

# ScPoEconometrics

## Regression Discontinuity Design

Florian Oswald, Gustave Kenedi and Pierre Villedieu  
SciencesPo Paris  
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# Recap from last week

- **Differences-in-differences** policy evaluation method
- Main estimation equation:

$$Y_{it} = \alpha + \beta TREAT_i + \gamma POST_t + \delta(TREAT_i \times POST_t) + \varepsilon_{it}$$

- Key assumption: **parallel trends**



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## Today: *Regression Discontinuity Design*

- Life is full of random rules which assign some treatment
- Exploits knowledge of assignment rule
- Key assumption: variable which assigns treatment cannot be manipulated by individuals
- *Empirical application*: effect of alcohol consumption on mortality



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- **RDD** exploits this precise information about allocation to treatment!



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We will focus our analysis on the following discontinuity:

- In the US, the legal drinking age is 21 years old ([Carpenter and Dobkin, 2009](#)).



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  - Because there may be unobserved selection into alcohol consumption that may also be a determinant of mortality.
- In the US, alcohol consumption is prohibited before the age of 21.
- Debate on whether the minimum legal drinking age (MLDA) should be lowered to 18, as was the case in the Vietnam-era.



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- ➡ *Regression discontinuity design* exploits this allocation to treatment!



# Carpenter and Dobkin's data

- Let's take a closer at the data used in the paper

```
# install package containing data
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##       <dbl>   <dbl>     <dbl>     <dbl>     <dbl>     <dbl>     <dbl>
## 1     19.1    92.8     16.6     76.2     0.639    16.3     11.2
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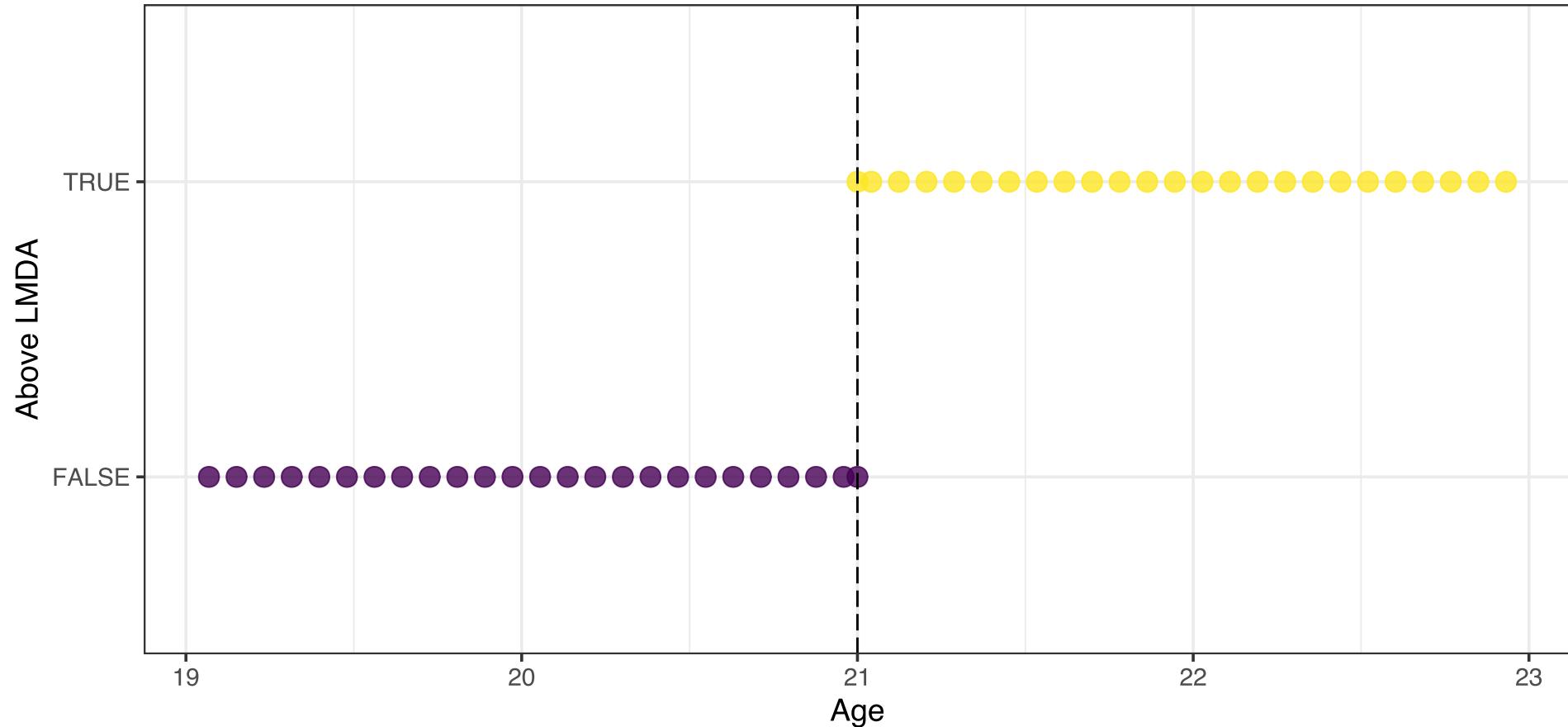
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- This dataset contains aggregate death rates (and their causes) for different age groups (`agecell`) between 19 and 23 years old.



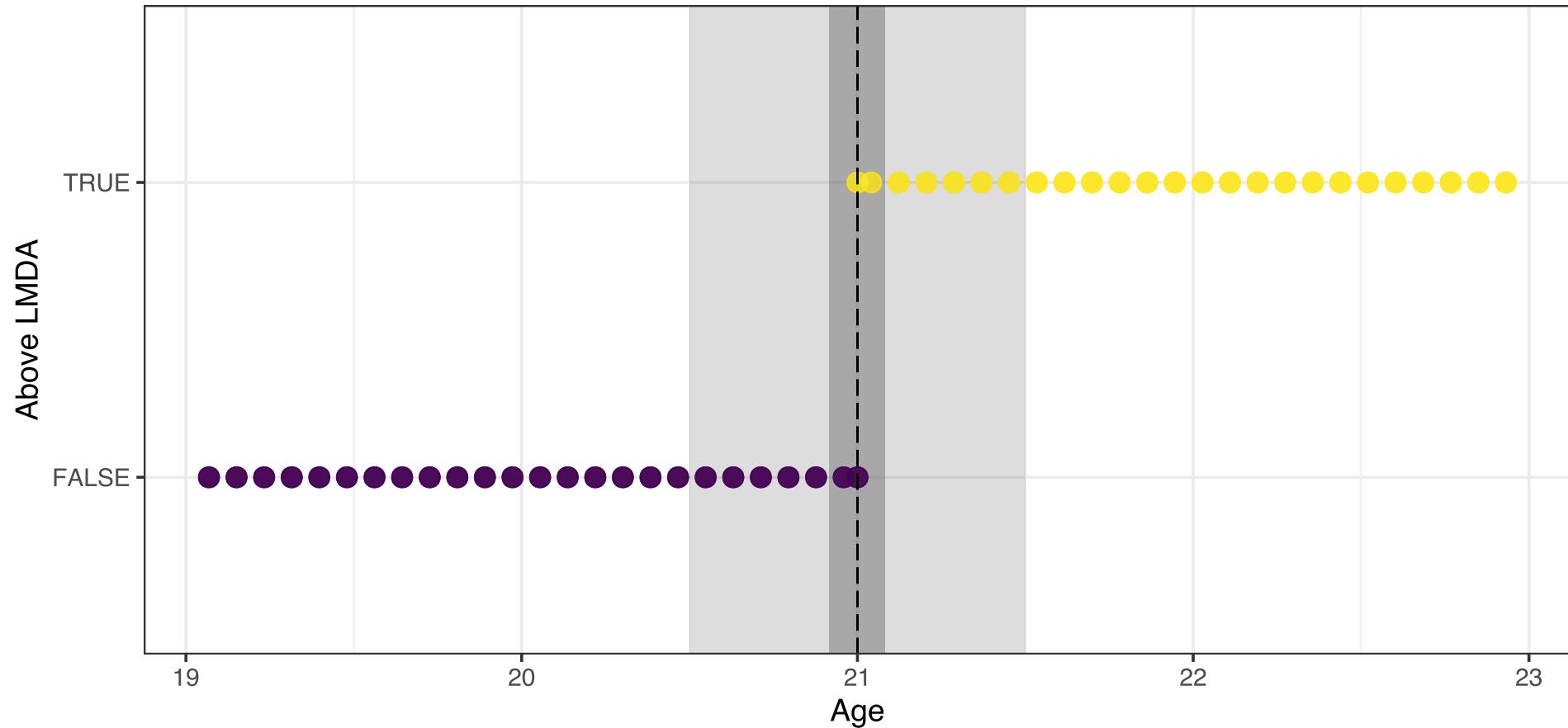
# Sharp Discontinuity at Cutoff



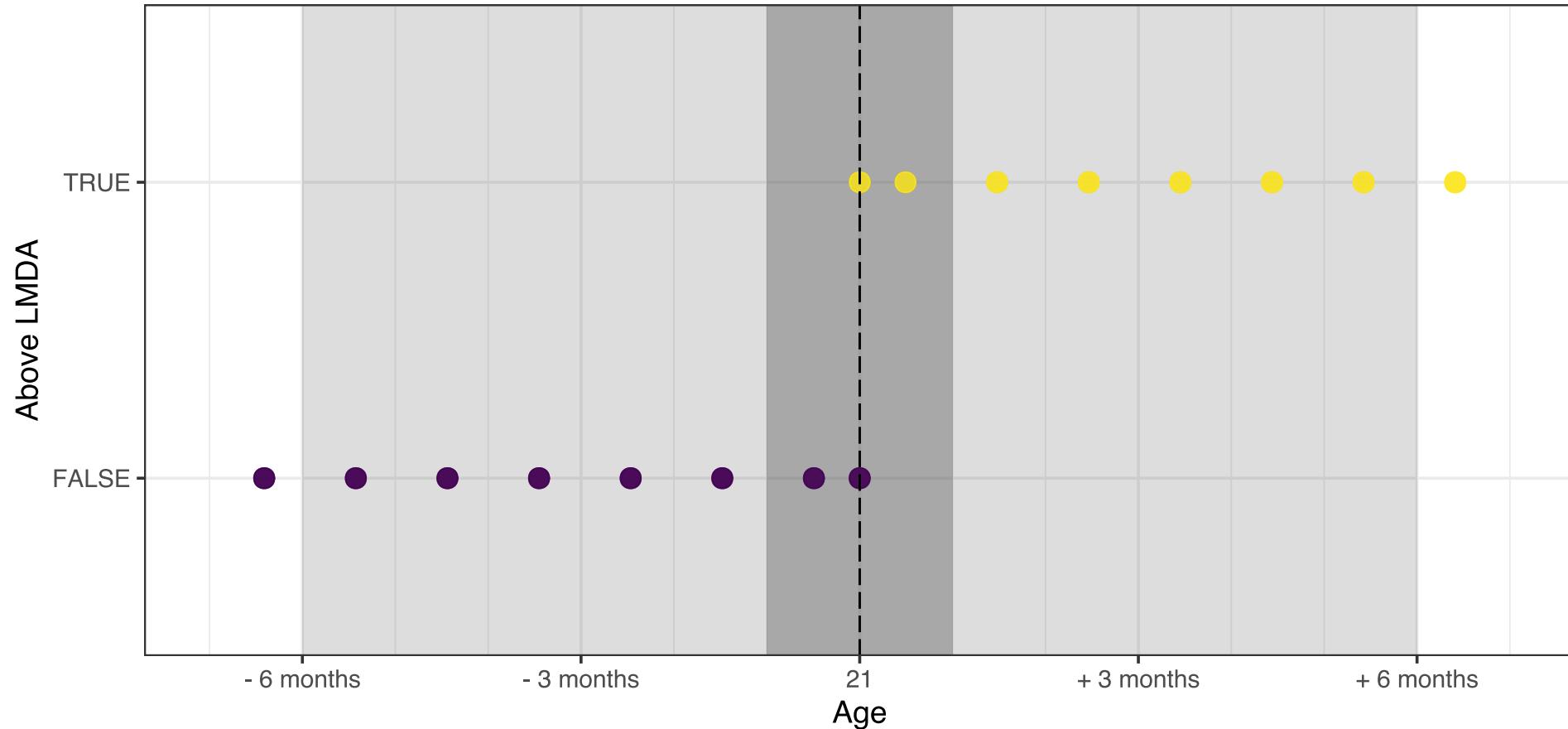
At the threshold, the probability of being treated jumps from 0 to 1.



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- The **cutoff** age, 21, separates those who can drink legally and those who can't:

$$D_a = \begin{cases} 1 & \text{if } a \geq 21 \\ 0 & \text{if } a < 21 \end{cases}$$

## Key features of RD designs

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## Key features of RD designs

1. Treatment status is a **deterministic** function of  $a \rightarrow$  we know the assignment rule
2. Treatment status is a **discontinuous** function of  $a \rightarrow$  there is some cutoff level

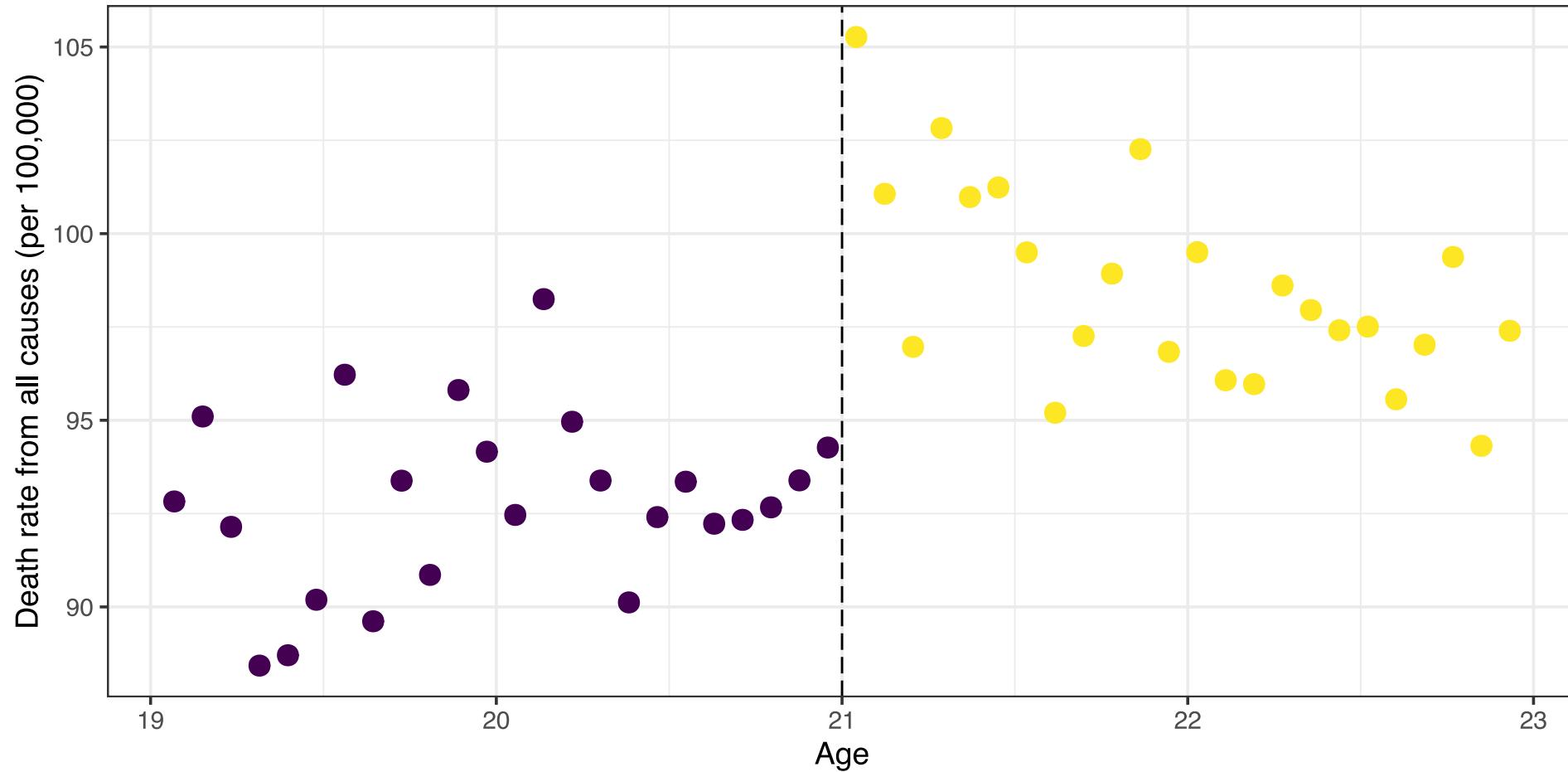


# Task 1 (10 minutes)

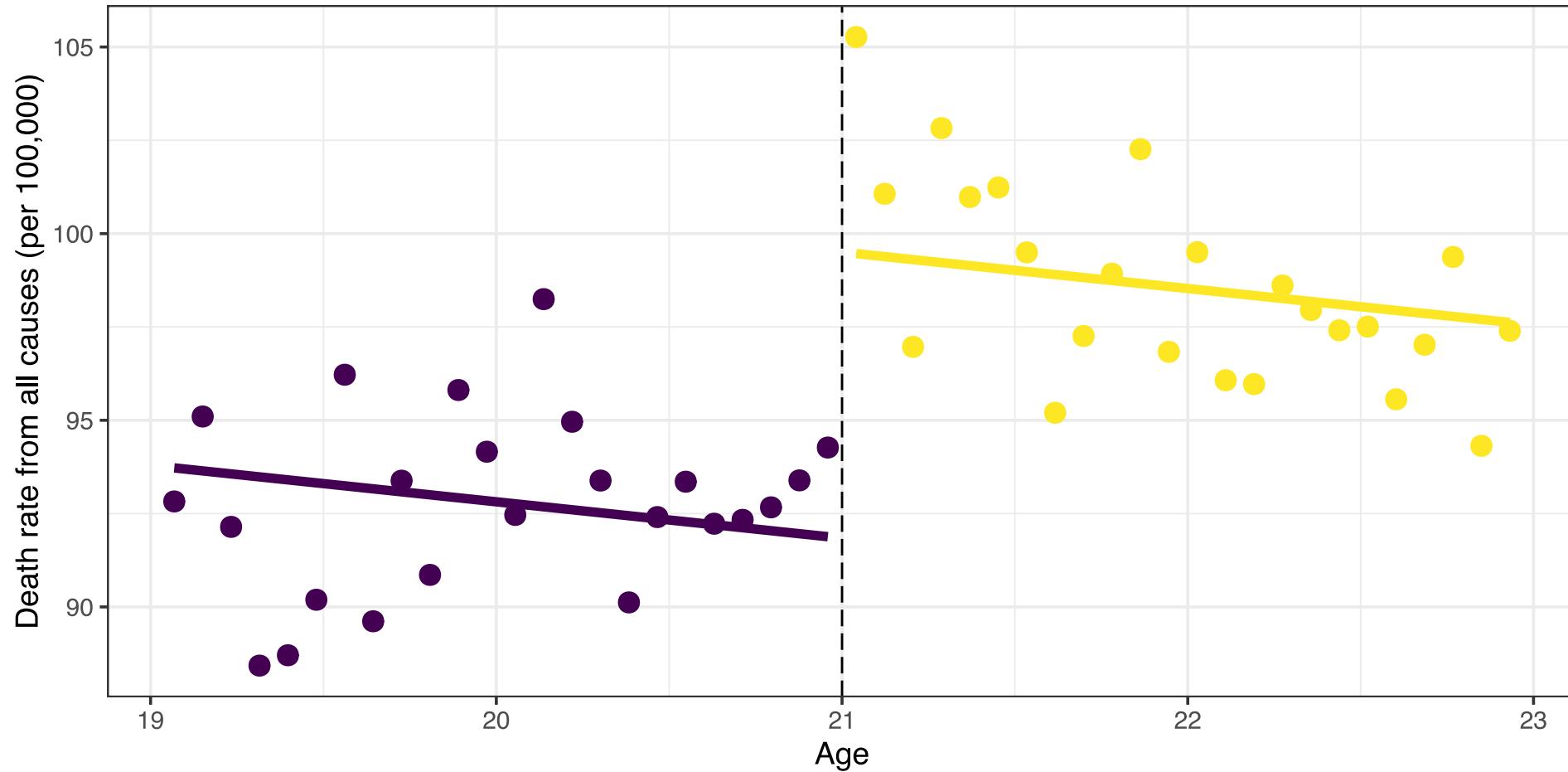
1. Import the dataset following the code from slide 7. How many age cells are there?
2. Create a dummy variable for individuals over 21 years old.
3. Plot the death rate for all causes (`all`) as a function of age (`agecell`) colouring observations above and below 21 years old. Does anything seem striking?
4. Add a regression line to the plot. What do you observe?
5. Do the same for motor vehicle-related causes (`mva`) and alcohol-related causes (`alcohol`) as a function of age.



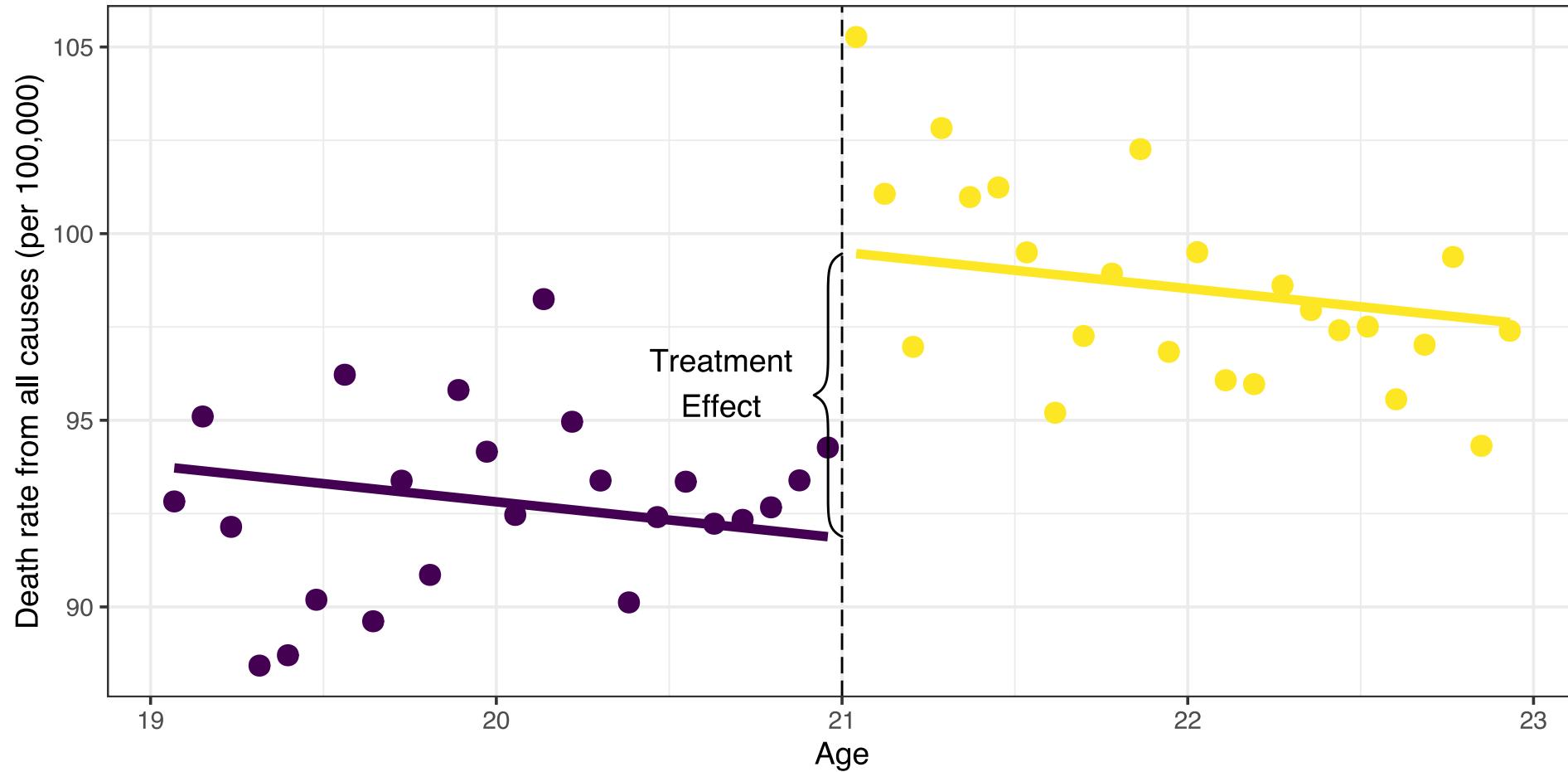
# Graphical Results: All Death Rates



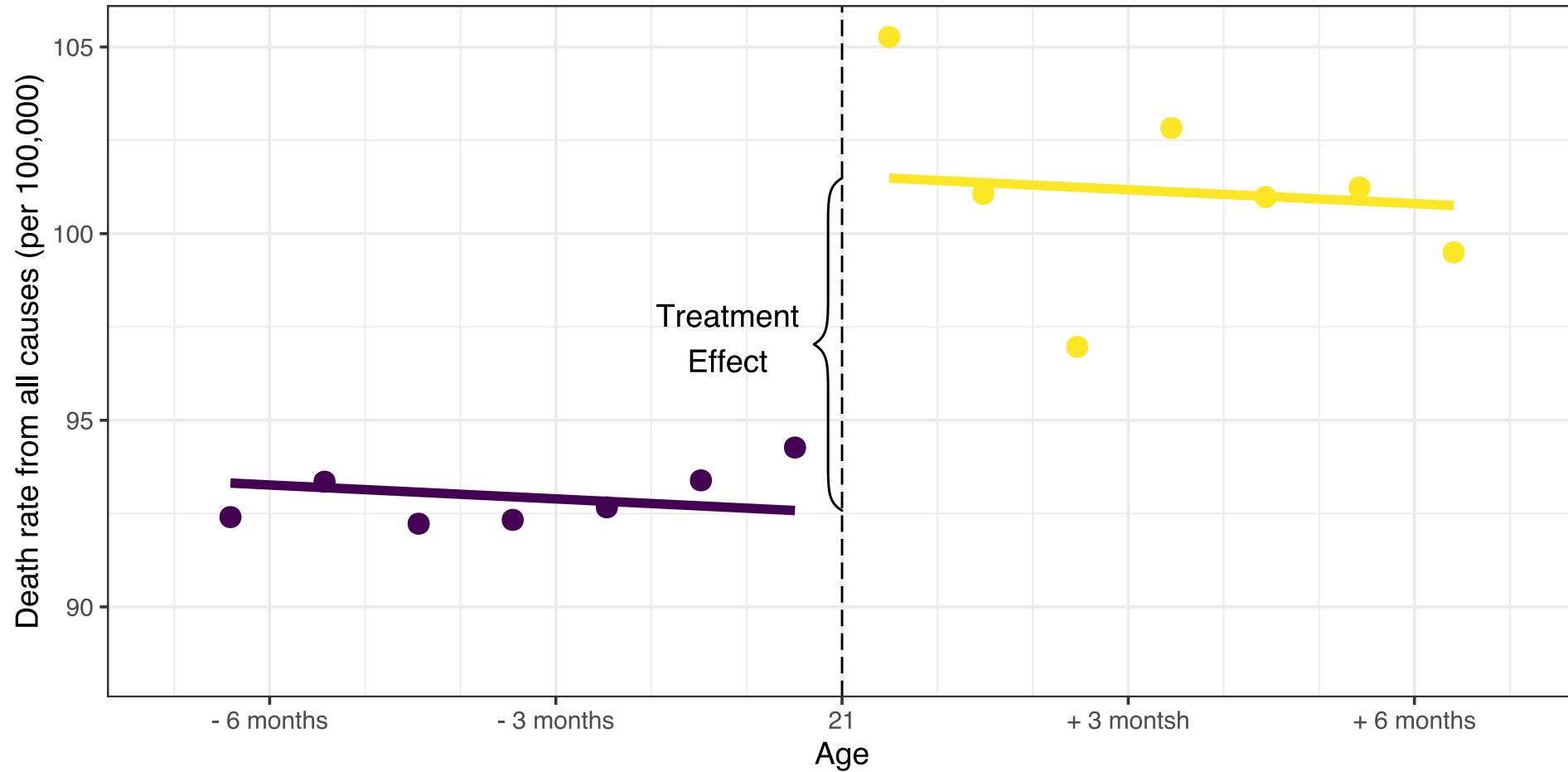
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- Limited *external validity* → you cannot extrapolate to the entire population.
- Using the 21 year old alcohol restriction age in the RD context will only tell you the effect of this restriction on death rates but not the general effect of alcohol consumption.
- One may easily argue that all results from quantitative empirical analyses have a local nature.



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→  $\delta$  captures the **jump in death rate** between individuals above and below 21 years old.

- The RDD estimator exploits a discontinuity at  $a = 21$  in the conditional expectation function:

$$\underbrace{\lim_{c \rightarrow 21^+} \mathbb{E}[DEATHRATE_a | a = c]}_{\alpha + \delta} - \underbrace{\lim_{c \rightarrow 21^-} \mathbb{E}[DEATHRATE_a | a = c]}_{\alpha} = \delta$$



# Task 2 (5 minutes)

1. Estimate the following model on all death causes.

$$DEATHRATE_a = \alpha + \delta D_a + \beta a + \varepsilon_i,$$

Does the RDD coefficient correspond to the graphical illustration?

2. How do you interpret each coefficient?
3. What is the causal effect of legal access to alcohol on death rates?



# Estimation #1: Simple Linear Model

$$DEATHRATE_a = \alpha + \delta D_a + \beta a + \varepsilon_a,$$

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mlda <- mlda %>%
  mutate(over21 = (agecell >= 21),
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rdd <- lm(all ~ agecell_21 + over21, mlda)

library(broom)
tidy(rdd)
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## # A tibble: 3 x 5
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This is a big effect considering the average death rate for individuals between 19 and 22 is:

```
mean(mlda$all, na.rm = TRUE)
## [1] 95.67272
```



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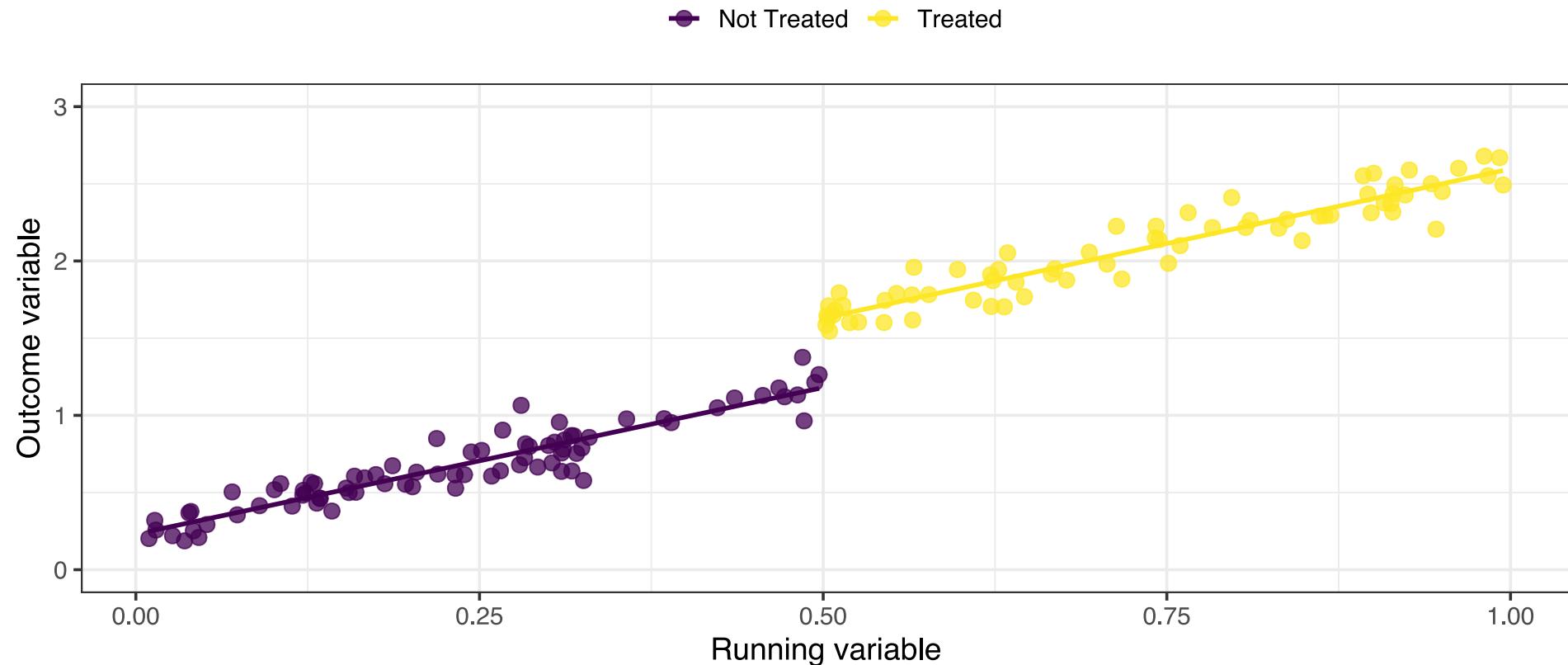


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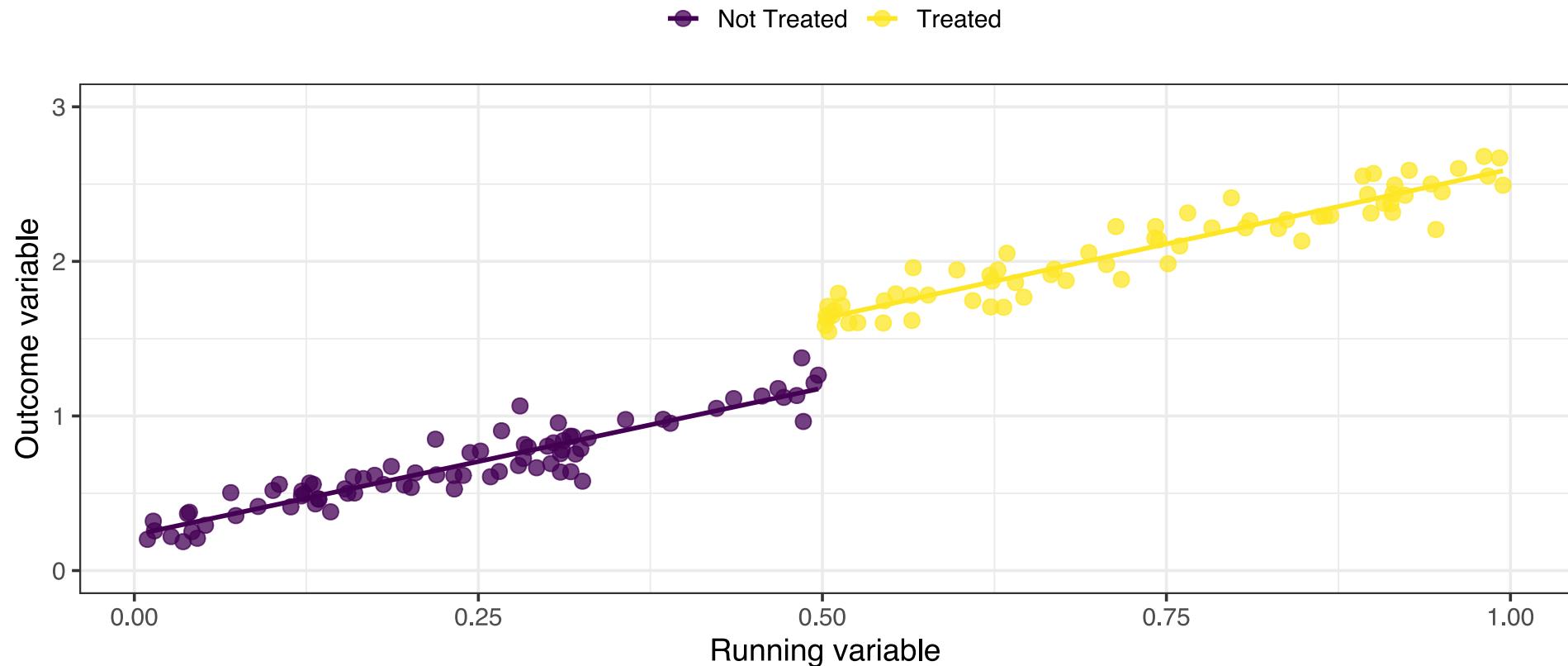
- The *functional form* used to approximate the lines really matters!
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  - an overly flexible specification reduces precision and runs the risk of overfitting.



# Simulations - Linear Relationship and Clear Discontinuity



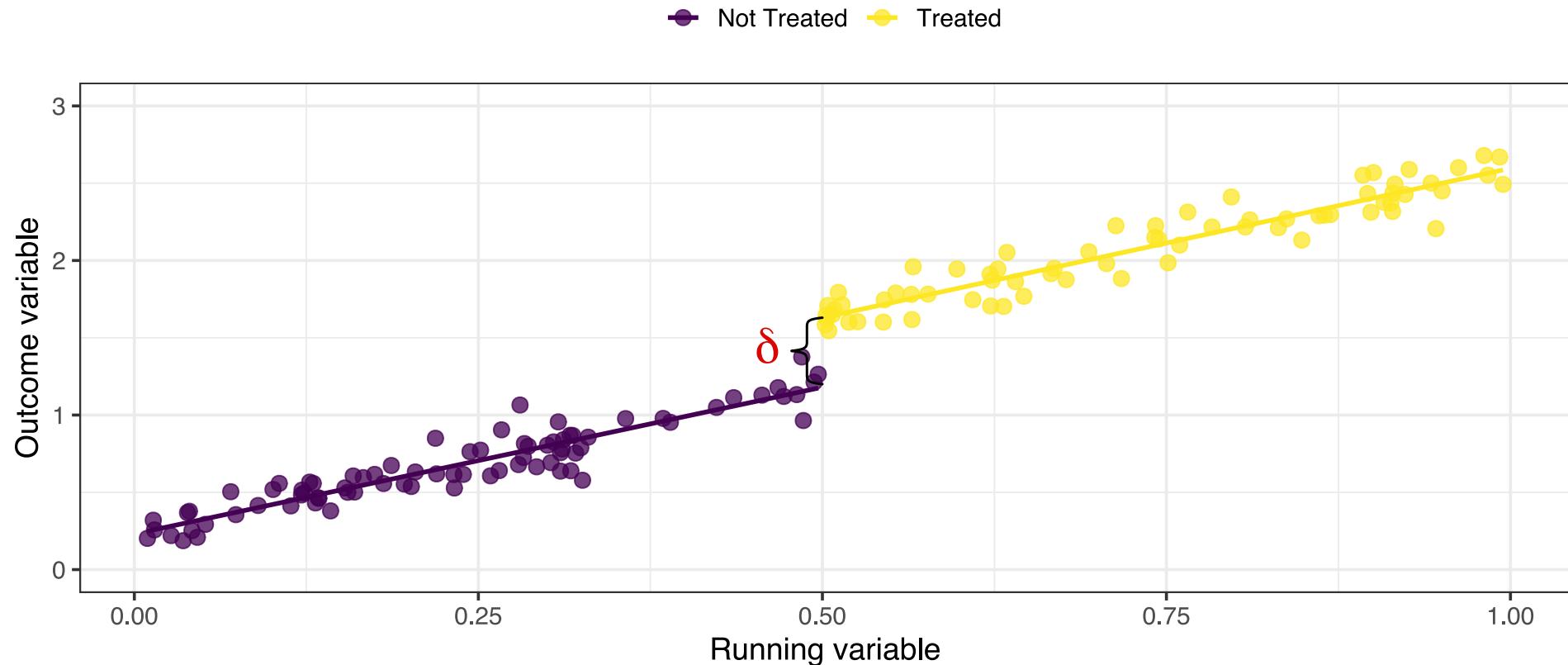
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$$outcome_i = \alpha + \delta treatment_i + \beta running_i + e_i,$$



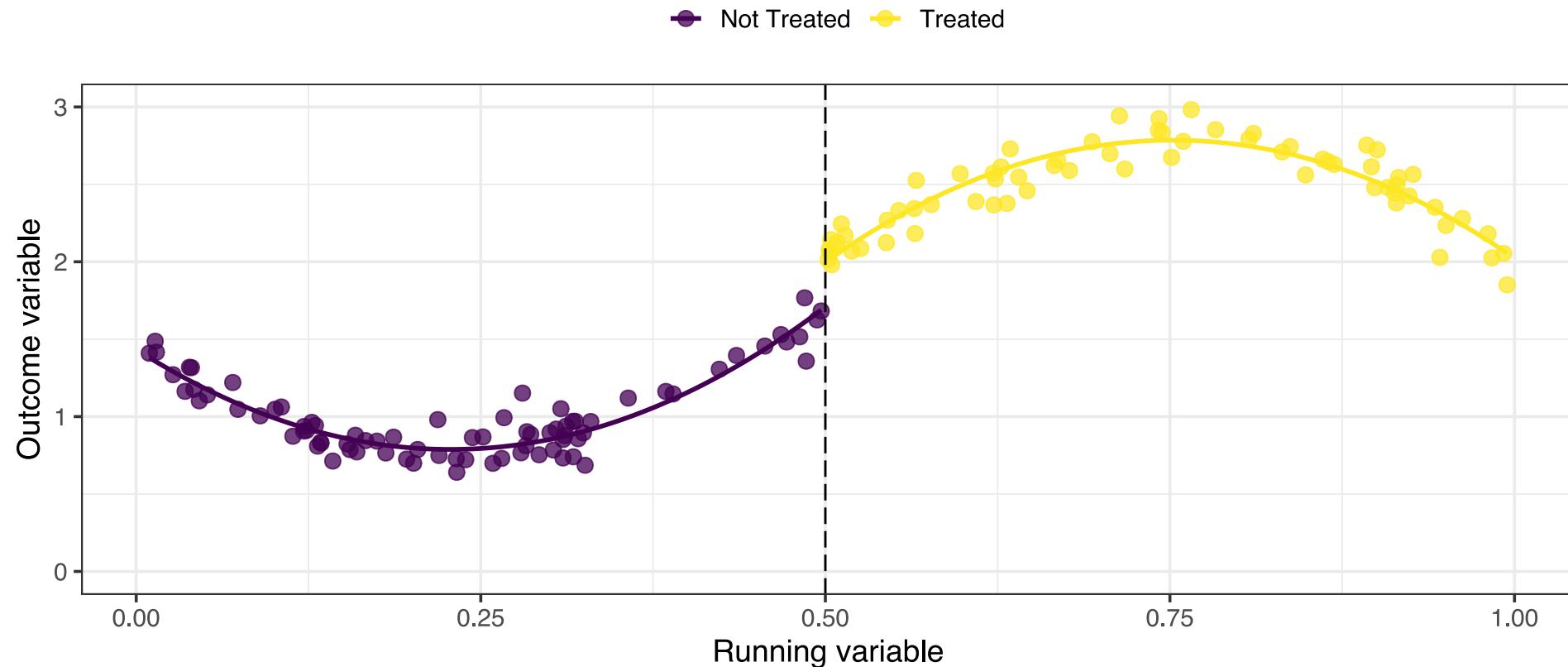
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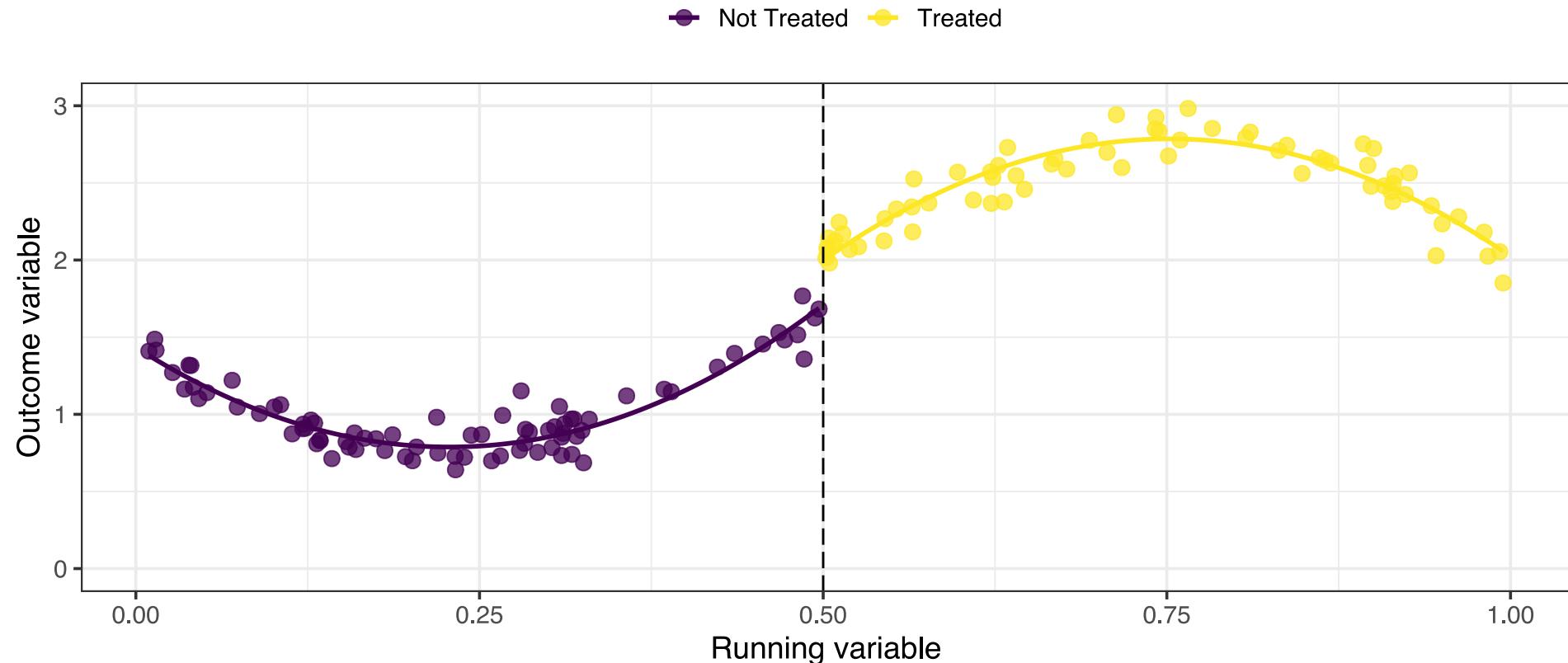
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# Simulations - Quadratic Relationship and Clear Discontinuity



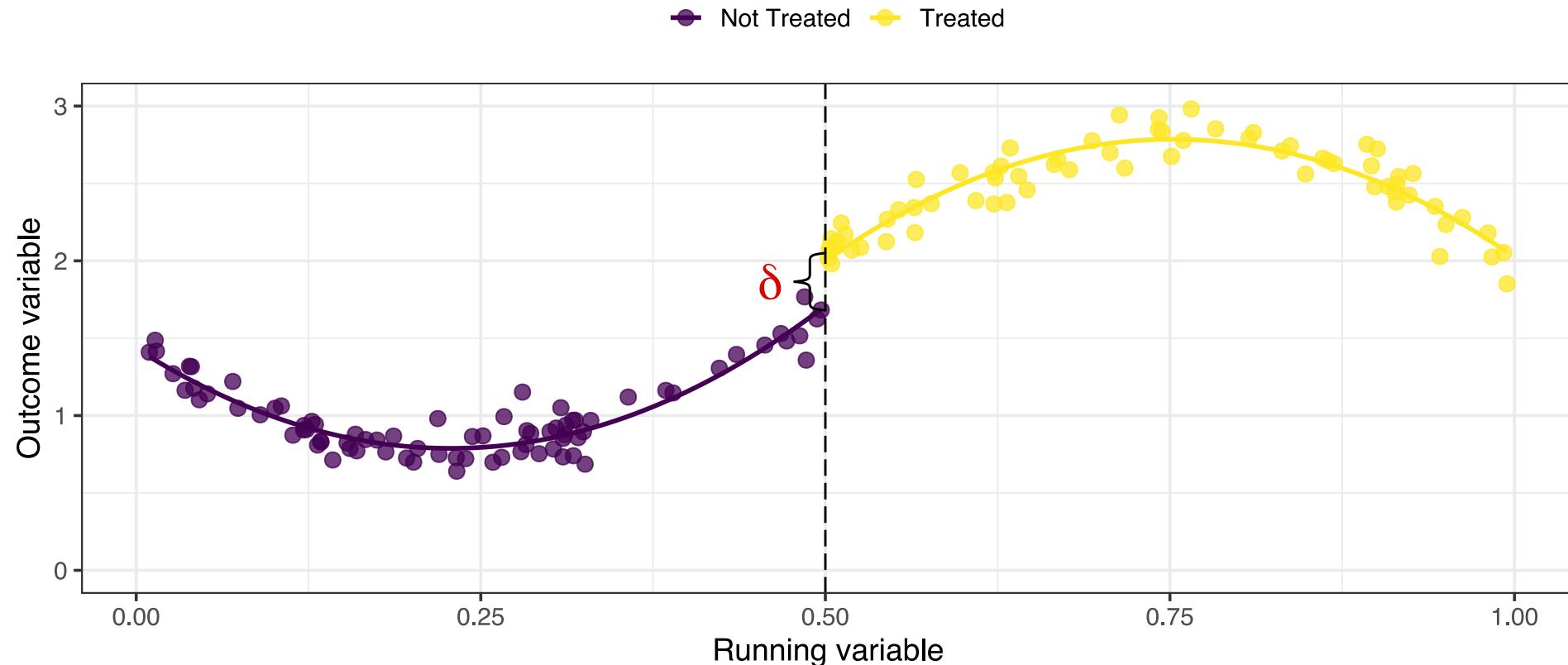
# Simulations - Quadratic Relationship and Clear Discontinuity



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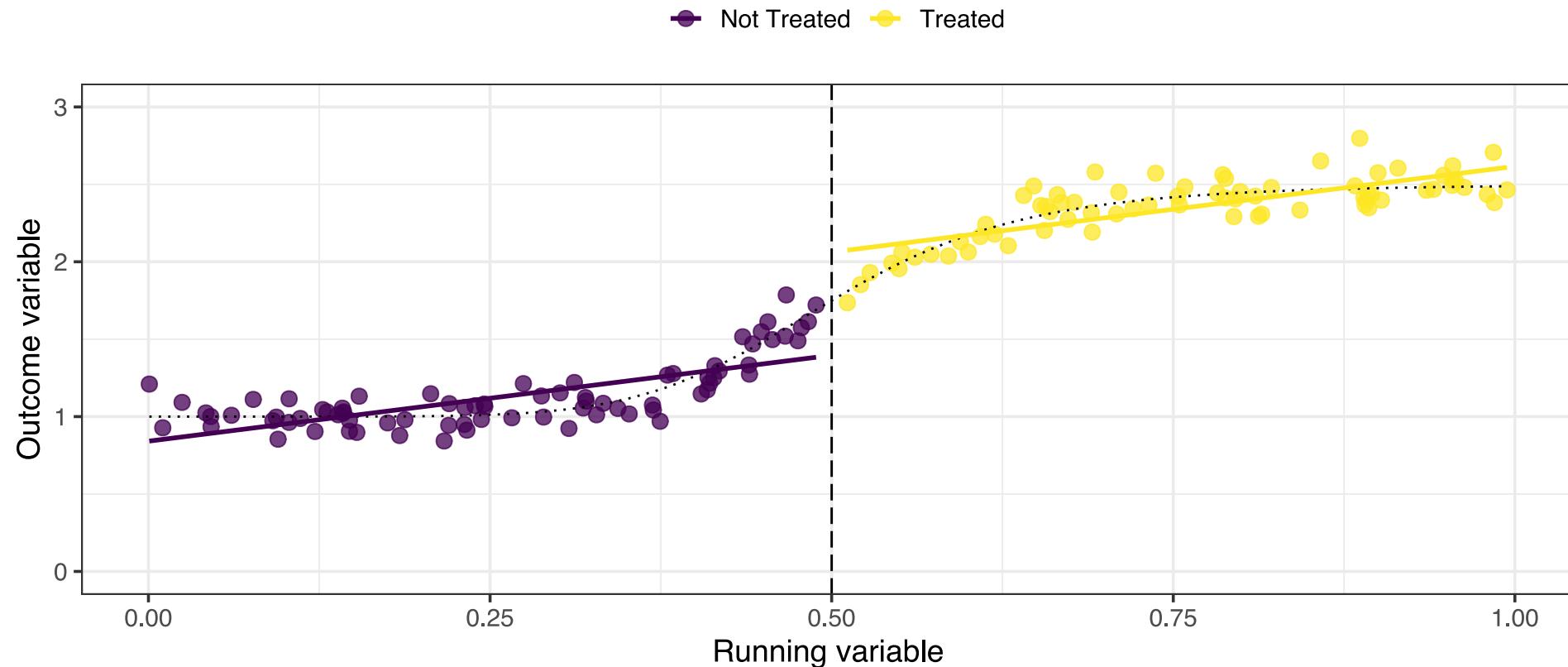
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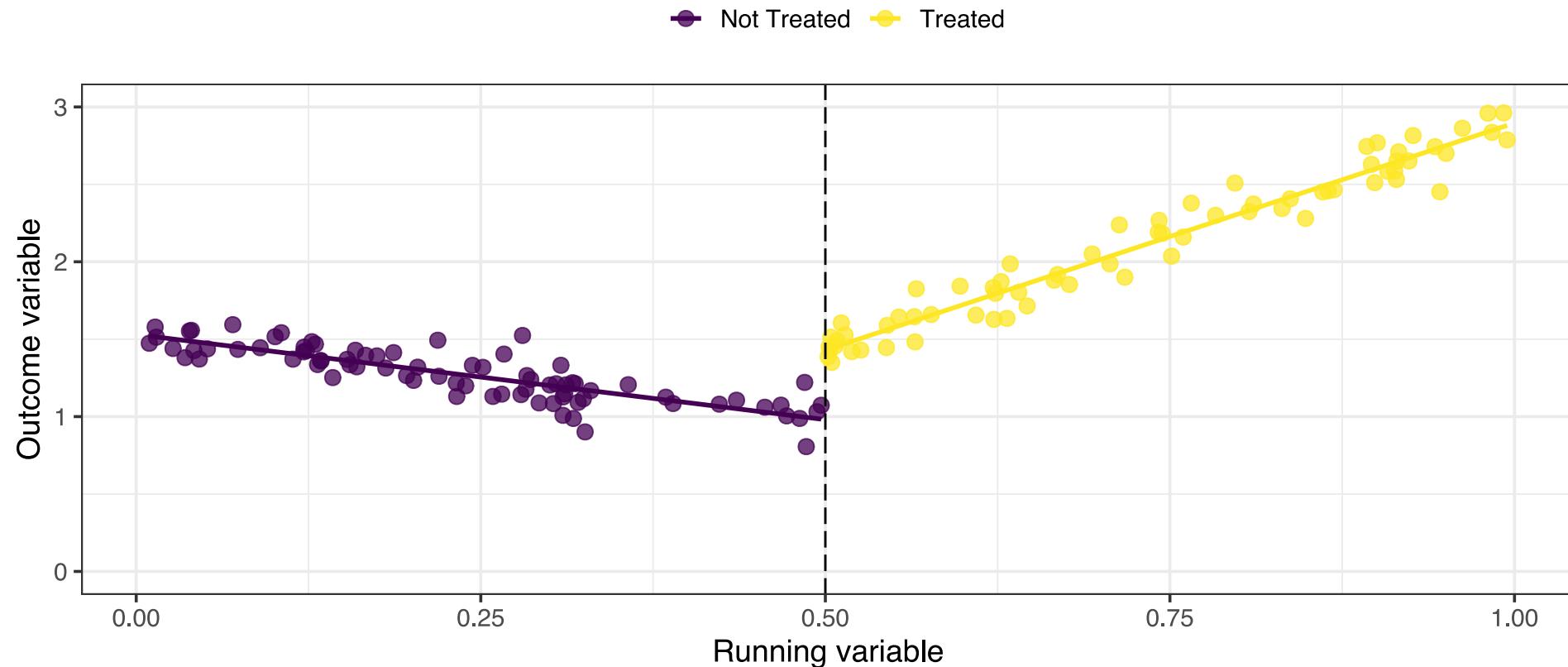
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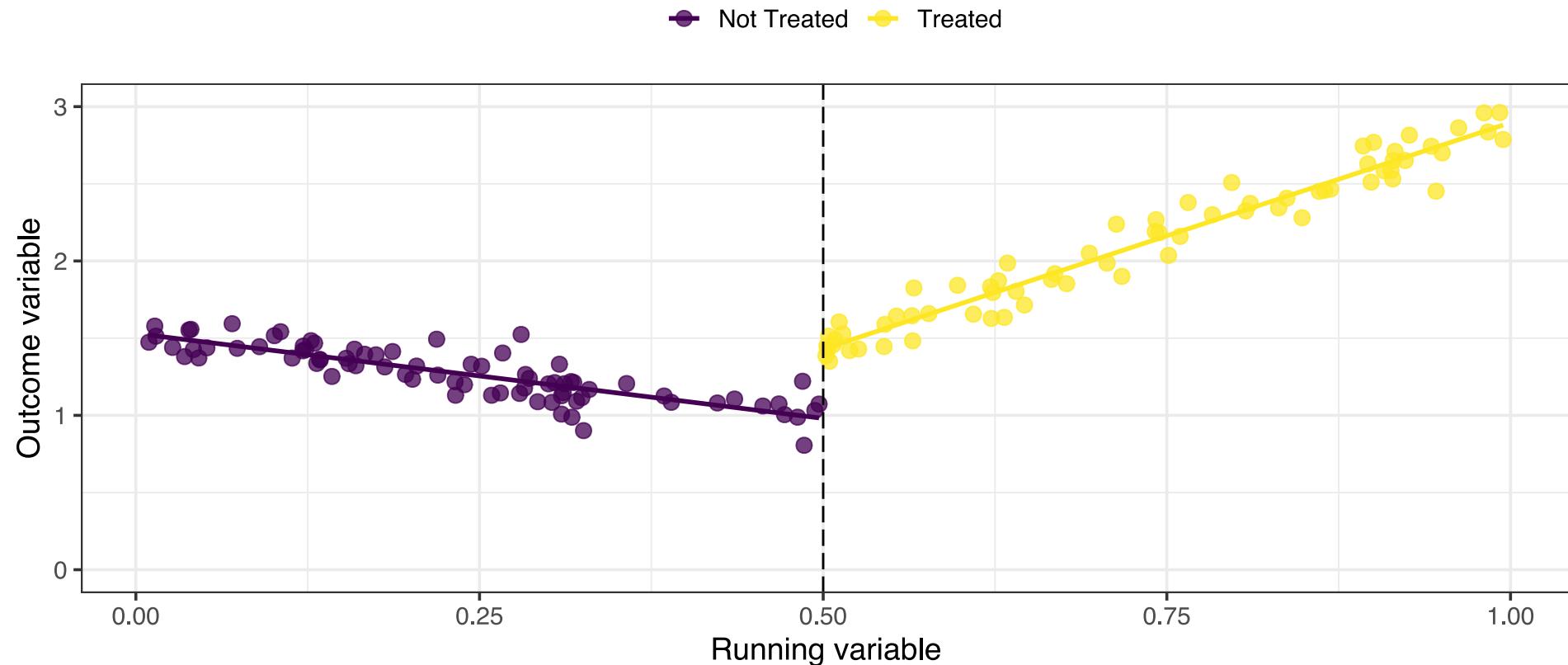
# Simulations - Linear Relationship but NO Discontinuity



# Simulations - Different Slopes



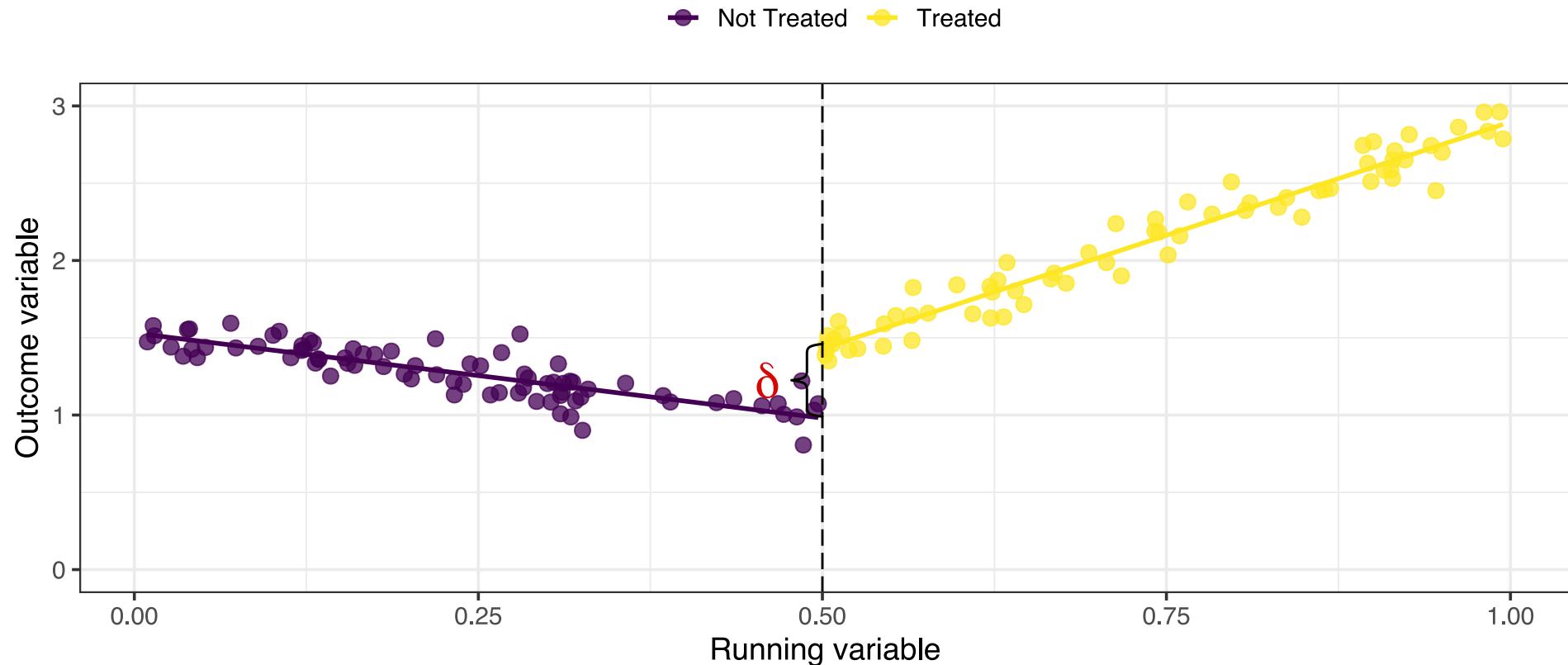
# Simulations - Different Slopes



$$\begin{aligned} \text{outcome}_i &= \alpha + \delta \text{treatment}_i + \beta (\text{running}_i - \text{cutoff}) + \\ &\gamma \text{treatment}_i * (\text{running}_i - \text{cutoff}) + e_i, \end{aligned}$$



# Simulations - Different (Linear) Slopes



$$\begin{aligned} \text{outcome}_i &= \alpha + \delta \text{treatment}_i + \beta (\text{running}_i - \text{cutoff}) + \\ &\gamma \text{treatment}_i * (\text{running}_i - \text{cutoff}) + e_i, \end{aligned}$$



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# How to Choose Appropriate Functional Form?

- Essential to **visualise** the data!
- Coefficients across models shouldn't vary too much.
- Should we expect the relationship between the outcome variable and the running variable to be nonlinear? Should we expect it to differ around the cutoff?

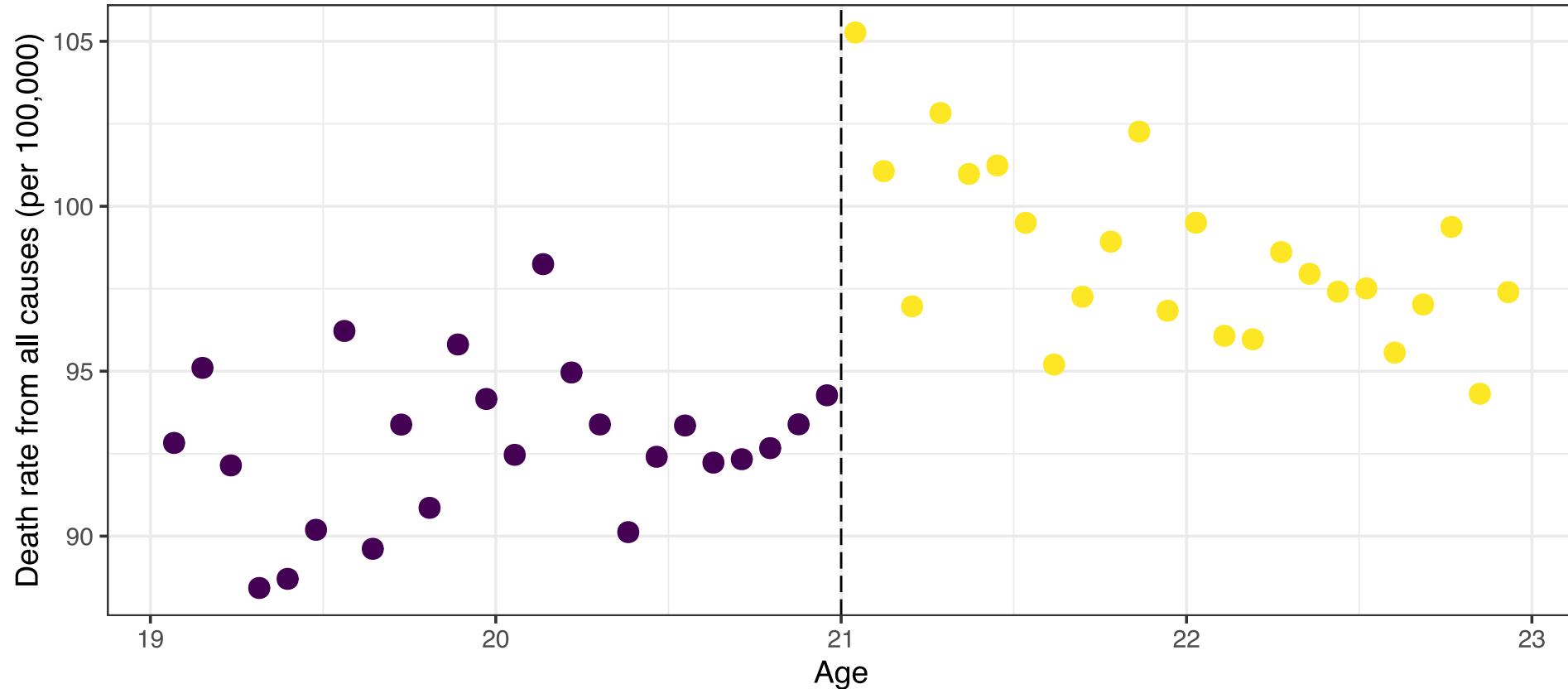


# How to Choose Appropriate Functional Form?

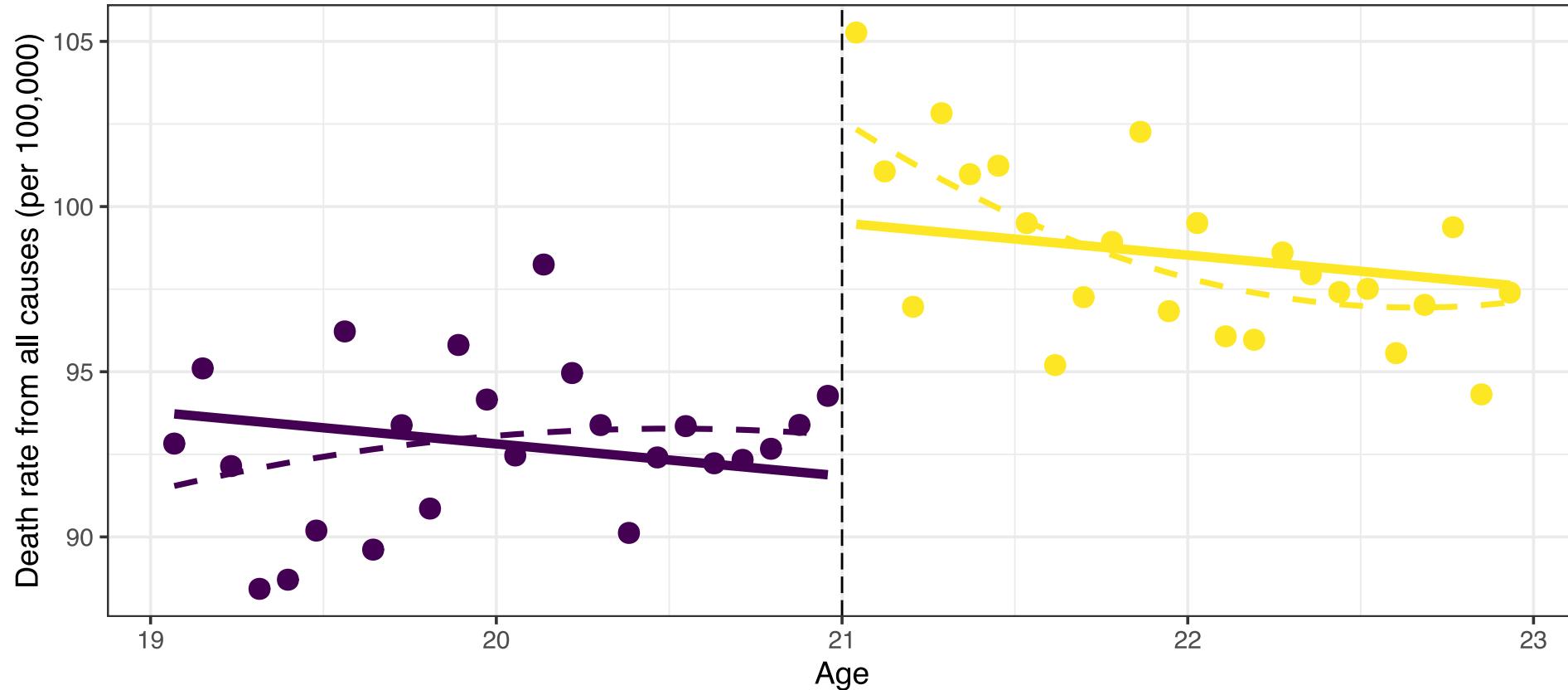
- Essential to **visualise** the data!
- Coefficients across models shouldn't vary too much.
- Should we expect the relationship between the outcome variable and the running variable to be nonlinear? Should we expect it to differ around the cutoff?
- **Gelman and Imbens (2019)**, "Why High-Order Polynomials Should Not Be Used in Regression Discontinuity Designs":  
*"We recommend researchers [...] use estimators based on local linear or quadratic polynomials or other smooth functions."*



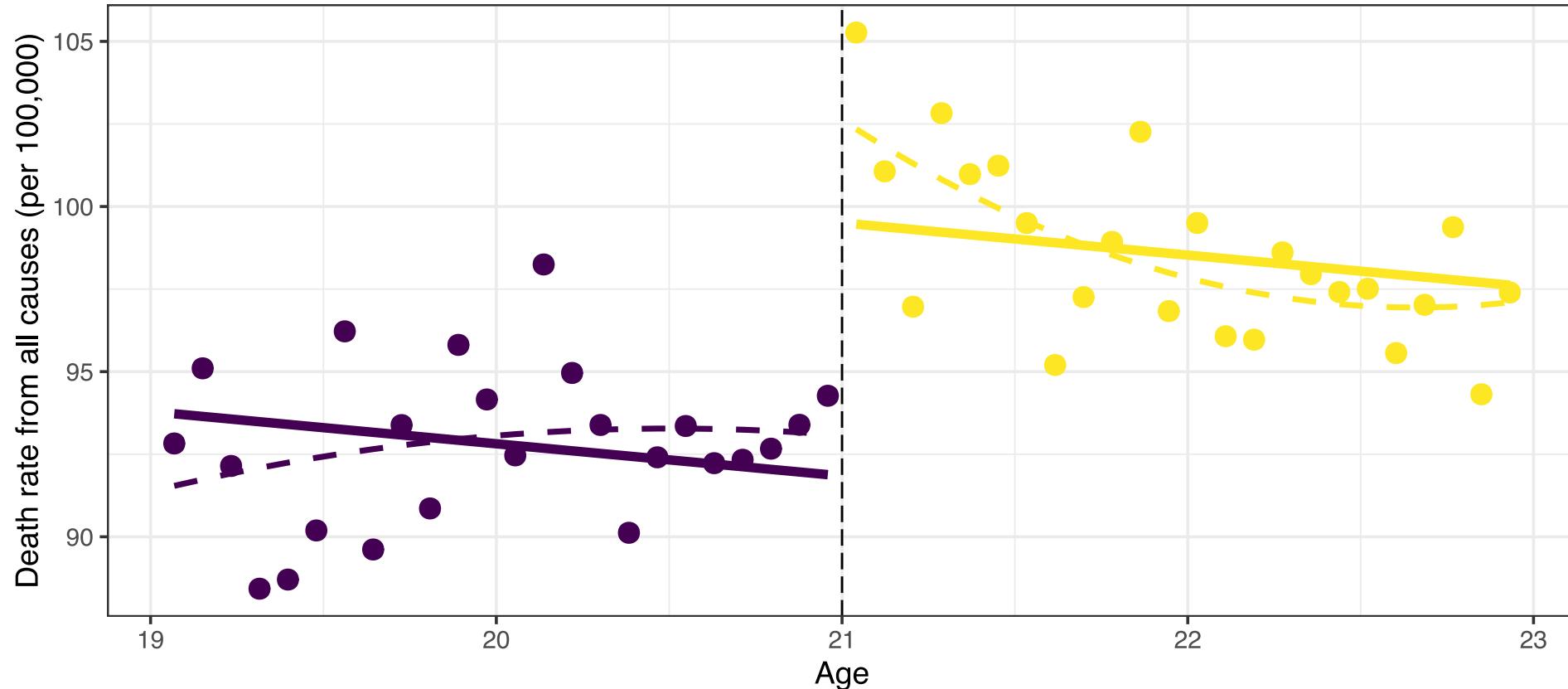
# Going Back to our Example: Nonlinearities / $\neq$ Slopes?



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Gap between the lines is roughly the same for both specifications.



# Task 3 (15 minutes)

1. Estimate the following *quadratic* model on all death causes. Does the RDD coefficient differ from the linear model?

$$DEATHRATE_a = \alpha + \delta D_a + \beta a + \beta a^2 + \varepsilon_a,$$

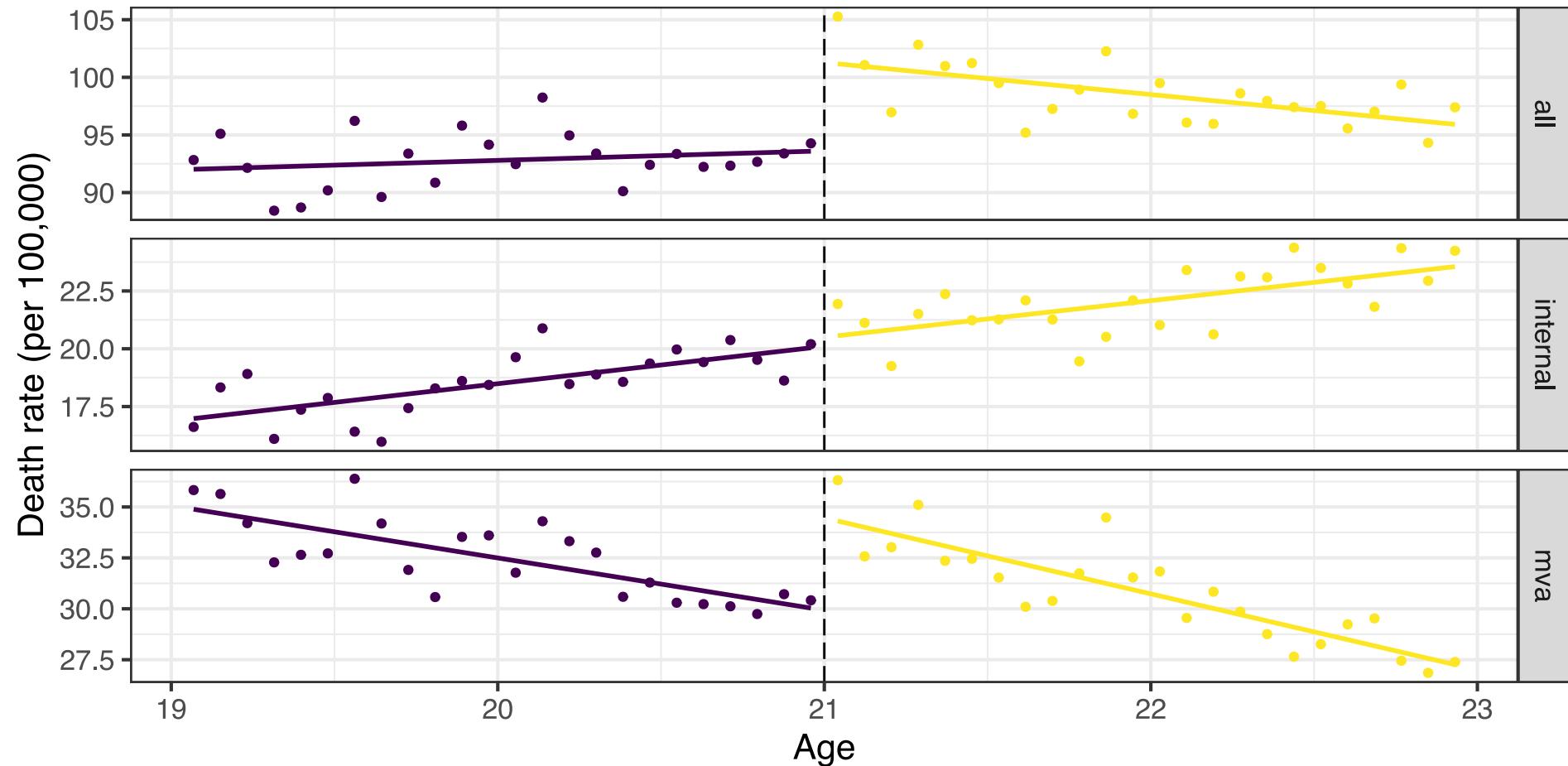
2. Recall that the regression model allowing for different slopes on each side of the cutoff is:

$$DEATHRATE_a = \alpha + \delta D_a + \beta(a - 21) + \gamma D_a * (a - 21) + \varepsilon_a,$$

- Why do we need to subtract the `cutoff` from `running_i`? (Hint: compute  $\mathbb{E}(DEATHRATE_a | a = 21)$ )
- Should we expect the relationship between death rates and age to change at 21?
- Estimate this model. How different is the RDD coefficient from the other models you have estimated?
- Re-run these models (linear, quadratic, different slopes) for the following death causes: motor vehicle accidents (`mva`), alcohol-related (`alcohol`), and internal (`internal`).



# Graphical Representation of the Regression Results



# Nonparametric Estimation

- Give more weight to observations close to the cutoff level



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2 settings:

- How much more weight?



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2 settings:

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  - depends on the chosen *bandwidth*.

Luckily there's an R package that chooses these settings optimally based on fancy algorythms: `rdrobust`.



# Identifying Assumptions

# RDD Assumptions

| Key assumption: *Potential outcomes are smooth at the threshold.*



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→ assignment variable cannot be manipulated!



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  - Knowing the cutoff value in itself does not violate the assumption, only ability to manipulate running variable does.



# RDD Assumptions

| Key assumption: **Potential outcomes are smooth at the threshold.**

If the assumption holds, we have:

$$\begin{aligned} & \lim_{r \rightarrow c^+} \mathbb{E}[Y_i | R_i = r] - \lim_{r \rightarrow c^-} \mathbb{E}[Y_i | R_i = r] \\ &= \lim_{r \rightarrow c^+} \mathbb{E}[Y_i^1 | R_i = r] - \lim_{r \rightarrow c^-} \mathbb{E}[Y_i^0 | R_i = r] \\ &= \mathbb{E}[Y_i^1 | R_i = c] - \mathbb{E}[Y_i^0 | R_i = c] \\ &= \mathbb{E}[Y_i^1 - Y_i^0 | R_i = c] \end{aligned}$$



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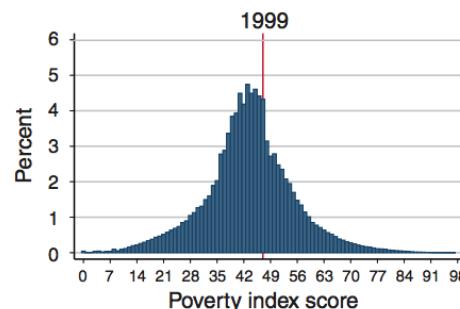
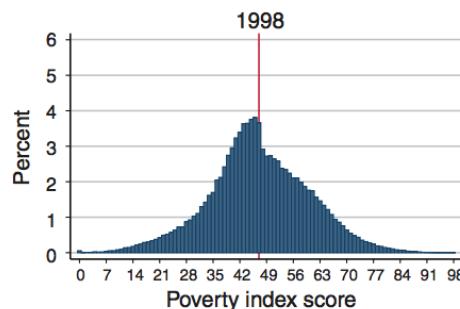
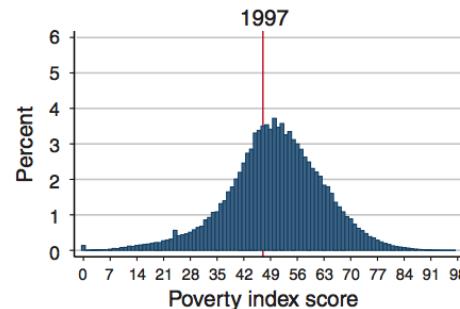
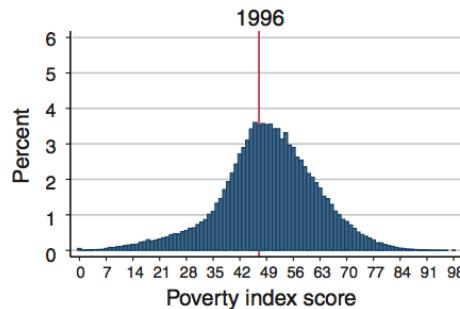
If the assumption holds, we have:

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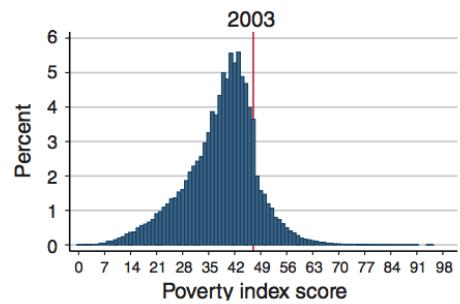
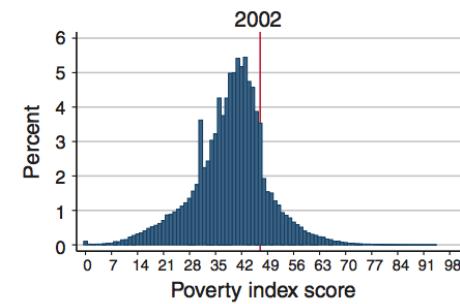
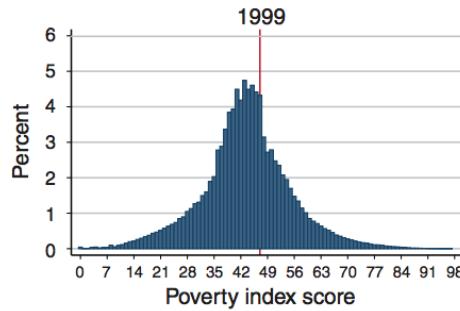
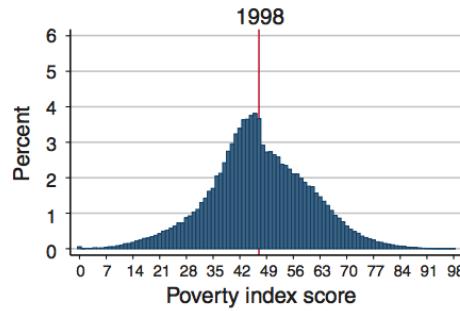
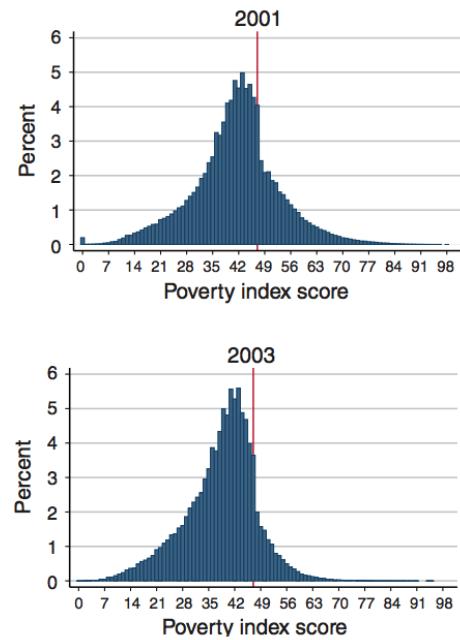
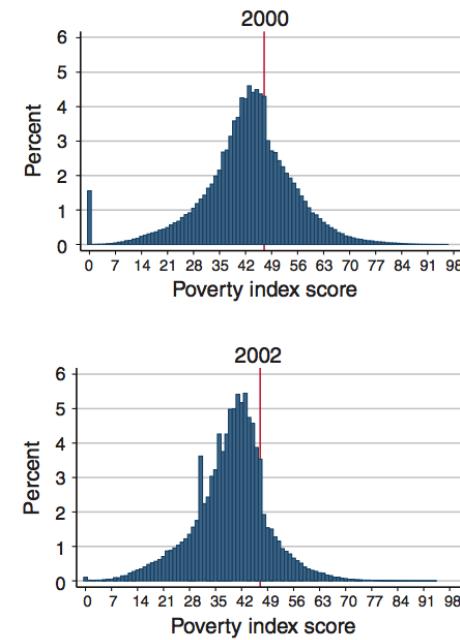
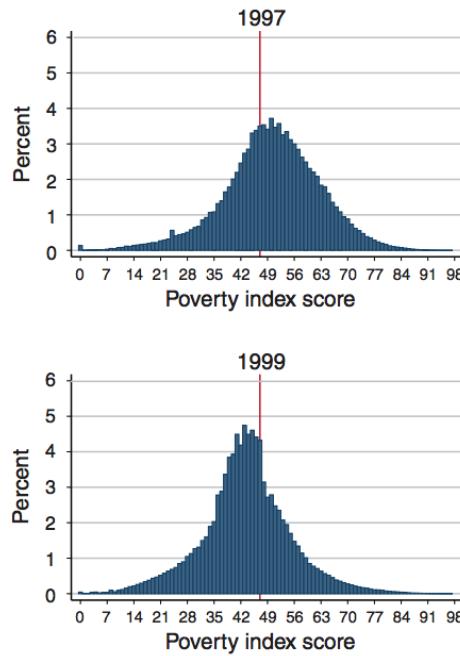
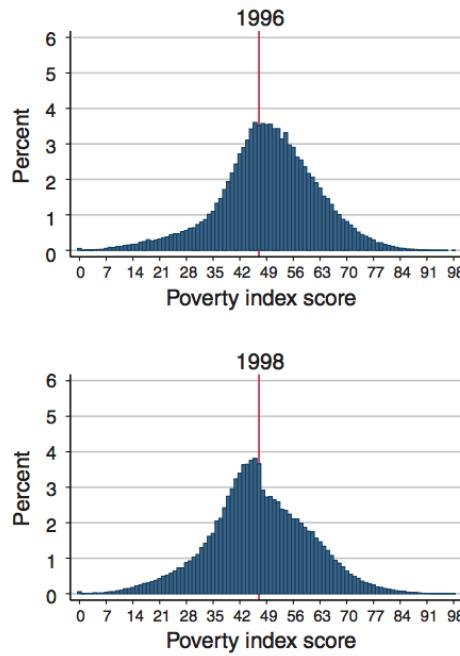
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What happens when threshold for eligibility to social assistance programs becomes known?



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What happens when threshold for eligibility to social assistance programs becomes known?



# Noncompliance

What if the running variable does not *fully* determine assignment to treatment?

→ *Fuzzy RDD*

- Even if all observations that satisfy the treatment condition are not treated, there is still a jump in the probability of being treated.
- For you, just know that problem of imperfect determination of allocation to treatment can still be solved



# 5 Steps for Conducting RDD in Practice<sup>1</sup>

Step #1: *Is assignment to treatment rule-based?*



<sup>1</sup> Taken from Andrew Heiss' wonderful course on RDD.

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Step #5: *How big is the gap?*

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END

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✉ florian.oswald@sciencespo.fr

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🔗 Book

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