SciencesPo
DEPARTMENT OF ECONOMICS

ScPoEconometrics

Regression Inference

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SciencesPo Paris
2021-11-10

Quick "Quiz" on Last Week's Material

1. From your ***computer*** ☞ connect to ***www.wooclap.com/SCPOCIHT***

   OR

2. From your ***phone*** ☞ flash QR code below

Today - Statistical inference in the regression framework

- Fully understand a ***regression table***
- Compare ***theory-based*** and ***simulation-based*** inference
- ***Classical Regression Model*** assumptions
- Empirical applications:
    - Class size and student performance
    - Returns to education by gender

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## Call:
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- What if we drew another random sample of schools from Tennessee and redid the experiment, would we find a different value for b_1?

- We know the answer is *yes*, but how different is this estimate likely to be?

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- You will often find $\hat{\beta}_k$ rather than b_k, both refer to sample estimate of β_k.
- Let's bring what we know about ***confidence intervals***, ***hypothesis testing*** and ***standard errors*** to bear on those $\hat{\beta}_k$!

Understanding Regression Tables

Here is our `tidy` regression:

```
library(broom)
tidy(lm(math ~ small, star_df))
```

```
## # A tibble: 2 x 5
##   term        estimate std.error statistic     p.value
##   <chr>          <dbl>     <dbl>     <dbl>       <dbl>
## 1 (Intercept)   484.        1.15    421.   0
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`std. error`	Standard error of b_k
`statistic`	Observed test statistic associated to $H_0 : \beta_k = 0, H_A : \beta_k \neq 0$
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- Let's focus on the `small` coefficient and make sense of each entry.

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- We'd run 1,000 regressions and obtain 1,000 estimates of β_k, b_k.
- The standard error of b_k quantifies how much variation in b_k one would expect across (*an infinity of*) samples.

Standard Error of b_{small}

- From the table, we get $\hat{\text{SE}}(b_{\text{small}}) = 1.68$
  - Notice that we write $\hat{\text{SE}}$ and not SE because 1.68 is an estimate of the real standard error of b_{small} we get from our sample.

Standard Error of b_{small}

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  - Notice that we write $\hat{\text{SE}}$ and not SE because 1.68 is an estimate of the real standard error of b_{small} we get from our sample.
- Let's simulate the sampling distribution of b_{small} to see where it comes from.

Task 1

10:00

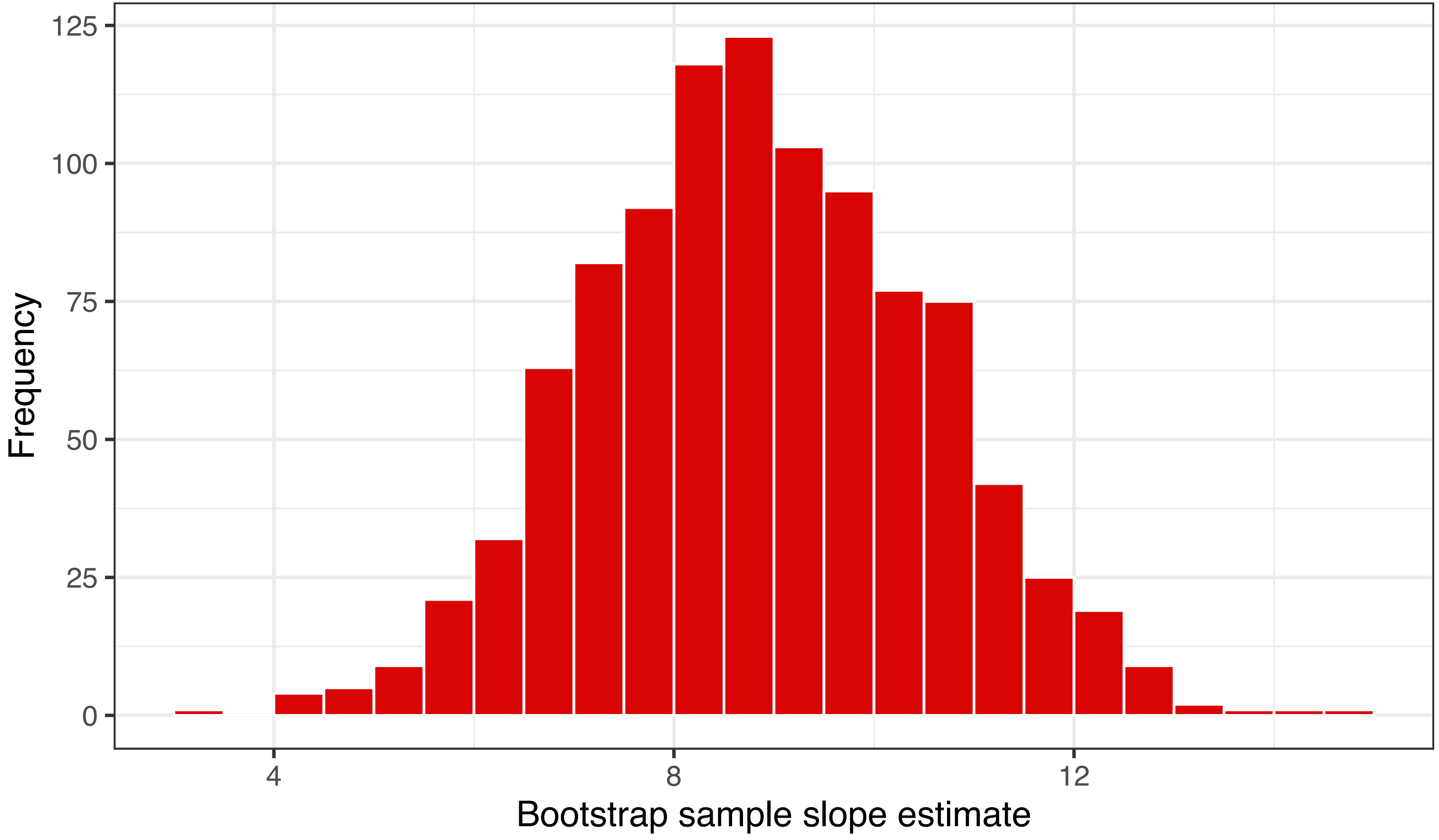
As we did for the sampling distribution of the proportion of *green pasta*, we want to generate the bootstrap distribution of b_{small}.

1. Copy the loading and cleaning code from slide 3 and run it.

2. Generate the bootstrap distribution of b_{small} based on 1,000 samples drawn from `star_df`.
   Hint 1: use the appropriate functions and arguments from the `infer` package so use the help pages.
   Hint 2: in `calculate` set `stat` to `slope` and `order` to `c("TRUE","FALSE")`.

3. Plot this simulated sampling distribution and compute the mean and standard error of b_{small}.

Bootstrap Distribution



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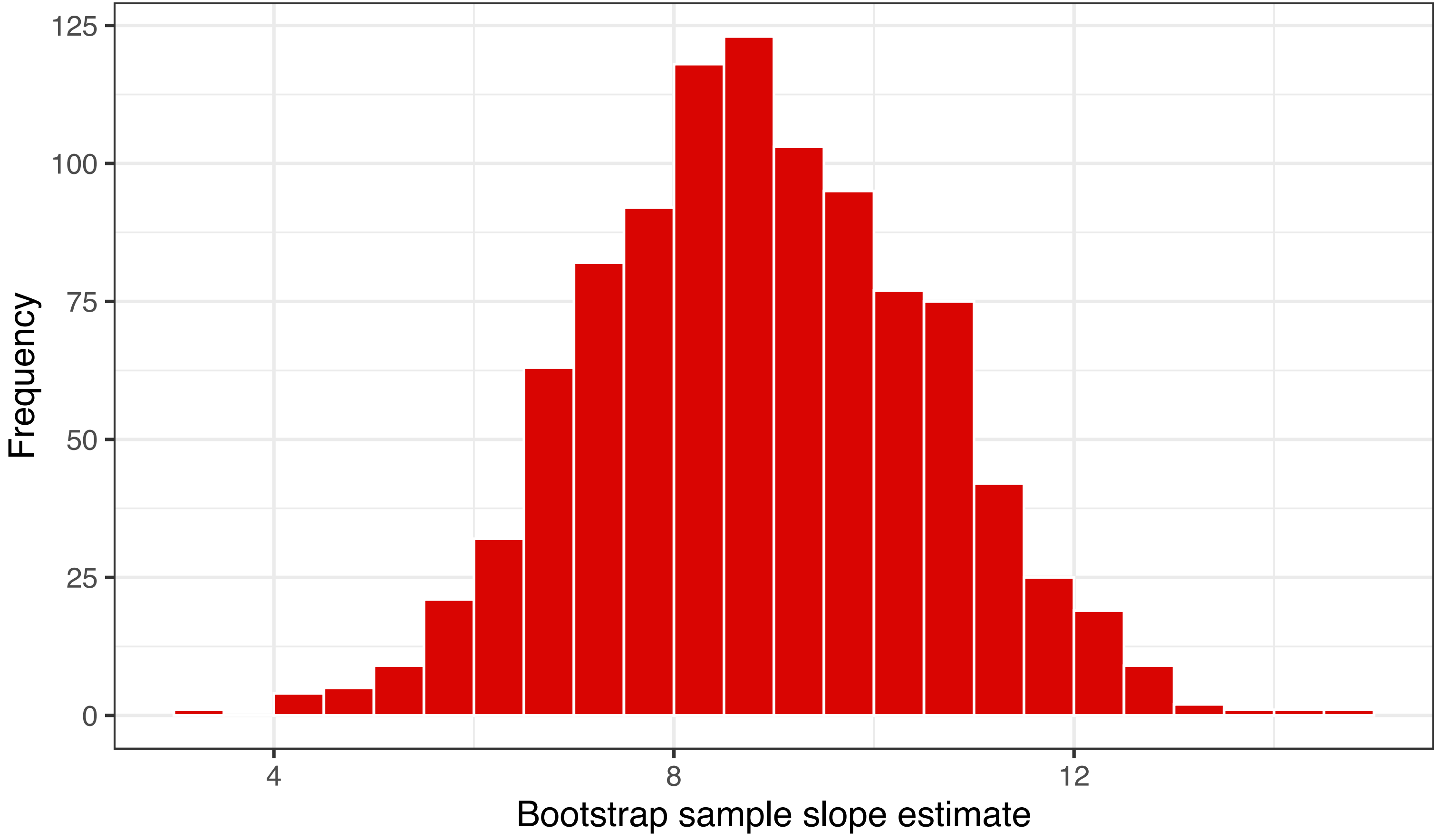

standard error: 1.66 → very close to the one in the table (1.68)!

Testing $\beta_k = 0$ vs $\beta_k \neq 0$

By default, the regression output provides the results associated with the following hypothesis test:

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- ***Important:*** This is a ***two-sided*** test!

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- The **p-value** measures the area outside of $\pm$ *observed test statistic* under the *null distribution*.
- Finally, we check if we can reject H_0 at the usual **significance levels**: α = 0.1, 0.05, 0.01.

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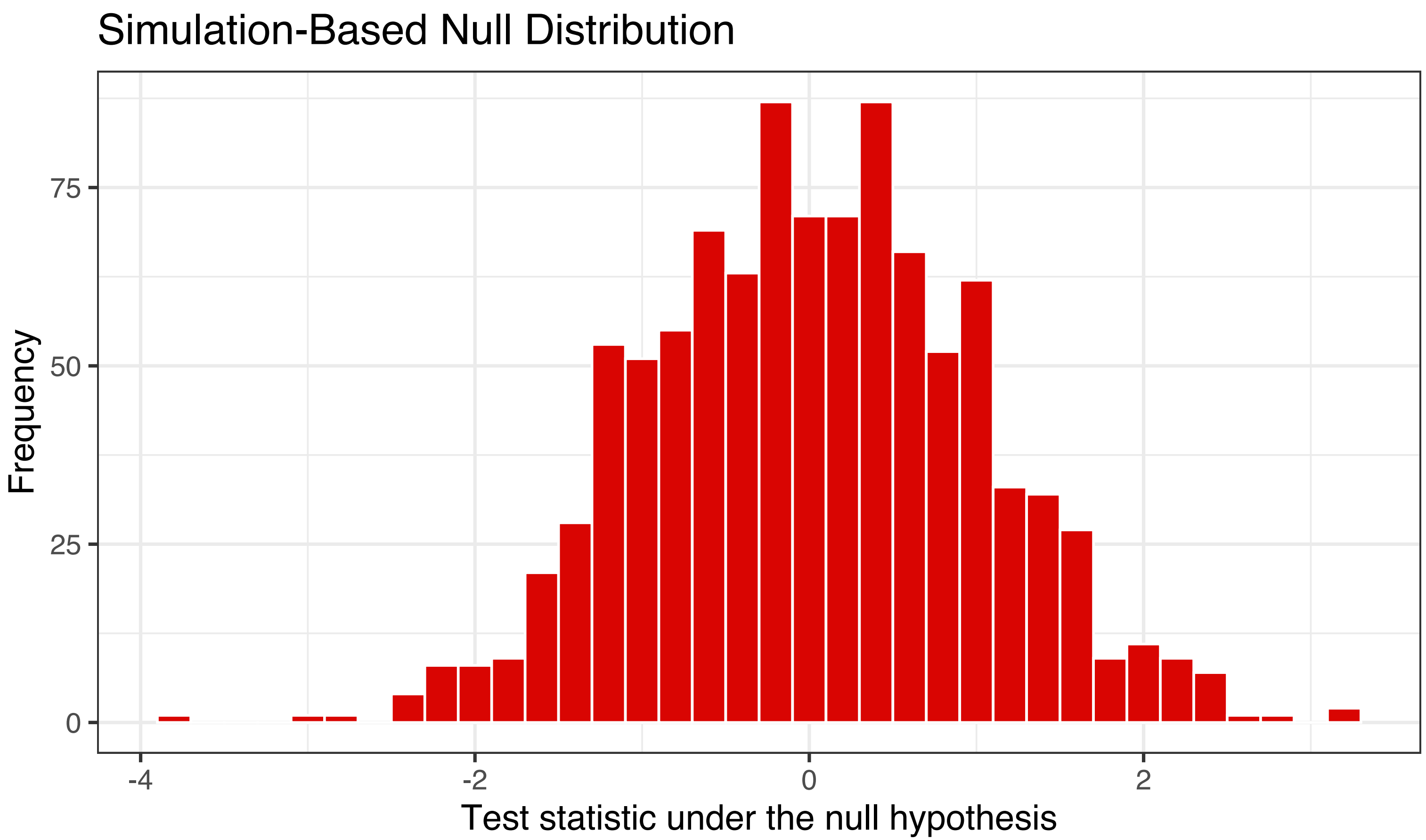
- We can compute the distribution of our test statistic $\frac{b_{\text{small}}}{\hat{SE}(b_{\text{small}})}$ under the null:

```
null_distribution <- null_distribution %>%
  mutate(test_stat = stat/sd(bootstrap_distrib$stat))
```

- Remember we got $\hat{SE}(b_{\text{small}})$ = 1.66 from our bootstrap distribution.

Testing $\beta_{\text{small}} = 0$ vs $\beta_{\text{small}} \neq 0$



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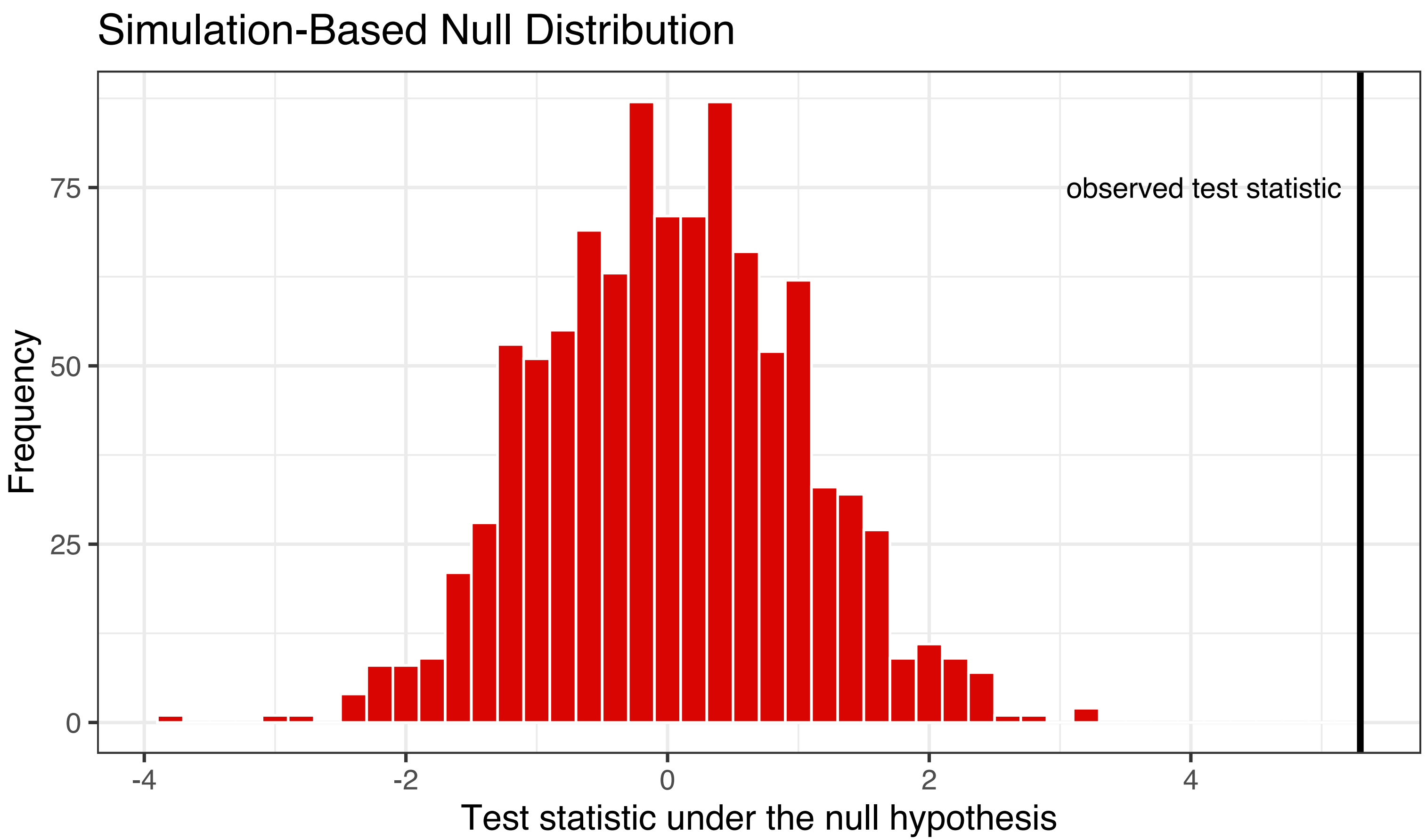

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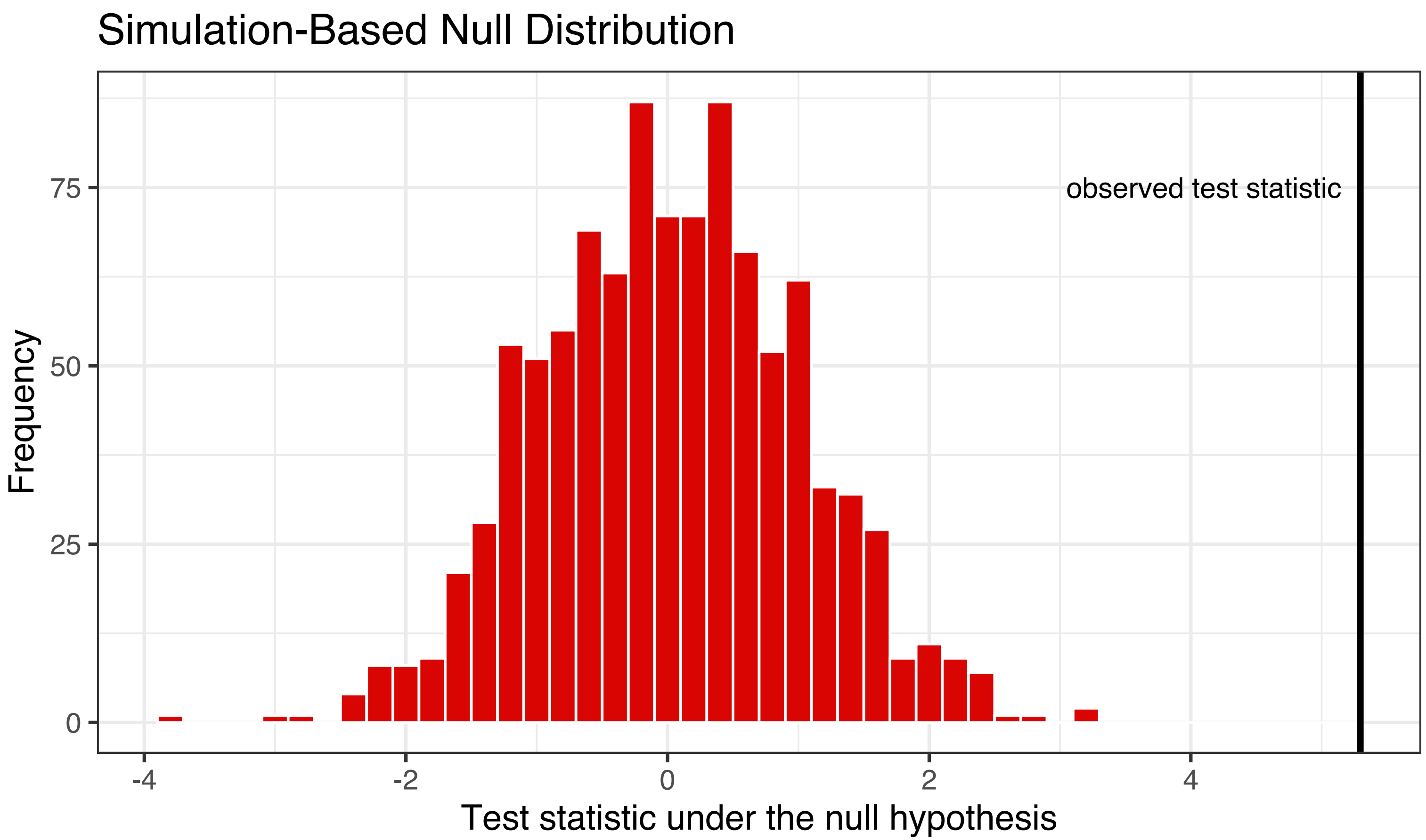

Very unlikely to obtain b_{small} = 8.8951932 when H_0 is true.

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- To decide if we reject H_0, recall we are considering a **two-sided test** here: *more extreme* means inferior to -5.351 **or** superior to 5.351.

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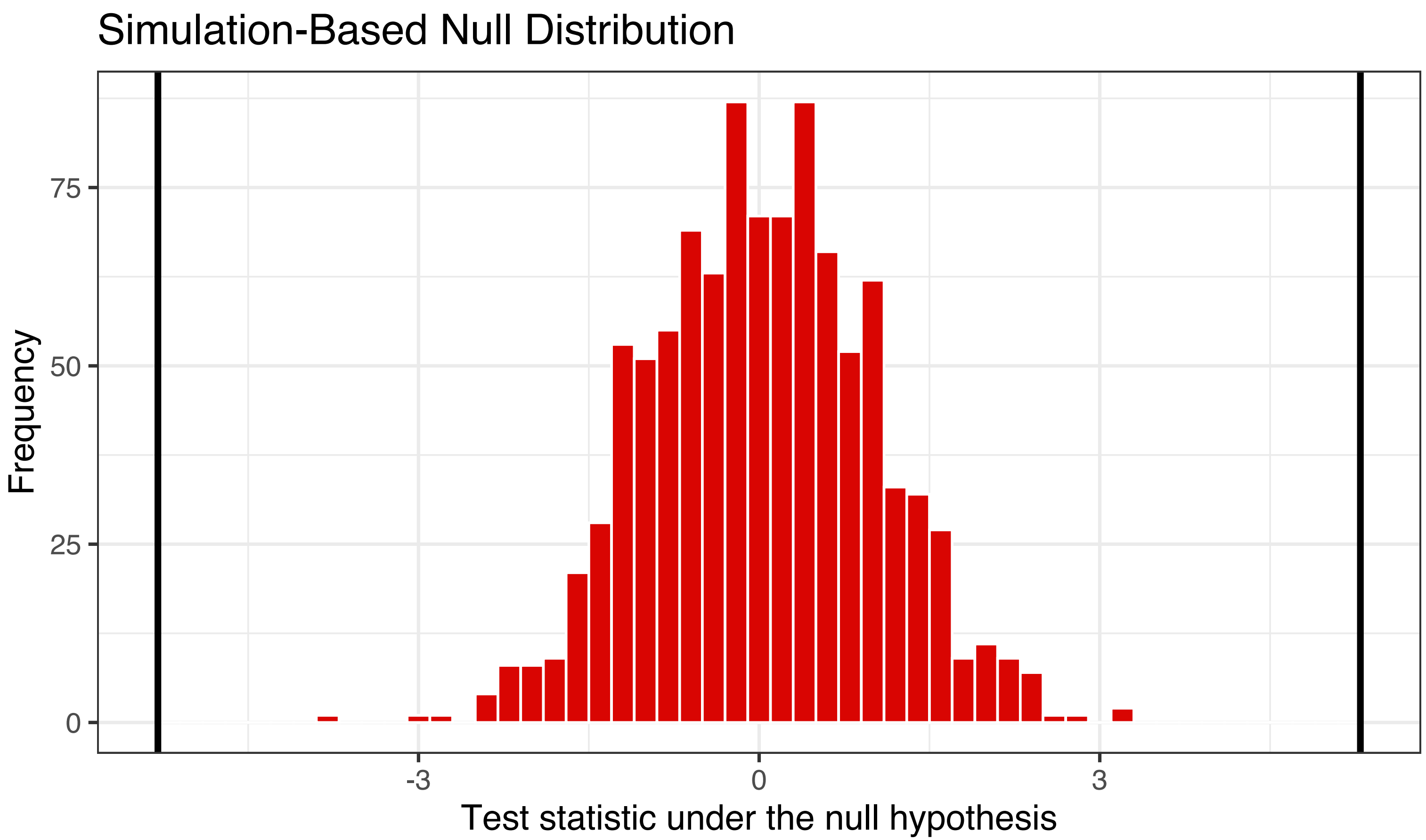

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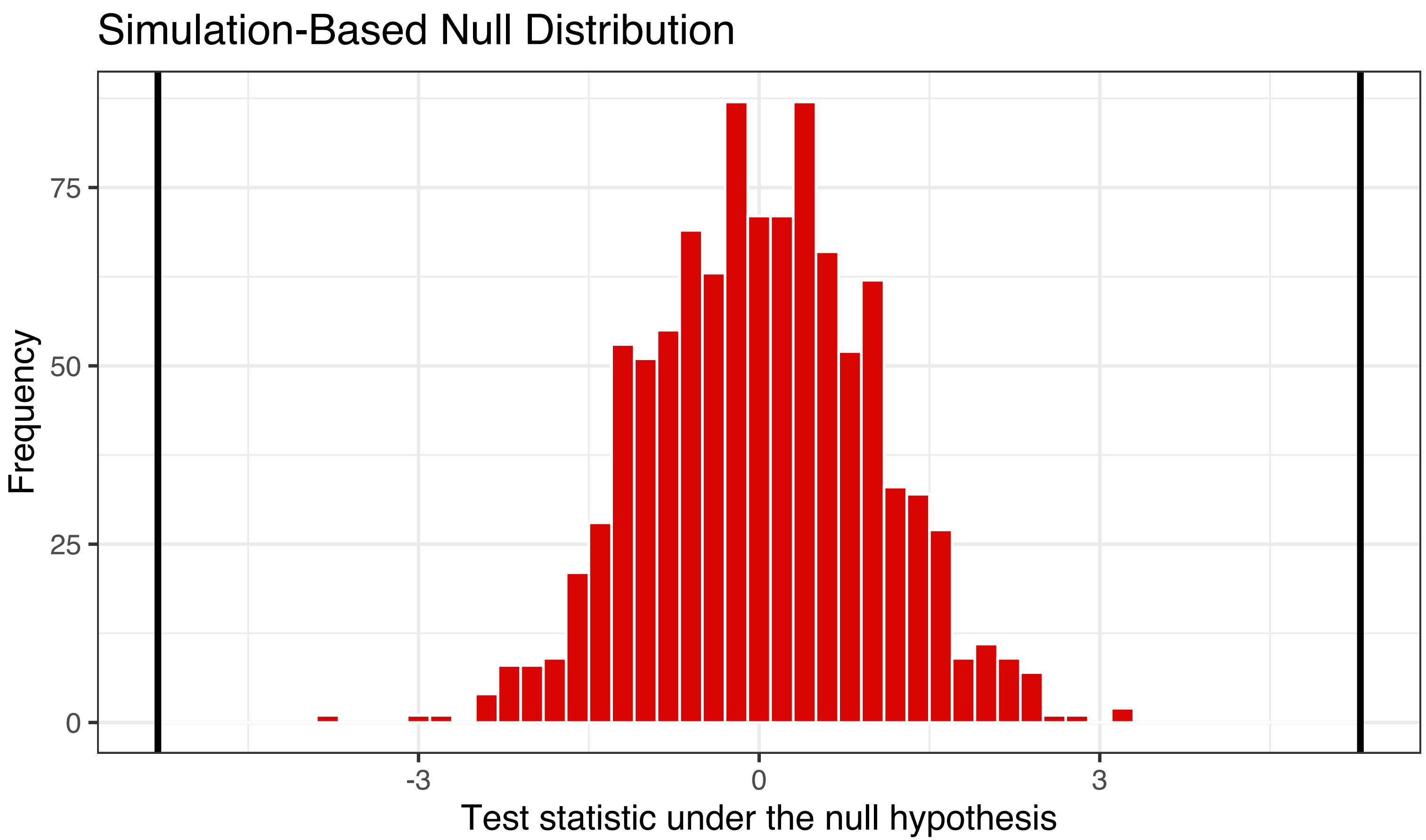

What does the p-value correspond to?

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- ***Question:*** Can we reject the null hypothesis at the 5% level?

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- *Answer:*

  - Since the *p-value* is equal to 0 it means that we would reject H_0 at any significance level: the p-value would always be inferior to α.
  - In other words, we can say that b_{small} is **statistically different from 0** at any significance level.
  - We also say that b_{small} is *statistically significant* (at any significance level).

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- Theoretical inference is based on **large sample approximations**.
  - One can show that sampling distributions *converge* to suitable distributions $\rightarrow$ *Central Limit Theorem*
- Let's briefly look into the theory-based approach.

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- Theory-based approach uses one fundamental result: the sampling distribution of the sample statistic $\frac{b-\beta}{\hat{\text{SE}}(b)}$ *converges* to a ***standard normal distribution*** as the sample size gets larger and larger.

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- We don't need to simulate any sampling distribution here, we derive it from theory and use it to construct confidence intervals or to conduct hypothesis tests.

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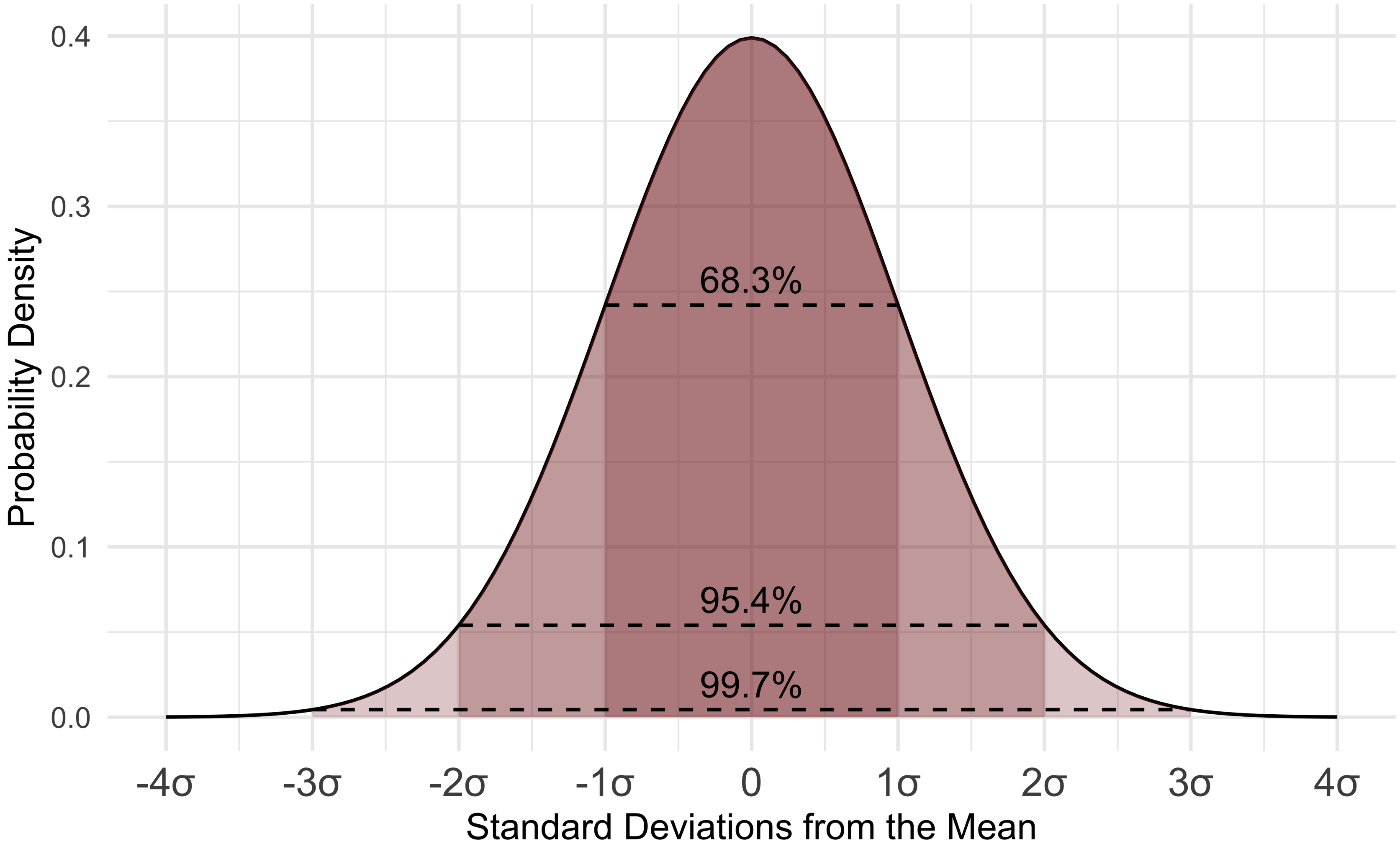
    - It is also obtained through a theoretical formula (which you can find in the book!) but we'll leave it aside.

- A ***standard normal distribution*** is a *normal distribution* with *mean* 0 and *standard deviation* 1.

- We don't need to simulate any sampling distribution here, we derive it from theory and use it to construct confidence intervals or to conduct hypothesis tests.

- Note that if $\frac{b-\beta}{\hat{\text{SE}}(b)}$ *converges* to a ***standard normal distribution***, then b converges to a ***normal distribution*** with mean β and standard deviation $\hat{\text{SE}}(b)$.

Normal Distribution: A Refresher



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tidy(lm(math ~ small, star_df),
     conf.int = TRUE, conf.level = 0.95) %>%
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  select(term, conf.low, conf.high)
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- This can easily be generalized to any confidence level by taking the appropriate quantile of the normal distribution.

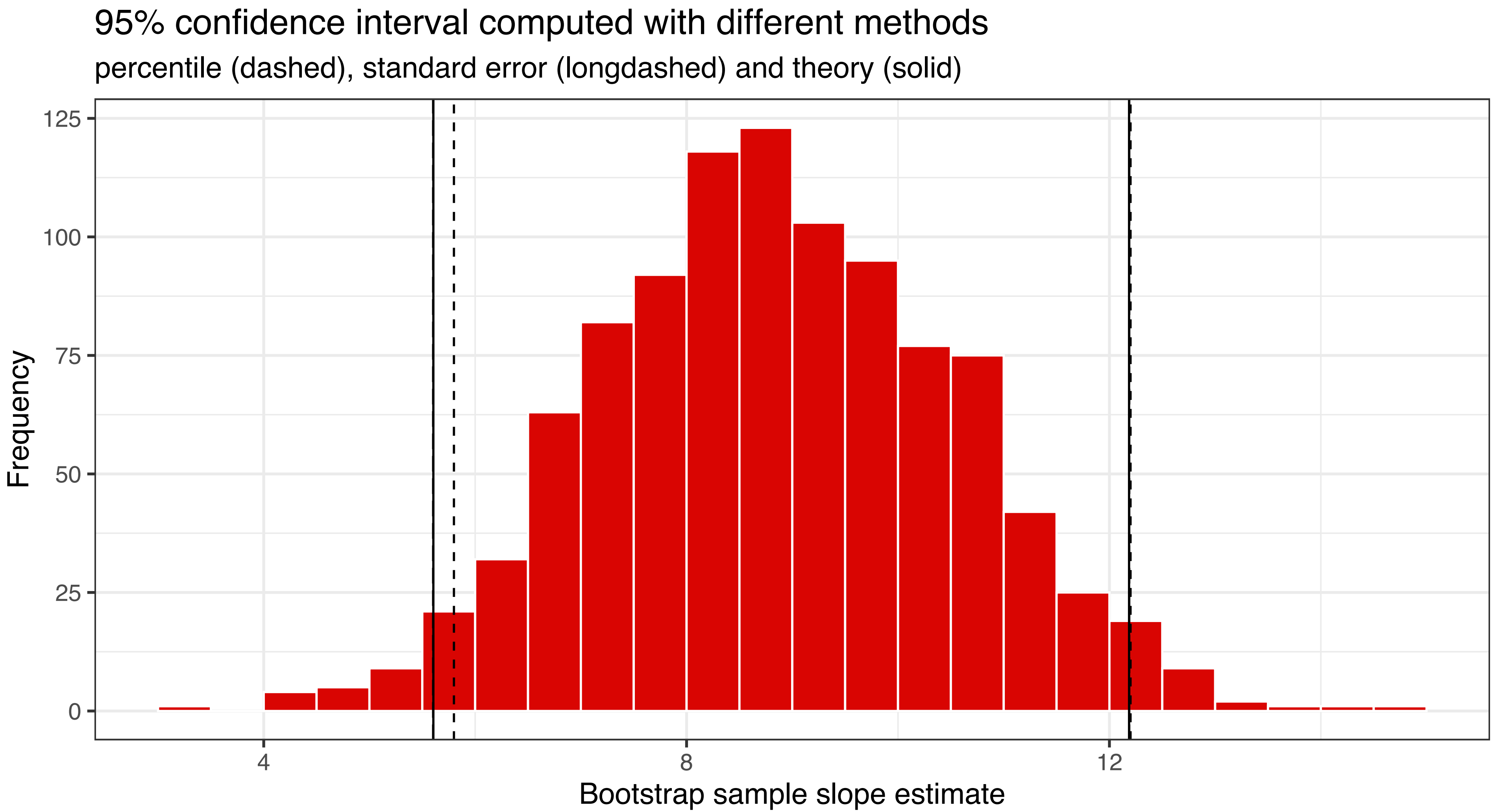
Task 2

05:00

1. Using the bootstrap distribution you generated in Task 1, compute the 95% confidence interval using the *percentile method*.

2. How similar is it to the confidence intervals obtained in the previous slide?

Confidence Intervals: Visually



Theory-Based Inference: Hypothesis Testing

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- Common rule of thumb: if the *estimate* is ***twice the size of the standard error***, then it is significant at the 5% level. Why?

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    - The classical regression model applies to **correctly specified linear regressions**: the model needs to be linear in parameters, include all relevant variables, and variables cannot be collinear.

CRM Assumptions

1. ***Mean Independence:*** the mean of the residuals conditional on x should be zero, $E[\varepsilon|x] = 0$. Notice that this also means that $Cov(\varepsilon, x) = 0$, i.e. that the errors and our explanatory variable(s) should be *uncorrelated*.

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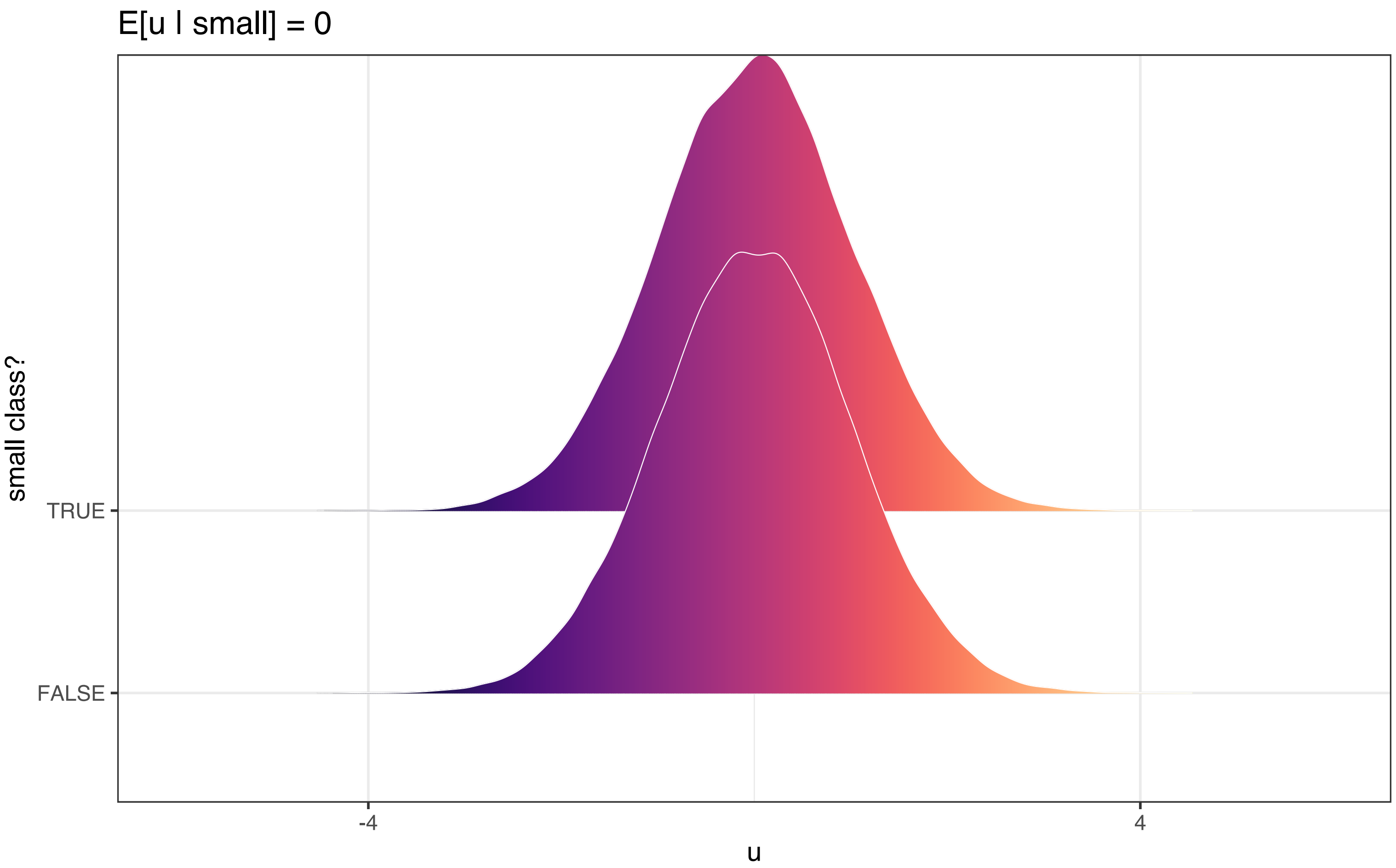

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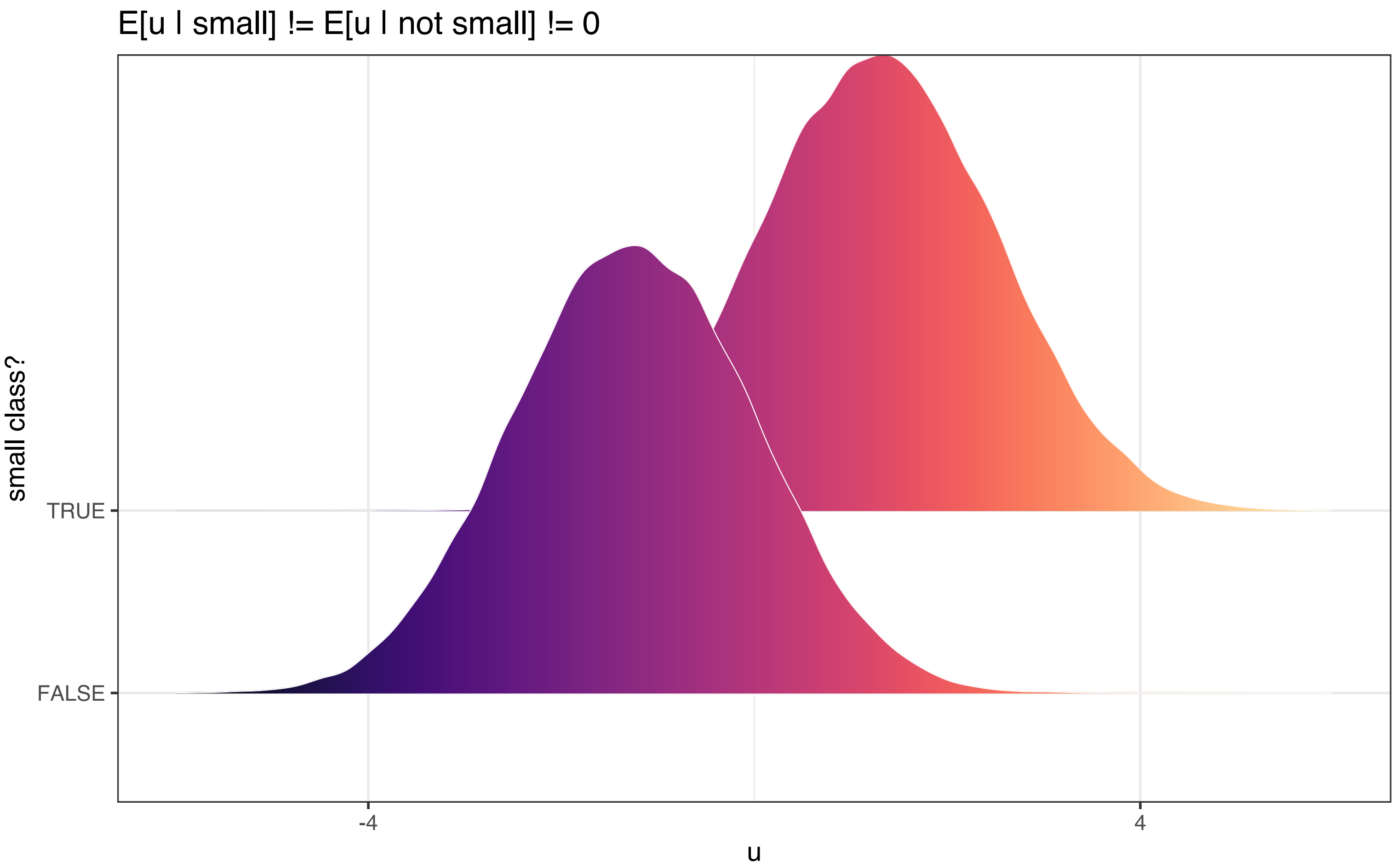

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- For example, imagine you are interested in the effect of education on wage

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  - Under the exogeneity assumption β_1 denotes the causal effect of education in the population.

- Suppose there is *unobserved* ability a_i.

  - High ability means higher wage.
  - It *also* means school is easier, and so i selects into more schooling.

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  - Thus, we have:

$$\mathbb{E}(b_1) = \beta_1 + OVB > \beta_1$$

- *Interpretation*: taking repeated sample from the population and computing b_1 each time, we would **systematically overestimate** the effect of education on wage.

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- Violating this assumption would make your sample less representative of the underlying population. It will lead to **biased** estimates of β_k.

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☞ Takeaway: **if assumptions violated, inference is invalid!**

Task 3.1

10:00

Let's go back to our question of returns to education and gender.

1. Load the data `CPS1985` from the `AER` package and look back at the `help` to get the definition of each variable: `?CPS1985`

2. Create the `log_wage` variable equal to the log of `wage`.

3. Regress `log_wage` on `gender` and `education`, and save it as `reg1`.

   - Interpret each coefficient.

   - Are the coefficients statistically significant? At which significance level?

4. Regress the `log_wage` on `gender`, `education` and their interaction `gender*education`, save it as `reg2`.

   - How do you interpret the coefficient associated to $female * education$?

   - Can we reject the nullity of this coefficient at the 5% level? At 10%?

Task 3.2

10:00

1. Produce a scatterplot of the relationship between the log wage and the level of education.

2. Add the *regression line* with `geom_smooth`. What does this line represents?

3. Let's illustrate what the shaded area stands for.

    1. Draw one bootstrap sample from our `cps` data.

    2. Regress the `log_wage` on `gender`, `education` and their interaction `gender*education`, save it as `reg_bootstrap`.

    3. From `reg_bootstrap` extract and save the value of the intercept for men as `intercept_men_bootstrap` and the value of the slope for men as `slope_men_bootstrap`. Do the same for women.

    4. Add both predicted lines from this bootstrap sample to the previous plot (*Hint*: use `geom_abline` (x2))

Illustrating Uncertainty

Let's repeat the procedure you just made 100 times!

```
library(AER)
data("CPS1985")
cps = CPS1985 %>% mutate(log_wage = log(wage))

set.seed(1)
bootstrap_sample = cps %>%
    rep_sample_n(size = nrow(cps), reps = 100, replace

ggplot(data=cps,aes(y = log_wage, x = education, colou
  geom_point(size = 1, alpha = 0.7) +
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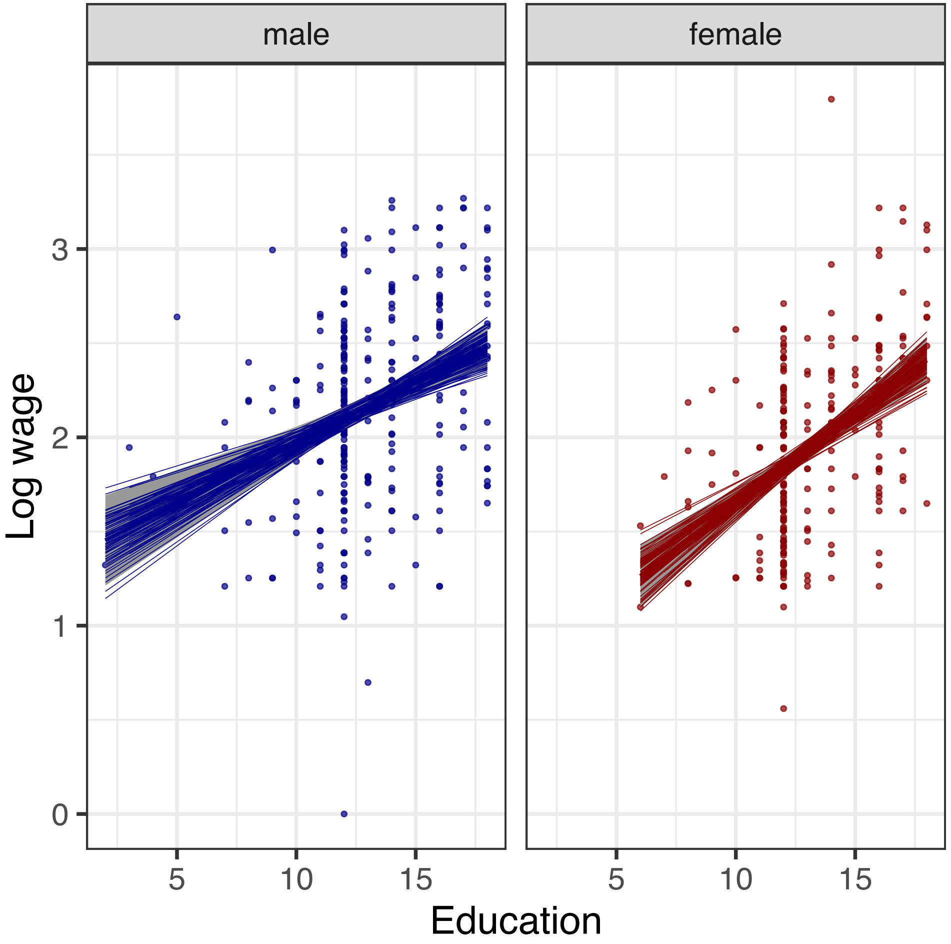

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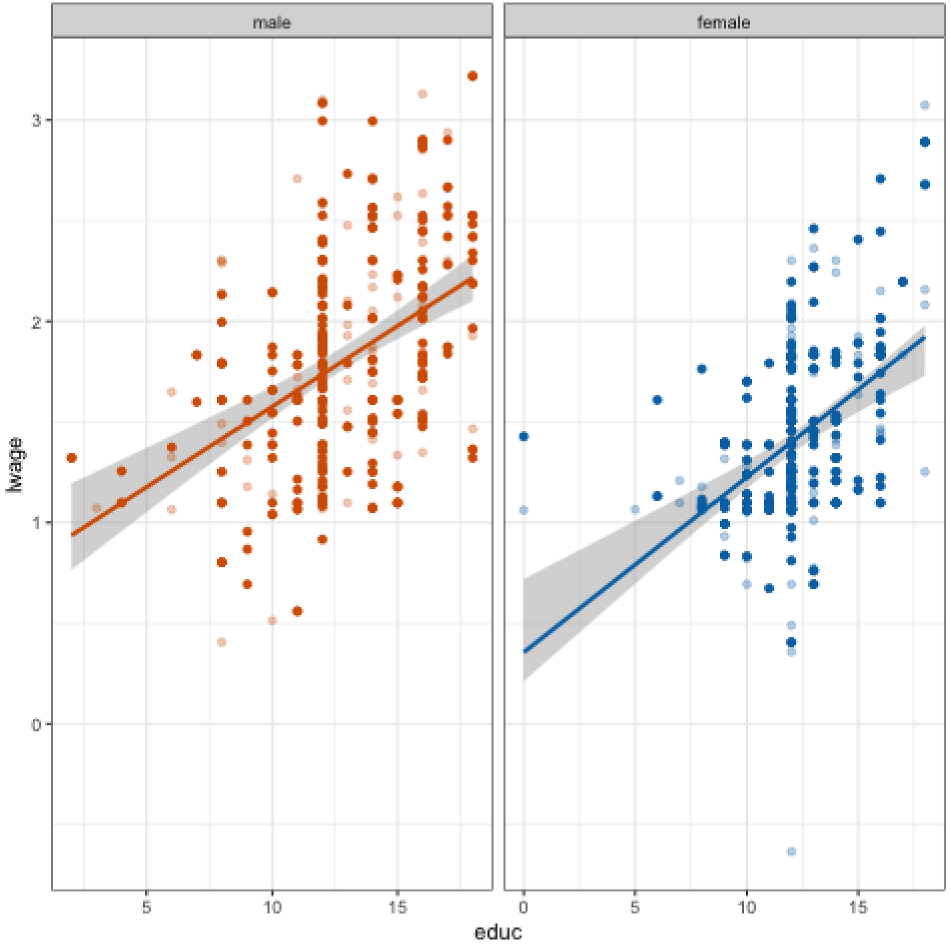

Even better : `ungeviz` and `gganimate` bring you moving lines!

- We took 20 bootstrap samples from our data
- You can see how different data points are included in each bootstrap sample.
- Those different points imply different regression lines.
- On average, 95% of these lines should fall into the shaded area.
- You should remember those moving lines when looking at the shaded area!

THANKS

To the amazing moderndive team!

Big Thanks to ungeviz and gganimate for their awesome packages!

SciencesPo
DEPARTMENT OF ECONOMICS

SEE YOU NEXT WEEK!

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	Slides
	Book
	@ScPoEcon
	@ScPoEcon