Unleashing Potentials
Shaping the Future

Unit 4 – Hypothesis Testing

- Fundamentals of Hypothesis Testing:
- One-Sample Tests

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Learning Objectives

In this chapter, you learn:

- The basic principles of hypothesis testing
- How to use hypothesis testing to test a mean or proportion
- The assumptions of each hypothesis-testing procedure, how to evaluate them, and the consequences if they are seriously violated
- How to avoid the pitfalls involved in hypothesis testing
- The ethical issues involved in hypothesis testing

A hypothesis is a claim (assumption) about a population parameter:

- population mean

Example: The mean monthly cell phone bill of this city is μ = $42

- population proportion

Example: The proportion of adults in this city with cell phones is π = 0.68

The Null Hypothesis, H_0

- States the claim or assertion to be tested

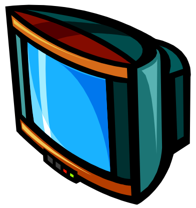
Example:

The average number of TV sets in U.S. Homes is equal to three (H_0: $\mu = 3$)

- Is always about a population parameter, not about a sample statistic

$$H_0 : \mu = 3$$

$$\cancel{H_0 : \bar{X} = 3}$$



The Null Hypothesis, H_0

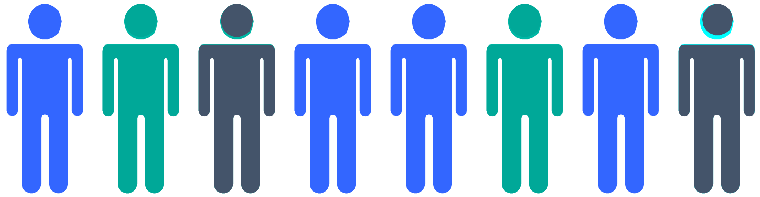
- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty
- Refers to the status quo
- Always contains "=" , "≤" or "≥" sign
- May or may not be rejected

The Alternative Hypothesis, H_1

- Is the opposite of the null hypothesis
  - e.g., The average number of TV sets in U.S. homes is not equal to 3 ($H_1: \mu \neq 3$)
- Challenges the status quo
- Never contains the "=" , "≤" or "≥" sign
- May or may not be proven
- Is generally the hypothesis that the researcher is trying to prove

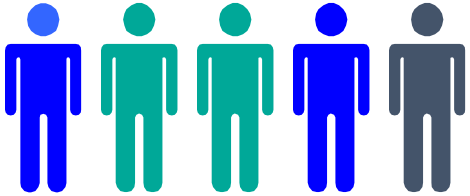
Hypothesis Testing Process

Claim: the population mean age is 50. (Null Hypothesis: $H_0: \mu = 50$)



Population

Now select a random sample



Sample

Suppose the sample mean age is 20: $\bar{X} = 20$

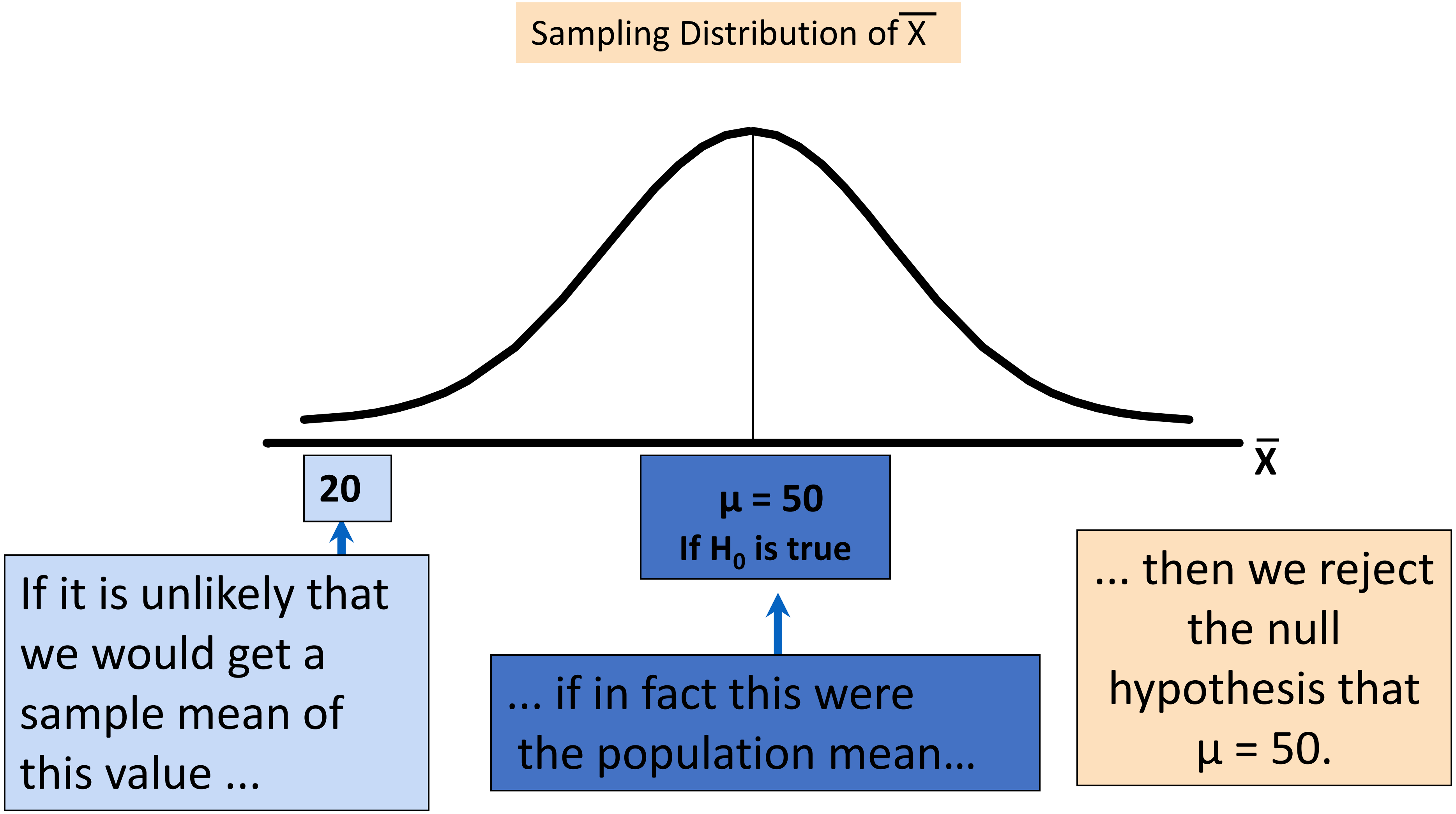
Is $\bar{X} = 20$ likely if $\mu = 50$?

If not likely,

REJECT

Null Hypothesis

Reason for Rejecting H_0



Level of Significant, α

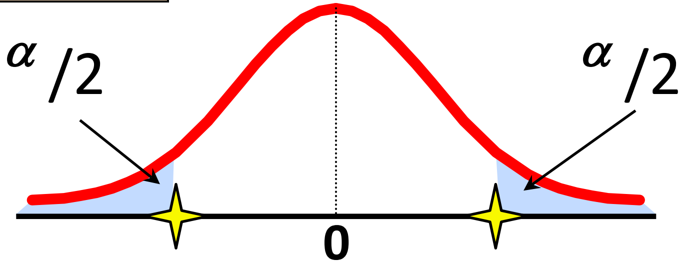
- **Defines the unlikely values of the sample statistic if the null hypothesis is true**
  - Defines rejection region of the sampling distribution
- Is designated by α , (level of significance)
  - Typical values are 0.01, 0.05, or 0.10
- Is selected by the researcher at the beginning
- Provides the critical value(s) of the test

Level of Significance and the Rejection Region

Level of significance = α

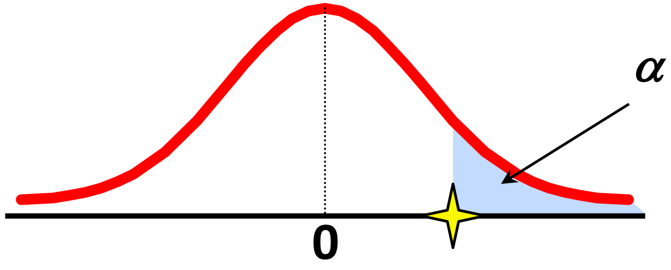
H_0: $\mu = 3$ H_1: $\mu \neq 3$

Two-tail test



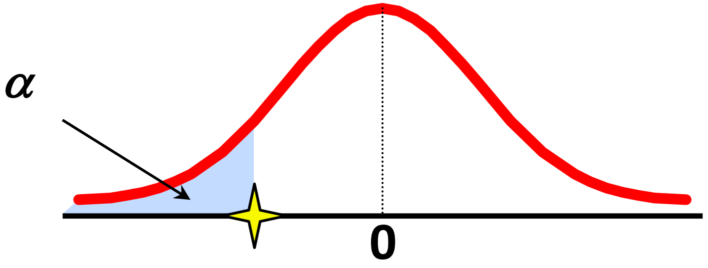
H_0: $\mu \leq 3$ H_1: $\mu > 3$

Upper-tail test



H_0: $\mu \geq 3$ H_1: $\mu < 3$

Lower-tail test



Represents critical value

Rejection region is shaded

Errors in Making Decisions

- **Type I Error**
  - Reject a true null hypothesis
  - Considered a serious type of error

The probability of Type I Error is α

  - Called level of significance of the test
  - Set by the researcher in advance
- **Type II Error**
  - Fail to reject a false null hypothesis

The probability of Type II Error is β

Outcomes and Probabilities

Key:
Outcome
(Probability)

Possible Hypothesis Test Outcomes

	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject H_0	No error $(1 - \alpha)$	Type II Error (β)
Reject H_0	Type I Error (α)	No Error $(1 - \beta)$

Type I & II Error Relationship

- Type I and Type II errors cannot happen at the same time
  - Type I error can only occur if H_0 is true
  - Type II error can only occur if H_0 is false

If Type I error probability (α) ↑,
then Type II error probability (β) ↓

Factors Affecting Type II Error

- All else equal,
  - β ↑ when the difference between hypothesized parameter and its true value ↓
  - β ↑ when α ↓
  - β ↑ when σ ↑
  - β ↑ when n ↓

Hypothesis Tests for the Mean

- **Hypothesis Tests for μ**
  - **σ Known (Z test)**
  - **σ Unknown (t test)**

- Convert sample statistic ($\bar{X}$) to a Z test statistic

Hypothesis Tests for μ

σ Known (Z test)

σ Unknown (t test)

The test statistic is:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

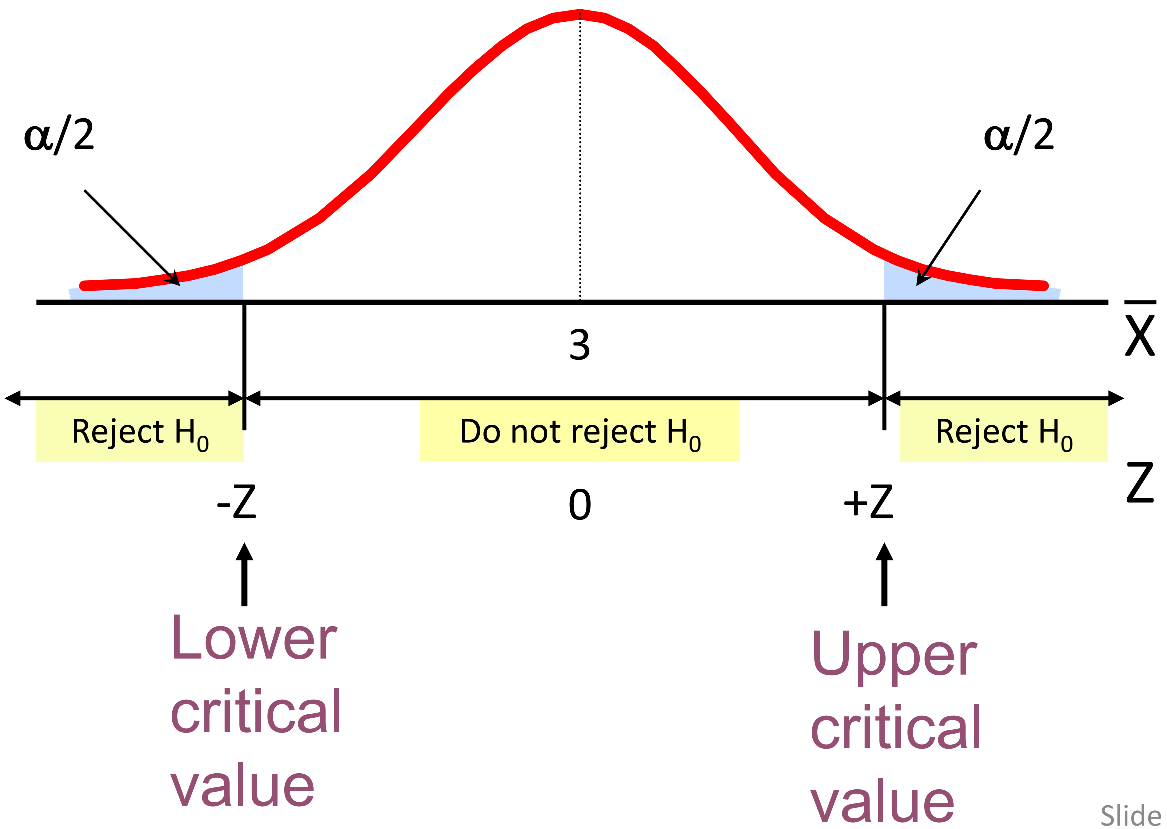
For a two-tail test for the mean, σ known:

- Convert sample statistic ($\bar{X}$) to test statistic (Z statistic)
- Determine the critical Z values for a specified level of significance α from a table or computer
- Decision Rule: If the test statistic falls in the rejection region, reject H_0 ; otherwise do not reject H_0

$H_0: \mu = 3$

$H_1: \mu \neq 3$

There are two cutoff values (critical values), defining the regions of rejection



1. State the null hypothesis, H_0 and the alternative hypothesis, H_1
2. Choose the level of significance, α, and the sample size, n
3. Determine the appropriate test statistic and sampling distribution
4. Determine the critical values that divide the rejection and nonrejection regions
5. Collect data and compute the value of the test statistic
6. Make the statistical decision and state the managerial conclusion. If the test statistic falls into the nonrejection region, do not reject the null hypothesis H_0. If the test statistic falls into the rejection region, reject the null hypothesis. Express the managerial conclusion in the context of the problem

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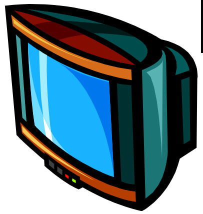
Test the claim that the true mean # of TV sets in US homes is equal to 3.

(Assume σ = 0.8)

1. State the appropriate null and alternative hypotheses
   - H_0: μ = 3 H_1: μ ≠ 3 (This is a two-tail test)
2. Specify the desired level of significance and the sample size
   - Suppose that α = 0.05 and n = 100 are chosen for this test
3. Determine the appropriate technique
   - σ is known so this is a Z test.

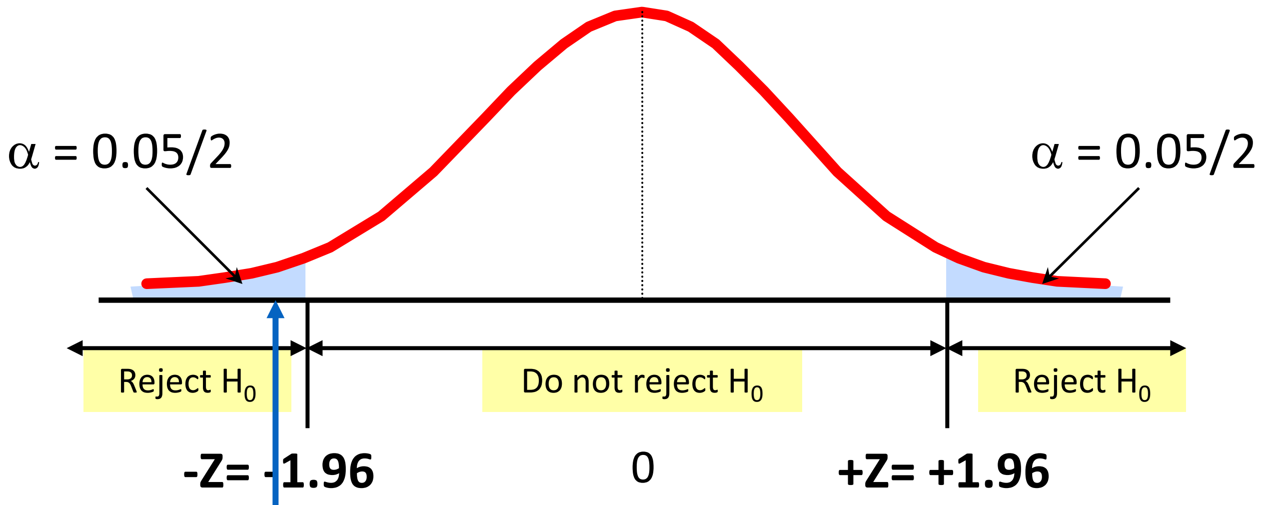


4. Determine the critical values
   - For α = 0.05 the critical Z values are ±1.96
5. Collect the data and compute the test statistic
   - Suppose the sample results are n = 100, $\bar{X}$ = 2.84 (σ = 0.8 is assumed known)

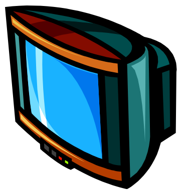
$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = -2.0$$

6. Is the test statistic in the rejection region?

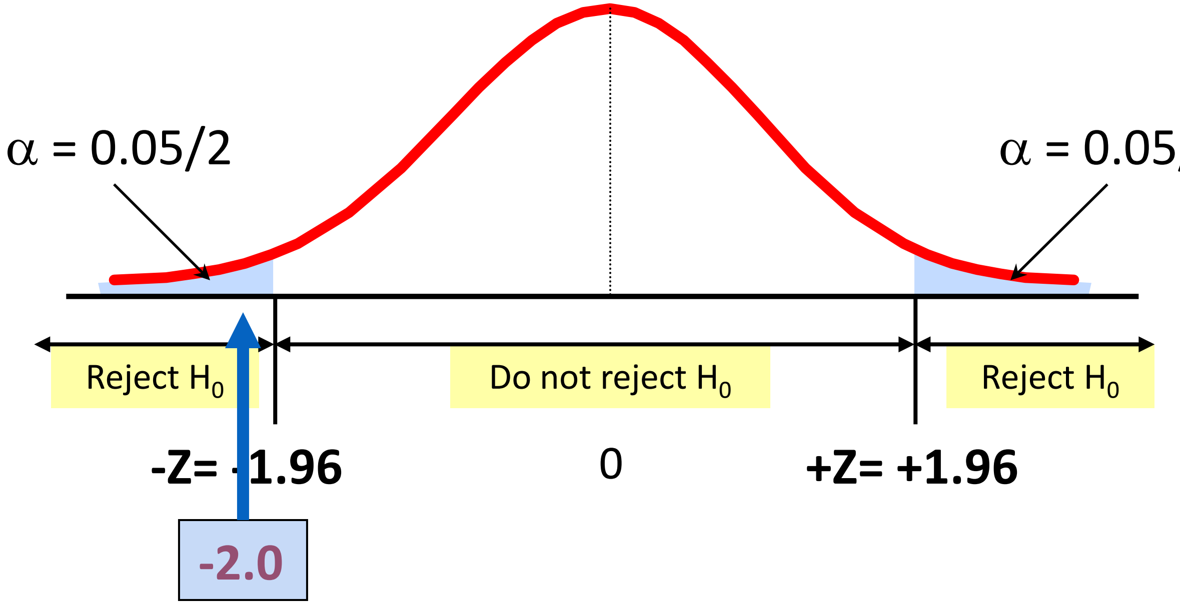
Reject H_0 if $Z < -1.96$ or $Z > 1.96$; otherwise do not reject H_0



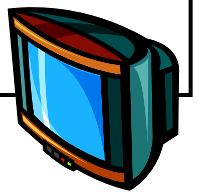
Here, $Z = -2.0 < -1.96$, so the test statistic is in the rejection region



6(continued). Reach a decision and interpret the result



Since $Z = -2.0 < -1.96$, we <u>reject the null hypothesis</u> and conclude that there is sufficient evidence that the mean number of TVs in US homes is not equal to 3



p-Value Approach to Testing

p-value: Probability of obtaining a test statistic more extreme (≤ or ≥) than the observed sample value given H_0 is true

- Also called observed level of significance
- Smallest value of α for which H_0 can be rejected

- Convert Sample Statistic (e.g., $\bar{X}$) to Test Statistic (e.g., Z statistic)
- Obtain the p-value from a table or computer
- Compare the p-value with α
  - If p-value $< \alpha$, reject H_0
  - If p-value $\geq \alpha$, do not reject H_0

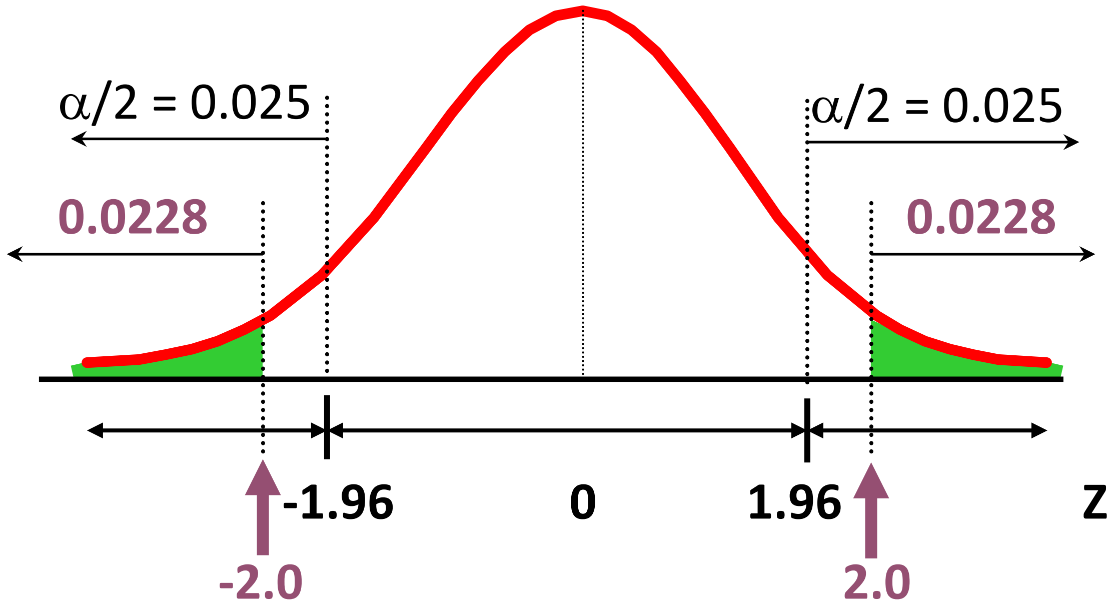
Example: How likely is it to see a sample mean of 2.84 (or something further from the mean, in either direction) if the true mean is $\mu = 3.0$?

$\bar{X} = 2.84$ is translated to a Z score of Z = -2.0

$$P(Z < -2.0) = 0.0228$$

$$P(Z > 2.0) = 0.0228$$

p-value = 0.0228 + 0.0228 = 0.0456



Compare the p-value with α

- If p-value < α , reject H_0
- If p-value $\geq$ α , do not reject H_0

Here: p-value = 0.0456 and α = 0.05

Since 0.0456 < 0.05, we reject the null hypothesis

Connection to Confidence Intervals

- For $\bar{X} = 2.84$, $\sigma = 0.8$ and $n = 100$, the 95% confidence interval is:

$$2.84 - (1.96)\frac{0.8}{\sqrt{100}} \quad \text{to} \quad 2.84 + (1.96)\frac{0.8}{\sqrt{100}}$$

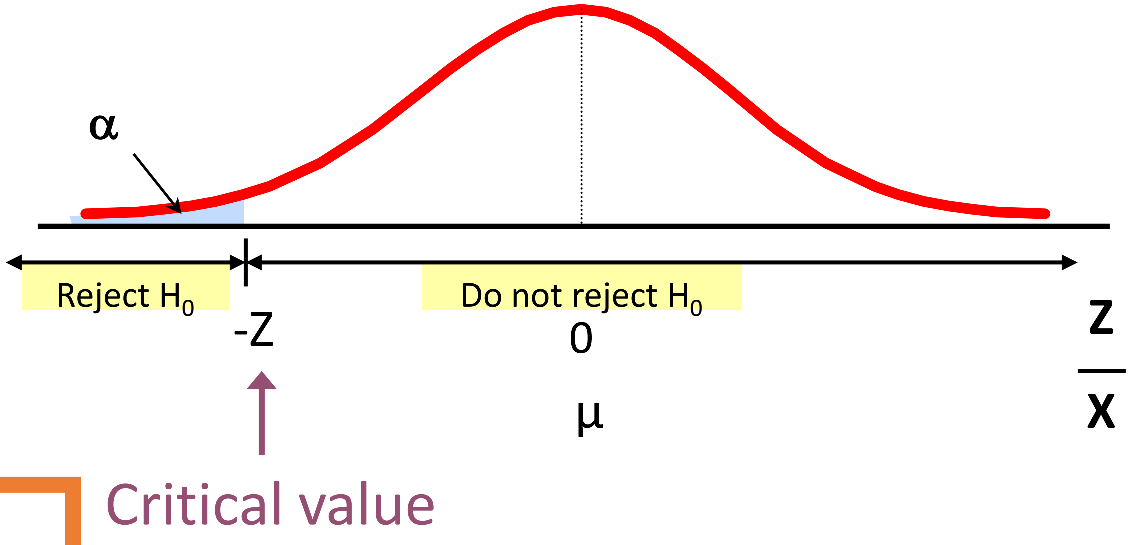
$$2.6832 \le \mu \le 2.9968$$

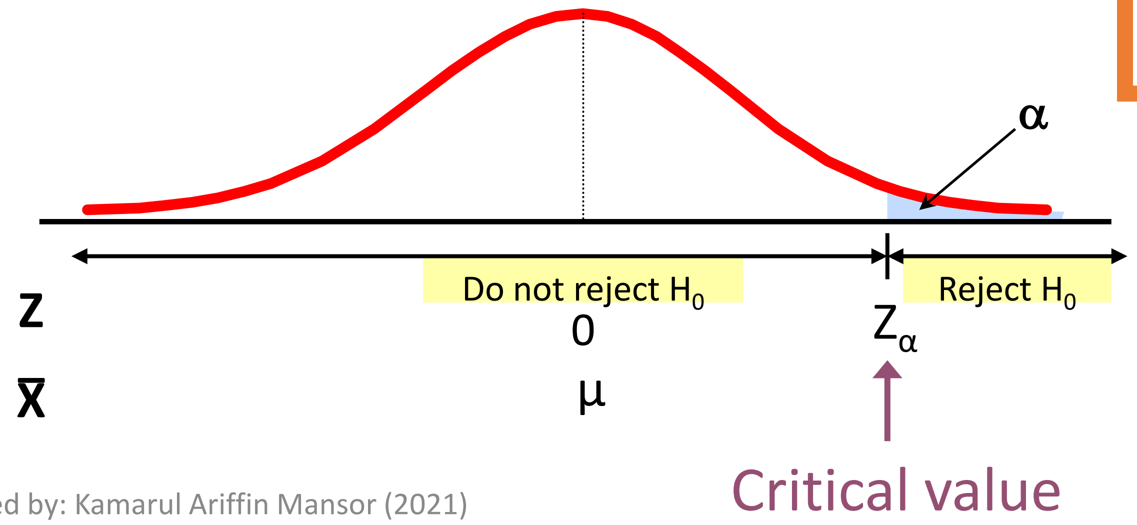
- Since this interval does not contain the hypothesized mean (3.0), we reject the null hypothesis at $\alpha = 0.05$

In many cases, the alternative hypothesis focuses on a particular direction. In this situation, there is only one critical value, since the rejection area is in only one tail.

$H_0: \mu \geq 3$
$H_1: \mu < 3$

This is a lower-tail test since the alternative hypothesis is focused on the lower tail below the mean of 3





$H_0: \mu \leq 3$
$H_1: \mu > 3$

This is an upper-tail test since the alternative hypothesis is focused on the upper tail above the mean of 3

Example: Upper-Tail Z Test for Mean (σ Known)

A phone industry manager thinks that customer monthly cell phone bills have increased, and now average over $52 per month. The company wishes to test this claim. (Assume $\sigma = 10$ is known)

Form hypothesis test:

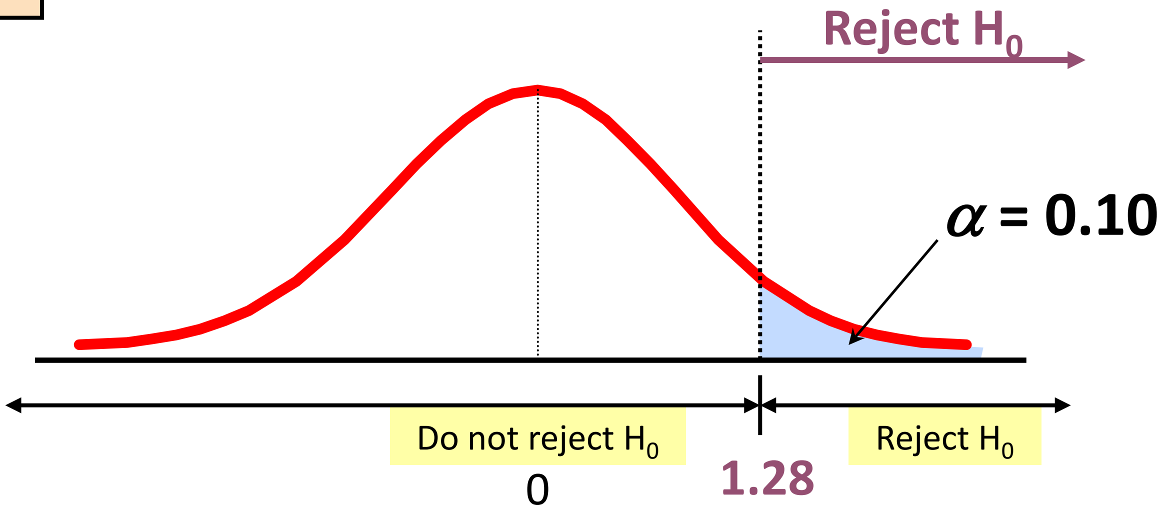
$H_0: \mu \leq 52$ the average is not over $52 per month

$H_1: \mu > 52$ the average **is** greater than $52 per month

(i.e., sufficient evidence exists to support the manager's claim)

Suppose that $\alpha = 0.10$ is chosen for this test

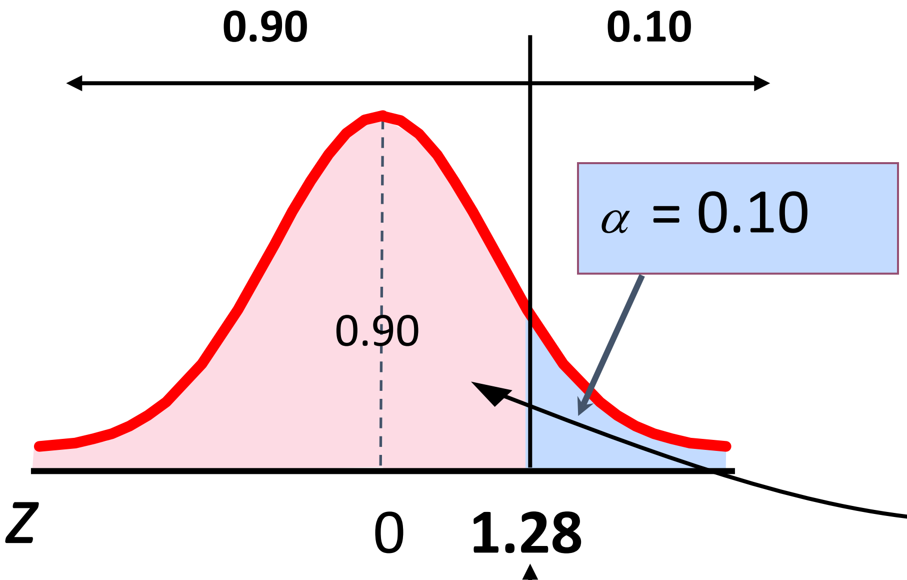
Find the rejection region:



Reject H_0 if $Z > 1.28$

Review: One-Tail Critical Value

What is Z given $\alpha = 0.10$?



Critical Value = 1.28

Standardized Normal Distribution Table (Portion)

Z	.07	.08	.09
1.1	.8790	.8810	.8830
1.2	.8980	.8997	.9015
1.3	.9147	.9162	.9177

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Example: Test Statistic

Obtain sample and compute the test statistic

Suppose a sample is taken with the following results: $n = 64$, $\bar{X} = 53.1$

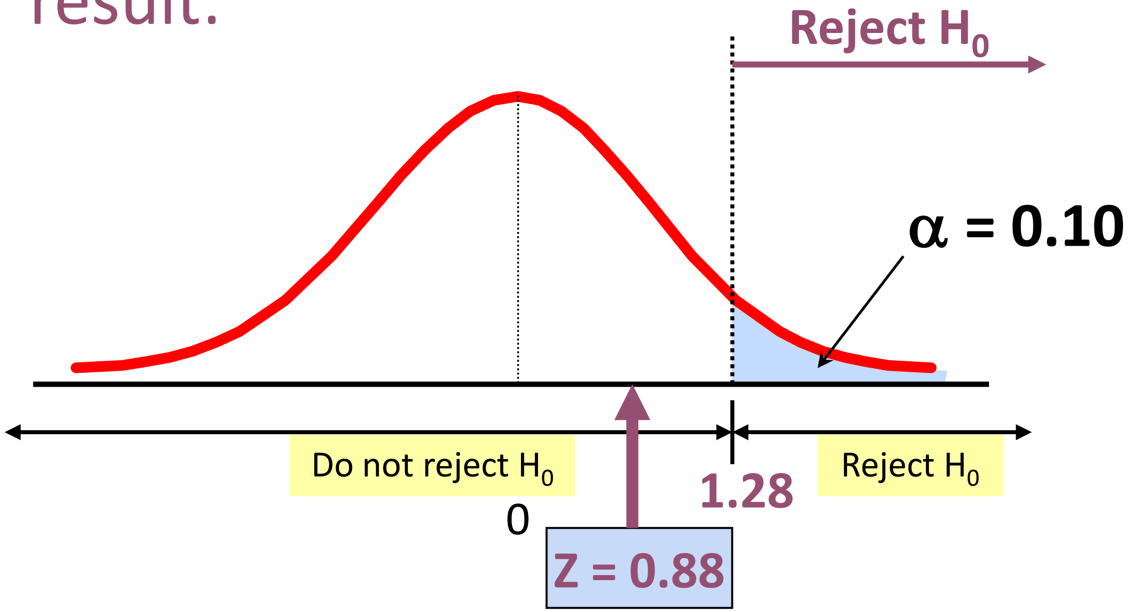
(σ=10 was assumed known)

Then the test statistic is:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{64}}} = 0.88$$

Example: Decision

Reach a decision and interpret the result:

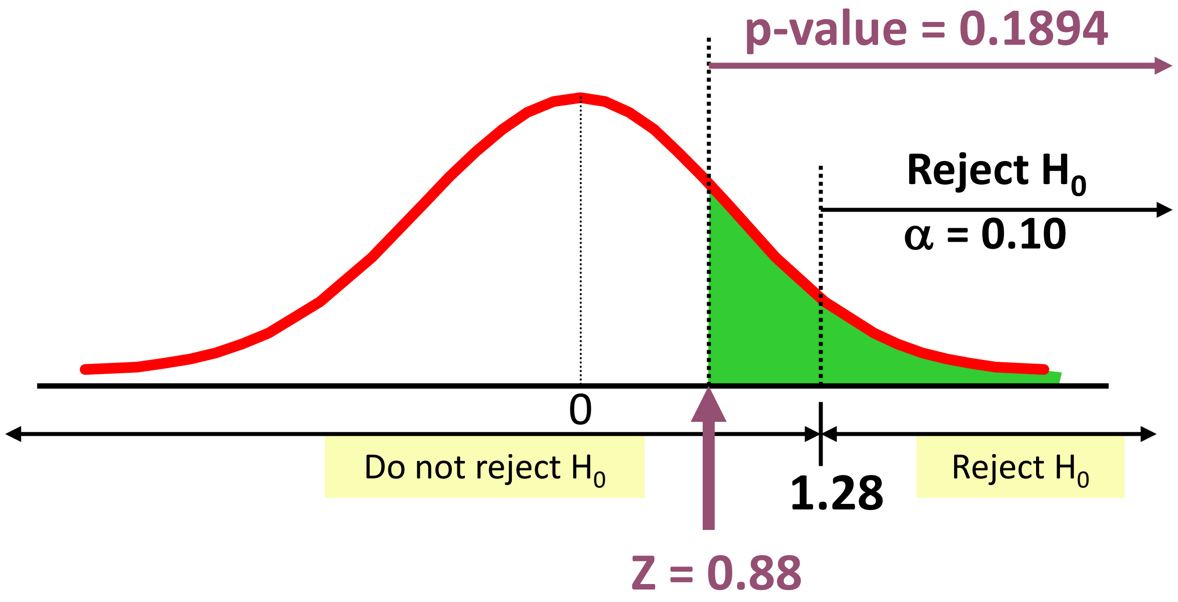


Do not reject H_0 since $Z = 0.88 \le 1.28$

i.e.: there is not sufficient evidence that the mean bill is over $52

Calculate the p-value and compare to α

(assuming that $\mu = 52.0$)



$$P(\bar{X} \geq 53.1)$$

$$= P\left(Z \geq \frac{53.1 - 52.0}{10/\sqrt{64}}\right)$$

$$= P(Z \geq 0.88) = 1 - 0.8106$$

$$= 0.1894$$

Do not reject H_0 since p-value = 0.1894 > α = 0.10

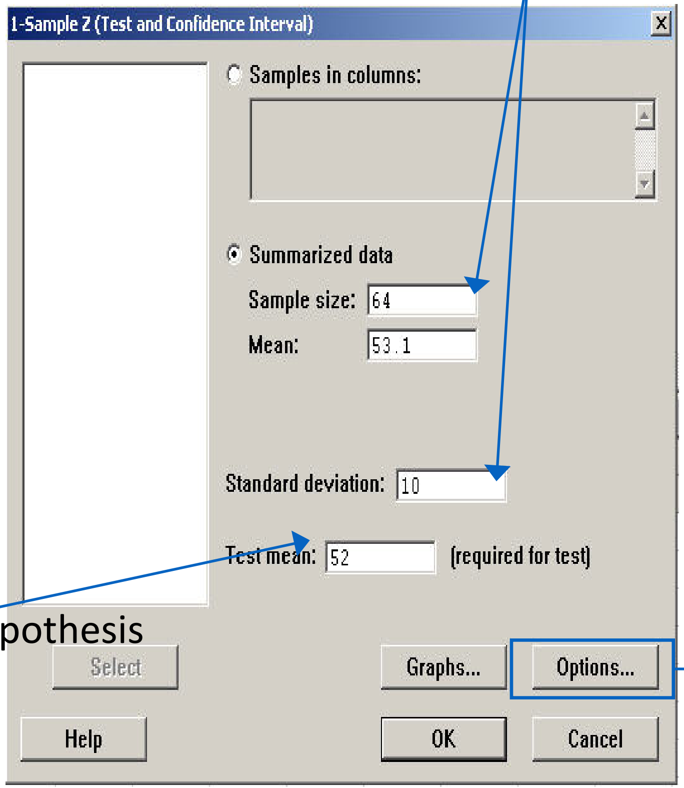
Hypothesis test for μ, σ Known (use Z):

Select:

Stat / Basic Statistics /

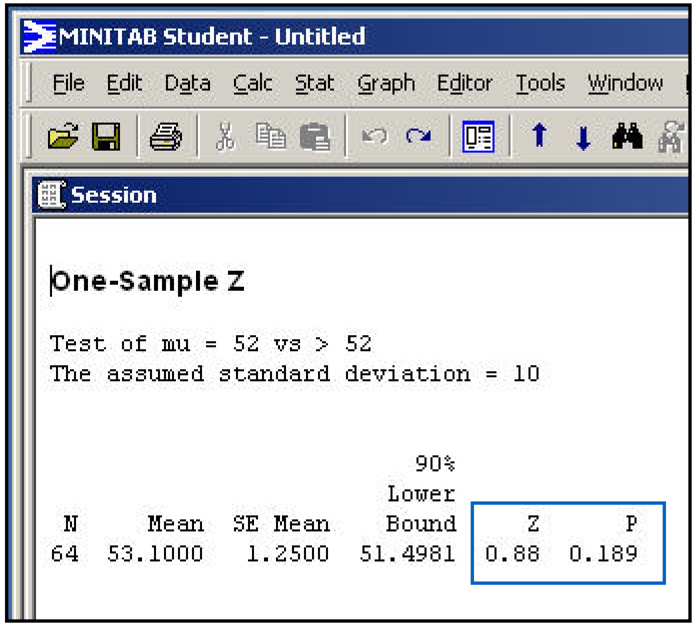
1-Sample Z…

Enter known values

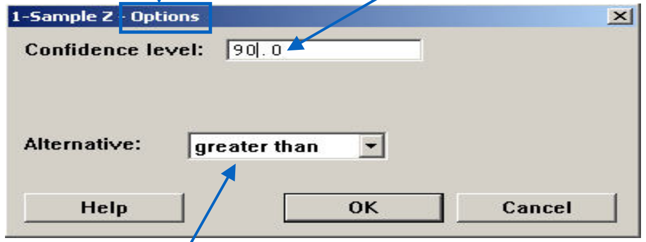


Value from null hypothesis

Minitab output:



Enter desired level of confidence



Choose desired alternative hypothesis

t Test of Hypothesis for the Mean (σ Unknown)

- Convert sample statistic ($\bar{X}$) to a t test statistic

Hypothesis Tests for μ

σ Known (Z test)

σ Unknown (t test)

The test statistic is:

$$t_{n-1} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

Example: Two-Tail Test (σ Unknown)

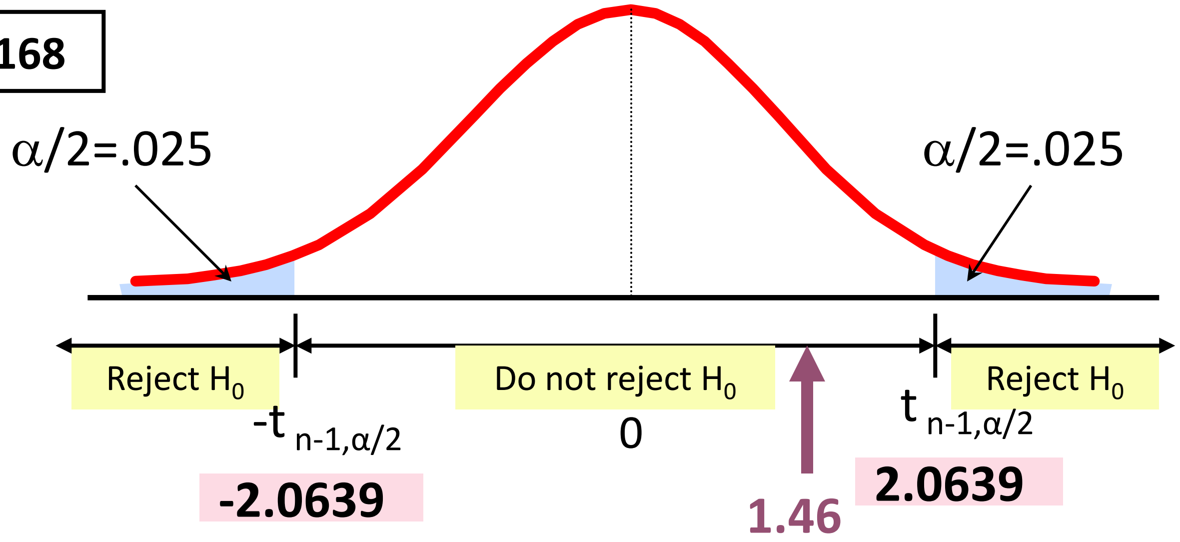
The average cost of a hotel room in New York is said to be \$168 per night. A random sample of 25 hotels resulted in $\bar{X}$ = \$172.50 and S = \$15.40.

Test at the α = 0.05 level.

(Assume the population distribution is normal)

SOLUTION

$H_0: \mu = 168 \quad H_1: \mu \neq 168$



- **α = 0.05**
- **n = 25**
- **σ is unknown, so use a t statistic**
- **Critical Value: $t_{24} = \pm 2.0639$**

$$t_{n-1} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

Do not reject H_0

not sufficient evidence that true mean cost is different than \$168

Connection to Confidence Intervals

For $\bar{X}$ = 172.5, s = 15.40 and n = 25, the 95% confidence interval is:

$$172.5 - (2.0639)\ 15.4/\sqrt{25} \quad \text{to} \quad 172.5 + (2.0639)\ 15.4/\sqrt{25}$$

$$166.14 \leq \mu \leq 178.86$$

Since this interval contains the Hypothesized mean (168), we do not reject the null hypothesis at α = 0.05

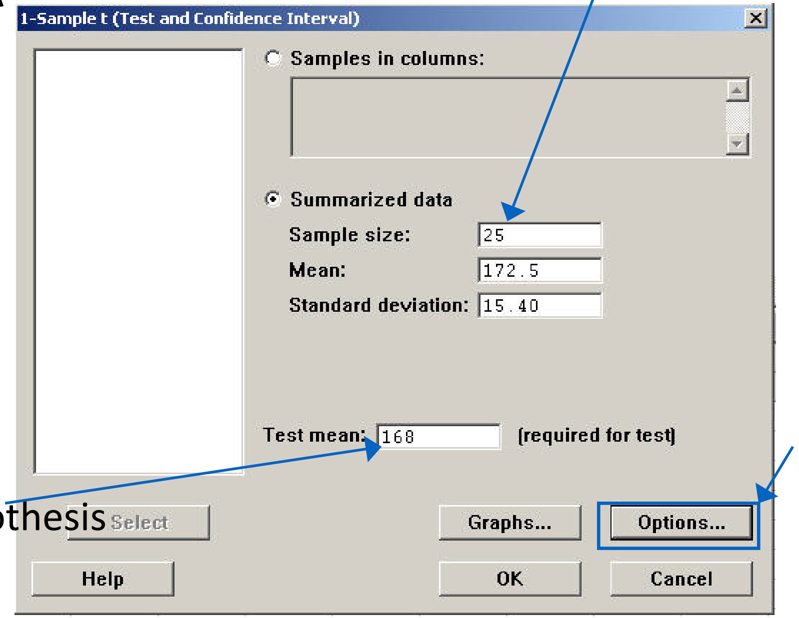
Hypothesis test for μ, σ Unknown (use t):

Select:

Stat / Basic Statistics /

1-Sample t…

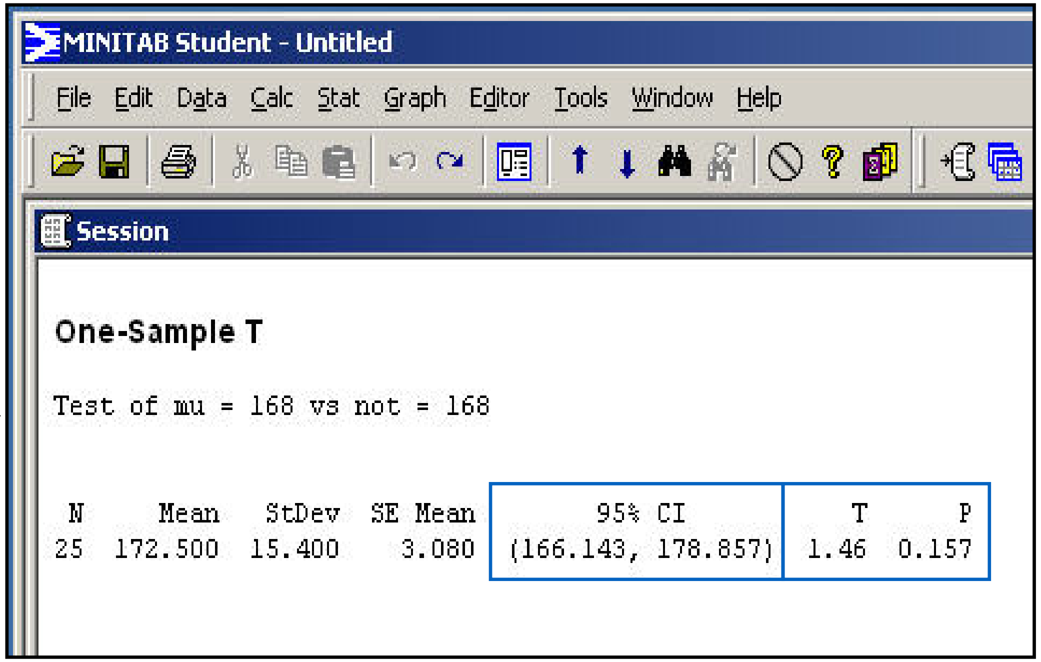
Enter sample results



Value from null hypothesis

Select needed options

Minitab output:



END OF SLIDES

PRESENTATIONS

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