Chaos Homework 12

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Note to grader
I was brave and did this week in python! If you want to run my code, all of the actual statements happen in the main function at the bottom. Feel free to comment out certain parts you don't need to see or just hit enter after the answer to every question to move on
You'll need to exit the graphs before hitting enter
There are also 3 files: helper-functions, homework-12, and a csv containing points to plot for question 2

1 Question 1

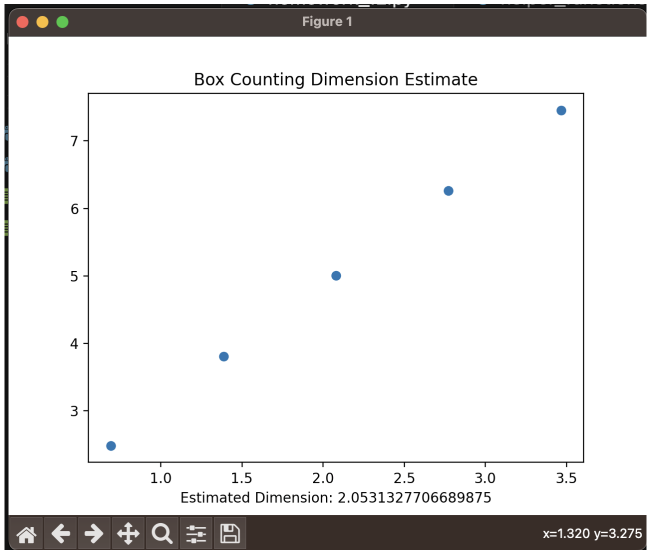
Compute the box counting dimension code to compute the fractal dimension of the Lorenz Attractor

The code for this question is located under the question 1 comment in my code (around line 300).
It calls 2 functions: iterate-lorenz-equations, which generate the orbit of an initial condition given some parameters (using rk2), and compute-boxcounting-dimension, which takes that orbit and box counts in 3d. It plots the set of points corresponding to each subdivision width and performs linear regression on those points to find the slope of the line. This slope is the estimated BCD

2 Question 2

Compute the lyapunov exponents for varying values of r
Here are the computed lyapunov exponents for the 3 different r values.
In the question 2 section of main, there is only 1 function called which is compute-lyapunov-exponents(v0, parameters). This takes in an initial condition and a set of parameters and returns their exponents

Inside the function, we are calculating the Jacobian, morphing the unit ball using the Jacobian "continuously" (in small time steps) using the function stepIt which performs a better version of rk2 (rk4) up until 1 full time step, and doing



the same thing to our v value.
We are then performing a similar orthogonalization process using QR decomp to find the eigenvalues of the upper-tringular matrix

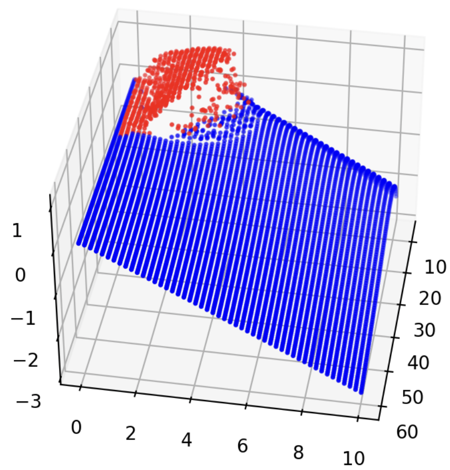
All of the code is pretty well commented!

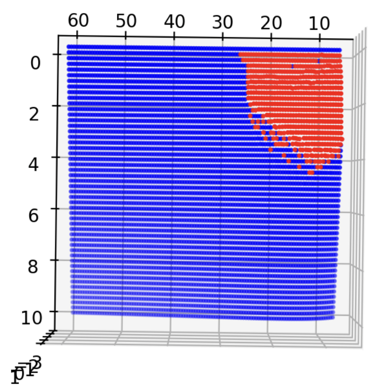
```
Calculating Lyapunov Exponents for varying R values. Sigma: 10, B: 8/3
R = 12:
 Exponents: [-0.4824 -0.4876 -12.55]

R = 24.5:
 Exponents: [0.7887 -0.0003756 -14.3]

R = 28:
 Exponents: [0.8957 -0.002411 -14.41]
```

3 Question 3

Iterate through possible values of sigma and b and plot their max-lyapunov exponent
For this graph I did a 3d plot with b and sigma values on the x, y axes and the z value being the maximum lyapunov exponent of the equations at those values Red points have a max exponent greater than 0, which implies chaos, and blue points do not

A top-down view yields a more informational view of the boundary:

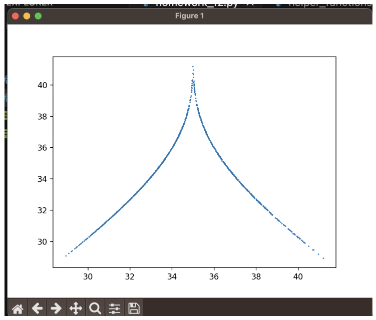


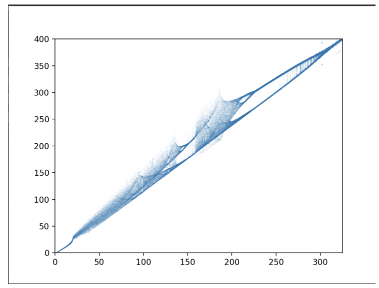


From this image, we can see there is a box-like region where chaos occurs, where b is less than 4.5 and sigma is less than 26
The boundary is not completely rectangular, but thats the only region where chaos occurs
Looking at the 3d graph, it is also interesting how the height of the exponents change inside the box
There is a "cliff" of chaos!

4 Question 4

The diagrams in question:





Interpreting the first figure, these are the maximal Z values plotted against themselves in the future. They form something which looks very similar to the tent map, where (I think) the big implication is that it has a fully connected transition graph. This means we can create orbits of any period, and chaotic orbits of the lorenz-equations in only the z values! Note: these orbits are continuous but we are looking at snapshots to do this

When we think of the Hula-Hoop experiment, the z values represent the deviation from linear conduction occuring vertically in the Hula-Hoop
This means the higher z is, the less linear the vertical temperature is
So the bifurcation diagram is showing orbits of the local maximum vertical temperature non-linearity
The state of the z value will oscilate between 2 points, then 4, then 8... in a normal bifurcation style
Eventually we might even get chaotic orbits of z, meaning that the deviation from linear temperature conduction is unpredictable (and maybe that we don't know what the vertical temp looks like at a point in time?)
But an important note is that as r continues to increase, we go back to a period 2 orbit!
This means as energy keeps getting added, eventually the temperature differentials simmer back down into a period 2 orbit!

5 Question 5

Here are the heatmaps I produced. The code is housed in generate-heat-map() which takes in a z param and a r parameter and finds the real-ev of the Jacobian at points in the x-y plane. plot-heat-map() then plots the 3 z-slices for a given r value

If I lived in the lorenz equations, the weather is most predictable where the eigenvalues are negative
This would be at low Z values where r is 12
In addition, I would want to live on the top right or bottom left edge of this x-y

plane, as the diagonal has a lot of uncertainty, shown as yellow in the plot!