Unleashing Potentials
Shaping the Future

Chapter 4: Hypothesis Testing

Unit 2
Testing the Difference Between Two Means

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Outline

1. Testing the Difference Between Two Means: Using the *z* Test
2. Testing the Difference Between Two Means of Independent Samples: Using the *t* Test
3. Testing the Difference Between Two Means: Dependent Samples

Learning Objectives

1. Test the difference between sample means, using the *z*-test.
2. Test the difference between two means for independent samples, using the *t* test.
3. Test the difference between two means for dependent samples.

Assumptions:

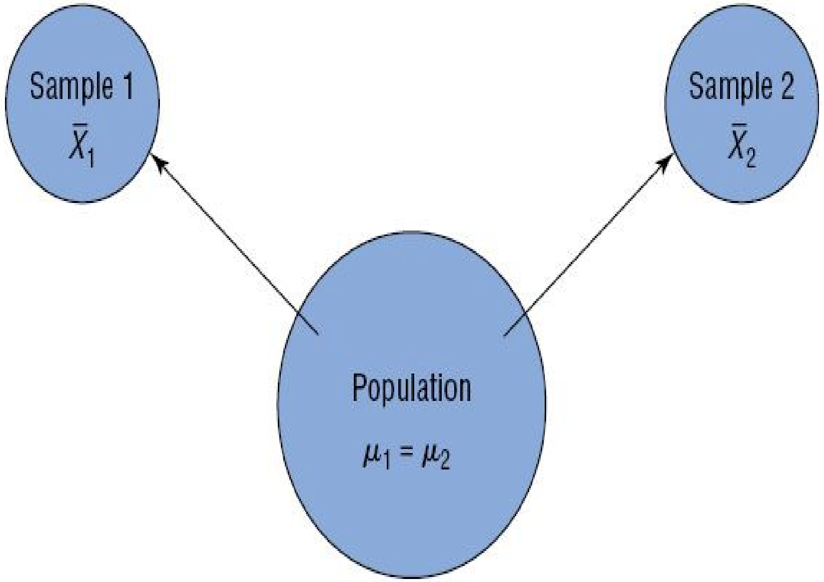
1. Both samples are random samples.
2. The samples must be independent of each other. That is, there can be no relationship between the subjects in each sample.
3. The standard deviations of both populations must be known; and if the sample sizes are less than 30, the populations must be normally or approximately normally distributed.

Large Sample Case

Formula for the z test for comparing two means from independent populations

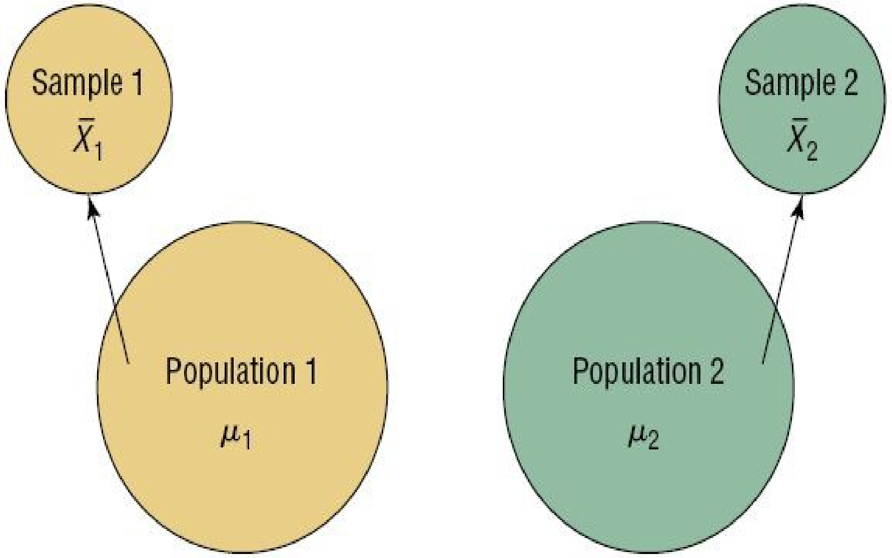
$$z = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Hypothesis Testing Situations in the Comparison of Means



(a) Difference is not significant

Do not reject H_0: $\mu_1 = \mu_2$ since $\bar{X}_1 - \bar{X}_2$ is not significant.



(b) Difference is significant

Reject H_0: $\mu_1 = \mu_2$ since $\bar{X}_1 - \bar{X}_2$ is significant.

Example 1: Leisure Time

A study using two random samples of 35 people each found that the average amount of time those in the age group of 26–35 years spent per week on leisure activities was 39.6 hours, and those in the age group of 46–55 years spent 35.4 hours. Assume that the population standard deviation for those in the first age group found by previous studies is 6.3 hours, and the population standard deviation of those in the second group found by previous studies was 5.8 hours.

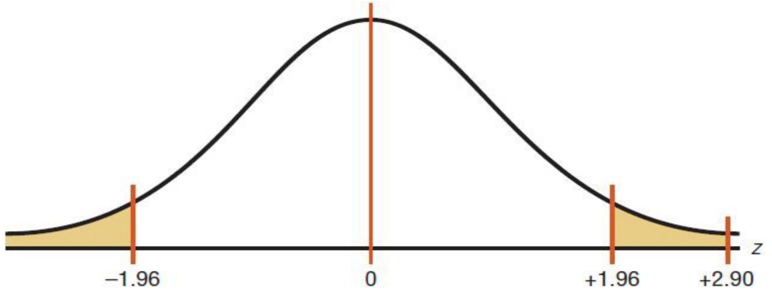
At $\alpha = 0.05$, can it be concluded that there is a significant difference in the average times each group spends on leisure activities?

Example 1: Leisure Time (Solution)

Step 1 State the hypotheses and identify the claim.

$$H_0: \mu_1 = \mu_2 \quad \text{and} \quad H_1: \mu_1 \neq \mu_2 \text{ (claim)}$$

- Since we have two groups…so we have μ1 and μ2.
- Above is how we stated the hypothesis (two-tailed testing)



Step 2 Find the critical values. Since $\alpha = 0.05$, the critical values are $+1.96$ and -1.96.

To find the critical value..same as for one sample. In this problem α need to divide by 2 (two tailed). Using table 4, since n > 30.

Step 3 Compute the test value.

$$z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(39.6 - 35.4) - 0}{\sqrt{\frac{6.3^2}{35} + \frac{5.8^2}{35}}} = \frac{4.2}{1.447} = 2.90$$

Step 4: Make the decision.

Reject the null hypothesis at $\alpha = 0.05$, since Z = 2.90 > CV = 1.96

Step 5: Summarize the results.

There is enough evidence to support the claim that the means are not equal.

That is, the average of the times spent on leisure activities is different for the groups.

Formula for the *t* test for comparing two means from independent populations with unequal variances

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad ; \quad df = n_1 + n_2 - 2$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

Assumptions

1. The samples are random samples.
2. The sample data are independent of one another.
3. When the sample sizes are less than 30, the populations must be normally or approximately normally distributed.

Example 2: Weight of Newborn Infants

A researcher wishes to see if the average weights of newborn male infants are different from the average weights of newborn female infants.

She selects a random sample of 10 male infants and finds the mean weight is 7 pounds 11 ounces and the standard deviation of the sample is 8 ounces.

She selects a random sample of 8 female infants and finds that the mean weight is 7 pounds 4 ounces, and the standard deviation of the sample is 5 ounces.

Can it be concluded at $\alpha = 0.05$ that the mean weight of the males is different from the mean weight of the females?

Assume that the variables are normally distributed.

Example 2: Weight of Newborn Infants (Solution)

Step 1: State the hypotheses and identify the claim.

H_0: $\mu_1 = \mu_2$ and H_1: $\mu_1 \neq \mu_2$ (claim)

$$7 \text{ lb } 11 \text{ oz} = 7 \times 16 + 11 = 123 \text{ oz}$$
$$7 \text{ lb } 4 \text{ oz} = 7 \times 16 + 4 = 116 \text{ oz}$$

Step 2: Find the critical values.

Since the test is two-tailed and α = 0.05, the critical values are +2.120 and -2.120.

Step 3: Find the Test Value.

Test value (t-statistic) = …..

Step 4: Make the decision.

……………………………………..

Step 5: Summarize the results.

There is not enough evidence to support the claim that the mean of the weights of the male infants is different from the mean of the weights of the female infants.

Hypothesis testing :

Decision rule when using a P-value

If P-value ≤ α reject the null hypothesis

If P-value >, α do not reject the null hypothesis

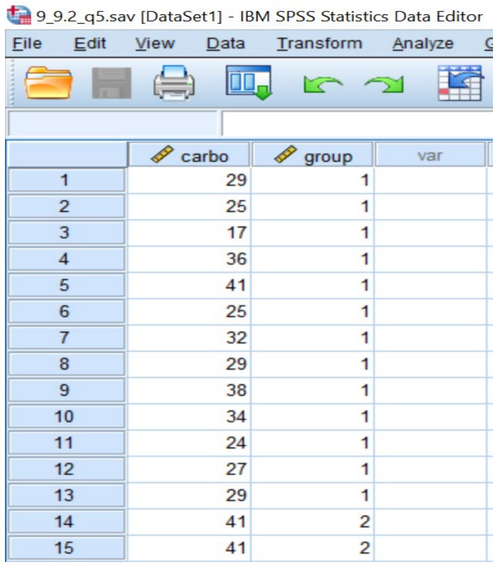
Right-tailed			Left-tailed		
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 > \mu_2$	or	$H_0: \mu_1 - \mu_2 = 0$ $H_1: \mu_1 - \mu_2 > 0$	$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 < \mu_2$	or	$H_0: \mu_1 - \mu_2 = 0$ $H_1: \mu_1 - \mu_2 < 0$

Exercise chapter 9 section 9.2 question no 5. (from textbook)

5. **Carbohydrates in Candies** The number of grams of carbohydrates contained in 1-ounce servings of randomly selected chocolate and nonchocolate candy is listed here. Is there sufficient evidence to conclude that the difference in the means is statistically significant? Use $\alpha = 0.10$.

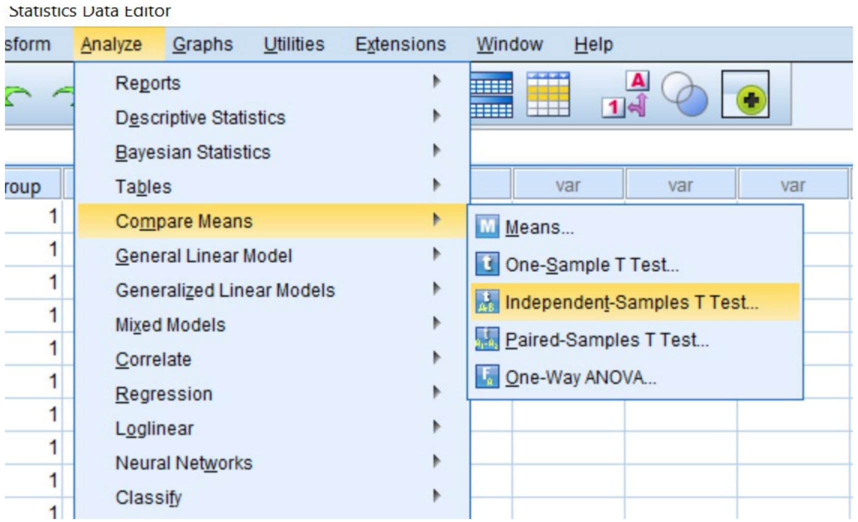
Chocolate:	29	25	17	36	41	25	32	29
	38	34	24	27	29			
Nonchocolate:	41	41	37	29	30	38	39	10
	29	55	29					

Test hypothesis using p-value method with SPSS

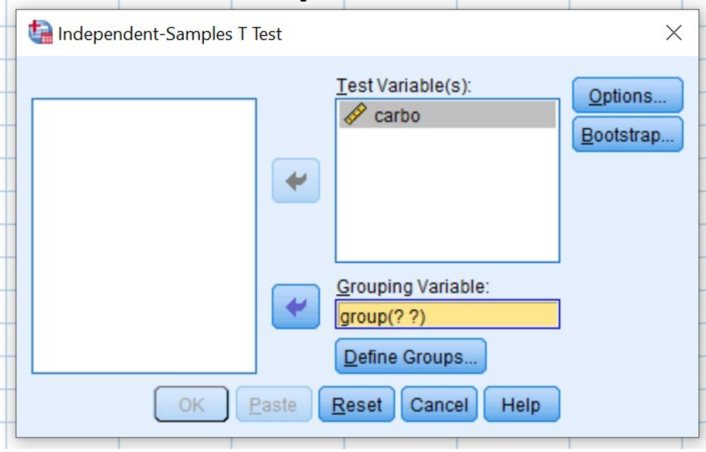


❶ Enter data in SPSS as shown.

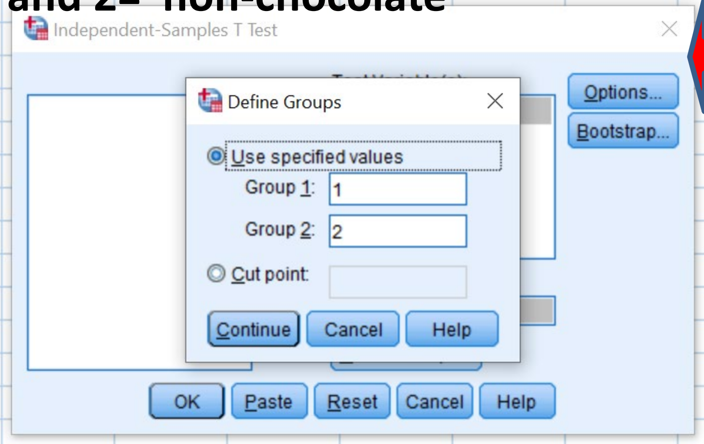
❷ Select analysis

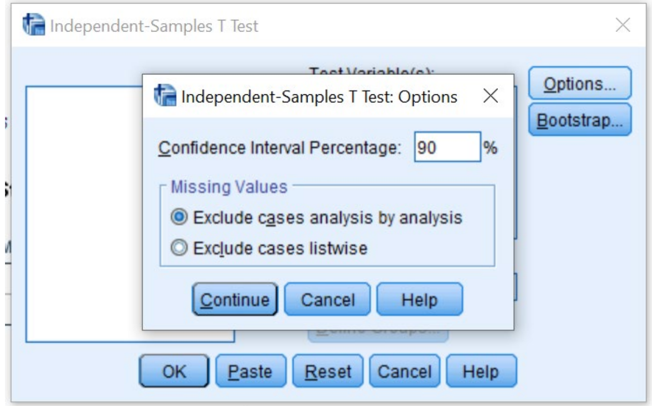


❸ Move test variable and grouping variable in respective boxes

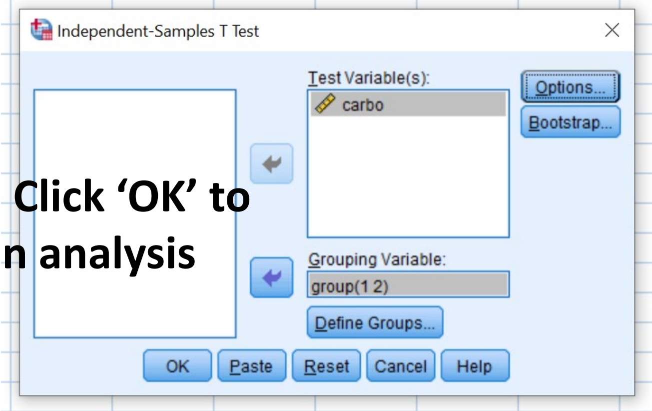


❹ Define group, 1="chocolate" and 2="non-chocolate"





❺ Set the confidence level



❻ Click 'OK' to run analysis

Test hypothesis using p-value method with SPSS

*Output1 [Document1] - IBM SPSS Statistics Viewer

File Edit View Data Transform Insert Format Analyze Graphs Utilities Extensions Window Help

Output
- Log
- T-Test
  - Title
  - Notes
  - Group Statistics
  - Independent Sam

```
T-TEST GROUPS=group(1 2)
  /MISSING=ANALYSIS
  /VARIABLES=carbo
  /CRITERIA=CI(.90).
```

T-Test

Group Statistics

❶

	group	N	Mean	Std. Deviation	Std. Error Mean
carbo	chocolate	13	29.69	6.499	1.802
	nonchocolate	11	34.36	11.201	3.377

Independent Samples Test

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		Levene's Test for Equality of Variances		t-test for Equality of Means					90% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
carbo	Equal variances assumed	1.836	.189	-1.274	22	.216	-4.671	3.666	-10.966	1.623
	Equal variances not assumed			-1.220	15.463	.241	-4.671	3.828	-11.369	2.026

Is there sufficient evidence to conclude that the difference in the means is statistically significant? Use $\alpha = 0.10$

SOLUTION :

- Since we have two groups, so we have μ_1= mean of carbohydrates for chocolate candy and μ_2=mean of carbohydrates for non chocolate candy.
- Situation is two-tailed hypothesis testing
- Now refer to table independent sample test, see column Levenes' Test For Equality of Variance . There is two column, first column F and second column Sig. See Sig value is 0.189, this is a p-value. We write down as: *p*-value = 0.189

❶ **Preliminary step**
Here you can find the mean sample and standard deviation for each group. The 'Std. Error Mean' column is simply, $\frac{s}{\sqrt{n}}$

❷ **Step 1 : Check for Levenes' Test**

Setup the hypothesis for Levenes' test

H_0: Equal variance assumed

H_1 : Equal variance is not assumed

Since p-value = 0.189 > alpha = 0.10.

Fail to reject H_0.

Conclusion : Equal variance assumed.

❸ **Step 2 :State the hypotheses**

H_0: $\mu_1 = \mu_2$

H_1: $\mu_1 \neq \mu_2$

❹ **Step 4: Test hypothesis and make decision**
Now refer to row equal variance assumed, there is another sig value. This is a second p-value. Now we are going to compare this *p*-value = 0.216 with alpha = 0.10. Since p-value = 0.216 > α = 0.10, we failed to reject H_0.

Conclusion :
No sufficient evidence to conclude that there is different carbohydrates in chocolate candy and in non-chocolate candy.

Test your SPSS skills
Exercise (from textbook)
Chapter 9, Section 9.2 Question No 6

6. **Weights of Vacuum Cleaners** Upright vacuum cleaners have either a hard body type or a soft body type. Shown are the weights in pounds of a random sample of each type. At $\alpha = 0.05$, can it be concluded that the means of the weights are different?

Hard body types				Soft body types			
21	17	17	20	24	13	11	13
16	17	15	20	12	15		
23	16	17	17				
13	15	16	18				
18							

Check your output:

At $\alpha = 0.05$, can it be concluded that the means of the weights are different?

Group Statistics

	Body type	N	Mean	Std. Deviation	Std. Error Mean
Weights of Vacuum Cleaner	Hard body type	17	17.41	2.451	.594
	Soft body type	6	14.67	4.761	1.944

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Weights of Vacuum Cleaner	Equal variances assumed	2.005	.171	1.830	21	.081	2.745	1.500	-.374	5.864
	Equal variances not assumed			1.351	5.963	.226	2.745	2.033	-2.236	7.726

END OF SLIDES

PRESENTATIONS

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