THE UNIVERSITY OF ARIZONA

INITIAL DATA FOR GENERAL-RELATIVISTIC SIMULATIONS OF CHARGED BLACK HOLES

International Alpine School,
Domodossola – Italy
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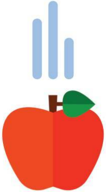
In the beginning there were…



$$S = \kappa \int \sqrt{-g} \left(\mathcal{R} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)$$

Kaluza - Klein

Mamjundar - Papapetrou

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Kerr - Newman

No-hair theorem

EINSTEIN - MAXWELL'S THEORY

Kaluza - Klein

Mamjundar - Papapetrou

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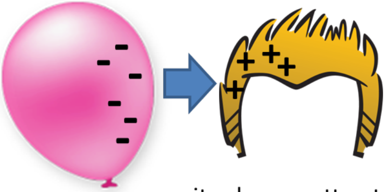
Kerr - Newman

No-hair theorem

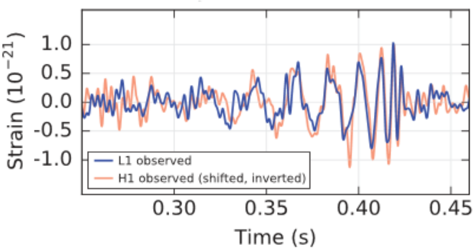
...little consideration for **dynamical** spacetimes...

ASTROPHYSICALLY RELEVANT BLACK HOLES



$$\frac{Q}{M} \approx 10^{-13} \left(\frac{a}{M}\right)^{\frac{1}{2}} \left(\frac{M}{M_\odot}\right)^{\frac{1}{2}}$$

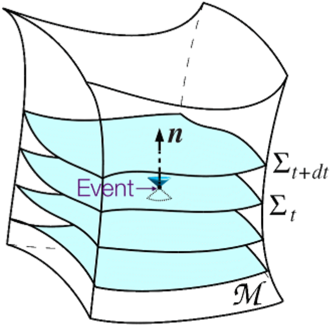
→ Dynamical formation of naked singularities
→ High-energy head-on collision
  → Does matter *matter*?
→ Coalescence and merger
  → Gravitational+EM-waves extraction



Can any imprint of charge end up here?

GOAL: Numerically solution of Einstein-Maxwell's equations

HOW: With a 3+1 decomposition, slice after slice



Numerical relativity = *know-how* to be successful in this feat

$$\text{EOM} \xrightarrow{\text{3+1 split}} \begin{array}{l} \text{Constraint eqs. } (\star) \\ \text{Evolution eqs. } (\bullet) \end{array}$$

Note: the problem is well-posed[1]

1. Generate initial data compatible with $(\star)$
2. Evolve them with $(\bullet)$

Here we focus on 1.

[1] Alcubierre et al, 2009

3+1 DECOMPOSITION

Given Σ_t with n^μ normal unit vector:

$$\begin{tikzcd} & \gamma_{ij} \\ g_{\mu\nu} \arrow[ur] \arrow[dr] & \\ & K_{ij} \end{tikzcd}$$

$$\begin{tikzcd} & E^i \\ F_{\mu\nu} \arrow[ur] \arrow[dr] & \\ & B^i \end{tikzcd}$$

3+1 DECOMPOSITION

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$$\begin{tikzcd} & \gamma_{ij} \\ g_{\mu\nu} \arrow[ur] \arrow[dr] & \\ & K_{ij} \end{tikzcd} \qquad \begin{tikzcd} & {}^{(3)}\mathcal{R} + K^2 - K_{ij}K^{ij} = 16\pi\mathcal{E} \\ \mathcal{G}_{0\mu} = 8\pi\, T^{\mathsf{EM}}_{0\mu} \arrow[ur] \arrow[dr] & \\ & D_j(K^{ij} - \gamma^{ij}K) = 8\pi S^i \end{tikzcd}$$

$$\begin{tikzcd} & E^i \\ F_{\mu\nu} \arrow[ur] \arrow[dr] & \\ & B^i \end{tikzcd} \qquad \begin{tikzcd} & D_i E^i = 0 \\ \nabla_\mu T^{0\mu}_{\mathsf{EM}} = 0 \arrow[ur] \arrow[dr] & \\ & D_i B^i = 0 \end{tikzcd}$$

Given Σ_t with n^μ normal unit vector:

$$\begin{tikzcd}[row sep=small] & \gamma_{ij} & \textbf{(6)} \\ g_{\mu\nu} \arrow[ur] \arrow[dr] & & \\ & K_{ij} & \textbf{(6)} \end{tikzcd}$$

$$\begin{tikzcd}[row sep=small] & {}^{(3)}\mathcal{R} + K^2 - K_{ij}K^{ij} = 16\pi\mathcal{E} & \textbf{(1)} \\ \mathcal{G}_{0\mu} = 8\pi\, T^{\text{EM}}_{0\mu} \arrow[ur] \arrow[dr] & & \\ & D_j(K^{ij} - \gamma^{ij}K) = 8\pi S^i & \textbf{(3)} \end{tikzcd}$$

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18 degrees of freedom

6 constraints

BOWEN-YORK'S APPROACH (1980)

physical fields = **conformal factor** $\times$ **background fields**

$$\begin{aligned}
\gamma_{ij} &= \psi^4 \bar{\gamma}_{ij} \\
E^i &= \psi^6 \bar{E}^i \\
B^j &= \psi^6 \bar{B}^i \\
K_{ij} &= K_{ij}(\psi, V^i, W^i)
\end{aligned}
\qquad
(\star) \Rightarrow
\qquad
\begin{aligned}
(\nabla^2 V)^i + \frac{1}{3} V^{k,i}{}_{,k} + 8\pi \bar{S}^i &= 0 \\
8\nabla^2\psi + \psi^{-7} k(V^i) + 16\pi\bar{\mathcal{E}} &= 0 \\
\partial_i \bar{E}^i &= 0 \\
\partial_i \bar{B}^i &= 0
\end{aligned}$$

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1. Choose $\bar{\gamma}_{ij}$ (e.g. flat) and $\bar{E}^i$, $\bar{B}^i$ (e.g. Kerr-Newman)

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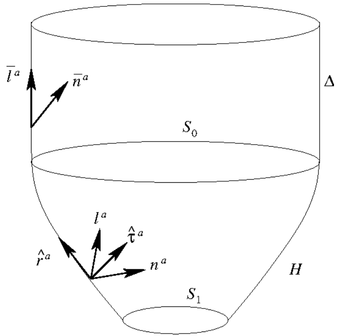
1. Choose $\bar{\gamma}_{ij}$ (e.g. flat) and $\bar{E}^i$, $\bar{B}^i$ (e.g. Kerr-Newman)
2. Solve for V^i
3. Solve for ψ
4. Revert to the physical fields



Local definition of $M_\mathcal{S}$, $Q_\mathcal{S}$, $J_\mathcal{S}$ (known *after-the-fact*) $\Rightarrow$ Iterative scheme for EM fields

NUMERICAL IMPLEMENTATION

`AHFinderDirect` + **`TwoPunctures`** + **`QuasiLocalMeasures`** on **`Cactus`** + **`Carpet`** infrastructure

Challenges:

- → Singularities
- → Domain (in the spectral method)

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$$\psi^2 = (1 + u + \eta)^2 - \phi^2 \qquad V^i = V^i_{\mathrm{GR}} + V^i_{\mathrm{EM}}$$

$$\eta = \sum_n \frac{m_n}{r_n} \quad \phi = \sum_n \frac{q_n}{r_n} \qquad V^i_{\mathrm{GR}} = V^i_{\mathrm{GR}}(P^j_n, J^j_n)$$

AHFinderDirect + **TwoPunctures** + **QuasiLocalMeasures** on **Cactus** + **Carpet** infrastructure

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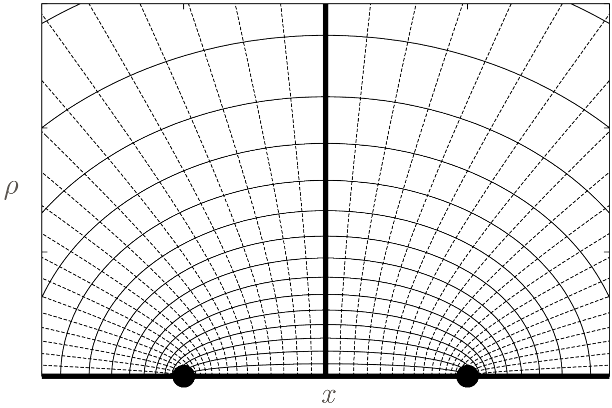
Numerical **Numerical**

$$\psi^2 = (1 + u + \eta)^2 - \phi^2 \qquad V^i = V^i_{\mathrm{GR}} + V^i_{\mathrm{EM}}$$

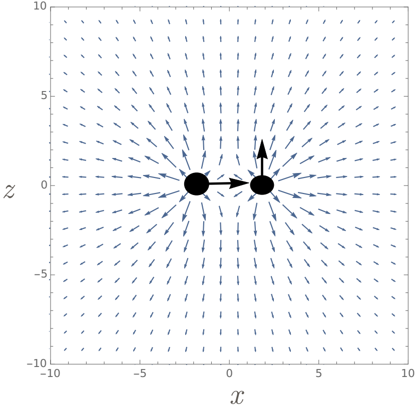
Analytical

$$\boxed{\eta = \sum_n \frac{m_n}{r_n} \quad \phi = \sum_n \frac{q_n}{r_n} \qquad V^i_{\mathrm{GR}} = V^i_{\mathrm{GR}}(P^j_n, J^j_n)}$$



[1]Ansorg et al, 2004

EXAMPLE OF ELECTRIC FIELD



Sanity checks and convergence tests are currently begin finalized

PROS AND CONS OF APPROACH + IMPLEMENTATION

Pros:

- → Perfectly fits in the numerical relativity ecosystem
- → Ready to be attached to a evolution code
- → Built on the battle-tested case with $Q = 0$
- → Perks of spectral method

Cons:

- → Drawbacks of conformal flatness
- → Difficult and expensive to go near extremality
- → Problems with coordinates singularities
- → No guarantee of convergence

CONCLUSIONS

In this talk:

- → Dynamical electrovacuum spacetimes
- → Bowen-York initial data for charged black holes
- → Physical parameters with dynamical horizons
- → Numerical implementation of the formalism

Next step: evolution of binary black hole systems

arXiv: gr-qc/1807XXX

Thanks for the attention! Questions?