SciencesPo
DEPARTMENT OF ECONOMICS

ScPoEconometrics

Regression Inference

Florian Oswald, Gustave Kenedi and Pierre Villedieu
SciencesPo Paris
2020-04-15

Recap from last week

- ***Confidence interval***: a plausible range of values for the population parameter.
- ***Hypothesis testing***: null hypothesis (H_0) vs alternative hypothesis (H_A), (observed) test statistic, null distribution.
- ***p-value***: probability of observing a test statistic as or more extreme than the observed test statistic assuming the null hypothesis is true.

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Today: Statistical inference in the regression framework

- Fully understand a regression table
- Compare theory-based and simulation-based inference
- *Classical Regression Model* assumptions

- Empirical applications:
  - Class size and student performance
  - Returns to education by gender

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```r
lm(math ~ small, star_df)
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##
## Call:
## lm(formula = math ~ small, data = star_df)
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## Coefficients:
## (Intercept)    smallTRUE
##     484.446        8.895
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- What if we drew another random sample of schools from Tennessee and redid the experiment, would we find a different value for b_1?

- We know the answer is *yes*, but how different is this estimate likely to be?

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- You will often find $\hat{\beta}_k$ rather than b_k, both refer to sample estimate of β_k.
- Let's bring what we know about ***confidence intervals***, ***hypothesis testing*** and ***standard errors*** to bear on those $\hat{\beta}_k$!

Understanding Regression Tables

Here is our `tidy` regression:

```r
library(broom)
tidy(lm(math ~ small, star_df))
```

```
## # A tibble: 2 x 5
##   term        estimate std.error statistic     p.value
##   <chr>          <dbl>     <dbl>     <dbl>       <dbl>
## 1 (Intercept)   484.        1.15    421.   0
## 2 smallTRUE       8.90      1.68      5.30 0.000000123
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- There are 3 new columns here: `std.error`, `statistic`, `p.value`.

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Entry	Meaning
`std. error`	Standard error of b_k
`statistic`	Observed test statistic associated to $H_0 : \beta_k = 0, H_A : \beta_k \neq 0$
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- Let's focus on the `small` coefficient and make sense of each entry.

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- We'd run 1000 regressions and would get 1000 estimates of β_k, b_k.
- The standard error of b_k quantifies how much variation in b_k one would expect across (*an infinity of*) samples.

Standard Error of b_{small}

- From the table, we get $\hat{\text{SE}}(b_{\text{small}}) = 1.68$
    - Notice that we write $\hat{\text{SE}}$ and not SE because 1.68 is an estimate of the real standard error of b_{small} we get from our sample.

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- `R` obtains this estimated standard error from ***theory*** which we will see in a few slides.

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- `R` obtains this estimated standard error from ***theory*** which we will see in a few slides.
- Let's simulate the bootstrap distribution of b_{small} to see how close the simulated value is to the theoretical value.

Task 1 (10 min)

As we did for the bootstrap distribution of the proportion of *green pasta*, we want to generate the bootstrap distribution of b_{small}.

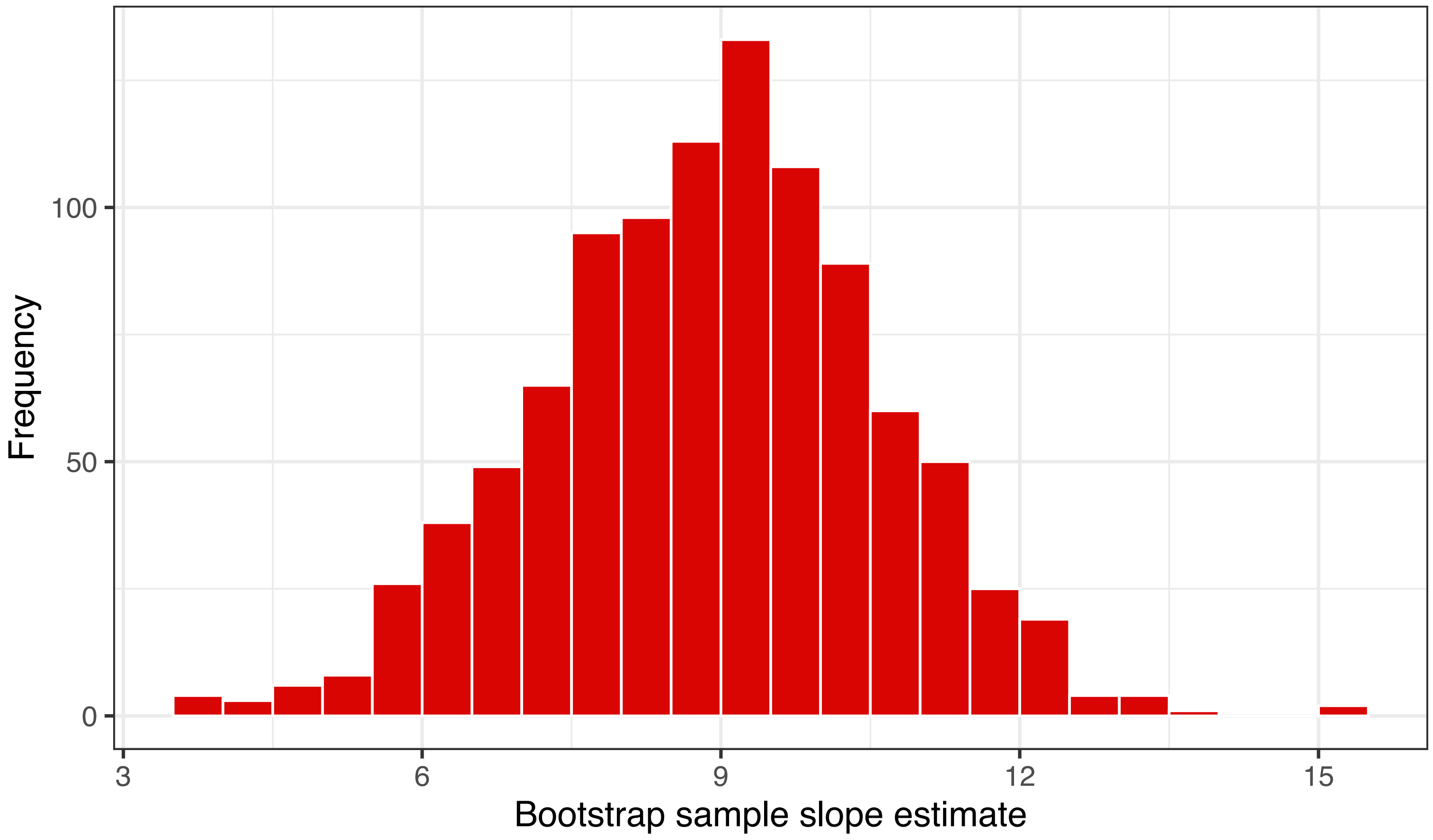
1. Copy the loading and cleaning code from slide 3 and run it.

2. Generate the bootstrap distribution of b_{small} based on 1000 samples drawn from `star_df`.
   Hints:

   - `specify()`: use the `explanatory` and `response` arguments and set them equal to the relevant variable.
   - `generate()`: 1000 replicates of type bootstrap.
   - `calculate()`: set the `stat` argument to `slope` and add the following argument: `order = c("TRUE", "FALSE")`.

3. Plot this bootstrap distribution and compute the mean and the standard error of b_{small}.

Bootstrap Distribution



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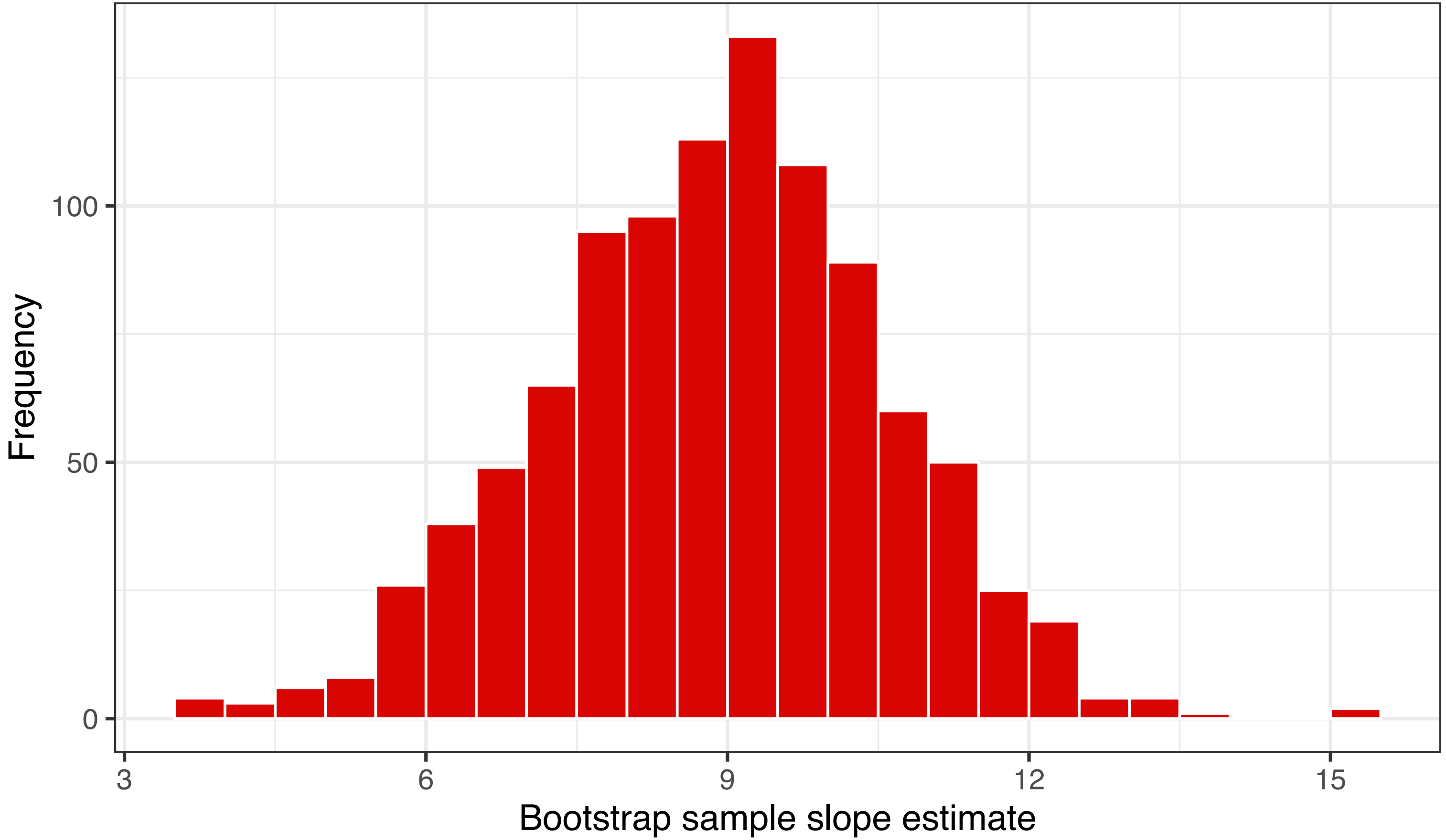

standard error: 1.69 $\rightarrow$ very close to the one in the table (1.68)!

Testing $\beta_k = 0$ vs $\beta_k \neq 0$

By default, the regression output provides the results associated with the following hypothesis test:

$$
\begin{aligned}
H_0 : &\beta_k = 0 \\
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- If H_0 is false, then there **is** a true relationship.
- ***Important:*** This is a ***two-sided*** test!

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- The **p-value** measures the area outside of $\pm$ *observed test statistic* under the *null distribution*.
- Finally, we check if we can reject H_0 at the usual **significance levels**: α = 0.1, 0.05, 0.01.

Testing $\beta_{\text{small}} = 0$ vs $\beta_{\text{small}} \neq 0$

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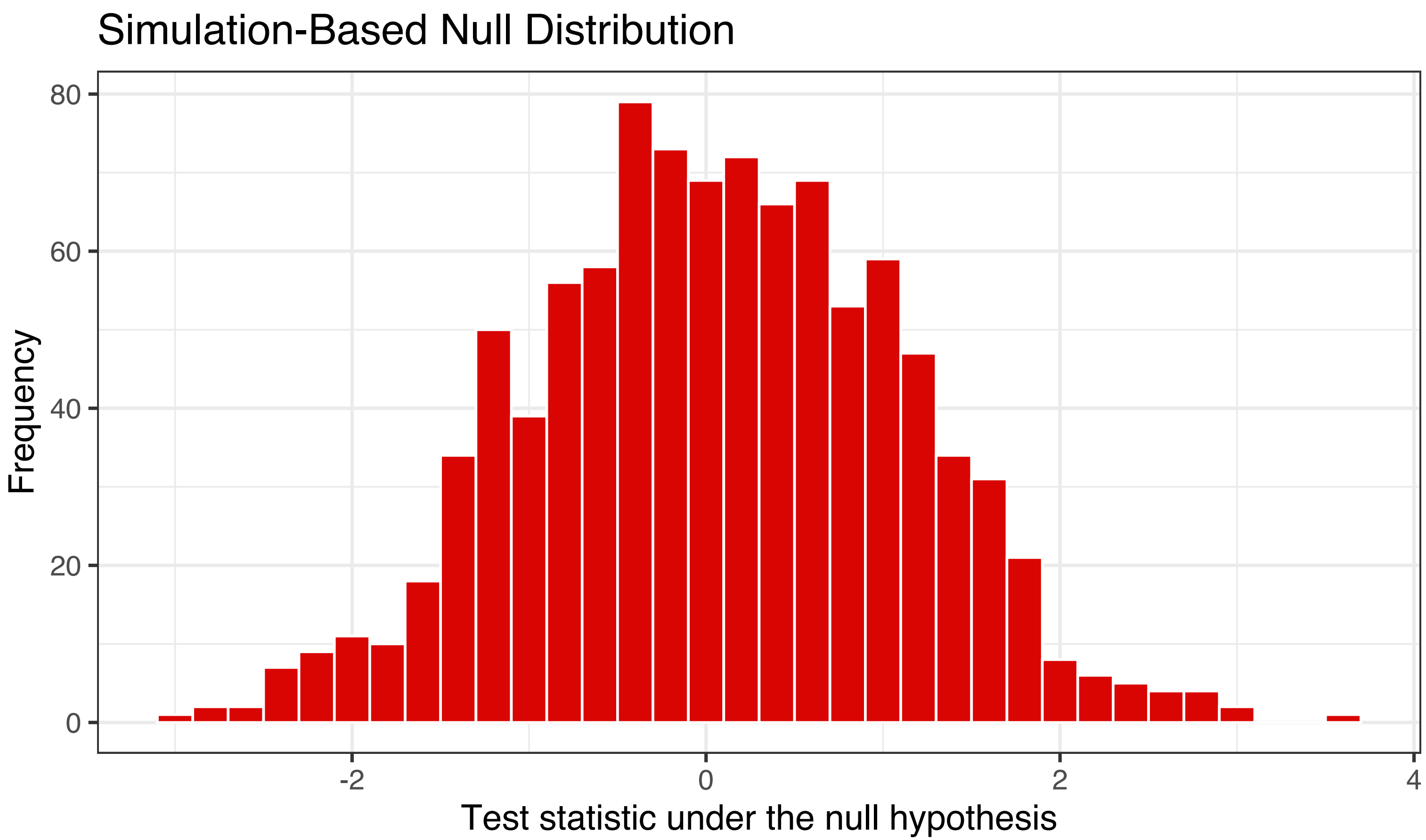
- We can compute the distribution of our test statistic $\frac{b_{\text{small}}}{\hat{SE}(b_{\text{small}})}$ under the null:

```
null_distribution <- null_distribution %>%
  mutate(test_stat = stat/sd(bootstrap_distrib$stat))
```

- Remember we got $\hat{SE}(b_{\text{small}})$ = 1.69 from our bootstrap distribution.

Testing $\beta_{\text{small}} = 0$ vs $\beta_{\text{small}} \neq 0$



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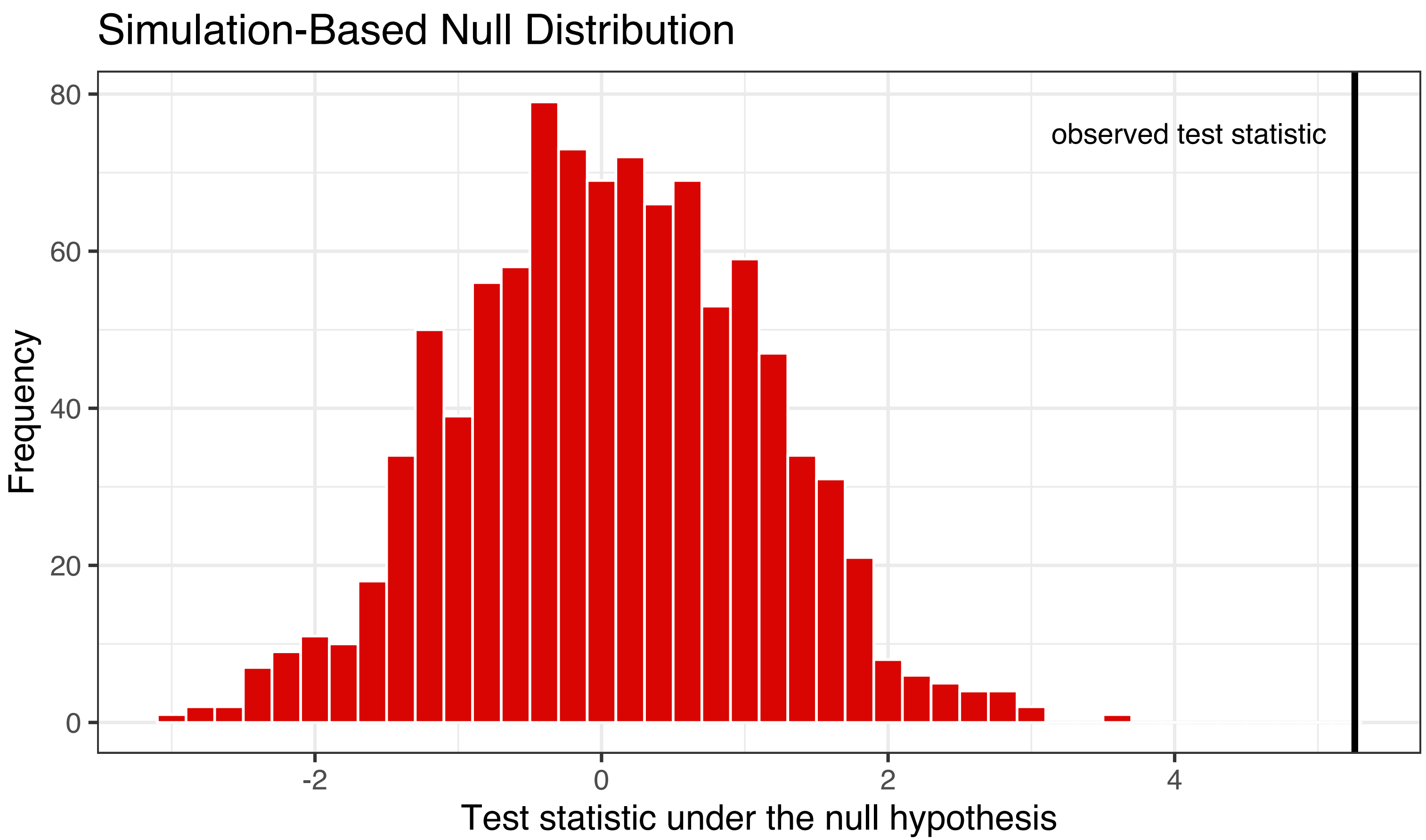

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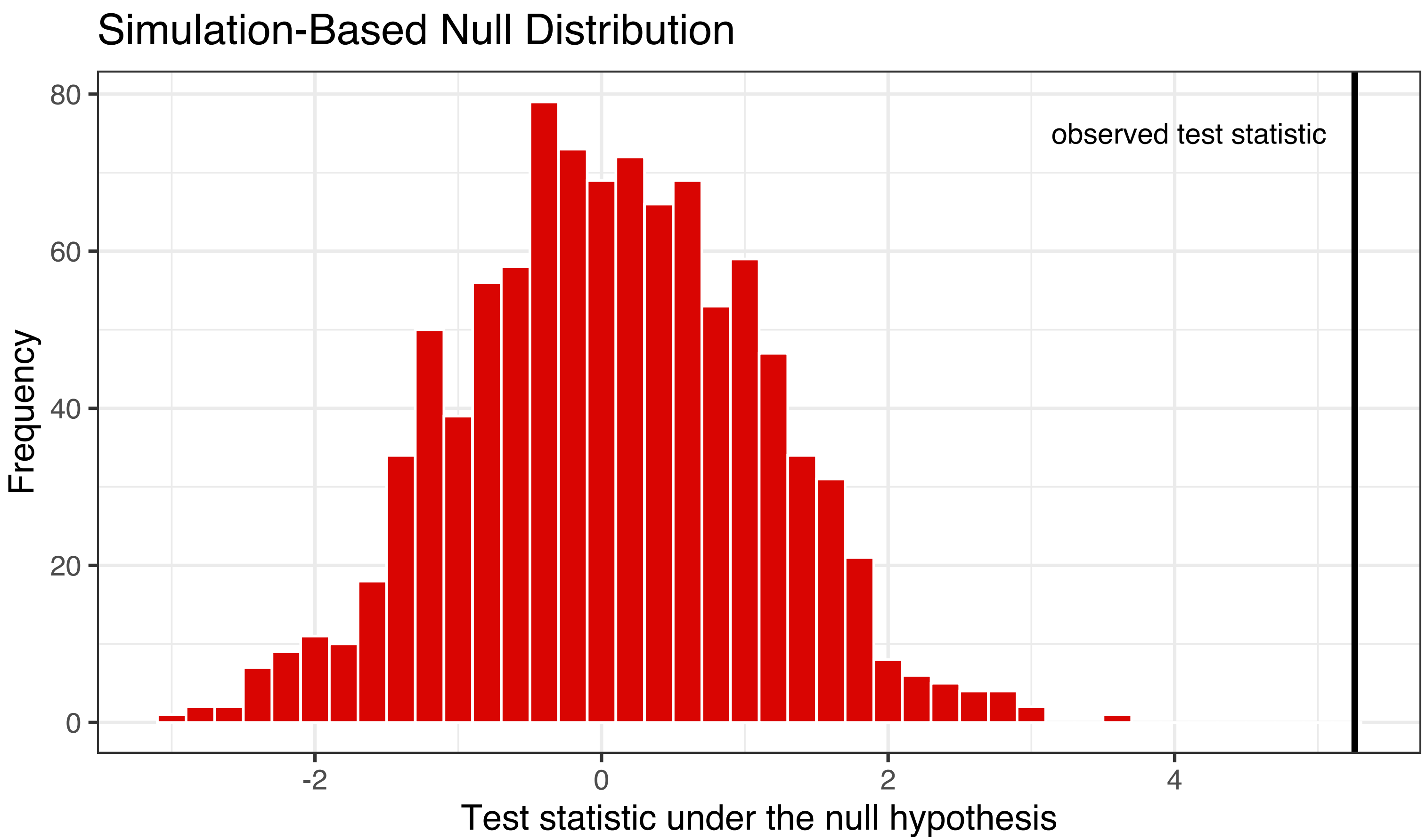

Very unlikely to obtain b_{small} = 8.9 when H_0 is true.

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- To decide if we reject H_0, recall we are considering a **two-sided test** here: *more extreme* means inferior to -5.257 **or** superior to 5.257.

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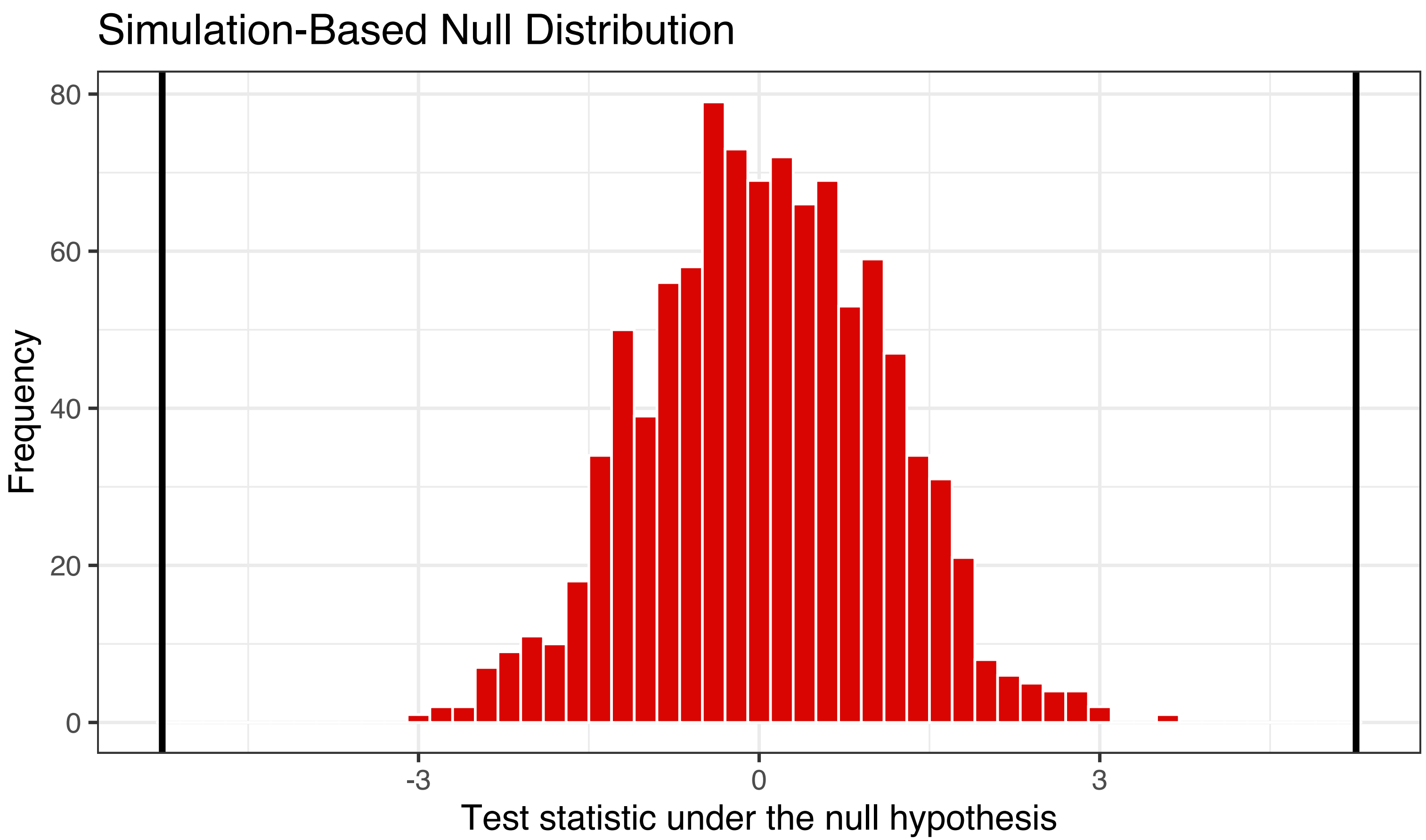

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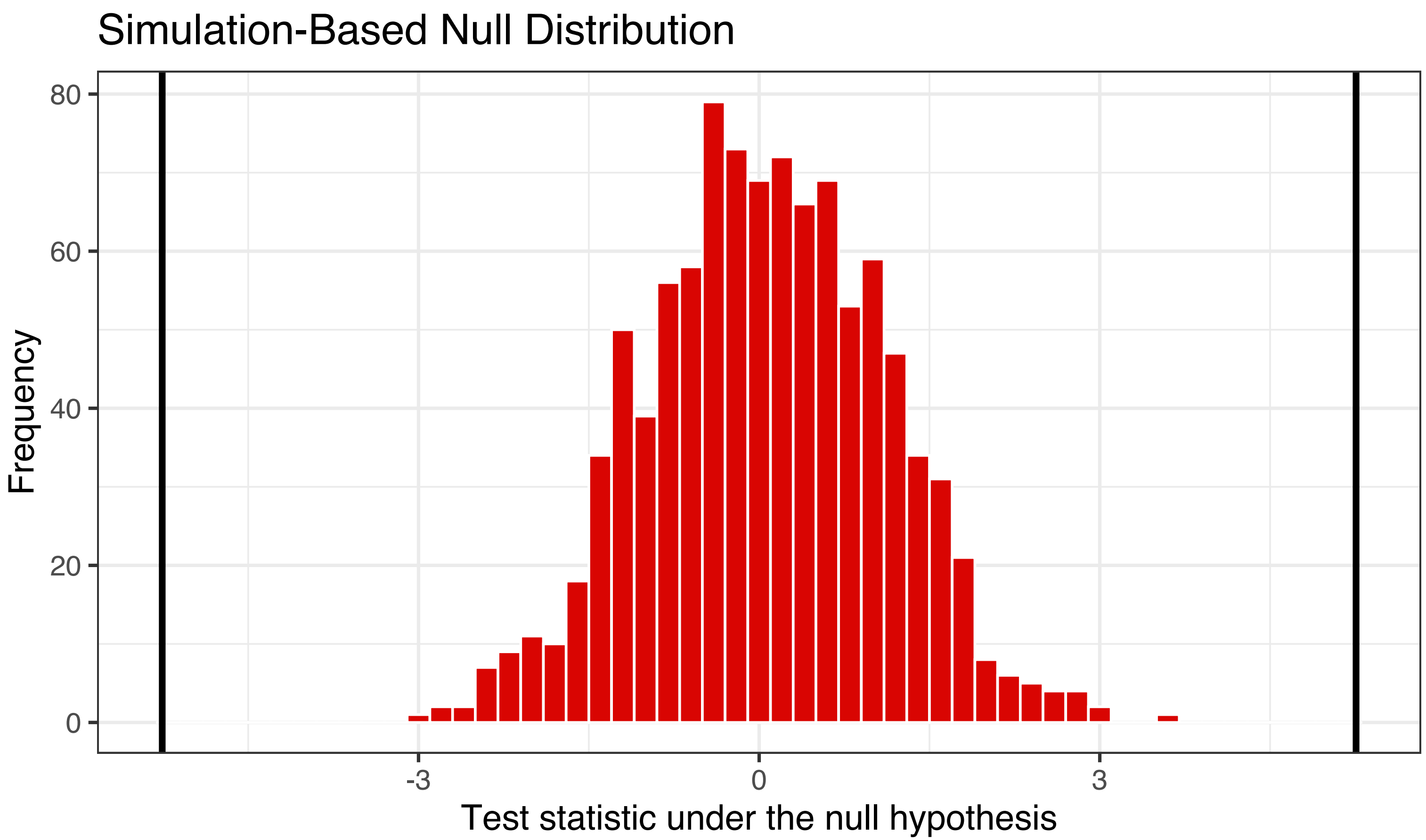

What does the p-value correspond to?

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- Computing the *p-value* we get:

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p_value = mean(abs(null_distribution$test_stat) >= observed_stat)
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- ***Question:*** Can we reject the null hypothesis at the 5% level?

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- ***Answer:***

    - Since the *p-value* is equal to 0 it means that we would reject H_0 at any significance level: the p-value would always be inferior to α.
    - In other words, we can say that b_{small} is **statistically different from 0** at any significance level.
    - We also say that b_{small} is *statistically significant* (at any significance level).

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- The values reported by statistical packages in `R` are instead obtained from theory.
- Theoretical inference is based on **large sample approximations**.
    - One can show that sampling distributions *converge* to suitable distributions.
- Let's briefly look into the theory-based approach.

Regression Inference: Theory

- Theory-based approach uses one fundamental result: the sampling distribution of the sample statistic $\frac{b-\beta}{\hat{\text{SE}}(b)}$ *converges* to a ***standard normal distribution*** as the sample size gets larger and larger.
    - $\hat{\text{SE}}(b)$ is the sample estimate of the standard deviation of b.
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- A ***standard normal distribution*** is a *normal distribution* with *mean* 0 and *standard deviation* 1.

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- A ***standard normal distribution*** is a *normal distribution* with *mean* 0 and *standard deviation* 1.

- We don't need to simulate any sampling distribution here, we derive it from theory and use it to construct confidence intervals or to conduct hypothesis tests.

Regression Inference: Theory

- Theory-based approach uses one fundamental result: the sampling distribution of the sample statistic $\frac{b-\beta}{\hat{\text{SE}}(b)}$ *converges* to a ***standard normal distribution*** as the sample size gets larger and larger.

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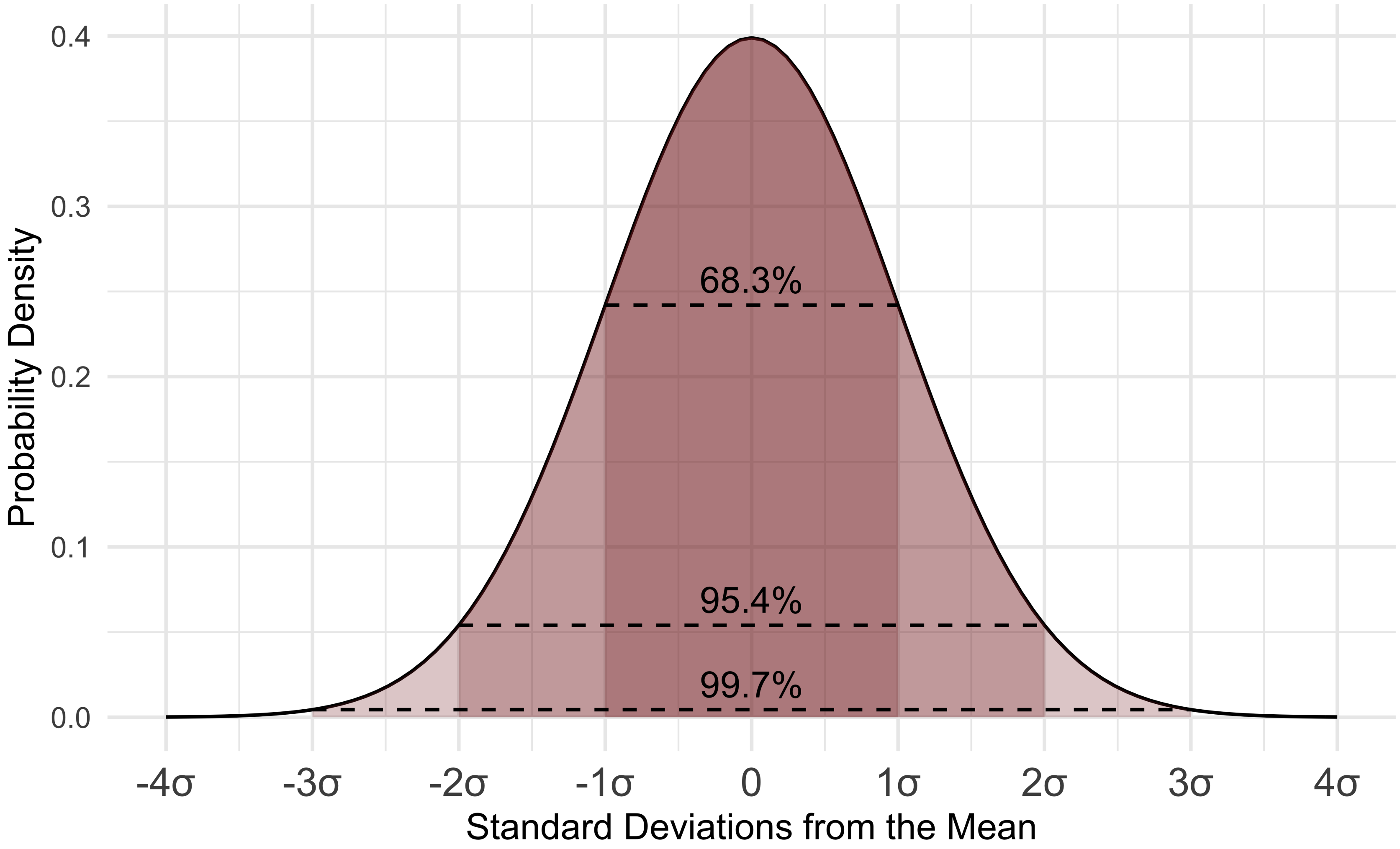
- Note that if $\frac{b-\beta}{\hat{\text{SE}}(b)}$ *converges* to a ***standard normal distribution***, then b converges to a ***normal distribution*** with mean β and standard deviation $\hat{\text{SE}}(b)$.

Normal Distributions: A Refresher



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```
tidy(lm(math ~ small, star_df),
     conf.int = TRUE, conf.level = 0.95) %>%
  filter(term == "smallTRUE") %>%
  select(term, conf.low, conf.high)
```

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## # A tibble: 1 x 3
##   term      conf.low conf.high
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- This can easily be generalized to any confidence level by taking the appropriate quantile of the normal distribution.

Task 2 (5 min)

1. Using the bootstrap distribution you generated in Task 1, compute the 95% confidence interval using the *percentile method*.

2. How similar is it to the confidence intervals obtained in the previous slide?

Theory-Based Inference: Hypothesis Testing

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- The ***p-value*** associated to our test is then equal to the area of the *standard normal distribution* outside $\pm$ the observed value of $\frac{b}{\hat{\text{SE}}(b)}$.

- Common rule of thumb: if the *estimate* is ***twice the size of the standard error***, then it is significant at the 5% level. Why?

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  - We already mentioned the distinction between the sample estimate b_k (or $\hat{\beta}_k$) and the population parameter β_k.

  - In the same way, we distinguish e, the sample error, from ε the error term from the true population model:

$$y_i = \beta_0 + \beta_1 x_{1,i} + \ldots + \beta_k x_{k,i} + \varepsilon_i$$

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4. ***Homoskedasticity:*** the variance of the error term ε is the same for each value of x: $Var(\varepsilon|x) = \sigma^2$.

5. ***Normally distributed errors:*** the error term is normally distributed, i.e. $\varepsilon \sim \mathcal{N}(0, \sigma^2)$

   - This last assumption allows avoiding large sample approximations, but it is never used in practice since samples are sufficiently large $(n \geq 30)$.

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The CRM assumption #2 is also know as the (strict) **exogeneity assumption**.

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  - Under the exogeneity assumption β_1 denotes the causal effect of education in the population.
- Suppose there is *unobserved* ability a_i.
  - High ability means higher wage.
  - It *also* means school is easier, and so i selects into more schooling.

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- Then regressing the wage on education we will attribute to `educ` part of the effect on wages that is actually *caused* by ability a_i!
    - Remember the formula of the **omitted variable bias**:

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    - Thus, we have:

$$\mathbb{E}(b_1) = \beta_1 + OVB > \beta_1$$

- *Interpretation*: taking repeated sample from the population and computing b_1 each time, we would **systematically overestimate** the effect of education on wage.

Breaking the other assumptions

- You can find examples associated to the other assumptions in our book!
- Takeaway: if assumptions violated, inference is invalid!

Task 3.1 (10 min)

Let's go back to our question of returns to education and gender.

1. Load the data `CPS1985` from the `AER` package and look back at the `help` to get the definition of each variable: `?CPS1985`

2. Create the `log_wage` variable equal to the log of `wage`.

3. Regress `log_wage` on `gender` and `education`, and save it as `reg1`.

    - Interpret each coefficient.

    - Are the coefficients statistically significant? At which significance level?

4. Regress the `log_wage` on `gender`, `education` and their interaction `gender*education`, save it as `reg2`.

    - How do you interpret the coefficient associated to $female * education$?

    - Can we reject the nullity of this coefficient at the 5% level? At 10%?

Task 3.2 (10 min)

1. Produce a scatterplot of the relationship between the log wage and the level of education, by gender (`color`).

2. Add a *linear regression line* to the plot using the `geom_smooth` layer. What does this line represents?

3. Let's illustrate what the shaded area stands for.

    1. Draw one bootstrap sample from our `cps` data.

    2. Regress the `log_wage` on `gender`, `education` and their interaction `gender*education`, save it as `reg_bootstrap`.

    3. From `reg_bootstrap` extract and save the value of the intercept for men as `intercept_men_bootstrap` and the value of the slope for men as `slope_men_bootstrap`. Do the same for women.

    4. Add both predicted lines from this bootstrap sample to the previous plot (*Hint*: use `geom_abline` (x2))

Illustrating Uncertainty

Let's repeat the procedure you just made 100 times!

```
library(AER)
data("CPS1985")
cps = CPS1985 %>% mutate(log_wage = log(wage))

set.seed(1)
bootstrap_sample = cps %>%
    rep_sample_n(size = nrow(cps), reps = 100, replace

ggplot(data=cps,aes(y = log_wage, x = education, colou
  geom_point(size = 1, alpha = 0.7) +
  geom_smooth(method = "lm", alpha = 2) +
  geom_smooth(data=bootstrap_sample,
              size = 0.2,
              aes(y = log_wage, x = education, group =
              method = "lm", se = FALSE) +
  facet_wrap(~gender) +
  scale_colour_manual(values = c("darkblue", "darkred"
  labs(x = "Education", y = "Log wage") +
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  theme_bw(base_size = 20)
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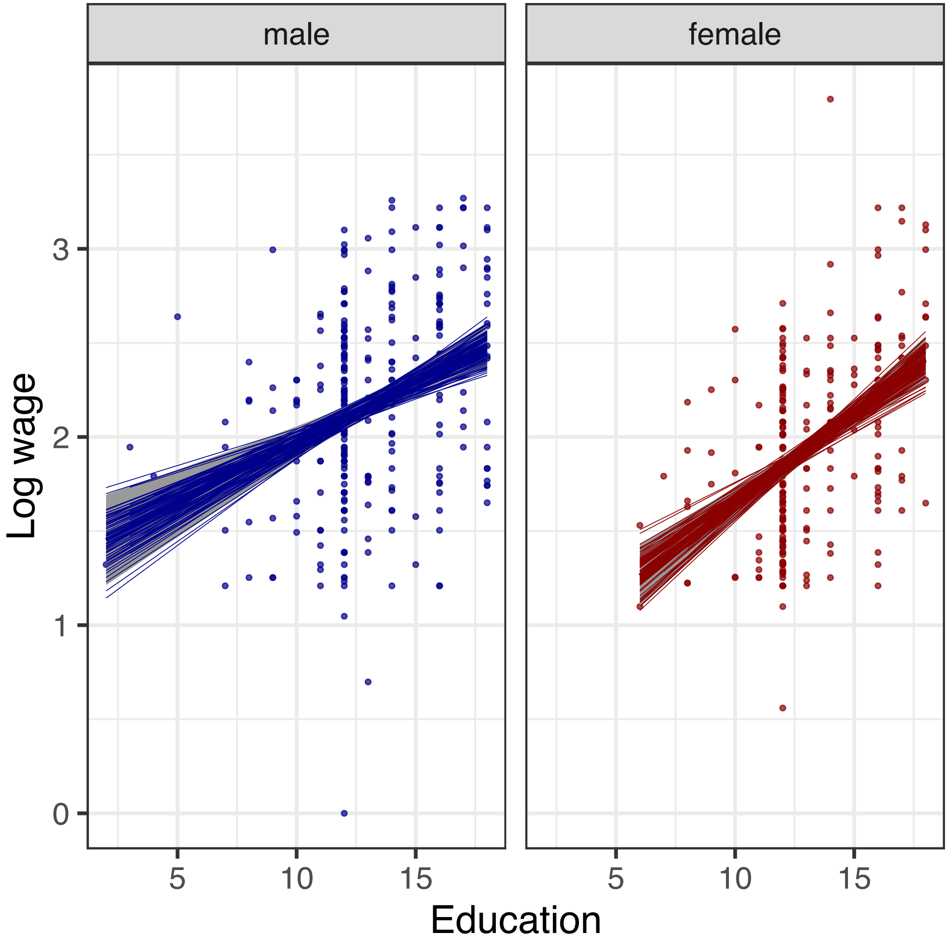

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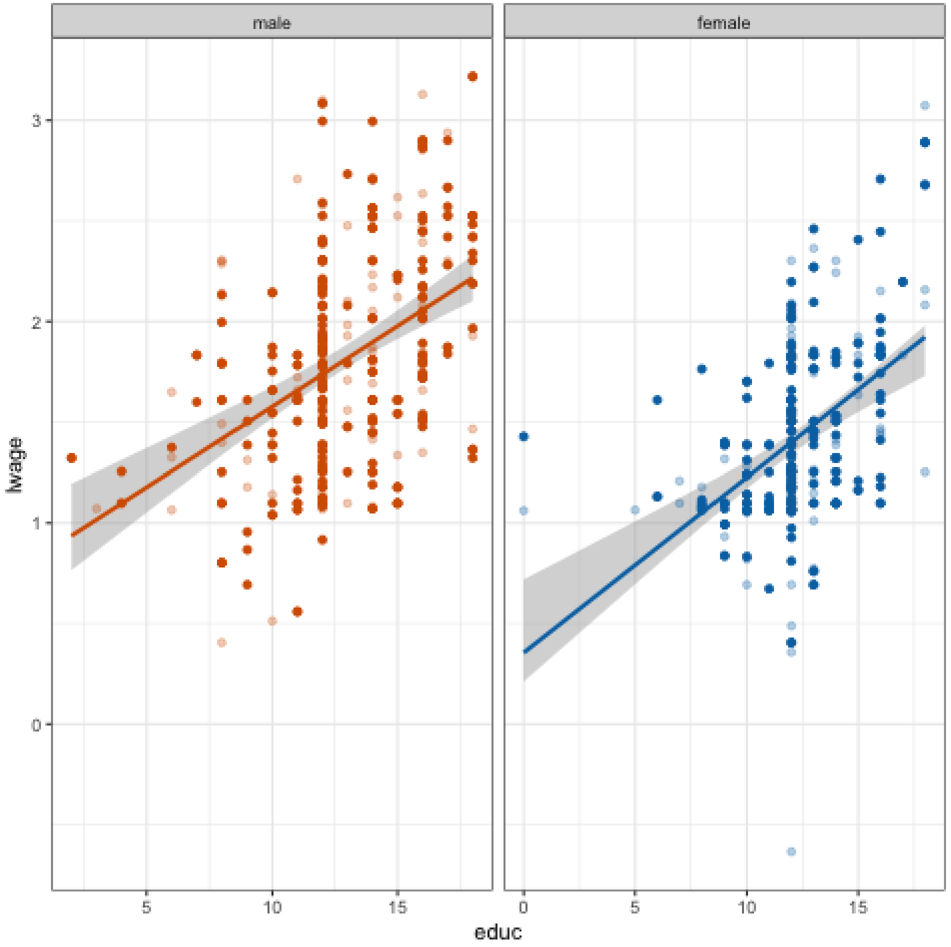

Even better : `ungeviz` and `gganimate` bring you moving lines!

- We took 20 bootstrap samples from our data
- You can see how different data points are included in each bootstrap sample.
- Those different points imply different regression lines.
- On average, 95% of these lines should fall into the shaded area.
- You should remember those moving lines when looking at the shaded area!

Teaser for next session (the last one 😢)

- Methods for program evaluation!

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Which do you prefer?

THANKS

To the amazing moderndive team!

Big Thanks 🎉 to ungeviz and 🎊 gganimate for their awesome packages!

SciencesPo
DEPARTMENT OF ECONOMICS

SEE YOU NEXT WEEK!

	florian.oswald@sciencespo.fr
	Slides
	Book
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