Chaos Homework 1

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1 Question 1

2 Question 2: $f(x) = 2x^2 - 5x$

<u>Part A: Solving for period 2 orbit</u>
$f^2(x) = 2(2x^2 - 5x)^2 - 5(2x^2 - 5x)$
$f^2(x) = 8x^4 - 40x^3 + 40x^2 + 25x$
Set this equal to x to solve for fixed points
$8x^4 - 40x^3 + 40x^2 + 24x = 0$
$(8x)(x-3)(x^2 - 2x - 1) = 0$ Note: this factoring took more work than shown
Solutions to equation: $0, 3, 1 + \sqrt(2), 1 - \sqrt(2)$

Since $0, 3$ are fixed points of $f(x)$, the points in the period 2 orbit are $1 + \sqrt(2), 1 - \sqrt(2)$

<u>Part B: Determining stability of orbit</u>
To do this we will determine if the absolute value of the derivative of $f^2(1+\sqrt{2})$ is greater than or less than 1

$f'(x) = 4x - 5$

$f^{2'}(x) = f'(f(x)) * f'(x)$
$f^{2'}(1 + \sqrt{2}) = f'(1 - \sqrt{2}) * f'(1 + \sqrt{2})$ Note: substitution comes from the fact that it is a period 2 orbit so $p_1 \rightarrow p_2$

$= 4(1 - \sqrt{2}) - 5 * 4(1 + \sqrt{2}) - 5$
$= -31$
$|-31| > 1$ therefore the period 2 orbit is unstable (it is repelling)

3 Question 3

$G(x) = 4x(1 - x)$
<u>Part A: Finding fixed points and orbits of G</u>

Fixed points of G are found by solving the equation for x
$4x(1-x) = x$
$-4x^2 + 4x - x = 0$
$(x)(-4x+3) = 0$

Therefore the fixed points of G are: $0, \frac{3}{4}$

To find the period 2 points of G we will solve for x in $G^2(x) = x$
$-64x^4 + 128x^3 - 80x^2 + 16x = x$
$-64x^4 + 128x^3 - 80x^2 + 15x = 0$
$(x)(x - \frac{3}{4})(-64x^2 + 80x - 20) = 0$
$(-64x^2 + 80x - 20) = 0$
$x = \frac{-20 \pm \sqrt{400-320}}{-32}$
$x = \frac{5 \pm \sqrt{5}}{8}$
These are the 2 period 2 points of G: $\frac{5 \pm \sqrt{5}}{8}$

Demonstrating these points are sources:
$g'(x) = -8x + 4$
$g2'(x) = g'(g(x)) * g'(x)$

$g'(0) = -8(0) + 4 = 4$
Since $|4| > 1$, 0 is a source

$g'(\frac{3}{4}) = -8(\frac{3}{4}) + 4 = -2$
Since $|-2| > 1$, 0 is a source

$g^{2'}(\frac{5+\sqrt{5}}{8}) = g'(g(\frac{5+\sqrt{5}}{8})) * g'(\frac{5+\sqrt{5}}{8})$
$= g'(\frac{5-\sqrt{5}}{8}) * g'(\frac{5+\sqrt{5}}{8})$
$= (4 - 5\sqrt{5}) * (4 - 5 - \sqrt{5})$
$= -4$
Since $|-4| > 0$, both of the points are period-2 sources. Note: it was shown in class that points in the same orbit share the same slote which is why it was only calculated for $\frac{5+\sqrt{5}}{8}$

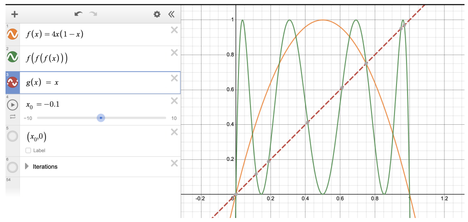

Figure 1: Test

Period	Fixed Points	Divisors	Points from period < k	Points only k	Orbits
1	2	1	0	2	2
2	4	1	2	2	1
3	8	1	2	6	2
4	16	1,2	4	12	3
5	32	1	2	30	6
6	64	1,2,3	10	54	9
7	128	1	2	126	18
8	256	1,2,4	16	240	30
9	512	1,3	8	504	56
10	1024	1,2,5	34	990	99

4 Question 4

When does $ax + b$ have an attracting fixed point
This function has an attracting fixed point when $|a| < 1$ and for all values of b
When does $ax + b$ have a repelling fixed point
This function has a repelling fixed point when $|a| > 1$ and for all values of b
When does $ax + b$ have a neutral fixed point
This function has a neutral fixed point when ($a = 1$ and $b = 0$) or when ($a = -1$ for all values of b)

5 Question 5

For which i is $x_i = \frac{3}{4}$
$x_6 = \frac{3}{4}$
Group the remaining fixed points into orbits