting in a higher number of photons at lower energies. This allows us to be able to detect the photons with the CCD camera as we predict they will now be in the range of UV frequencies. Additionally, we will benefit from higher resolution and we can make use of long exposure images to get a more complete picture.

We decided to use a scintillator instead of an MCP as the latter is a highly specialised component which, as high school students, was beyond our ability to tinker with. Furthermore, MCPs require a vacuum to function properly, an aspect that is exceedingly difficult to work with. The scintillator therefore vastly simplifies the camera to make it accessible to a greater proportion of people, including high school students like us. However, a problem we expect to encounter is the low spatial resolution of the EJ200 scintillator. Hopefully the experts at CERN/DESY can help us overcome this or improve our design.

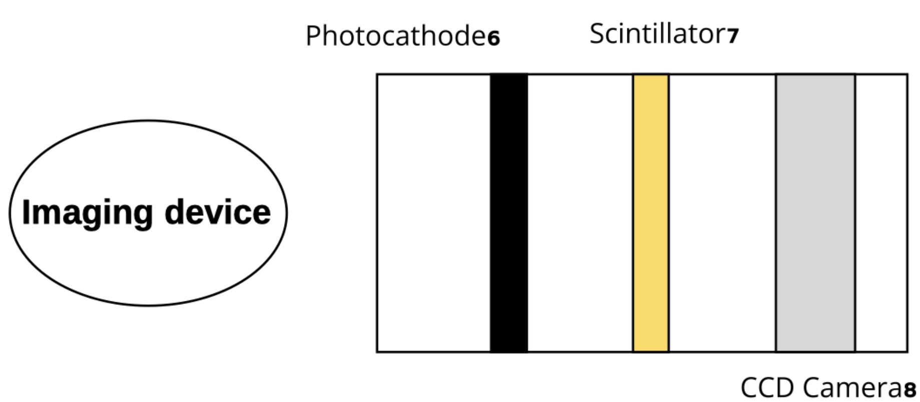



Figure 1: Camera design

[1] details in appendix on how we plan on funding the required equipment through science-based activities

# EXPERIMENTAL SETUP

We plan on carrying out two main experiments. For both, we will be using either the 0.5-4GeV high purity electron beam at CERN or the 1-6GeV electron beam at DESY. [8] Both at CERN and DESY the beam spot size at the focal point is circular with diameter of approximately 2cm. Particles are delivered one by one with a rate of one electron per 400ms at CERN (a bit slower at DESY).

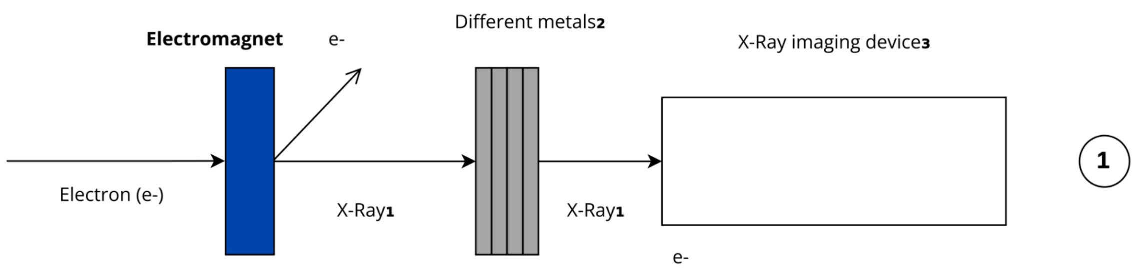



Figure 2: Synchrotron radiation set-up

- The beam passes through the electromagnet, which deflects the electrons according to the Lorentz force.
- The accelerated electrons emit synchrotron radiation in the x-ray frequency.
- The x-rays pass through samples of metal (including beryllium, iron and nickel) with varying thicknesses. This means more x-rays will be absorbed in the thicker areas. Additionally, the different elements have different absorption peaks which we will hopefully be able to see.
- The x-rays that are not absorbed then continue into the camera. Via the process detailed above, an image maintaining spatial resolution is created.
- The characteristics of the image can be analysed to reveal properties of the type of radiation. (See section titled Image Analysis).

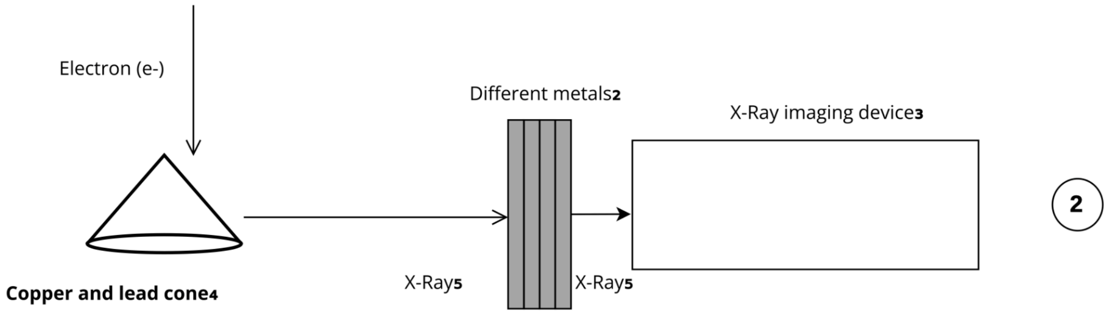



Figure 3: Bremsstrahlung set up

- The beam hits the slope of the rapidly spinning lead-coated copper cone.
- Because of lead's high density, the electrons are abruptly slowed down as they are deflected by the electric fields of the atomic nuclei.
- The deceleration of high-energy electrons causes them to release Bremsstrahlung radiation.
- The rest of the experiment is exactly the same as for synchrotron radiation.
- The spinning and copper core are for cooling purposes.

## SCHEDULE

Days 1-4: set up equipment and align camera, metal samples and cone/electromagnet

Day 5: control runs with no samples of metals to test endurance of the camera and any systematic errors

Day 6-7: synchrotron radiation with different samples of metals and changing parameters (i.e. initial energy of electron beam, strength of electromagnetic field)

Day 8-9: repeat of days 6 and 7 but with Bremsstrahlung

Day 10: image analysis and comparison

The remaining days could be used for repeated/extra runs to fill gaps in the data, but also to extend the reach of the analysis. For example, we want to use CERN/DESY's scintillators with a photomultiplier tube (PMT) instead of our camera to get a signal reading for the strength of the synchrotron radiation and Bremsstrahlung. This would allow us to compare them quantitatively as well as qualitatively. If possible, we would like to also compare the signal reading with a specialised SiPM to see differences in precision. Additionally, by filtering the x-rays to achieve monochromatic light, and using the available calorimeters, we could investigate the adequacy of both types of radiation for spectroscopy. We estimate that we could achieve a higher resolution (depending on the camera's resolution), allowing us to probe the atomic configuration of the metal samples.

## IMAGE ANALYSIS

For each of the properties of the rays we want to investigate, the following is what we will look for in the image:

Amount of x-ray photos released (i.e. intensity):

- A less intense beam of x-rays will generate a dimmer image.
- We expect the images from synchrotron radiation to be brighter than the ones generated with Bremsstrahlung.

Wavelength/energy:

- If we are able to analyse the maximum absorption for each sample of metal with both types of radiation, we can extrapolate to see which one releases higher energy photons.

Distribution:

- For a Gaussian distribution in space, the edges of the image will be less bright than the centre regardless of thickness of the metal samples.
- We expect this to be the case for synchrotron radiation, but we are unsure for Bremsstrahlung.

Collimation (how parallel the rays are):

- A highly collimated ray would yield an image which is true in size to the samples of metal. Otherwise, the image will be enlarged if the rays diverge.
- By comparing the magnification of the metals in images generated by

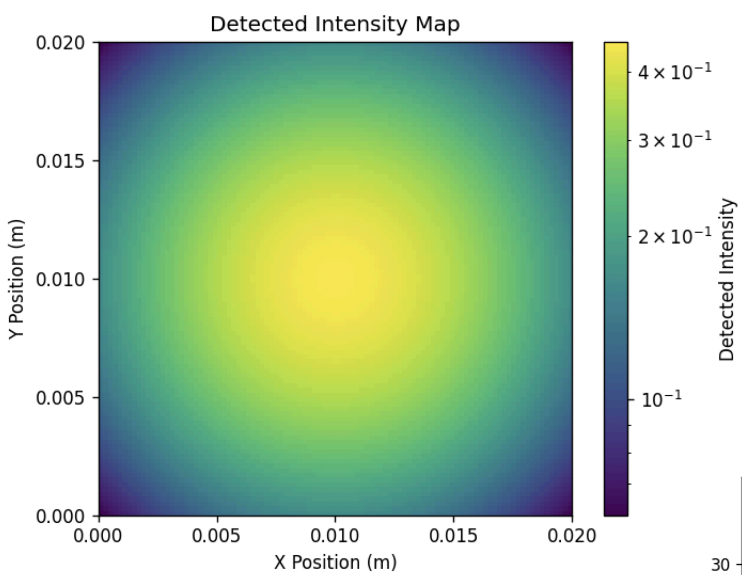
synchrotron radiation and Bremsstrahlung, we can see which type is more collimated. (Effect is amplified the further away we place the camera).

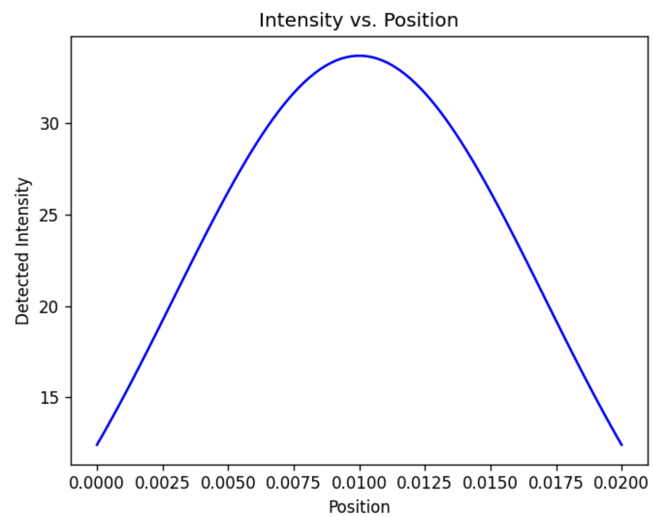
Polarisation:

- By using a polariser as one of the samples the x-rays will pass through, we can observe the polarisation of synchrotron radiation and Bremsstrahlung.

## SIMULATIONS

The pattern of light we would expect for a beam of photons with Gaussian distribution:




Figures 4 and 5: Intensity distribution we would expect from a beam of photons with Gaussian distribution of power or position.

For synchrotron radiation, we simulated expected results for the power of a photon using the following equations:

$$v = c\sqrt{1 - \left(\frac{m_e c^2}{E_k + m_e c}\right)^2}$$

where $v$ is velocity, $c$ is the speed of light, $m_e$ is the mass of an electron and $E_k$ is kinetic energy (i.e. the beam energy).

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where $\gamma$ is the Lorentz factor.

$$B = \frac{v}{c} \qquad r = \frac{m_e v}{Be}$$

where $B$ is the magnetic field strength and $r$ is the radius of the curved trajectory the electrons undertake in said magnetic field.

$$E = \frac{3hc\gamma^3}{2r} \qquad P = \frac{e^2 c}{6\pi\varepsilon_0}\left(\frac{E_e}{mc^2}\right)^4 \frac{\beta^4}{R^2}.$$

where $E$ is the energy of a photon, $h$ is Planck's constant;

and where $P$ is power of a photon of synchrotron radiation and $\varepsilon_0$ is the absolute dielectric permittivity of classical vacuum.

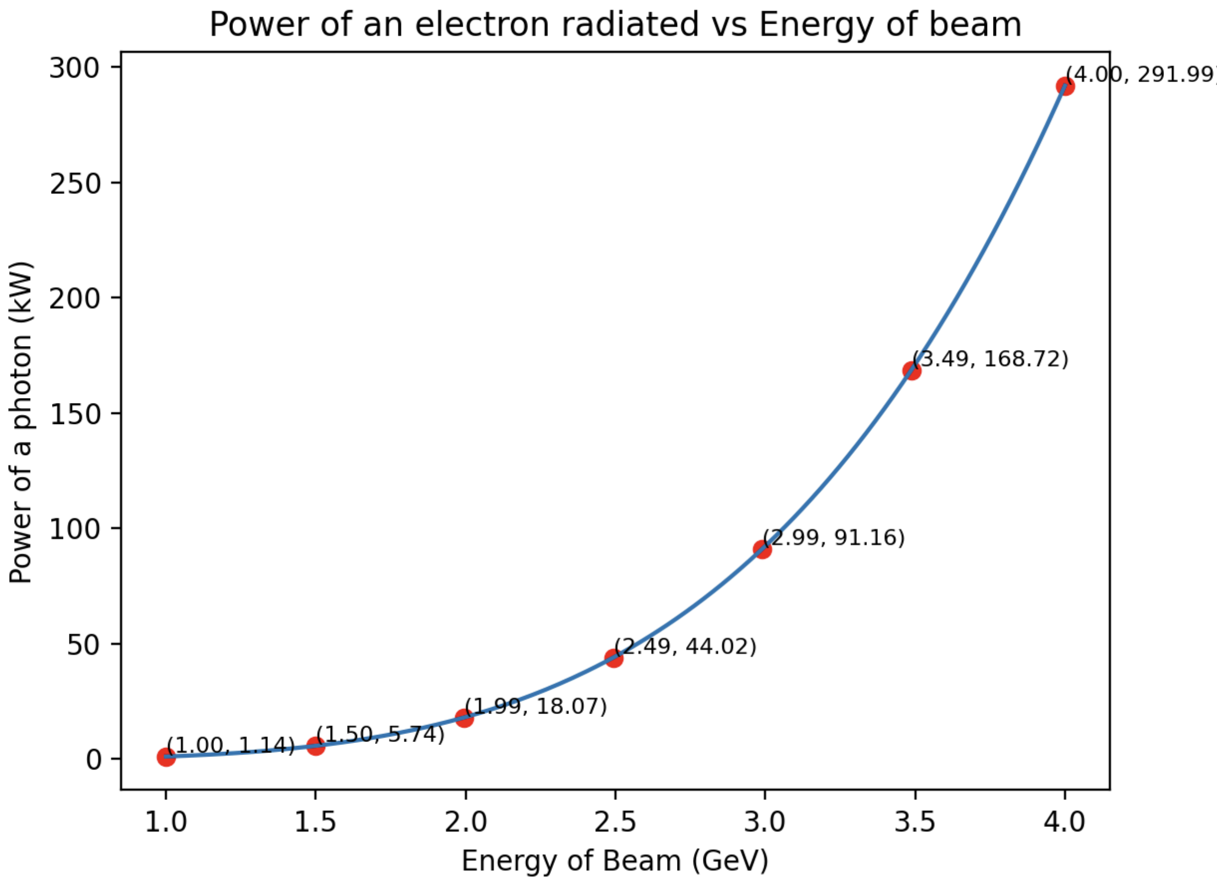



Figure 6: The power of a photon of synchrotron radiation in kilowatts depending on the energy of the beam in GeV. Strength of the magnetic field used: 700 mT

For Bremsstrahlung, we simulated expected results for emitted power per unit frequency of a single electron, which will collide against our lead target, using the following equations:

$$v = c\sqrt{1 - \left(\frac{m_e c^2}{E_k + m_e c}\right)^2} \qquad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

to find the velocity and thus the Lorentz factor of the electrons.

Using the formulae for energy of a photon and circular frequency,

$$f = \frac{E}{\hbar} \qquad w = 2\pi f$$

where $f$ is frequency, $\hbar$ is reduced Planck's constant and $w$ is circular frequency, we can find $b$, the impact parameter. *

$$b \approx \frac{v}{w} \qquad b_{\text{min}} = \frac{\hbar}{m_e v} \qquad b_{\text{max}} \approx \frac{2v^2}{\omega m_e}$$

Relativistically, it seems that the ions in the lead move rapidly towards the electron, and thus they appear as a "pulse of electromagnetic radiation" (a *virtual quanta*). [9] With this we can calculate:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega} = \frac{8Z^2e^6}{3\pi b^2 m_e^2 c^3 v^2}\left(\frac{b\omega}{\gamma^2 v}\right)^2 K_1^2\left(\frac{b\omega}{\gamma^2 v}\right)$$

where $W$ is electron energy, $Z$ is lead-82, and $K_1$ is the modified Bessel function of order one.

Finally, we integrate to find emitted power per unit frequency of a single electron for Bremsstrahlung,

$$\frac{\mathrm{d}W}{\mathrm{d}t\mathrm{d}\omega} = 2\pi c n_i \int_{b_{\text{min}}}^{\infty} \frac{\mathrm{d}W}{\mathrm{d}\omega} b \mathrm{d}b$$

where $n_i$ is the ion number density of lead. Here, where the lead coating on the copper cone is a result of electroplating, the value is approx 1.74e30 ions/m$^3$.

* Equation for $b_{max}$ derived from page 127, substitute $b_{min}$: *Chapter 7 Radiation from Charged Particle Interaction with Matter 7.1 Bremsstrahlung*. (2003). https://ocw.mit.edu/courses/22-105-electromagnetic-interactions-fall-2005/d6e432ae955f2a598e67c983e42704ab_chap7.pdf

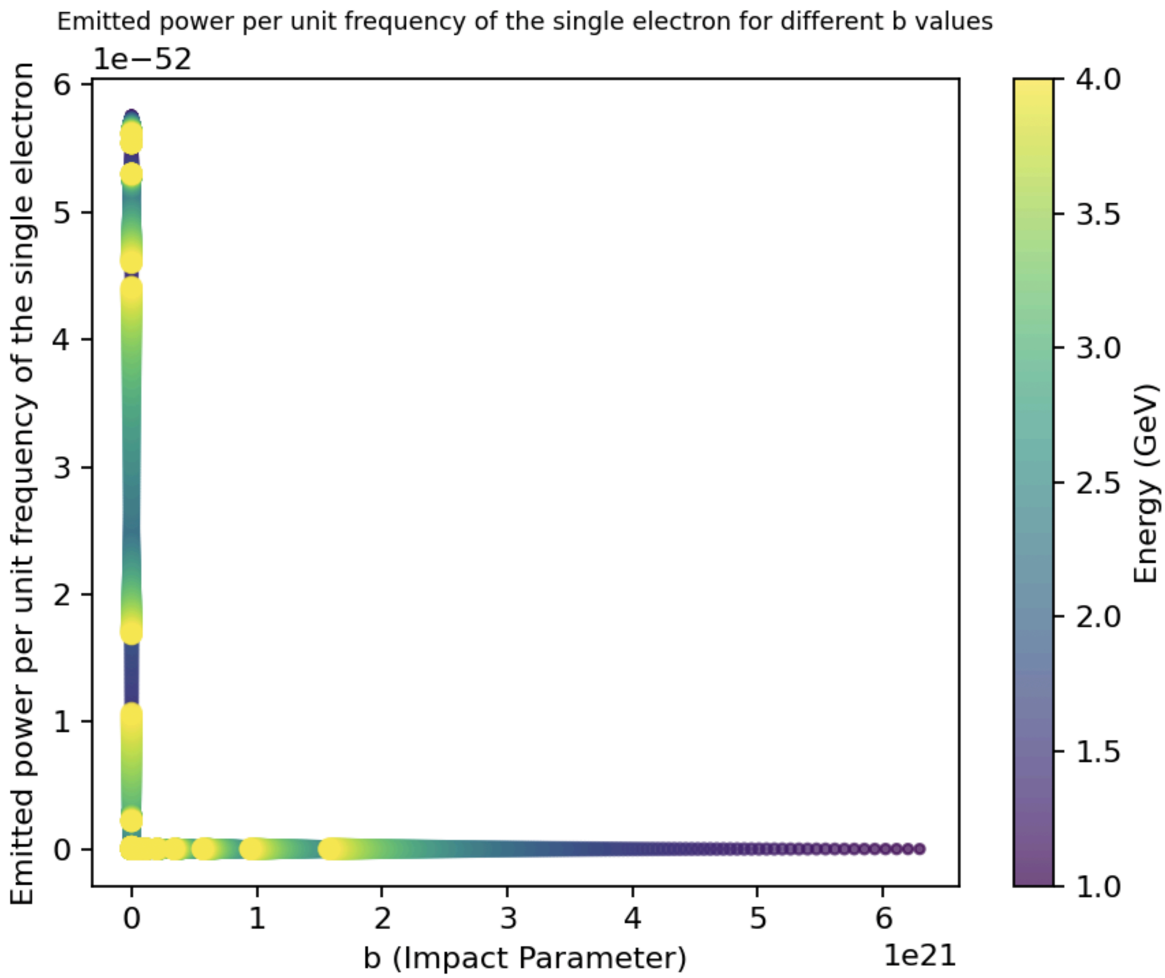



Figure 8: To get non-zero values for Power per unit frequency in the calculations, we divided the limits of the integral (from $b_{min}$ to infinity) into different sections. We noticed that for regions closer to $b_{min}$, we yielded greater values for Power per unit frequency. Therefore, we decided to divide the regions exponentially, making more and smaller regions closer to $b_{min}$ to obtain a more optimal computational integration. Based on the graph, we concluded that in the integration, power per unit frequency is derived from a small range of $b$ values and it is best to have more precise integrations over those areas.

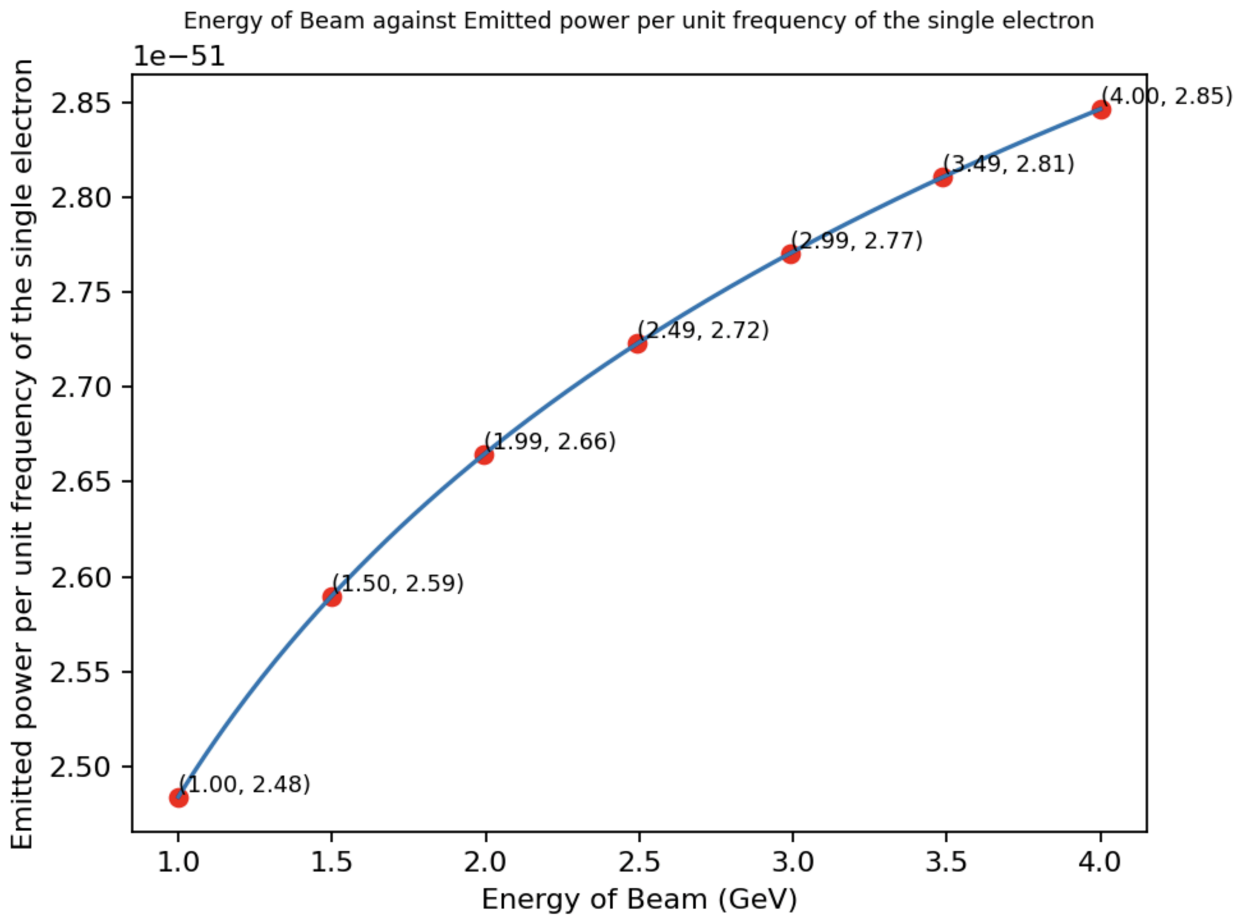



Figure 9: The power emitted in Bremsstrahlung by a single electron colliding against  a lead target (watts, W), depending on the initial energy of the beam.

## WHAT WE HOPE TO TAKE AWAY FROM THE EXPERIENCE

Creating this proposal has really enhanced our understanding of particle physics, our teamwork, our creativity and our problem solving skills. Further cultivating all these aspects, working with physicists at CERN or DESY would expand the horizons of our early scientific careers and provide us with a unique possibility to explore physics. We would also inspire physics students in Spain who tend to think that opportunities like BL4S are rare and almost impossible to achieve here.

## SCIENCE EDUCATION AND OUTREACH ACTIVITY

To spark awe and wonder for Experimental Science in younger students (aged 9 to 13), we have started a weekly club where we carry out simple practicals in both Physics and Chemistry. In doing so, we aim to explain intriguing concepts in a visual, applied context as many pupils struggle with understanding or seeing the relevance of pure theory. Furthermore, we believe fostering curiosity is the key to enable determined, enthusiastic and pioneering STEM students to realise their full potential. Our goal is to achieve an equal number of girls and boys, as we find that in our area, physics classes at a higher level are dominated by men.

For older students (aged 13 to 17), we plan to hold conversations at science events or open days at schools and charities in disadvantaged areas of Madrid to explain our journey delving into the practical aspects of physics at a particle accelerator. If given the chance, we will work with UNLOCK, a charity dedicated to helping teenagers with criminal records receive an education [10], to extend science even further. We intend to describe the process, results and physics behind it at a welcoming level, so everyone who is interested can grasp it. Hopefully, this will incentivise organisations to bolster students' love for science by funding projects like ours or including them as an option for pupils to access and take part in.

## ACKNOWLEDGEMENTS

First and foremost, we are indebted to our physics teacher Dr Tello, not only for inspiring us to take on this challenge that has shaped us and better prepared us for our futures, but also for guiding us through it. We extend our gratitude to Prof Antonio Hernando (UCM), Dr Miguel-Ángel García (CSIC) and Dr Hernandez (CIEMAT) for their aid and advice in adapting our proposal and overcoming obstacles. We would also like to thank Mr White, our other physics teacher, who helped us understand how Bremsstrahlung is emitted and its equations. Many thanks to Markus Joos and the rest of the BL4S team for swiftly answering all our queries. Thank you to Natalia Echeguren, our classmate, for her help organising and running the Science Experiment Club and BL4S video. Finally, we would like to appreciate our school, Runnymede College, for providing us with this thrilling opportunity and always pushing us to enrich our exploration of science.

## REFERENCES


[1] DESY. (n.d.). *How does a synchrotron radiation source work?* Photon-Science DESY. https://photon-science.desy.de/research/students__teaching/primers/synchrotron_radiation/index_eng.html

[2] Poore, R. (2020). *Synchrotron Radiation 101: How Light Sources Work and Their Applications*. RadiaSoft. https://www.radiasoft.net/blog/synchrotron-radiation-101-light-sources/

[3] Suortti, P., & Thomlinson, W. (2003). Medical applications of synchrotron radiation. *Physics in Medicine and Biology*, 48(13), R1–R35. https://doi.org/10.1088/0031-9155/48/13/201

[4] Botteon, A. (2012). *Synchrotron emission and astrophysical applications.* https://amslaurea.unibo.it/5626/1/botteon_andrea_tesi.pdf

[5] Reiger, F. (n.d.). *High energy astrophysics - Lecture 5*. Retrieved January 16, 2024, from https://www.mpi-hd.mpg.de/personalhomes/frieger/HEA5.pdf

[6] L'Annunziata, M. F. (2003). *Nuclear radiation, its interaction with matter and radioisotope decay*. Handbook of Radioactivity Analysis, 1–121. https://doi.org/10.1016/b978-012436603-9/50006-5

[7] Harvard Natural Sciences. (n.d.). *Bremsstrahlung and X-Rays*. Harvard Uni. https://sciencedemonstrations.fas.harvard.edu/presentations/bremsstrahlung-and-x-rays

[8] CERN (2023) *Beams and Detectors*. Beamline for Schools. https://beamline-for-schools.web.cern.ch/sites/default/files/Beams_Detectors_BL4S2024.pdf

[9] Zeković, V., Arbutina, B., Dobardžić, A., & Pavlović, M. Z. (2013). Relativistic Non-Thermal Bremsstrahlung Radiation. International Journal of Modern Physics A, 28(29), 1350141. https://doi.org/10.1142/s0217751x13501418

[10] Unlock. (n.d.). *Education*. Unlock. Retrieved February 18, 2024, from https://unlock.org.uk/topic/education/

# APPENDIX

1. **Fundraising for photocathode and/or other equipment**

In May or June, we will be organising a Science Fair aimed at children aged 8 to 12. There, we will have a competition for science projects like dioramas, posters, mini-experiments, building robots or structures… The entrance fee will be 5€, and the top three winners will get a prize (science books or science related things, amazon gift card…). The rest of the profit will go to funding the photocathode, Bremsstrahlung cone, CCD camera etc. We expect to make a profit of about 500€.

2. **Creation of Bremsstrahlung Cone**

For the creation of the cone for Bremsstrahlung, which requires specific dimensions and customisations, we got into contact with the Spanish National Research Council (CSIC). We received an offer for them to construct the cone in their ceramic and glass institute (ICV). The cone will be attached to the screw on the end of the motor to make it spin.

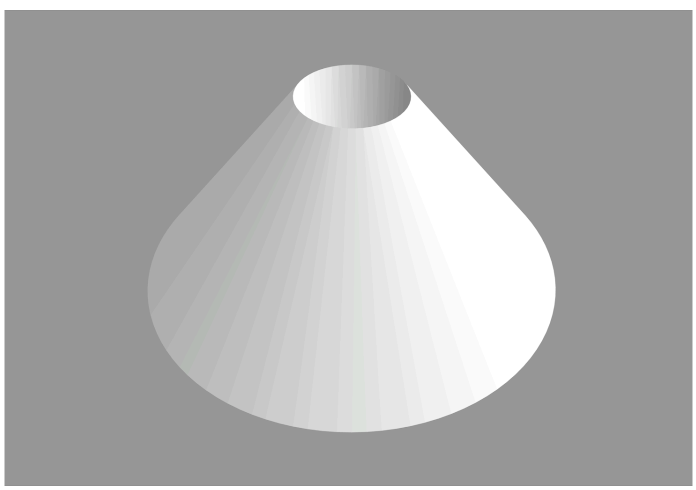


Figure 10: 3D model of the Bremsstrahlung cone — radius: 40mm, height: 40mm, angle of slope: 45º. The shape of the centre hole of the cone will depend on the motor screw and will be decided by the team at ICV CSIC to best suit its purpose.

## 3. Equipment Required

https://nerdytechy.com/arduino-brushless-motor-control-tutorial/

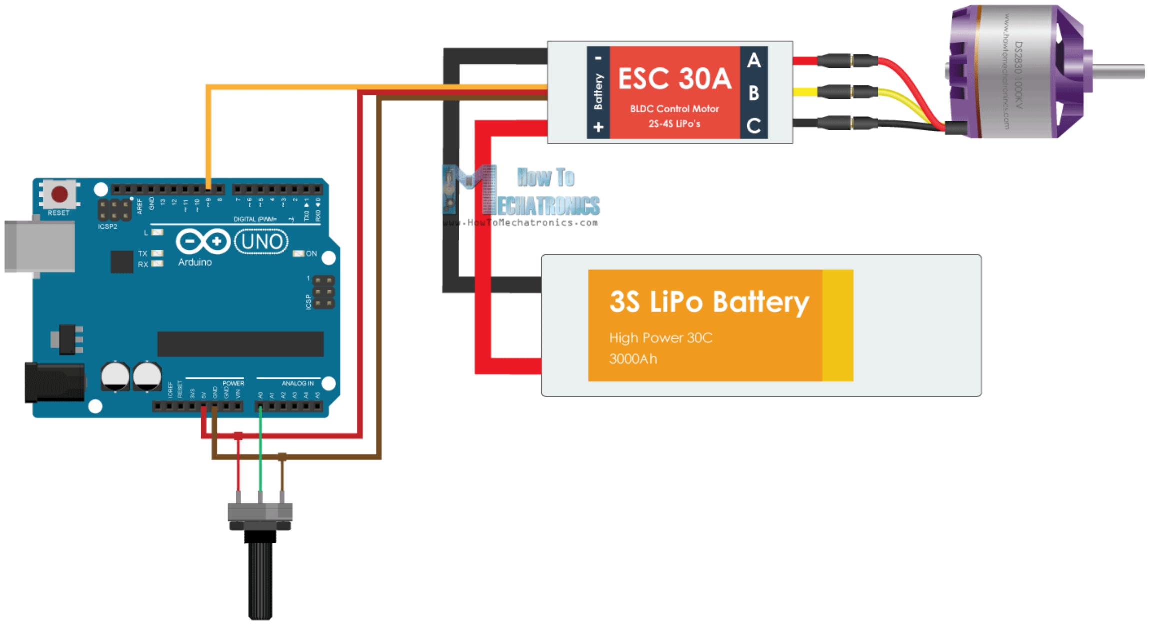



Figure 11: Wire connections to motor. We need the ESC 30A and brushless motor

**ESC 30A:** 12,77€
https://nl.banggood.com/HGLRC-30A-30AMP-2-5S-BLHeli_S-16_5-BB2-Brushless-ESC-Dshot600-Ready-for-RC-Drone-FPV-Racing-p-1398923.html?cur_warehouse=CN&rmmds=search

**brushless motor:** 24,06€
https://nl.banggood.com/FlysfishRC-Flash-2806_5-1350KV-1750KV-4-6S-Unibell-Brushless-Motor-for-Long-Range-RC-Drone-FPV-Racing-p-1989989.html?cur_warehouse=CN&ID=6291970521072&rmmds=search

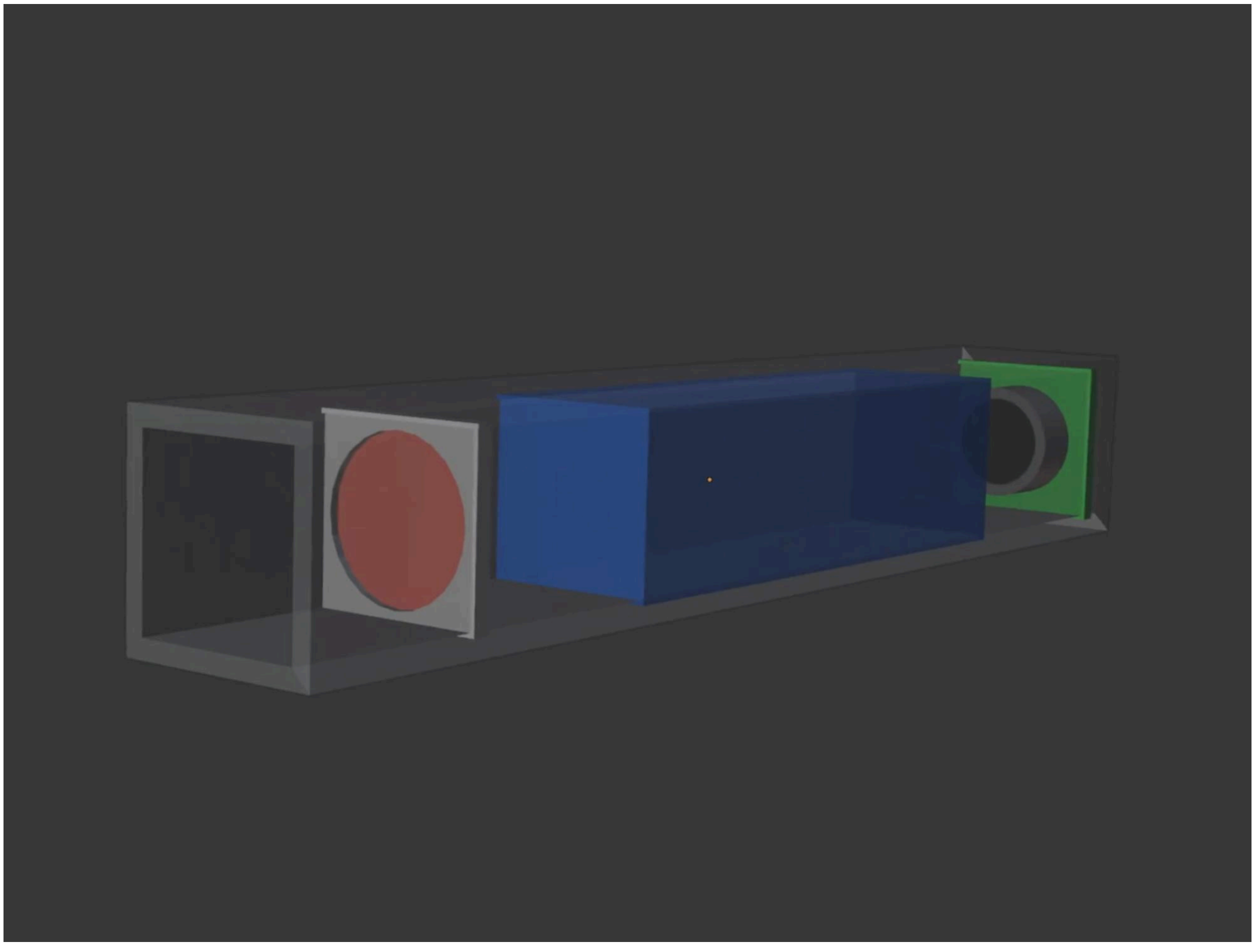
**metal samples for x-ray spectroscopy:**
50x50x10mm beryllium ≈ 30€
50x50x10mm iron ≈ 50€
50x50x10mm nickel ≈ 40€

**scrap metal for Bremsstrahlung cone:**
500g lead for electrolysis ≈ 5€
1.5kg copper ≈ 10€

**CCD camera:**
https://www.meetoptics.com/search?q=CCD%20Camera&Detection%20Wavelength%20Region__t_s=UV,DUV — we prefer the TDI board level camera C10000-A01 From HAMAMATSU Photonics: estimated price = 250-500€ (quota)

To include:

4. **3D model of camera design in CAD**



5. **<u>Simulation Code</u>**

We attach here the code for the simulations we made. We got results, but because we did this completely on our own, we are not fully sure of the accuracy. The opportunity to be able to discuss the code with experts at CERN/DESY and get feedback and improvements on it would be once in a lifetime for us.

Synchrotron Distribution Code

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.backends.backend_tkagg import FigureCanvasTkAgg
from matplotlib.colors import LogNorm
import matplotlib.ticker as ticker
import tkinter as tk

# Constants
BEAM_DIAMETER = 0.02  # Diameter of the electron beam (m)
INTERVAL = 100  # Interval of graph plot
METAL_DENSITY_FACTOR = 1.0  # Fixed Metal density

# Unknown values
METAL_THICKNESS = 0.01  # in meters
PHOTON_ATTENUATION_FACTOR = 0.01  # Fraction of X-rays passing through
metal plate
PHOTO_CATHODE_EFFICIENCY = 0.9  # Efficiency of the photo-cathode
THRESHOLD_INTENSITY = 0.01  # Threshold for photo-electron emission
SCINTILLATOR_EFFICIENCY = 0.8  # Efficiency of the scintillator
PHOSPHOR_SCREEN_EFFICIENCY = 0.7  # Efficiency of the phosphor screen


def xray_attenuation(thickness, density_factor):
    return np.exp(-thickness * density_factor * PHOTON_ATTENUATION_FACTOR)


def calculate_photo_cathode_efficiency(xray_intensity):
    return np.where(xray_intensity > THRESHOLD_INTENSITY, xray_intensity *
PHOTO_CATHODE_EFFICIENCY, 0.0)


def calculate_scintillator_efficiency(photoelectrons):
```

```
    return photoelectrons * SCINTILLATOR_EFFICIENCY


def calculate_phosphor_screen_efficiency(uv_photons):
    return uv_photons * PHOSPHOR_SCREEN_EFFICIENCY


def gaussian_beam(x, y):
    return np.exp(-(x**2 + y**2) / (2 * (BEAM_DIAMETER / 2)**2))


def create_tkinter_window(detected_intensity_map, positions_x):
    root = tk.Tk()
    root.title('Simulated Images')

    fig_xy, ax_xy = plt.subplots(dpi=60)
    ax_xy.set_title('Detected Intensity Map')
    ax_xy.set_xlabel('X Position (m)')
    ax_xy.set_ylabel('Y Position (m)')

    ax_xy.xaxis.set_major_locator(ticker.MultipleLocator(0.005))
    ax_xy.yaxis.set_major_locator(ticker.MultipleLocator(0.005))

    im_xy = ax_xy.imshow(detected_intensity_map, cmap='viridis', extent=[0,
BEAM_DIAMETER, 0, BEAM_DIAMETER],
                         norm=LogNorm(), origin='lower')
    fig_xy.colorbar(im_xy, ax=ax_xy, label='Detected Intensity')

    fig_intensity, ax_intensity = plt.subplots(dpi=60)
    ax_intensity.set_title('Intensity vs. Position')
    ax_intensity.set_xlabel('Position')
    ax_intensity.set_ylabel('Detected Intensity')

    intensity_profile = np.sum(detected_intensity_map, axis=0)  # Sum along the
y-axis
    ax_intensity.plot(positions_x, intensity_profile, label='Intensity Profile',
color='blue')

    canvas_xy = FigureCanvasTkAgg(fig_xy, master=root)
    canvas_xy.get_tk_widget().pack(side=tk.LEFT, fill=tk.BOTH, expand=1)  #
Display on the left
    canvas_xy.draw()

    canvas_intensity = FigureCanvasTkAgg(fig_intensity, master=root)
```

```
    canvas_intensity.get_tk_widget().pack(side=tk.RIGHT, fill=tk.BOTH, expand=1)
# Display on the right
    canvas_intensity.draw()

    tk.mainloop()


def simulate_image():
    positions_x = np.linspace(0, BEAM_DIAMETER, INTERVAL)
    positions_y = np.linspace(0, BEAM_DIAMETER, INTERVAL)
    x, y = np.meshgrid(positions_x, positions_y)

    detected_intensity_map = np.zeros((INTERVAL, INTERVAL))
    beam_distribution = gaussian_beam(x - BEAM_DIAMETER / 2, y -
BEAM_DIAMETER / 2)

    for i, pos_x in enumerate(positions_x):
        for j, pos_y in enumerate(positions_y):
            beam_intensity = beam_distribution[i, j]
            transmitted_intensity = beam_intensity *
xray_attenuation(METAL_THICKNESS, METAL_DENSITY_FACTOR)
            photo_cathode_efficiency =
calculate_photo_cathode_efficiency(transmitted_intensity)
            photoelectrons = transmitted_intensity * photo_cathode_efficiency
            uv_photons = calculate_scintillator_efficiency(photoelectrons)
            visible_light = calculate_phosphor_screen_efficiency(uv_photons)
            detected_intensity = calculate_photo_cathode_efficiency(visible_light)
            detected_intensity_map[i, j] = detected_intensity

    return detected_intensity_map, positions_x


# Display Graphs
detected_intensity_map, positions_x = simulate_image()
create_tkinter_window(detected_intensity_map, positions_x)
```

Synchrotron Radiation Code

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.constants import c, pi, h, m_e, e, epsilon_0, electron_volt
```

```
# Constants
MAGNETIC_FIELD_STRENGTH = 0.7  # in Tesla
INTERVALS = 200     # Graph Plot Intervals

def synchrotron_radiation(v, lorentz_factor):
    # E = lorentz_factor * m_e * c ** 2
    radius = (m_e * v) / (MAGNETIC_FIELD_STRENGTH * e)
    energy = (3 * h * c * lorentz_factor ** 3) / (2 * radius)

    B = v / c
    power = ((e ** 2 * c) / (6 * pi * epsilon_0)) * (lorentz_factor ** 4) * ((B ** 4) /
(radius ** 2))

    return energy, power


def intensity_energy(energy_gev, intensity):
    # Mark Points
    indices = np.linspace(0, INTERVALS - 1, 7, dtype=int)  # Get 10 evenly spaced
indices
    plt.scatter(energy_gev[indices], intensity[indices], color='red', label='Key
Points')

    for i in indices:
        plt.text(energy_gev[i], intensity[i], f'({energy_gev[i]:.2f}, {intensity[i]:.2f})',
fontsize=8, ha='left',
                 va='bottom')

    plt.plot(energy_gev, intensity)
    plt.title('Power of an electron radiated vs Energy of beam')
    plt.xlabel('Energy of Beam (GeV)')
    plt.ylabel('Power of a photon (kW)')
    plt.show()


energy_gev = np.linspace(1, 4, INTERVALS)
energy_j = energy_gev * electron_volt * 10e9

v = c * (1 - ((m_e * c ** 2) / (energy_j + m_e * c ** 2)) ** 2) ** .5
lorentz_factor = 1 / (1 - (v / c) ** 2) ** .5
energy, power = synchrotron_radiation(v, lorentz_factor)
intensity_energy(energy_gev, power * 1e-3)
```

Bremsstrahlung Code

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.backends.backend_tkagg import FigureCanvasTkAgg
from scipy.special import k1
from scipy.constants import c, pi, h, hbar, m_e, e, electron_volt
from scipy.integrate import quad
import tkinter as tk


# Constants
Z = 82  # Atomic number of lead
REGIONS = 100   # Regions during integration
DISTRIBUTION_POWER = 50    # The x^n at which the regions are distributed
INTERVALS = 200     # Graph Plot Intervals


def create_regions(b_min, b_max, num_regions):
    regions = []
    d = b_max - b_min
    quadratic_factor = d / (num_regions ** DISTRIBUTION_POWER)
    start = b_min
    for i in range(1, num_regions):
        end = b_min + quadratic_factor * i ** DISTRIBUTION_POWER
        regions.append([start, end])
        start = end

    # last region
    regions.append([start, np.inf])

    return regions


def integrand(b, w, v, lorentz_factor):
    energy_per_frequency = ((8 * Z ** 2 * e ** 6) / (
            3 * pi * b ** 2 * m_e ** 2 * c ** 3 * v ** 2)) * ((b * w) / ((lorentz_factor ** 2) * v)) ** 2 * (
                                (k1((b * w) / ((lorentz_factor ** 2) * v))) ** 2)

    return energy_per_frequency * b
```

```
def bremsstrahlung_radiation(energy, v, lorentz_factor):
    n_pb = 2.89e30
    frequency = energy / h
    w = 2 * pi * frequency
    b_min = hbar / (m_e * v)
    b_max = (2 * v ** 2) / (w * m_e)  # approximation

    regions = create_regions(b_min, b_max, REGIONS)
    regions_median = []
    power_per_frequency = []

    for i in regions:
        regions_median.append((i[0] + i[1]) / 2)
        integral = quad(integrand, i[0], i[1], args=(w, v, lorentz_factor),
epsabs=1.49e-8, epsrel=1.49e-8)[0]
        power_per_frequency.append(2 * pi * c * n_pb * integral)

    return regions_median, power_per_frequency


def intensity_energy(root, energy, power_per_frequency):
    fig_xy, ax_xy = plt.subplots(dpi=80)
    ax_xy.set_title('Energy of Beam against Emitted power per unit frequency of
the single electron', fontsize=8)
    ax_xy.set_xlabel('Energy of Beam (GeV)')
    ax_xy.set_ylabel('Emitted power per unit frequency of the single electron')
    ax_xy.plot(energy, power_per_frequency)

    # Mark Points
    indices = np.linspace(0, INTERVALS - 1, 7, dtype=int)  # Get 10 evenly spaced
indices
    ax_xy.scatter(energy[indices], power_per_frequency[indices], color='red',
label='Key Points')

    for i in indices:
        ax_xy.text(energy[i], power_per_frequency[i], f'({energy[i]:.2f},
{power_per_frequency[i] * 1e51:.2f})',
                   fontsize=8, ha='left',
                   va='bottom')
    # Embed the Matplotlib plots in Tkinter window
    canvas_xy = FigureCanvasTkAgg(fig_xy, master=root)
    canvas_xy.get_tk_widget().pack(side=tk.LEFT, fill=tk.BOTH, expand=1)  #
```

```
Display on the left
    canvas_xy.draw()


def alternate_graph(root, energy, power_per_frequency, b):
    fig_xy, ax_xy = plt.subplots(dpi=80)
    ax_xy.set_title('Emitted power per unit frequency of the single electron for
different b values', fontsize=8)
    ax_xy.set_xlabel('b (Impact Parameter)')
    ax_xy.set_ylabel('Emitted power per unit frequency of the single electron')

    # Plot the x-y graph
    scatter = ax_xy.scatter(b, power_per_frequency, s=10 * energy, c=energy,
cmap='viridis', alpha=0.7)
    fig_xy.colorbar(scatter, ax=ax_xy, label='Energy (GeV)')

    # Embed the Matplotlib plots in Tkinter window
    canvas_xy = FigureCanvasTkAgg(fig_xy, master=root)
    canvas_xy.get_tk_widget().pack(side=tk.RIGHT, fill=tk.BOTH, expand=1)  #
Display on the left
    canvas_xy.draw()


energy_gev = np.linspace(1, 4, INTERVALS)
energy_j = energy_gev * electron_volt * 10e9


v = c * (1 - ((m_e * c ** 2) / (energy_j + m_e * c ** 2)) ** 2) ** .5
lorentz_factor = 1 / (1 - (v / c) ** 2) ** .5

regions = np.array([])
energies = np.array([])
sliced_power_per_frequency = np.array([])
power_per_frequency = np.zeros_like(energy_gev)

for index, val in enumerate(energy_j):
    region, power_per_frequency_val = bremsstrahlung_radiation(val, v[index],
lorentz_factor[index])
    sliced_power_per_frequency = np.concatenate((sliced_power_per_frequency,
power_per_frequency_val))
    power_per_frequency[index] = np.sum(power_per_frequency_val)

    regions = np.concatenate((regions, region))
    energies = np.concatenate((energies, np.array([energy_gev[index] for i in
```

```
range(REGIONS)])))


# Display Graphs
root = tk.Tk()
root.title('Bremsstrahlung Graphs')
intensity_energy(root, energy_gev, power_per_frequency)
alternate_graph(root, energies, sliced_power_per_frequency, regions)
tk.mainloop()
```