Analysis In Several Variables Homework 1

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Not-Proofs:
a) Syllabus Read!
b) I don't need any exam accommodations
c) Sections read!
d) Any time Wednesday is great! Maybe like 1:00 pm?
e) I've gotten a few peoples phone numbers for study groups, but I don't think I need to be manually connected with anyone!
f) I am on the rowing team at UVM, and I have a very dumb orange cat named Butter. I got him over the summer!

1 Question 1

Prove $0 < 1$
Strategy: Show that the square of every number is greater than 0, and then we can use the fact that 1 * 1 = 1 to claim that 1 > 0

$1 * 1 = 1$ comes from axiom 5 which is that $\exists y$ s.t $1 * y = 1$ and that y is written as $\frac{1}{1}$
I think its safe to say that $\frac{1}{1} = 1$?

Showing the square of any number is positive
Fix $x \neq 0 \in \mathbb{R}$
One of the following holds:
$\underline{x > 0}$:
In this case we have $0 < x \Rightarrow 0 < x * x$ by Axiom 8
$0 < x^2$
$\underline{x < 0}$:
$x < 0$
$x - x < 0 - x$ Axiom 7
$0 < -x$ Axiom 4
$0 < (-x) * (-x)$ Axiom 8
$0 < (-x)^2$

Finally $1^2 > 0$ Since any number squared is greater than 0
$1 * 1 > 0$
$1 > 0$ As $1 * 1 = 1$
Done!

2 Question 2

Show that if $a, b < 0$, $ab > 0$
Strategy: Show $(-a) * (-b) = ab$
$a < 0$
$a - a < 0 - a$ Axiom 7
$0 < -a$ Axiom 4
Same holds for b
So we have:
$0 < (-a) * (-b)$ Axiom 8
$0 < (-a) * (-b) + a(b - b)$ Axiom 7
$0 < (-a)(-b) + a(-b) + ab$ Axioms 3 and 1
$0 < (-b)(a - a) + ab$ Axiom 3
$0 < (-b) * 0 + ab$ Axiom 4
$0 < 0 + ab$
$0 < ab$
Therefore $ab > 0$
Done!

Show $a < 0$ and $b > 0$ implies $ab < 0$
Step 1: Start by showing $-a = -1 * a$:
$-a = -a + (0 * a)$
$= -a + (1 - 1)a$ Axiom 4
$= -a + (1 * a) + (-1 * a)$ Axiom 3
$= 1 * (a - a) + (-1 * a)$ Axiom 3
$= 1 * 0 + (-1 * a)$ Axiom 4
$= 0 + (-1 * a)$ Plus.0
$= -1 * a$

Step 2: Show that if $x > 0, -1 * x < 0$
$x > 0$
$x - x > 0 - x$ Axiom 7
$0 > -x$ Axiom 4
$0 > -1 * x$ Shown Above
$-1 * x < 0$

Step 3
$-a > 0$ shown above
$-a * b > 0$ Axiom 8

$-1 * a * b > 0$ Above
$-1 * -1 * a * b < 0 * -1$ Shown Above
$1 * a * b < 0 * -1$ Axiom 4 (-1 is its own inverse)
$a * b < 0$
Done!

3 Question 3

Show that with $x, y, z \in \mathbb{R}$ and $x < y, z > 0, xz < yz$
Since $x < y$:
$x - x < y - x$ Axiom 7
$0 < y - x$ Axiom 4
$0 * z < (y - x) * z$ Axiom 8
$0 < (y - x) * z$
$0 < zy - zx$ Axiom 3
$0 + zx < zy - zx + zx$ Axiom 7
$zx < zy + (zx - zx))$ Axioms 3 and 1 and 4
$zx < zy + 0$ Axiom 4
$zx < zy$ Axiom 4
$zx < yz$ Axiom 1
Done!

4 Question 4

If $a, b \in \mathbb{R}$ with $b < a$, then $0 < \frac{a-b}{2}$
$b < a$
$b - b < a - b$ Axiom 7
$0 < a - b$ Axiom 4
Sidebar: Show $\frac{1}{2} > 0$

We know $2 * \frac{1}{2} = 1$ by Axiom 3
$1 > 0$ as shown earlier
$2 * \frac{1}{2} > 0$
We know that 2 is positive, as $2 = 1 + 1$ and $1 > 0$, therefore $1 + 1 > 0$ By Axiom 7
And since we showed above that $ab > 0$ iff ($a < 0$ and $b < 0$) or ($a > 0$ and $b > 0$) (not one positive one negative), we can conclude that $\frac{1}{2} > 0$

$0 < (a - b) * \frac{1}{2}$ Axiom 8
$0 < \frac{(a-b)}{2}$
Done!