SciencesPo
DEPARTMENT OF ECONOMICS

ScPoEconometrics

Regression Inference

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SciencesPo Paris
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Quick "Quiz" on Last Week's Material

1. From your ***computer*** ☞ connect to ***www.wooclap.com/SCPOCIHT***

   OR

2. From your ***phone*** ☞ flash QR code below

Today - Statistical inference in the regression framework

- Fully understand a ***regression table***
- Compare ***theory-based*** and ***simulation-based*** inference
- ***Classical Regression Model*** assumptions
- Empirical applications:
    - Class size and student performance
    - Returns to education by gender

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    - *Kindergarten* grade.

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- What if we drew another random sample of schools from Tennessee and redid the experiment, would we find a different value for b_1?

- We know the answer is *yes*, but how different is this estimate likely to be?

Regression Inference: b_k vs β_k

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- You will often find $\hat{\beta}_k$ rather than b_k, both refer to sample estimate of β_k.
- Let's bring what we know about ***confidence intervals***, ***hypothesis testing*** and ***standard errors*** to bear on those $\hat{\beta}_k$!

Understanding Regression Tables

Here is our `tidy` regression:

```r
library(broom)
tidy(lm(math ~ small, star_df))
```

```
## # A tibble: 2 x 5
##   term        estimate std.error statistic     p.value
##   <chr>          <dbl>     <dbl>     <dbl>       <dbl>
## 1 (Intercept)   484.        1.15    421.   0
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Entry	Meaning
`std. error`	Standard error of b_k
`statistic`	Observed test statistic associated to $H_0 : \beta_k = 0, H_A : \beta_k \neq 0$
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- Let's focus on the `small` coefficient and make sense of each entry.

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- We'd run 1,000 regressions and obtain 1,000 estimates of β_k, b_k.
- The standard error of b_k quantifies how much variation in b_k one would expect across (*an infinity of*) samples.

Standard Error of b_{small}

- From the table, we get $\hat{\text{SE}}(b_{\text{small}}) = 1.68$
    - Notice that we write $\hat{\text{SE}}$ and not SE because 1.68 is an estimate of the real standard error of b_{small} we get from our sample.
    - We would love to know the real standard error SE, but we have only one sample!

Standard Error of b_{small}

- From the table, we get $\hat{\text{SE}}(b_{\text{small}}) = 1.68$
  - Notice that we write $\hat{\text{SE}}$ and not SE because 1.68 is an estimate of the real standard error of b_{small} we get from our sample.
  - We would love to know the real standard error SE, but we have only one sample!
- Let's simulate the sampling distribution of b_{small} to see where it comes from.

Task 1

10:00

As we did for the sampling distribution of the proportion of *green pasta*, we want to generate the bootstrap distribution of b_{small}.

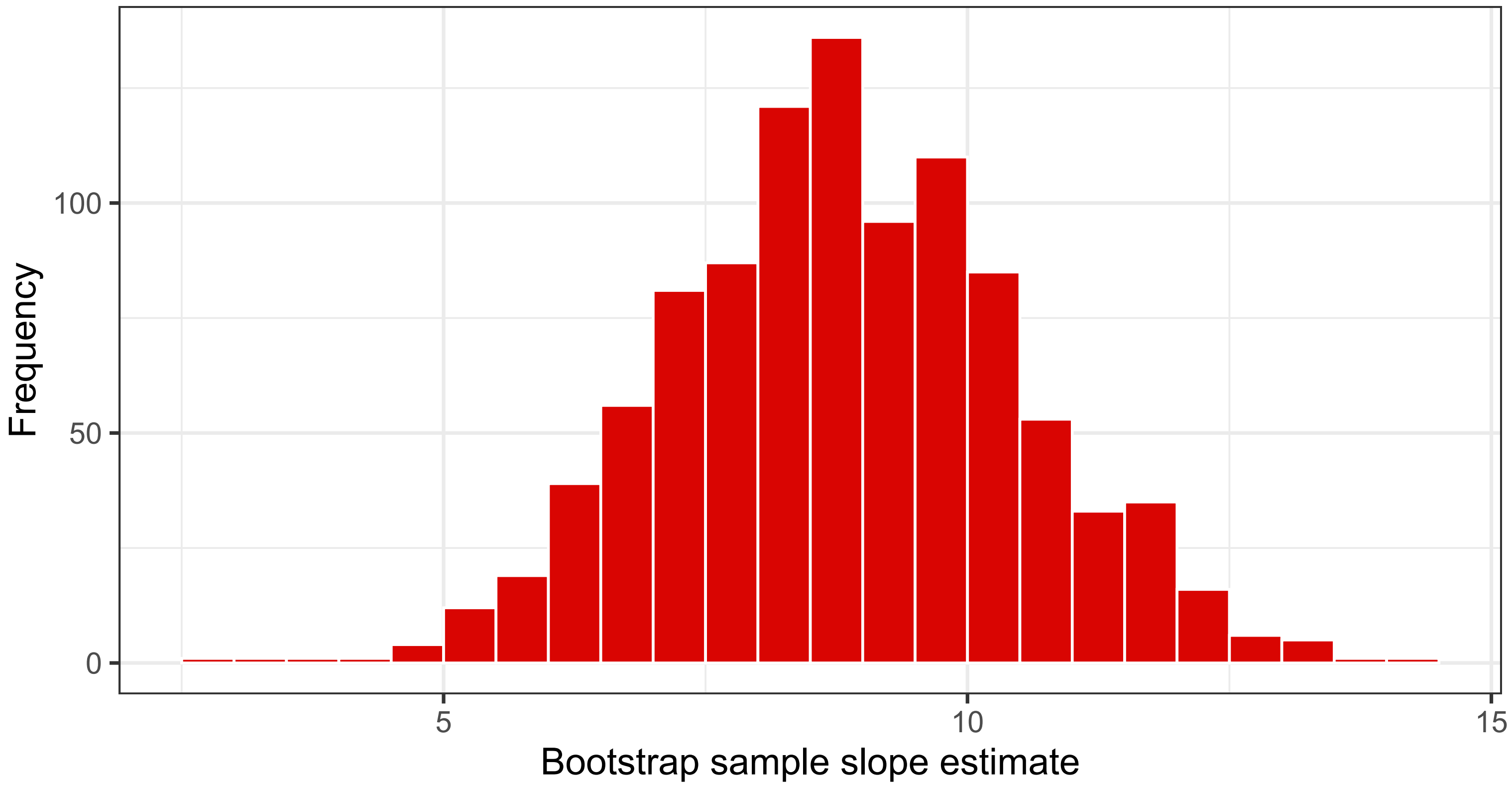
1. Copy the loading and cleaning code from slide 3 and run it.

2. Generate the bootstrap distribution of b_{small} based on 1,000 samples drawn from `star_df`. You can do so through the following code

```
bootstrap_distrib <- star_df %>%
    mutate(small=as.numeric(small)) %>%
    specify(response = math, explanatory = small) %>%
    generate(reps = 1000, type = "bootstrap") %>%
    calculate(stat = "slope")
```

1. Plot this simulated sampling distribution and compute the mean and standard error of b_{small}.

Bootstrap Distribution



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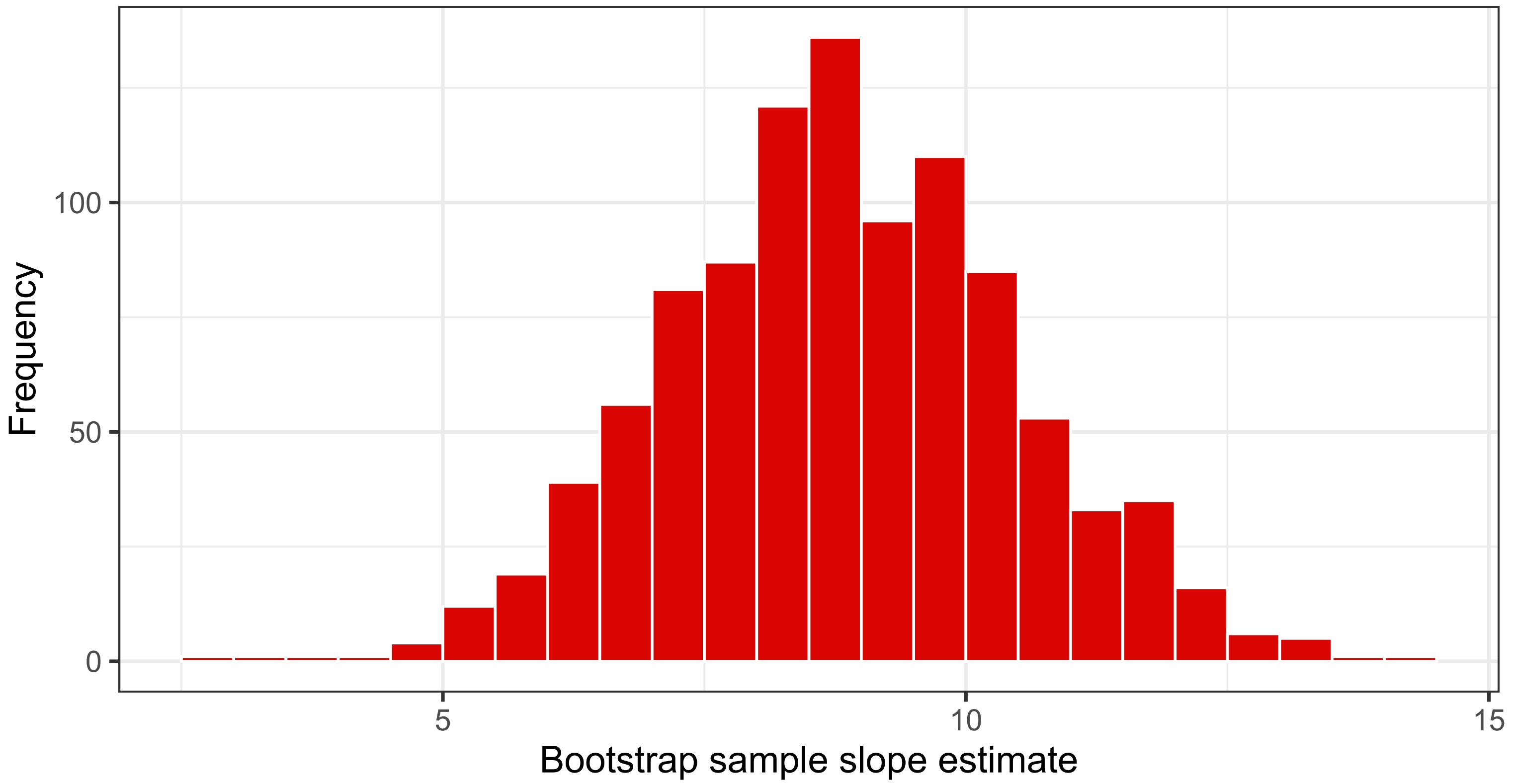

standard error: 1.66 $\rightarrow$ very close to the one in the table (1.68)!

Not exactly equal, because we used bootstrapping instead of the theory approach used by `R`.

Back to our regression results

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- We have made sense of the `std.error` column.
- The next two columns in our regression are `statistic` and `p.value`
- We know those terms from our previous class on hypothesis testing
- But which hypothesis test do they correspond to?

Testing $\beta_k = 0$ vs $\beta_k \neq 0$

By default, the regression output provides the results associated with the following hypothesis test:

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- If H_0 is false, then there **is** a true relationship.
- ***Important:*** This is a ***two-sided*** test!

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    - Why not just b? We'll come back and explain this formula later.

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- The **p-value** measures the area outside of $\pm$ *observed test statistic* under the *null distribution*.
- Finally, we check if we can reject H_0 at the usual **significance levels**: α = 0.1, 0.05, 0.01.

Testing $\beta_{\text{small}} = 0$ vs $\beta_{\text{small}} \neq 0$

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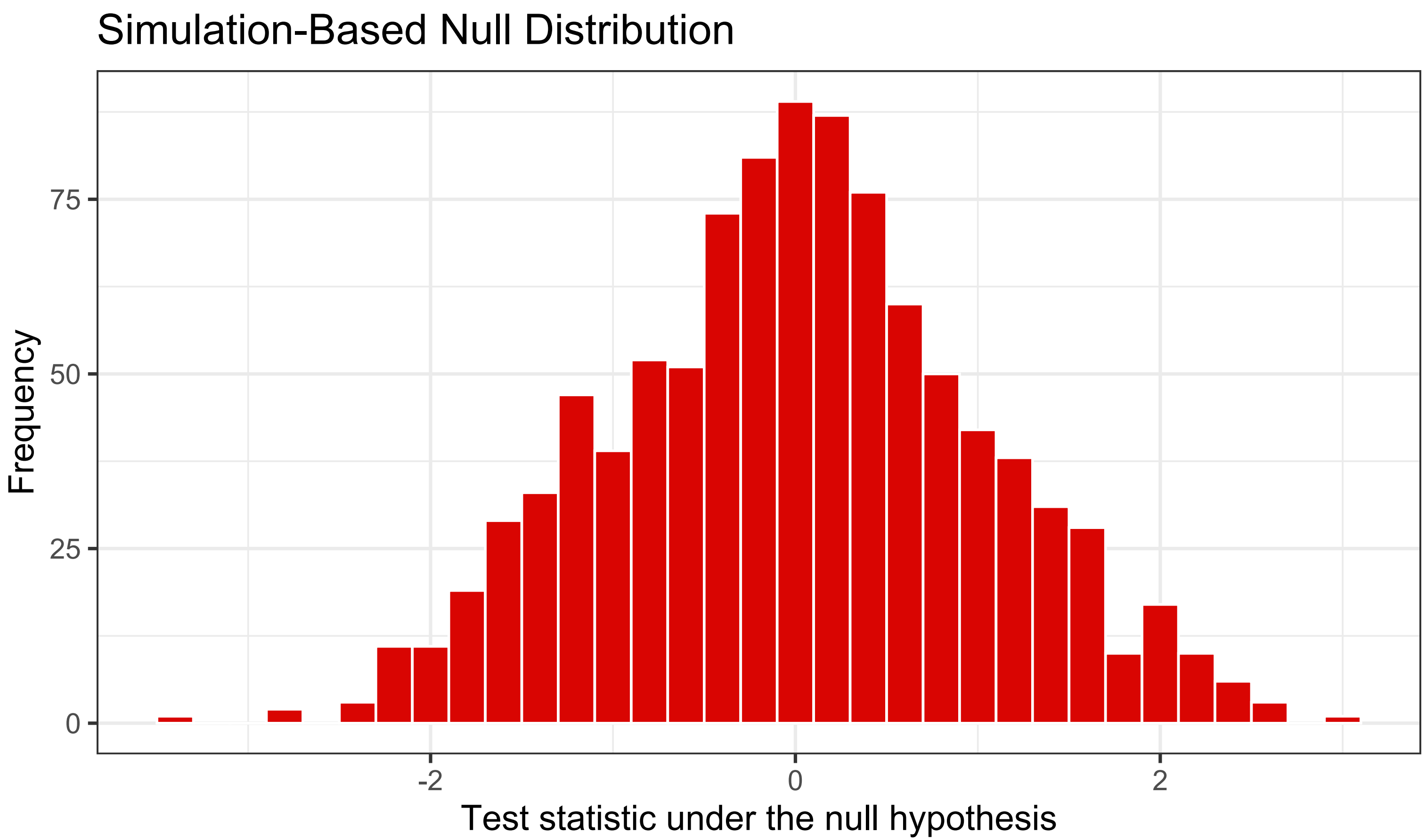
- We can compute the distribution of our test statistic $\frac{b_{\text{small}}}{\hat{SE}(b_{\text{small}})}$ under the null:

```
null_distribution <- null_distribution %>%
  mutate(test_stat = stat/sd(bootstrap_distrib$stat))
```

- Remember we got $\hat{SE}(b_{\text{small}})$ = 1.66 from our bootstrap distribution.

Testing $\beta_{\text{small}} = 0$ vs $\beta_{\text{small}} \neq 0$



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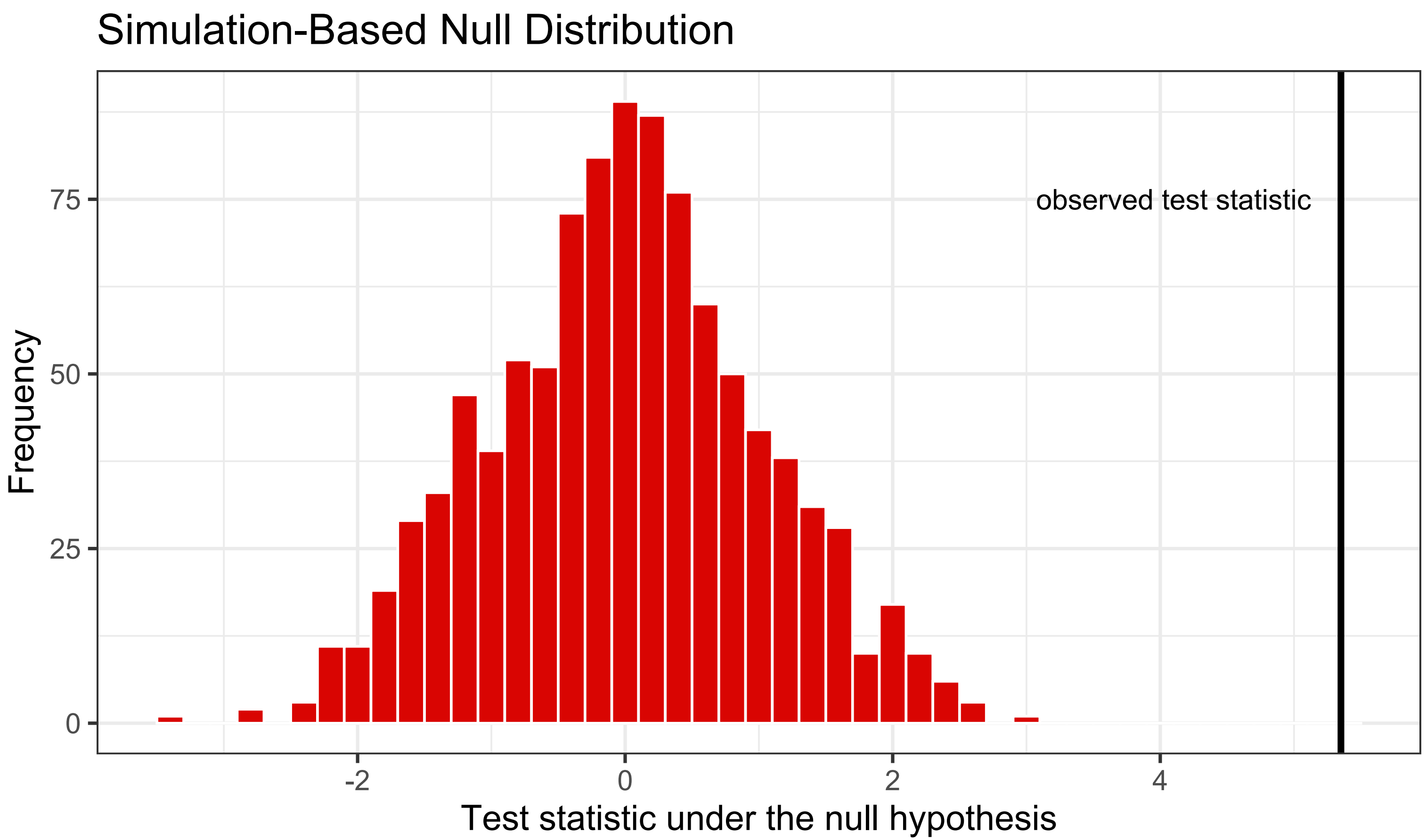

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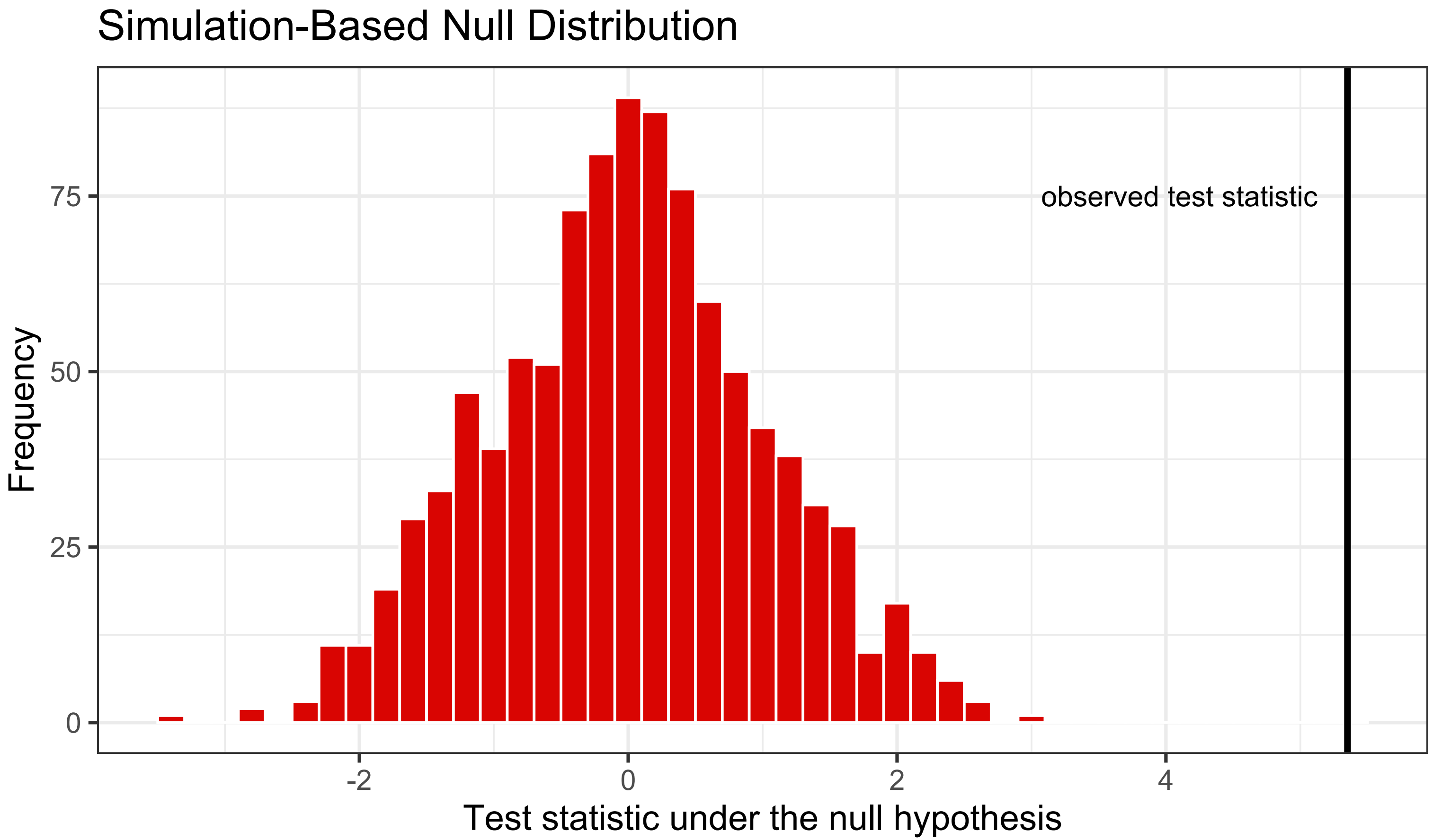

Very unlikely to obtain b_{small} = 8.8951932 when H_0 is true.

Testing $\beta_{\text{small}} = 0$ vs $\beta_{\text{small}} \neq 0$

- To decide if we reject H_0, recall we are considering a **two-sided test** here: *more extreme* means inferior to -5.351 **or** superior to 5.351.

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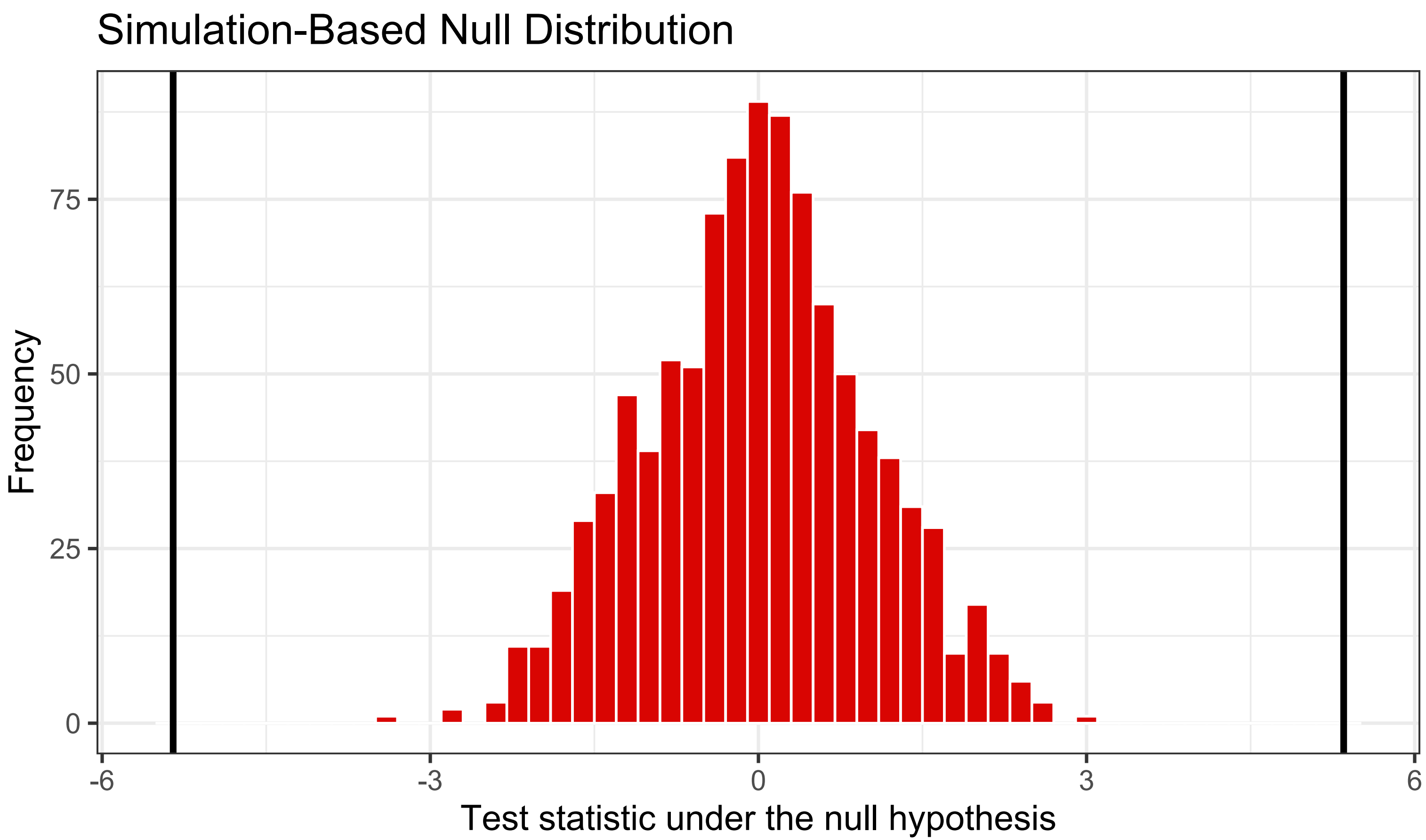

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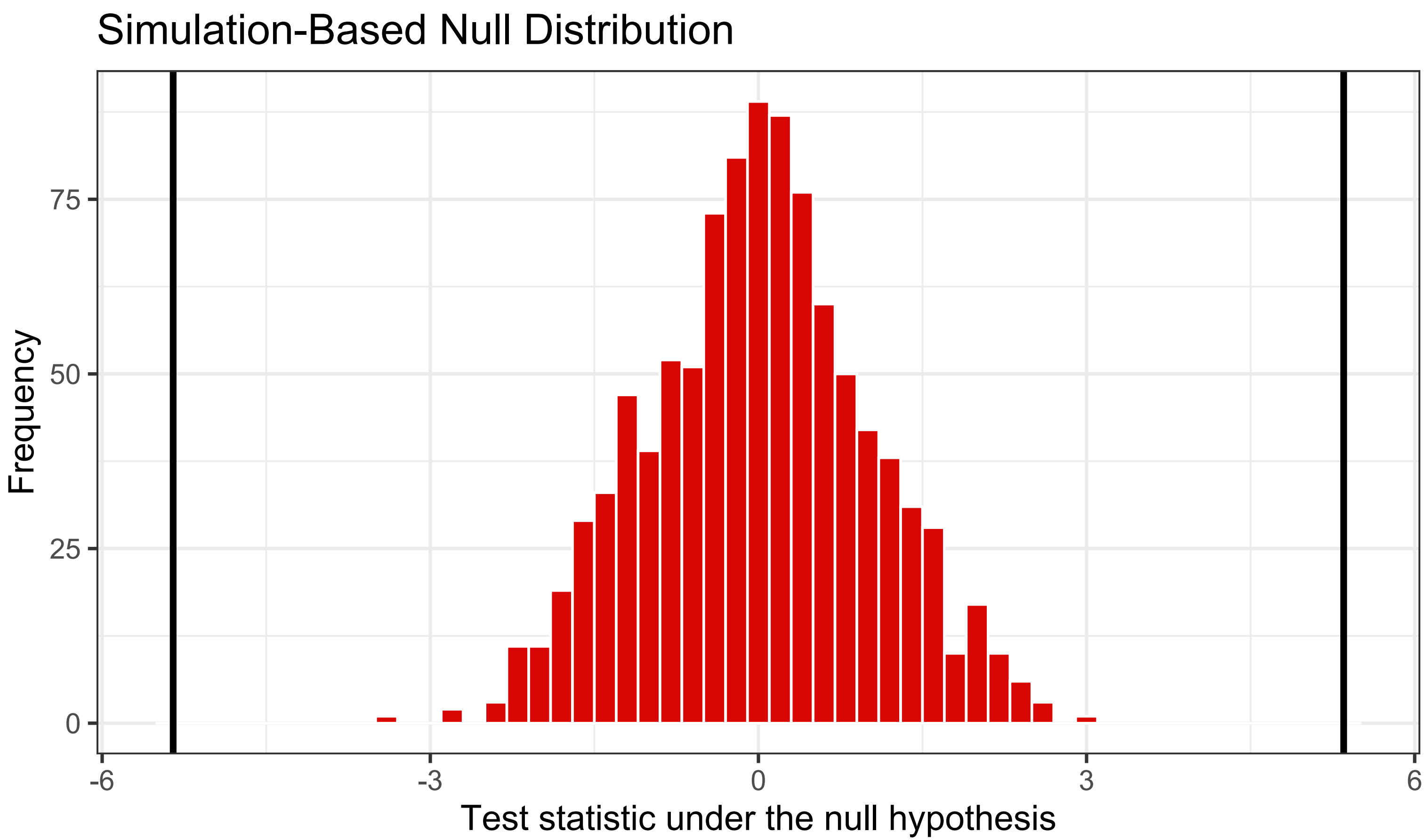

What does the p-value correspond to?

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- Computing the *p-value* we get:

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p_value
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- ***Question:*** Can we reject the null hypothesis at the 5% level?

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- ***Answer:***

    - Since the *p-value* is equal to 0 it means that we would reject H_0 at any significance level: the p-value would always be inferior to α.
    - In other words, we can say that b_{small} is **statistically different from 0** at any significance level.
    - We also say that b_{small} is *statistically significant* (at any significance level).

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- The values reported by statistical packages in `R` are instead obtained from theory.
- Theoretical inference is based on **large sample approximations**.
    - One can show that sampling distributions *converge* to suitable distributions $\rightarrow$ ***Central Limit Theorem***
- Let's briefly look into the theory-based approach.

Regression Inference: Theory

- Theory-based approach uses one fundamental result: the sampling distribution of the sample statistic $\frac{b-\beta}{\hat{\text{SE}}(b)}$ *converges* to a ***standard normal distribution*** as the sample size gets larger and larger.

  - $\hat{\text{SE}}(b)$ is the sample estimate of the standard deviation of b.
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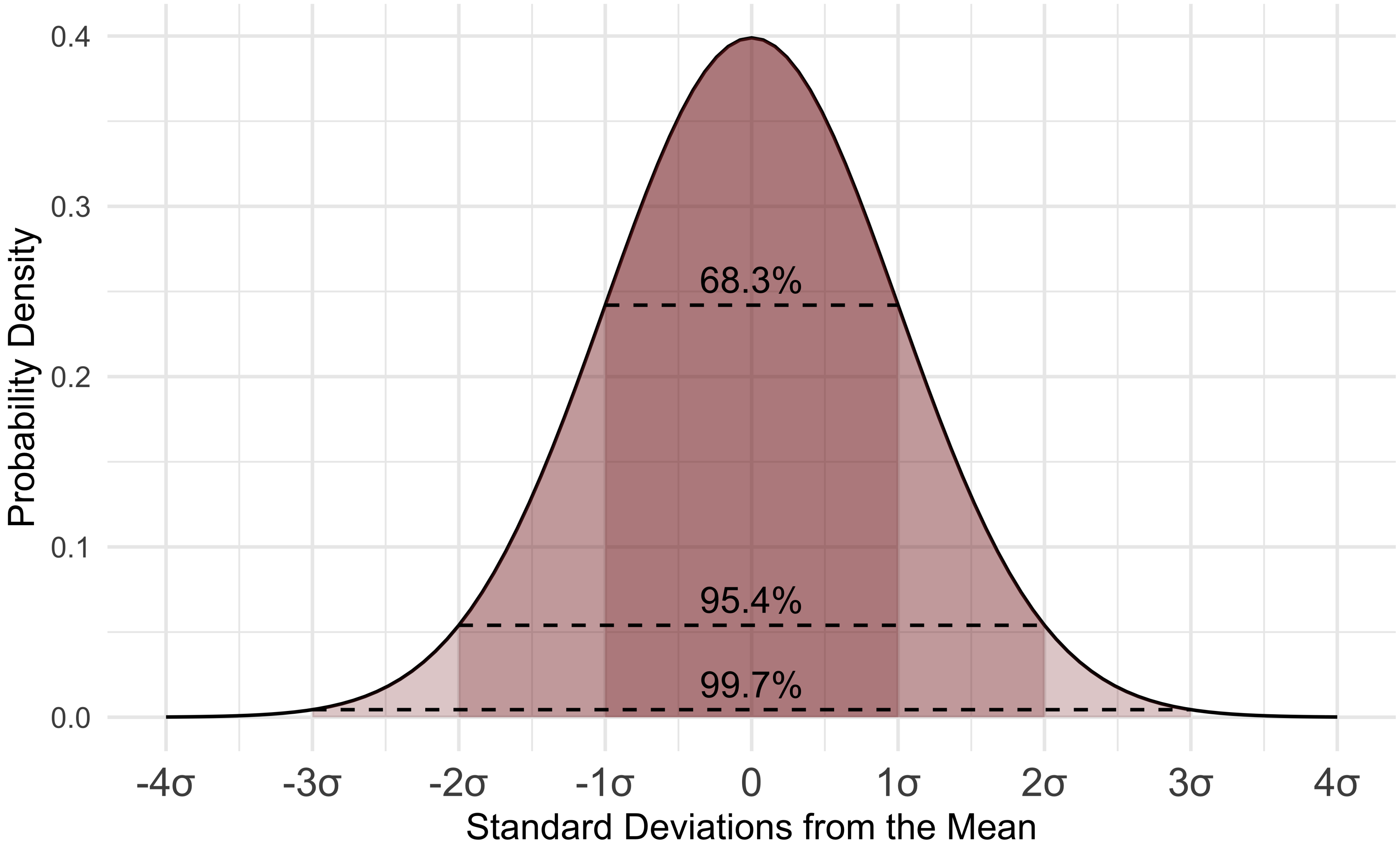
- We don't need to simulate any sampling distribution here, we derive it from theory and use it to construct confidence intervals or to conduct hypothesis tests.
- Note that if $\frac{b-\beta}{\hat{\text{SE}}(b)}$ *converges* to a ***standard normal distribution***, then b converges to a ***normal distribution*** with mean β and standard deviation $\hat{\text{SE}}(b)$.

Normal Distribution: A Refresher



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```
tidy(lm(math ~ small, star_df),
     conf.int = TRUE, conf.level = 0.95) %>%
  filter(term == "smallTRUE") %>%
  select(term, conf.low, conf.high)
```

```
## # A tibble: 1 x 3
##   term      conf.low conf.high
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- This can easily be generalized to any confidence level by taking the appropriate quantile of the normal distribution.

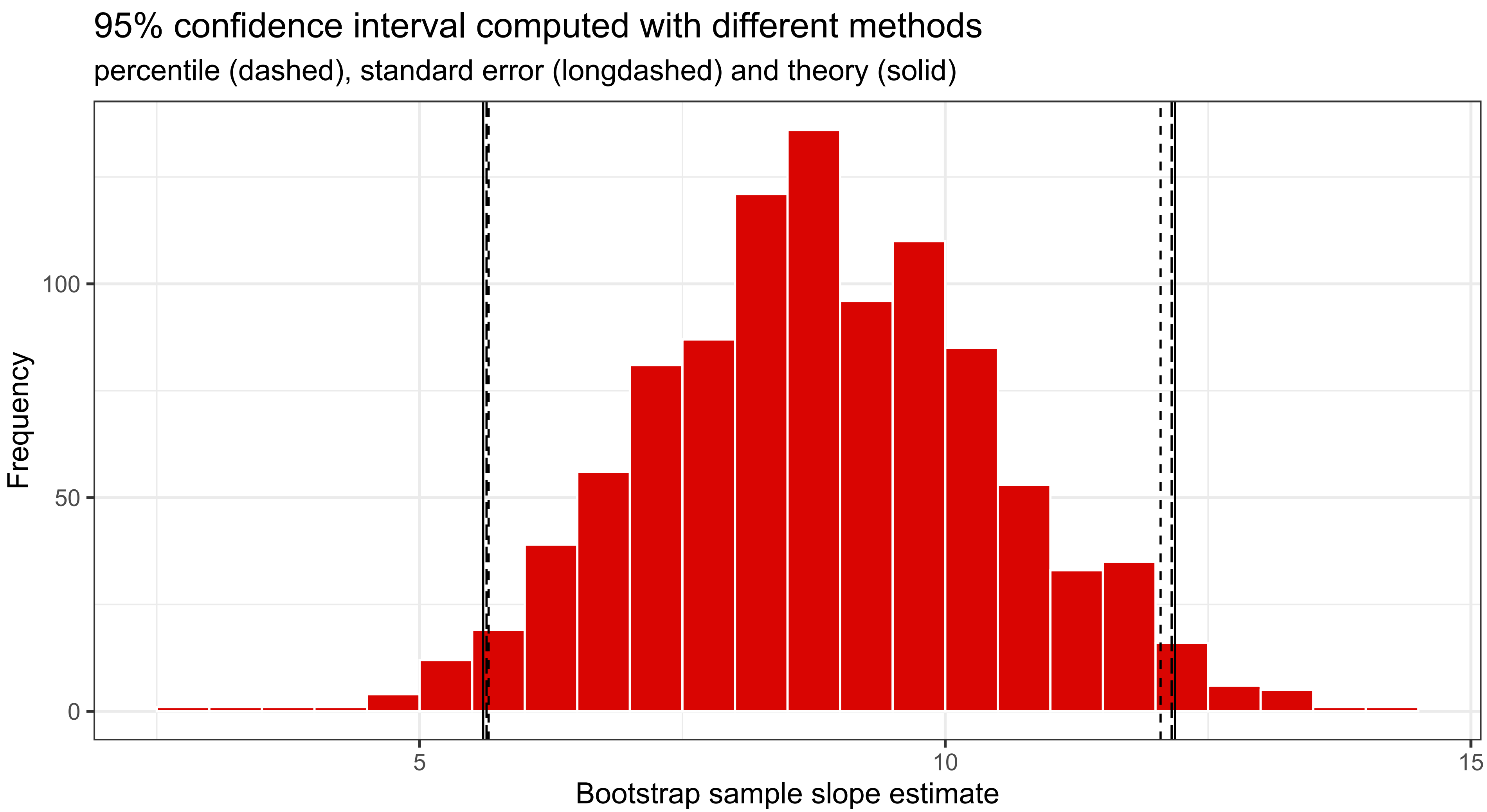
Task 2

05:00

1. Using the bootstrap distribution you generated in Task 1, compute the 95% confidence interval using the *percentile method*.

2. How similar is it to the confidence intervals obtained in the previous slide?

Confidence Intervals: Visually



Theory-Based Inference: Hypothesis Testing

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- The ***p-value*** associated to our test is then equal to the area of the *standard normal distribution* outside $\pm$ the observed value of $\frac{b}{\hat{\text{SE}}(b)}$.

- Common rule of thumb: if the *estimate* is *twice the size of the standard error*, then it is significant at the 5% level. Why?

Formatting a regression table

- Now that we have learned about all components of a regression table, let's finally learn how to create and read one!

```
reg_simple_math <- lm(math ~ small, data=star_df)
reg_gender_math <- lm(math ~ small + gender , data=star_df)
reg_simple_read <- lm(read ~ small, data=star_df)
reg_gender_read <- lm(read ~ small + gender , data=star_df)

export_summs(reg_simple_math, reg_gender_math,
             reg_simple_read, reg_gender_read,
             model.names = c("Math score", "Math Score",
                             "Reading score", "Reading score"),
             coefs=c("Small class" = "smallTRUE",
                     "Male gender" = "gendermale"))
```

Formatting a regression table

	Math score	Math Score	Reading score	Reading score
Small class	8.90 ***	8.94 ***	5.37 ***	5.41 ***
	(1.68)	(1.67)	(1.09)	(1.09)
Male gender		-8.56 ***		-7.49 ***
		(1.67)		(1.09)
N	3359	3359	3359	3359
R2	0.01	0.02	0.01	0.02
*** p < 0.001; ** p < 0.01; * p < 0.05.				

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- Each column corresponds to a regression. For the first regression we have:
  - the **name of the outcome variable** in blue
  - the **coefficient associated to being in a small class** β_{small} in green
  - its **estimated standard error** in yellow
  - the **number of observations** in purple
  - the **R-squared** in red
  - interpretation of the stars at the bottom

Classical Regression Model

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- The set of assumptions needed defines the *Classical Regression Model* (CRM).

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  - We already mentioned the distinction between the sample estimate b_k (or $\hat{\beta}_k$) and the population parameter β_k.

  - In the same way, we distinguish e, the sample error (*residual*), from ε, the error term from the true population model:

$$y_i = \beta_0 + \beta_1 x_{1,i} + \ldots + \beta_k x_{k,i} + \varepsilon_i$$

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  - The classical regression model applies to **correctly specified linear regressions**: the model needs to be linear in parameters, include all relevant variables, and variables cannot be collinear.

CRM Assumptions

1. ***Mean Independence:*** the mean of the residuals conditional on x should be zero, $E[\varepsilon|x] = 0$. Notice that this also means that $Cov(\varepsilon, x) = 0$, i.e. that the errors and our explanatory variable(s) should be *uncorrelated*.

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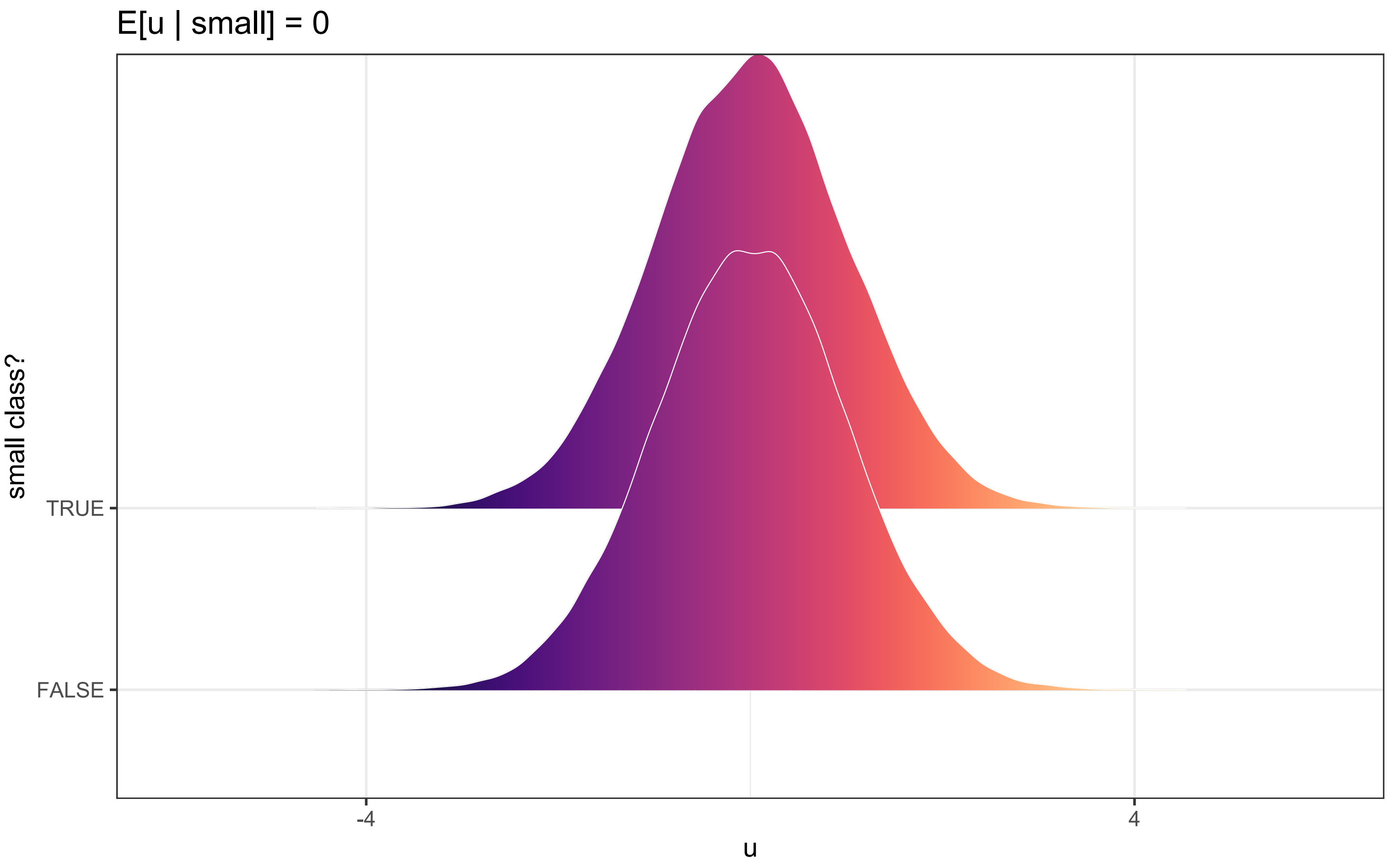

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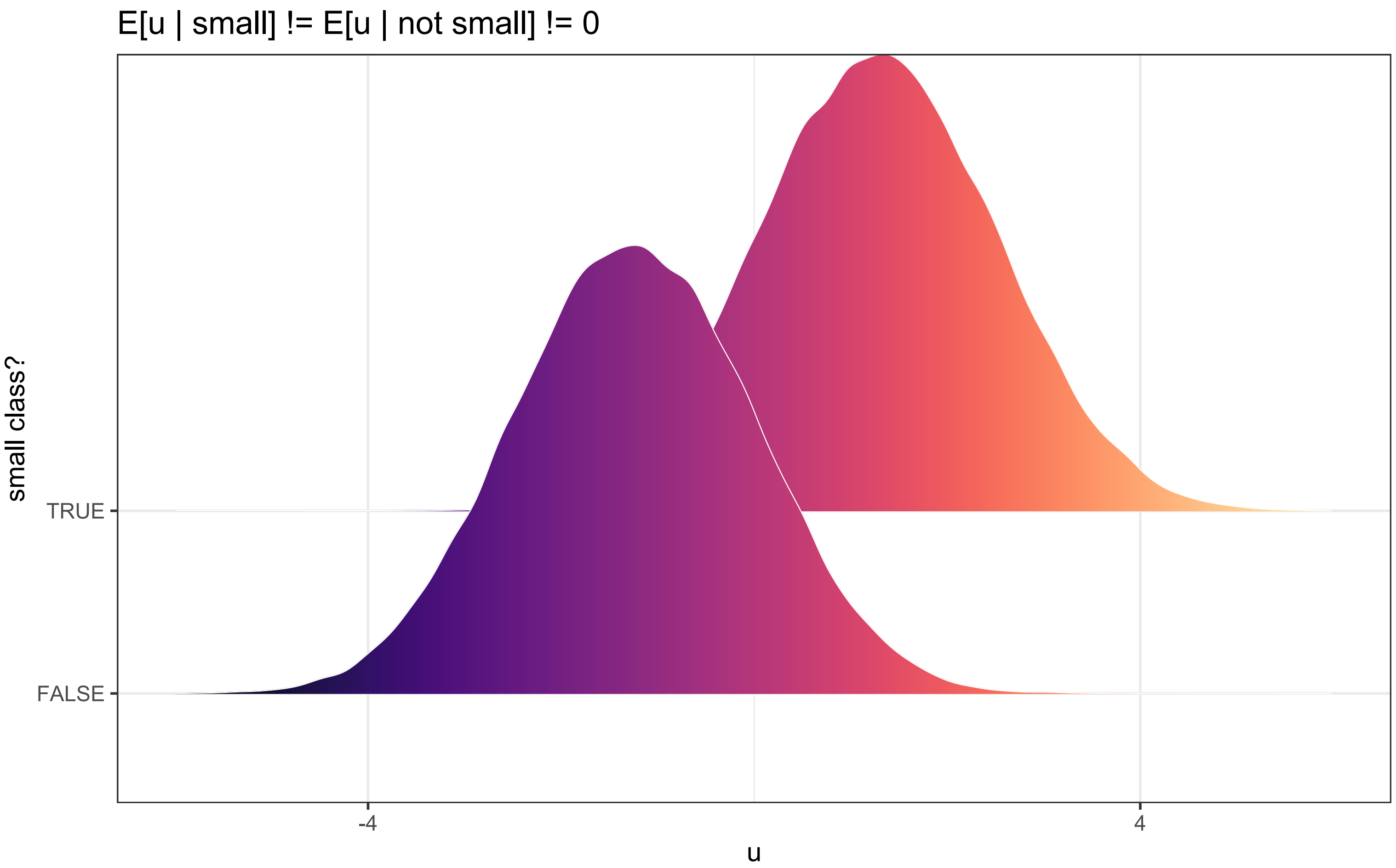

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  - Under the exogeneity assumption β_1 denotes the causal effect of education in the population.

- Suppose there is *unobserved* ability a_i.

  - High ability means higher wage.
  - It *also* means school is easier, and so i selects into more schooling.

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$$\text{OVB} = \text{multiple regression coefficient on omitted variable} \times \frac{Cov(x,z)}{Var(x)}$$

  - Thus, we have:

$$\mathbb{E}(b_1) = \beta_1 + OVB > \beta_1$$

- *Interpretation*: taking repeated sample from the population and computing b_1 each time, we would **systematically overestimate** the effect of education on wage.

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- Violating this assumption would make your sample less representative of the underlying population. It will lead to **biased** estimates of β_k.

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☞ Takeaway: **if assumptions violated, inference is invalid!**

Task 3.1

10:00

Let's go back to our question of returns to education and gender.

1. Load the data `CPS1985` from the `AER` package and look back at the `help` to get the definition of each variable: `?CPS1985`

2. Create the `log_wage` variable equal to the log of `wage`.

3. Regress `log_wage` on `gender` and `education`, and save it as `reg1`.

    - Interpret each coefficient.

    - Are the coefficients statistically significant? At which significance level?

4. Regress the `log_wage` on `gender`, `education` and their interaction `gender*education`, save it as `reg2`.

    - How do you interpret the coefficient associated to $female * education$?

    - Can we reject the nullity of this coefficient at the 5% level? At 10%?

Task 3.2

10:00

1. Produce a scatterplot of the relationship between the log wage and the level of education.

2. Add the *regression line* with `geom_smooth`. What does this line represents?

3. Let's illustrate what the shaded area stands for.

    1. Draw one bootstrap sample from our `cps` data.

    2. Regress the `log_wage` on `gender`, `education` and their interaction `gender*education`, save it as `reg_bootstrap`.

    3. From `reg_bootstrap` extract and save the value of the intercept for men as `intercept_men_bootstrap` and the value of the slope for men as `slope_men_bootstrap`. Do the same for women.

    4. Add both predicted lines from this bootstrap sample to the previous plot (*Hint*: use `geom_abline` (x2))

Illustrating Uncertainty

Let's repeat the procedure you just made 100 times!

```
library(AER)
data("CPS1985")
cps = CPS1985 %>% mutate(log_wage = log(wage))

set.seed(1)
bootstrap_sample = cps %>%
    rep_sample_n(size = nrow(cps), reps = 100, replace

ggplot(data=cps,aes(y = log_wage, x = education, colou
  geom_point(size = 1, alpha = 0.7) +
  geom_smooth(method = "lm", alpha = 2) +
  geom_smooth(data=bootstrap_sample,
              size = 0.2,
              aes(y = log_wage, x = education, group =
              method = "lm", se = FALSE) +
  facet_wrap(~gender) +
  scale_colour_manual(values = c("darkblue", "darkred"
  labs(x = "Education", y = "Log wage") +
  guides(colour=FALSE) +
  theme_bw(base_size = 20)
```

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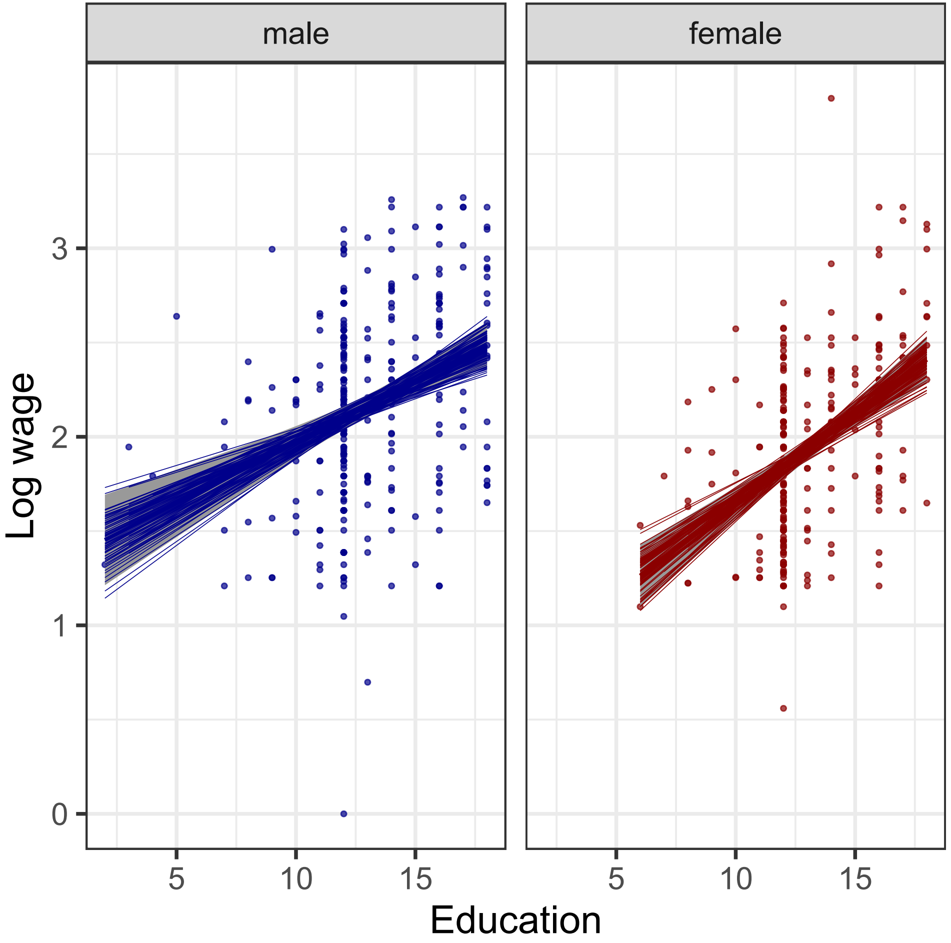

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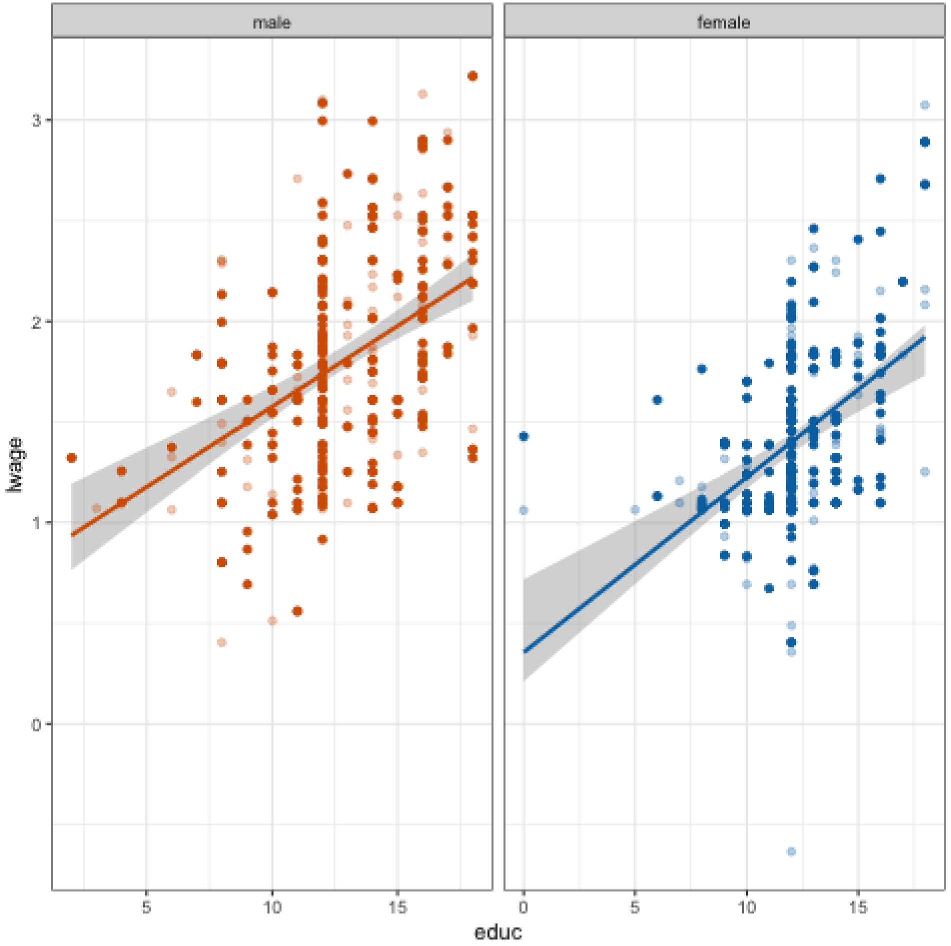

Even better : `ungeviz` and `gganimate` bring you moving lines!

- We took 20 bootstrap samples from our data
- You can see how different data points are included in each bootstrap sample.
- Those different points imply different regression lines.
- On average, 95% of these lines should fall into the shaded area.
- You should remember those moving lines when looking at the shaded area!

On the way to causality

☑ How to manage data? Read it, tidy it, visualise it!

☑ How to summarise relationships between variables? Simple and multiple linear regression, non-linear regressions, interactions...

☑ What is causality?

☑ What if we don't observe an entire population? Sampling!

☑ **Are our findings just due to randomness?** Confidence intervals and hypothesis testing, regression inference.

✗ How to find exogeneity in practice?

THANKS

To the amazing moderndive team!

Big Thanks 🎉 to ungeviz and 🎊 gganimate for their awesome packages!

SciencesPo
DEPARTMENT OF ECONOMICS

SEE YOU NEXT WEEK!

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