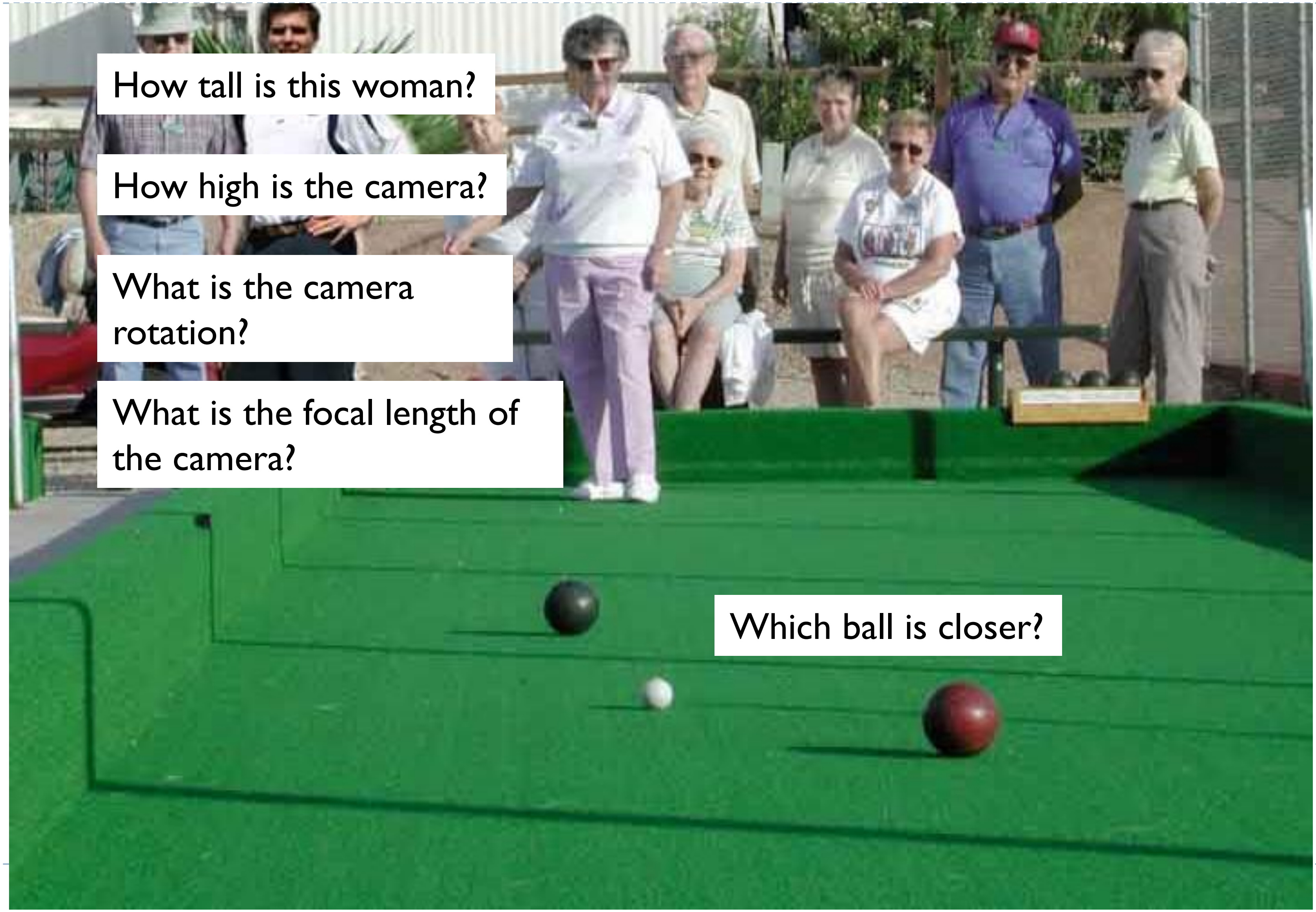
The pinhole camera

Lihi Zelnik-Manor, Computer Vision

Next two classes: Single-view Geometry



Today

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
- Other camera models

Today

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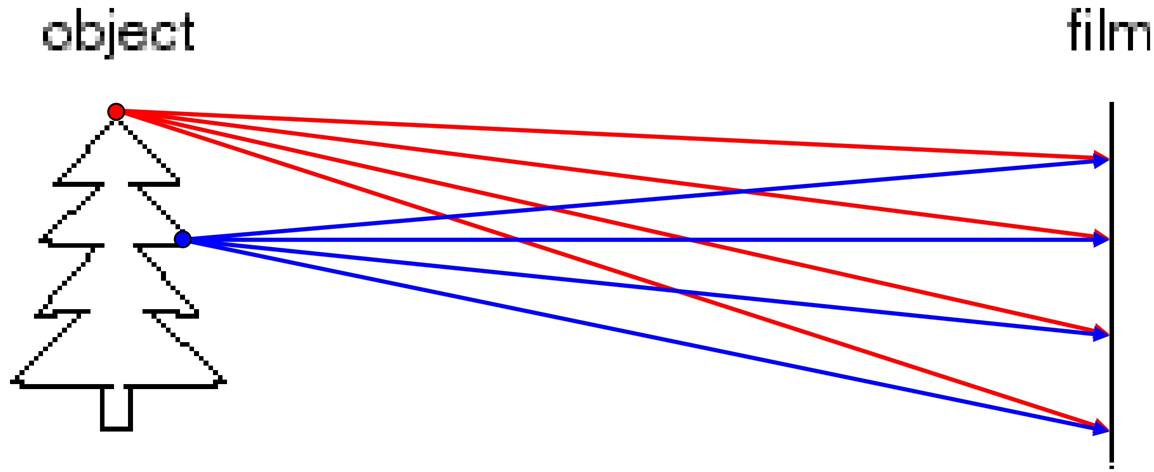
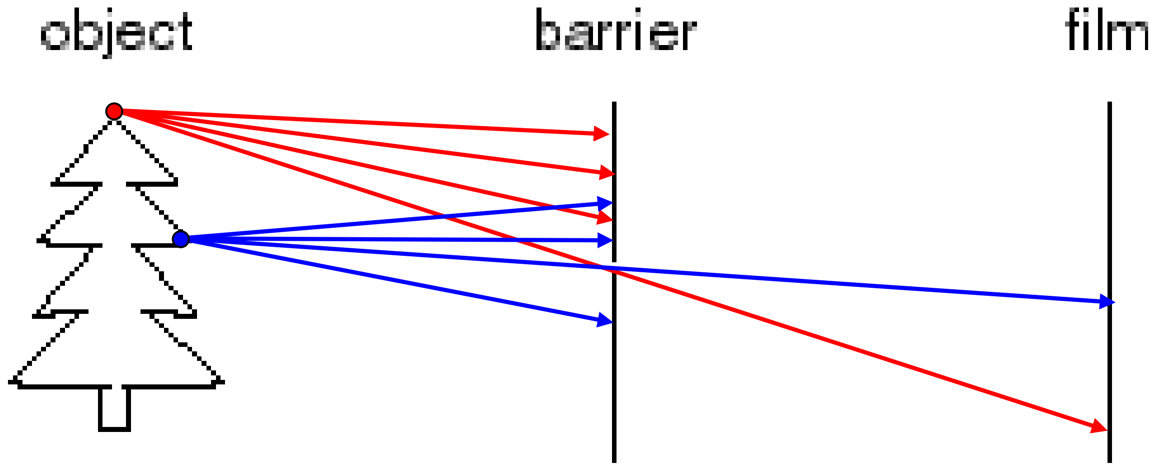
- Other camera models

How do we see the world?



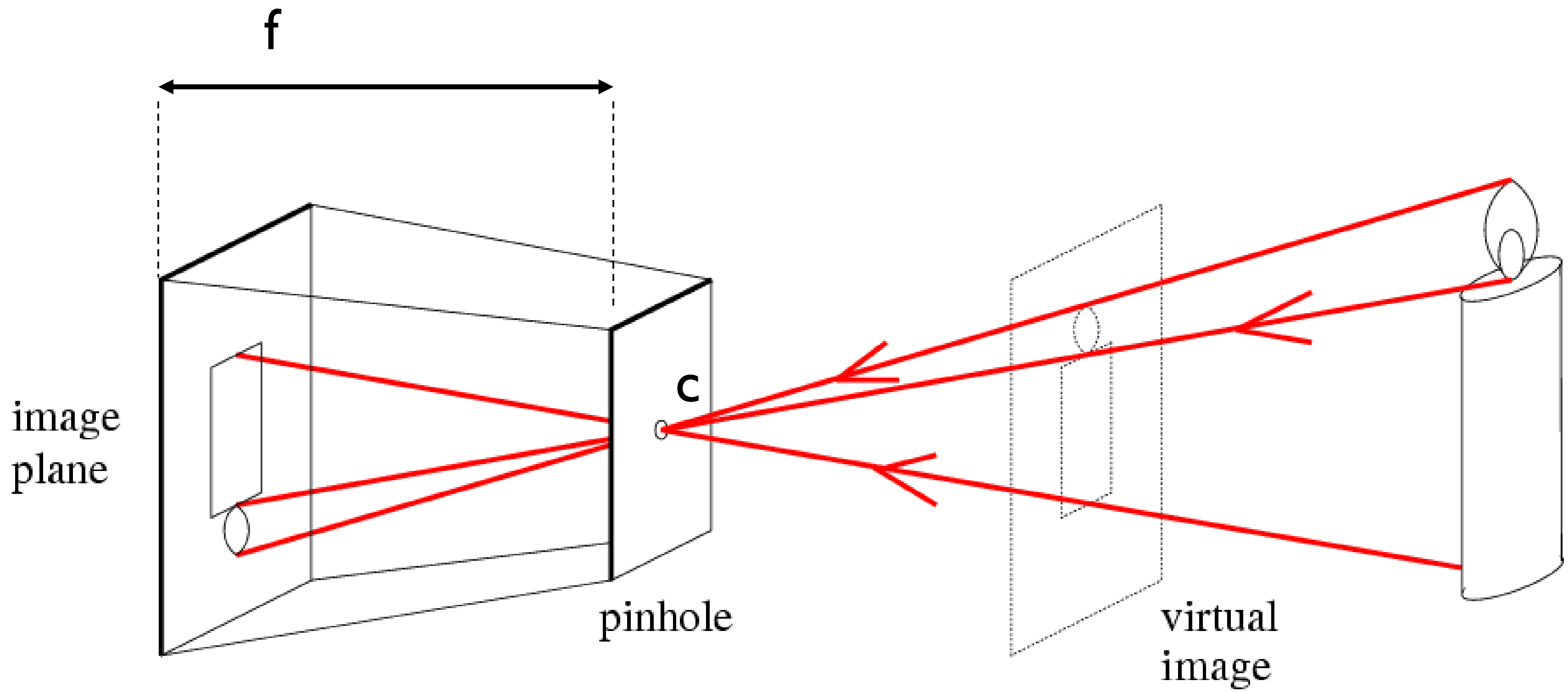
- Let's design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

Pinhole camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening is known as the **aperture**

Pinhole camera



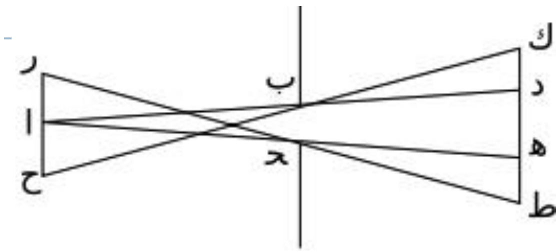
f = focal length

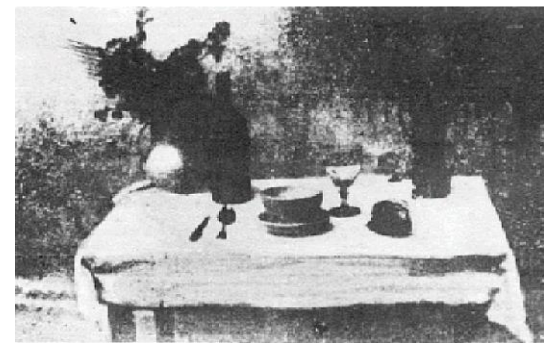
c = center of the camera

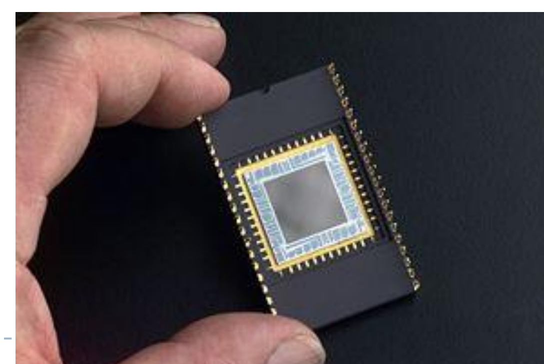
Figure from Forsyth

Historical context

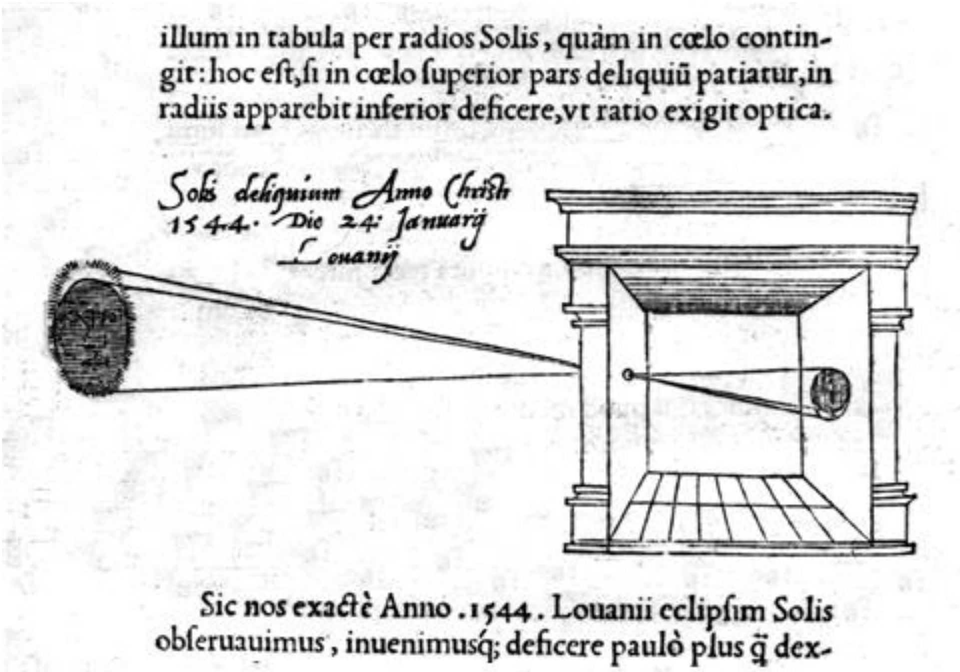
- **Pinhole model:** Mozi (470-390 BCE), Aristotle (384-322 BCE)
- **Principles of optics (including lenses):** Alhacen (965-1039 CE)
- **Camera obscura**: Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- **First photo**: Joseph Nicephore Niepce (1822)
- **Daguerréotypes** (1839)
- **Photographic film** (Eastman, 1889)
- **Cinema** (Lumière Brothers, 1895)
- **Color Photography** (Lumière Brothers, 1908)
- **Television** (Baird, Farnsworth, Zworykin, 1920s)
- **First consumer camera with CCD**: Sony Mavica (1981)
- **First fully digital camera**: Kodak DCS100 (1990)


Alhacen's notes


Niepce, "La Table Servie," 1822


CCD chip

Camera obscura

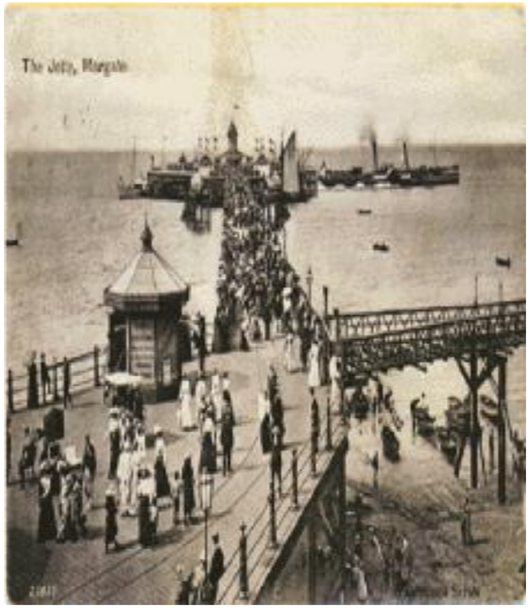

illum in tabula per radios Solis, quàm in cœlo contin-
git: hoc eſt, ſi in cœlo ſuperior pars deliquiũ patiatur, in
radiis apparebit inferior deficere, vt ratio exigit optica.

Sic nos exactè Anno .1544. Louanii eclipſim Solis
obſeruauimus, inuenimusq; deficere paulò plus q̃ dex-

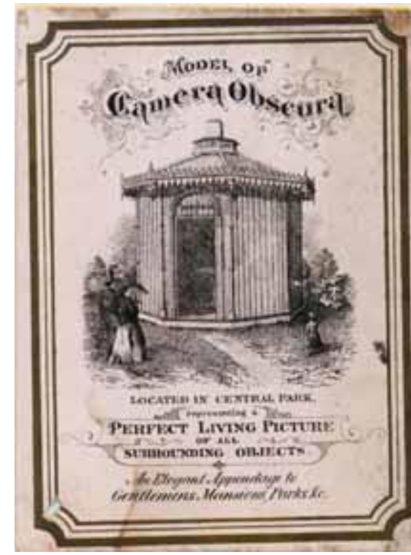
In Latin, means 'dark room'

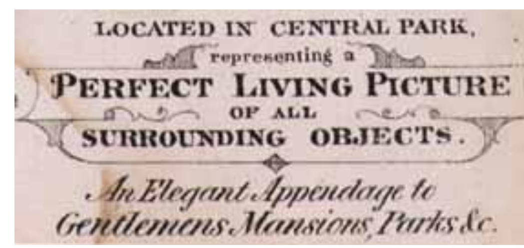
"**Reinerus Gemma-Frisius**, observed an eclipse of the sun at Louvain on January 24, 1544, and later he used this illustration of the event in his book De Radio Astronomica et Geometrica, 1545. It is thought to be the first published illustration of a camera obscura..." Hammond, John H., The Camera Obscura, A Chronicle

Camera obscura



Jetty at Margate England, 1898.





Around 1870s

An attraction in the late 19th century

Camera obscura at home

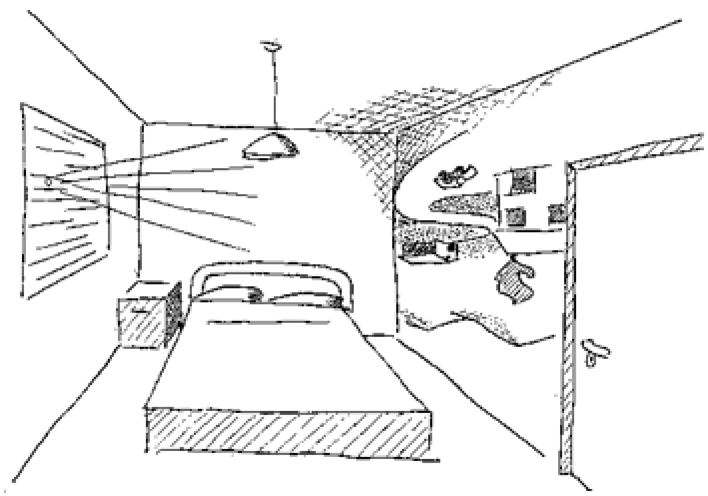


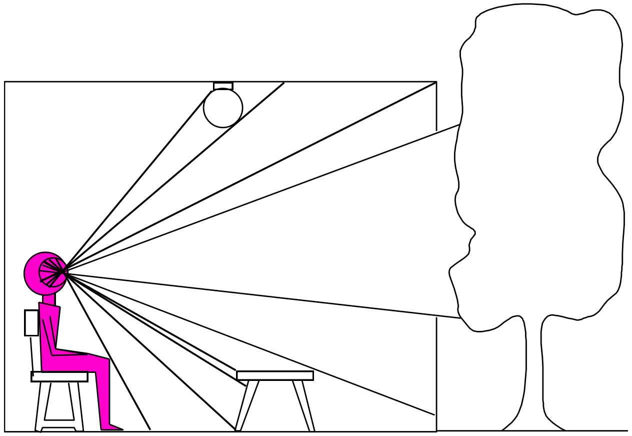
Figure 1 - A lens on the window creates the image of the external world on the opposite wall and you can see it every morning, when you wake up.

http://blog.makezine.com/archive/2006/02/how_to_room_sized_camera_obscu.html

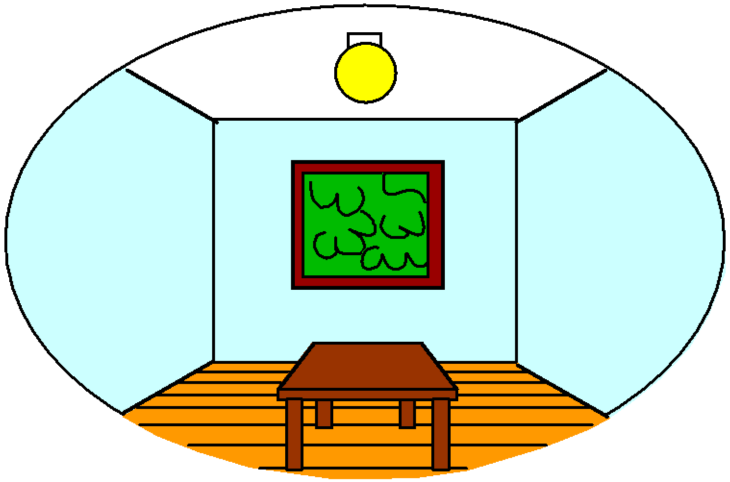
Dimensionality Reduction Machine (3D to 2D)

3D world

2D image



Point of observation



Slide source: Seitz

Projection can be tricky…

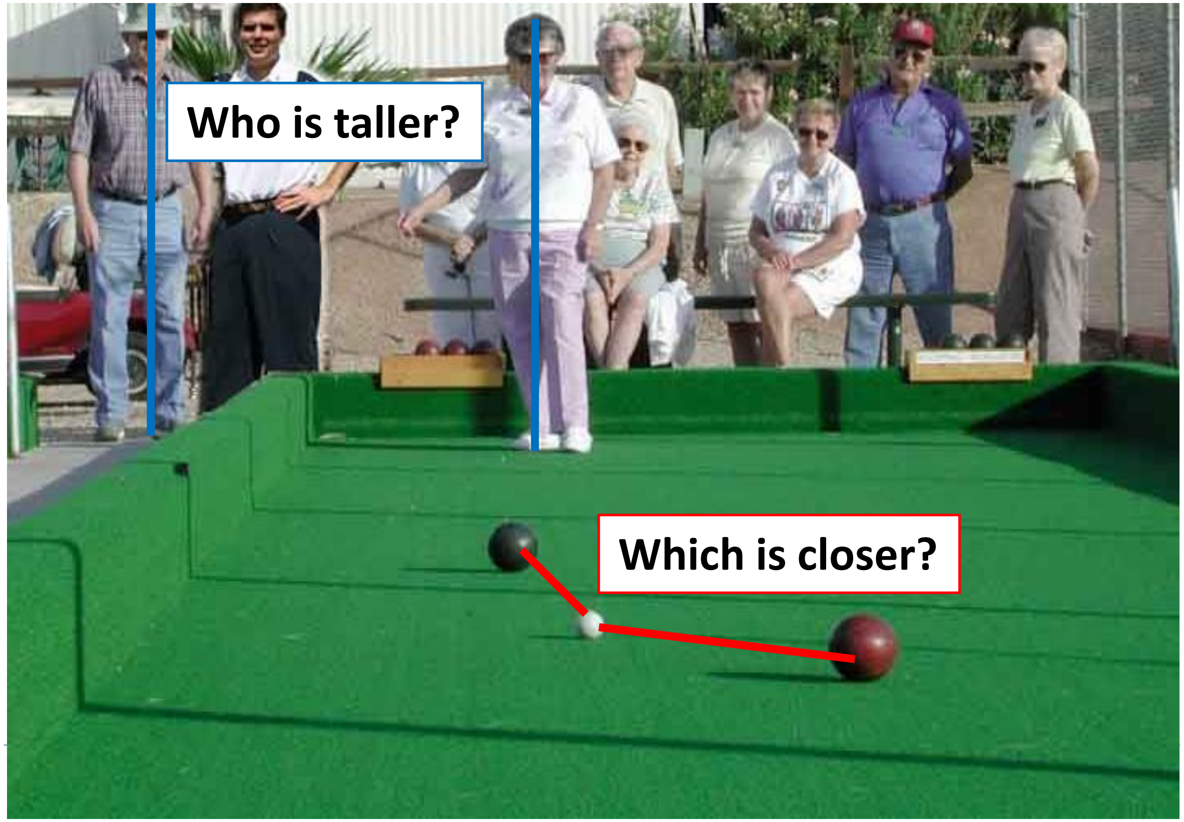
Slide source: Seitz

Projection can be tricky…

- Making of 3D sidewalk art: http://www.youtube.com/watch?v=3SNYtd0Ayt0

What is lost?

- Length



Length is not preserved

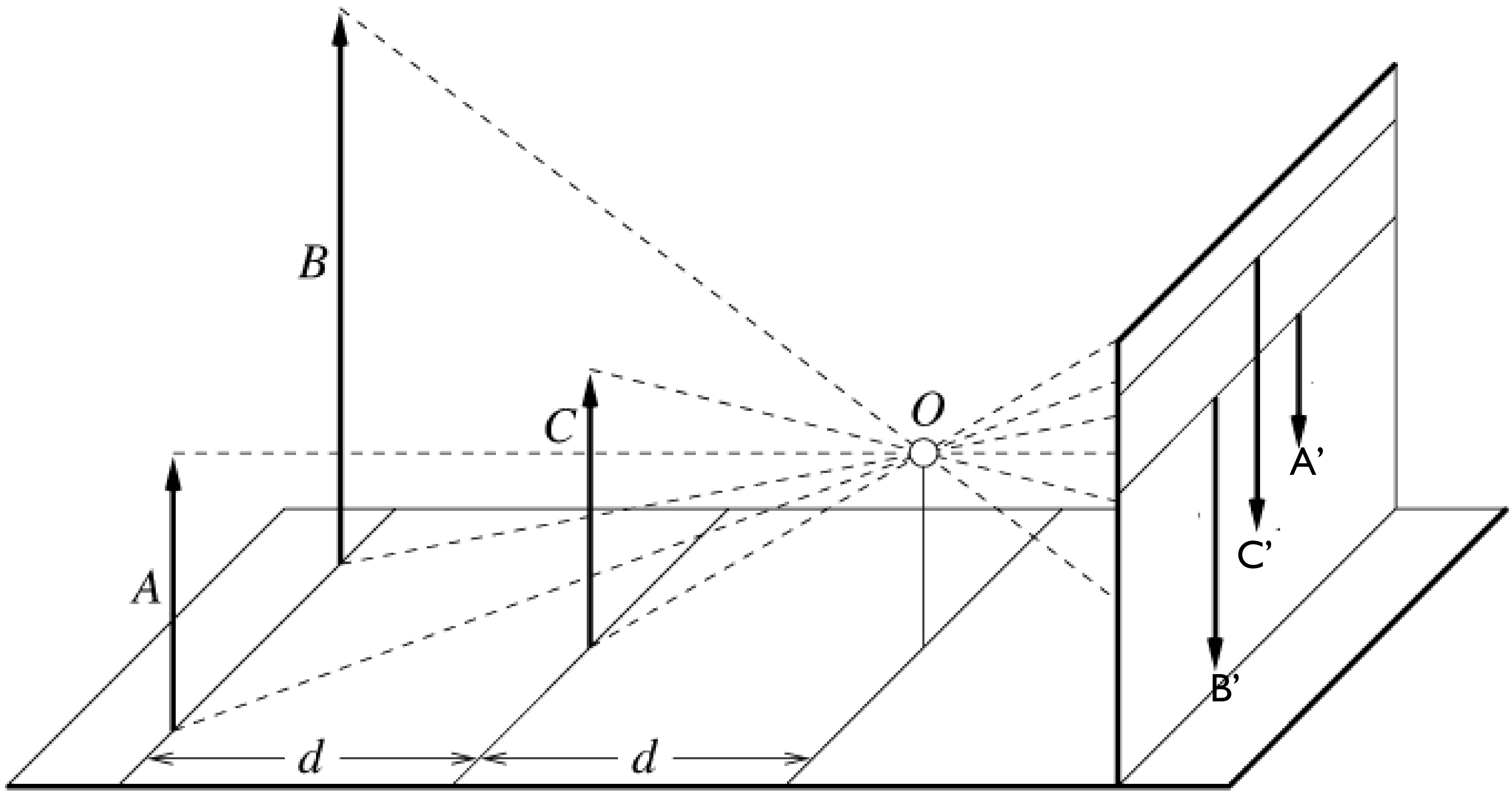
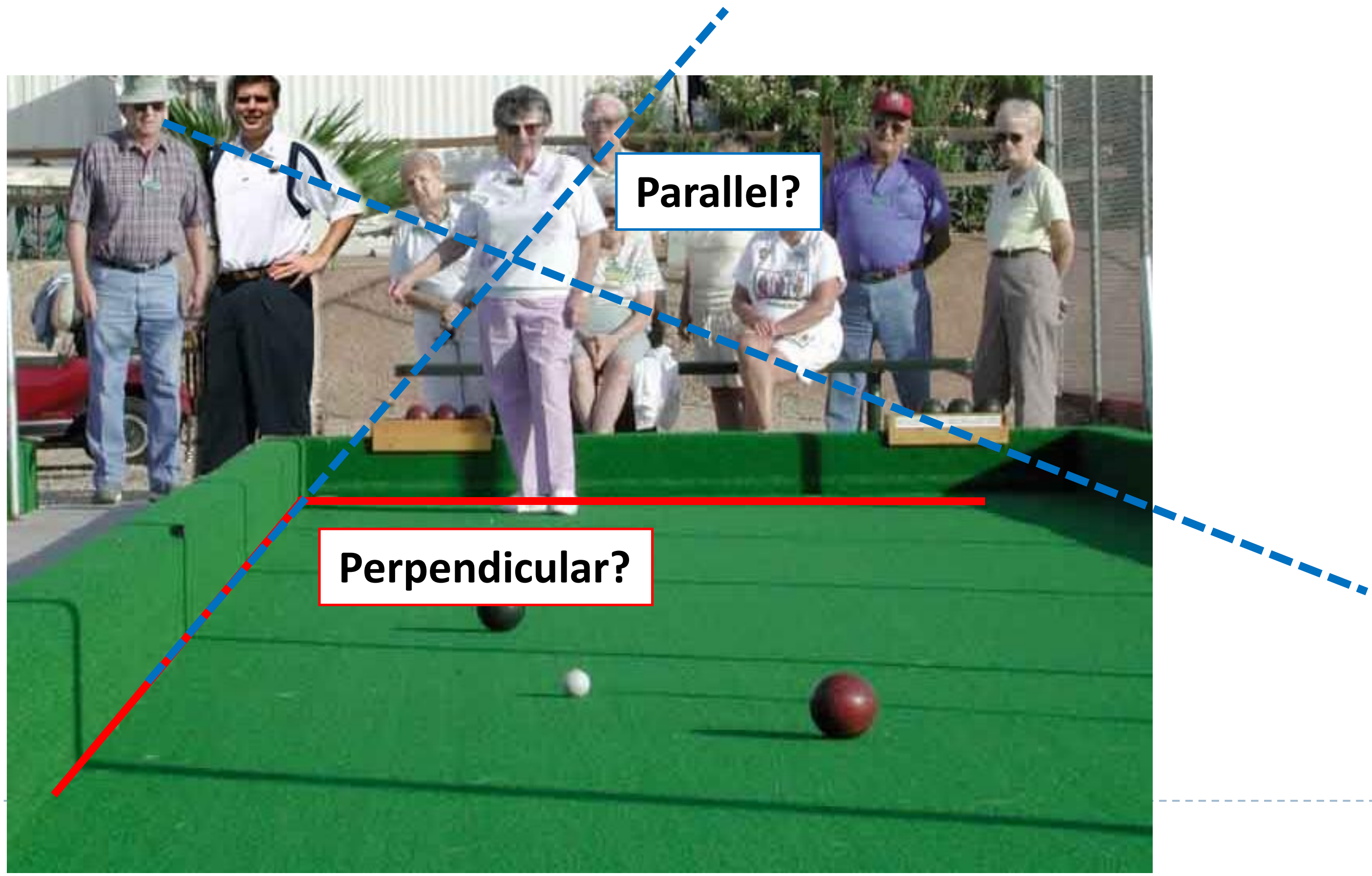


Figure by David Forsyth

Projective Geometry

What is lost?

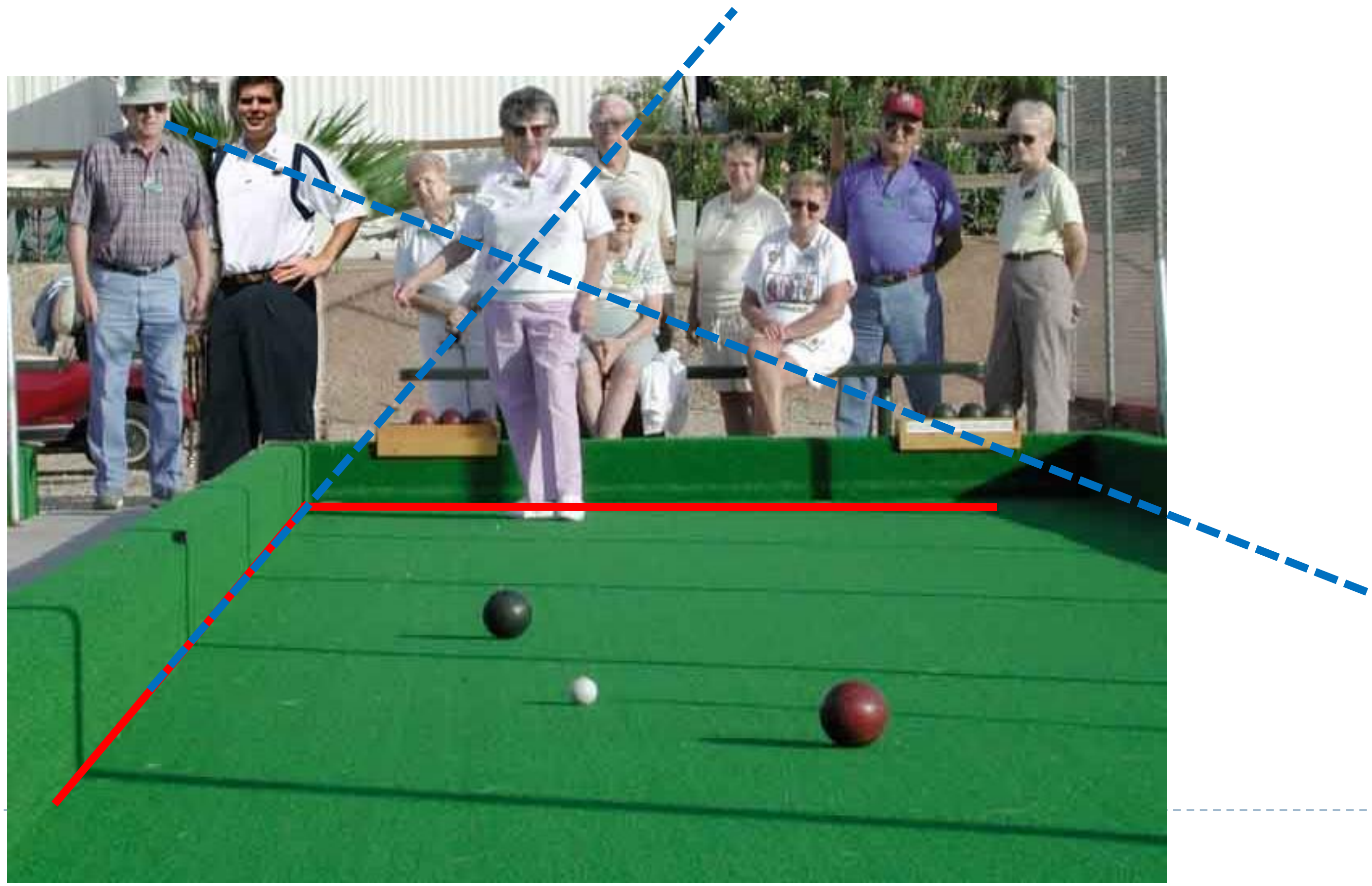
- Length
- Angles



Projective Geometry

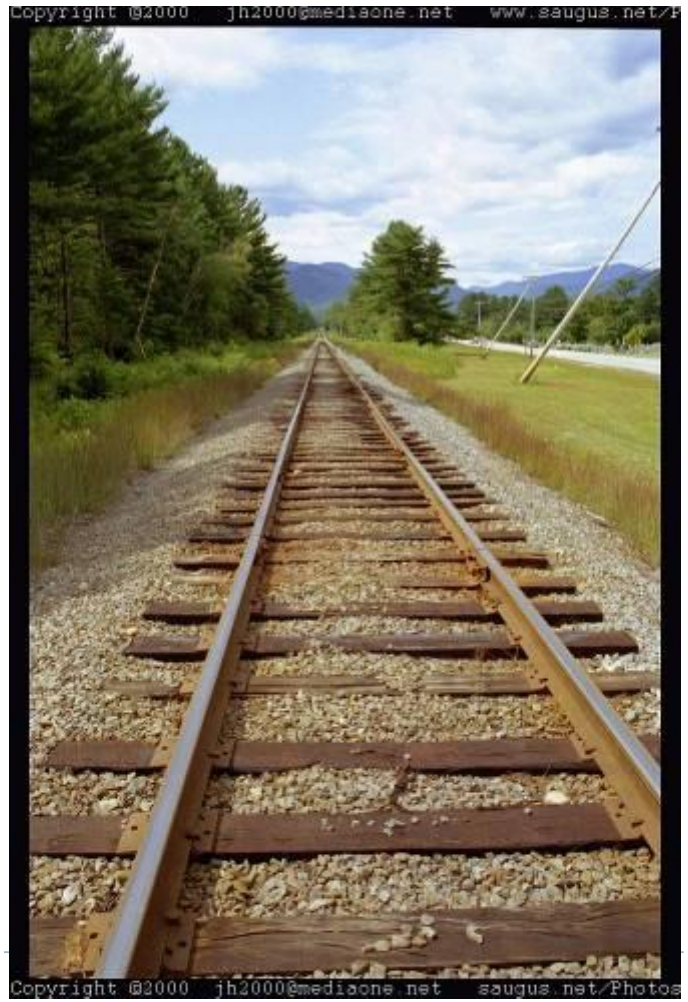
What is preserved?

- Straight lines are still straight

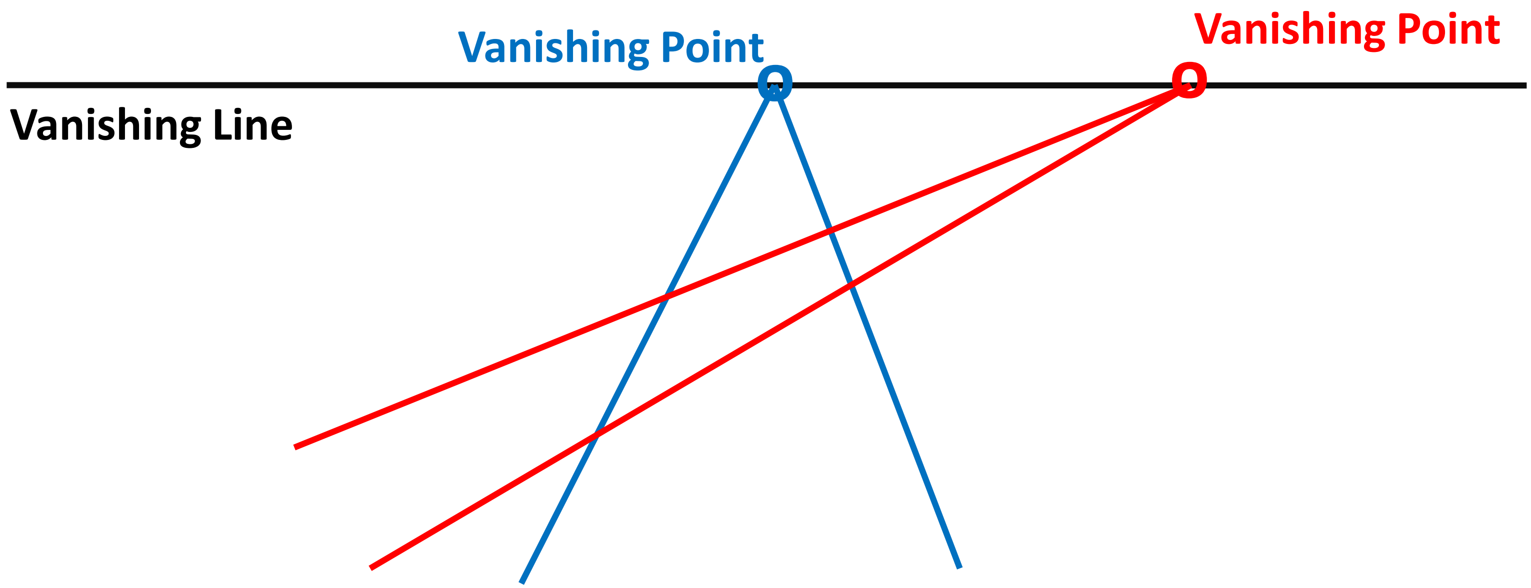


Vanishing points and lines

Parallel lines in the world intersect in the image at a "vanishing point"

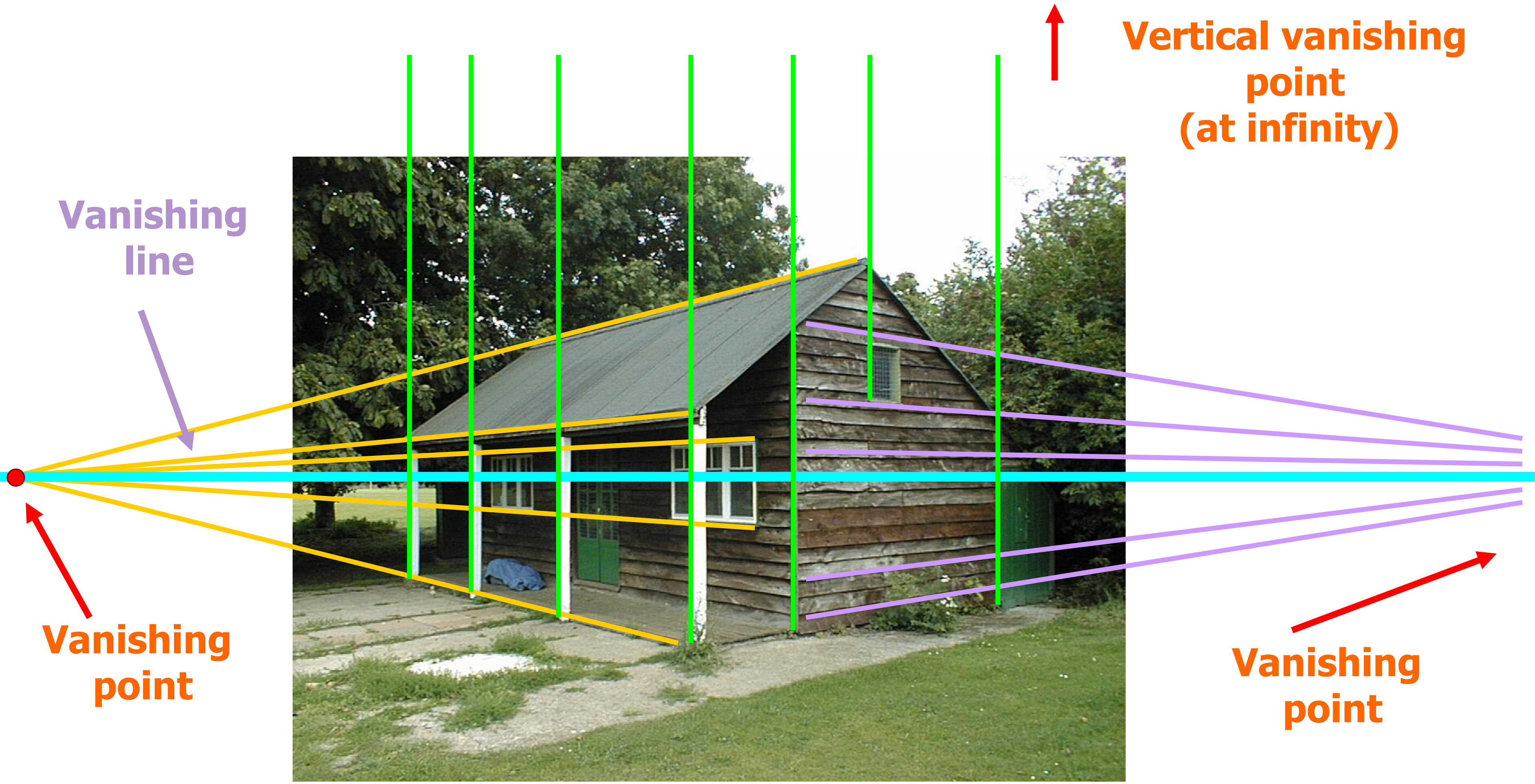


Vanishing points and lines



- The projections of parallel 3D lines intersect at a **vanishing point**
- The projection of parallel 3D planes intersect at a **vanishing line**
- If a set of parallel 3D lines are also parallel to a particular plane, their vanishing point will lie on the vanishing line of the plane
- Not all lines that intersect are parallel
- Vanishing point <-> 3D direction of a line
- Vanishing line <-> 3D orientation of a surface

Vanishing points and lines

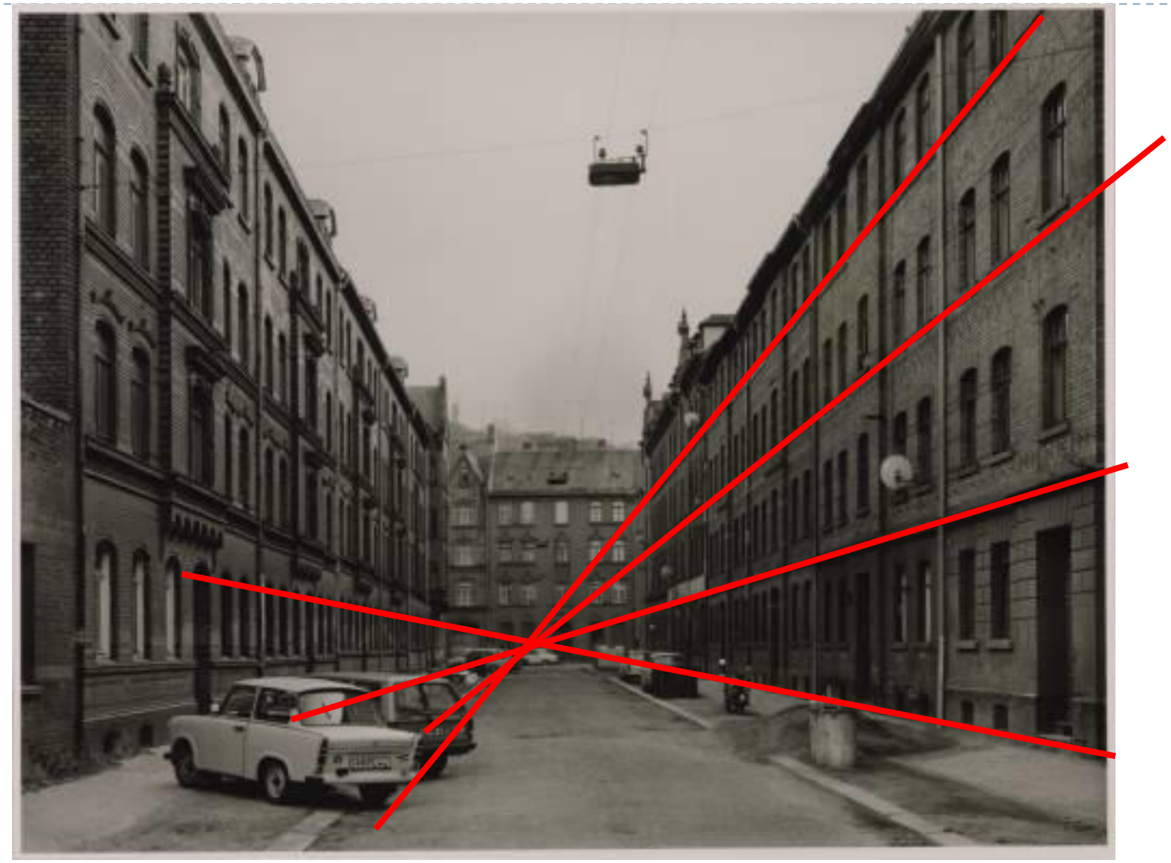


Slide from Efros, Photo from Criminisi

Vanishing points and lines

Photo from online Tate collection

Note on estimating vanishing points



Use multiple lines for better accuracy

… but lines will not intersect at exactly the same point in practice

One solution: take mean of intersecting pairs

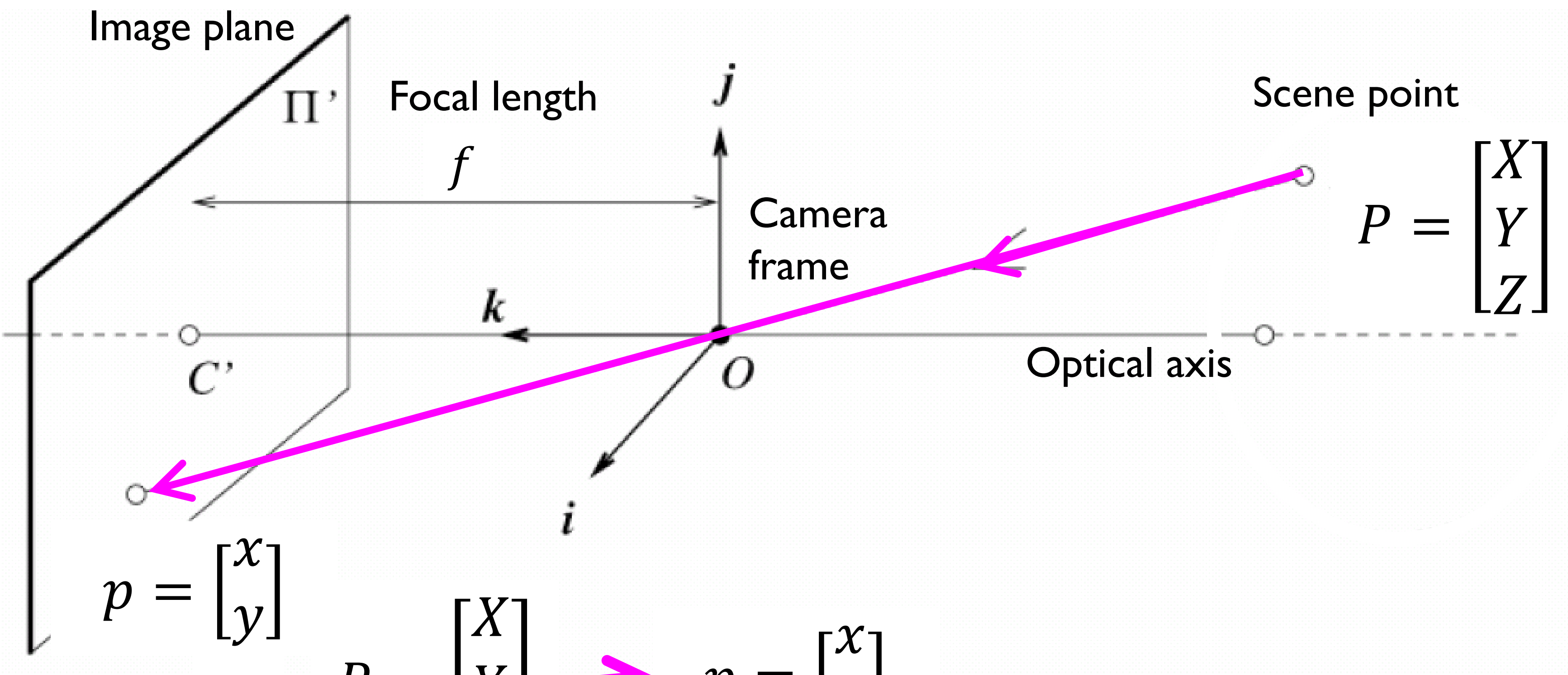
… bad idea!

- Instead, minimize angular differences

Vanishing objects

Projection equations of ideal Pinhole

- 3d world mapped to 2d projection in image plane



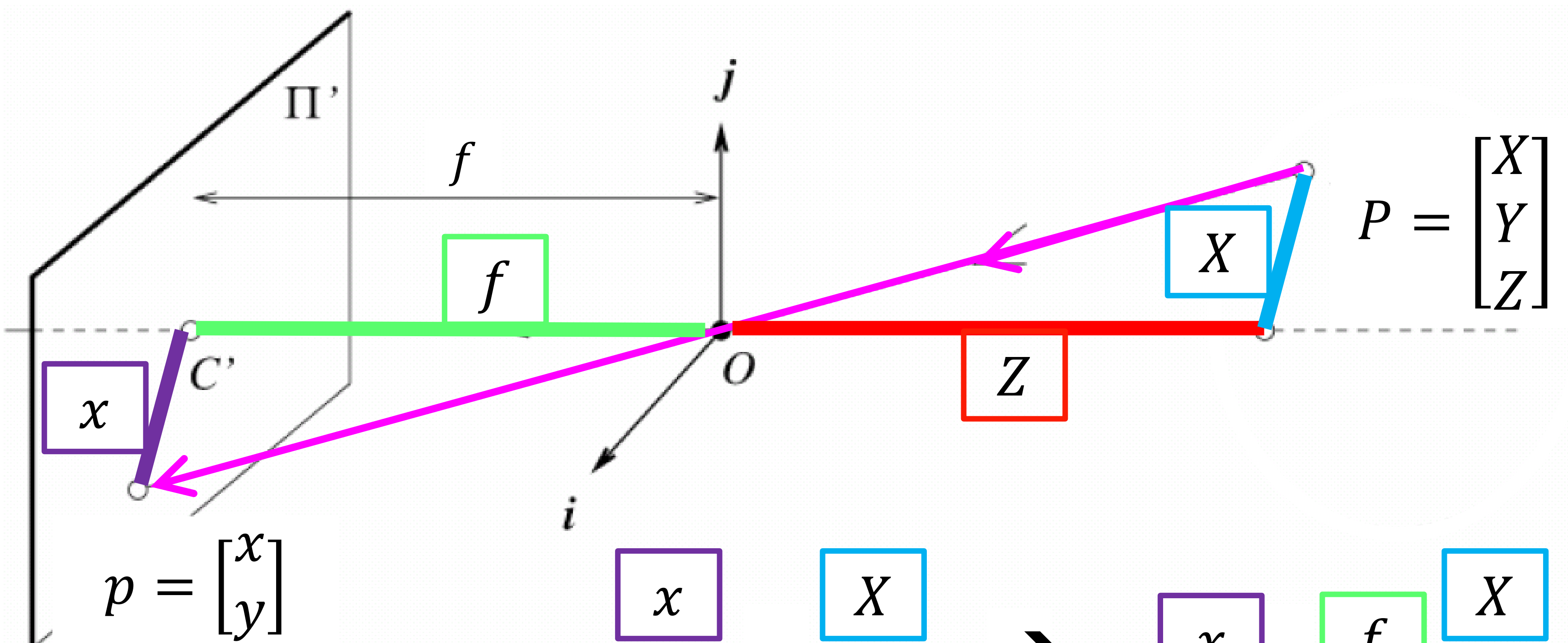
$$P = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$p = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow p = \begin{bmatrix} x \\ y \end{bmatrix}$$

Projection equations of ideal Pinhole

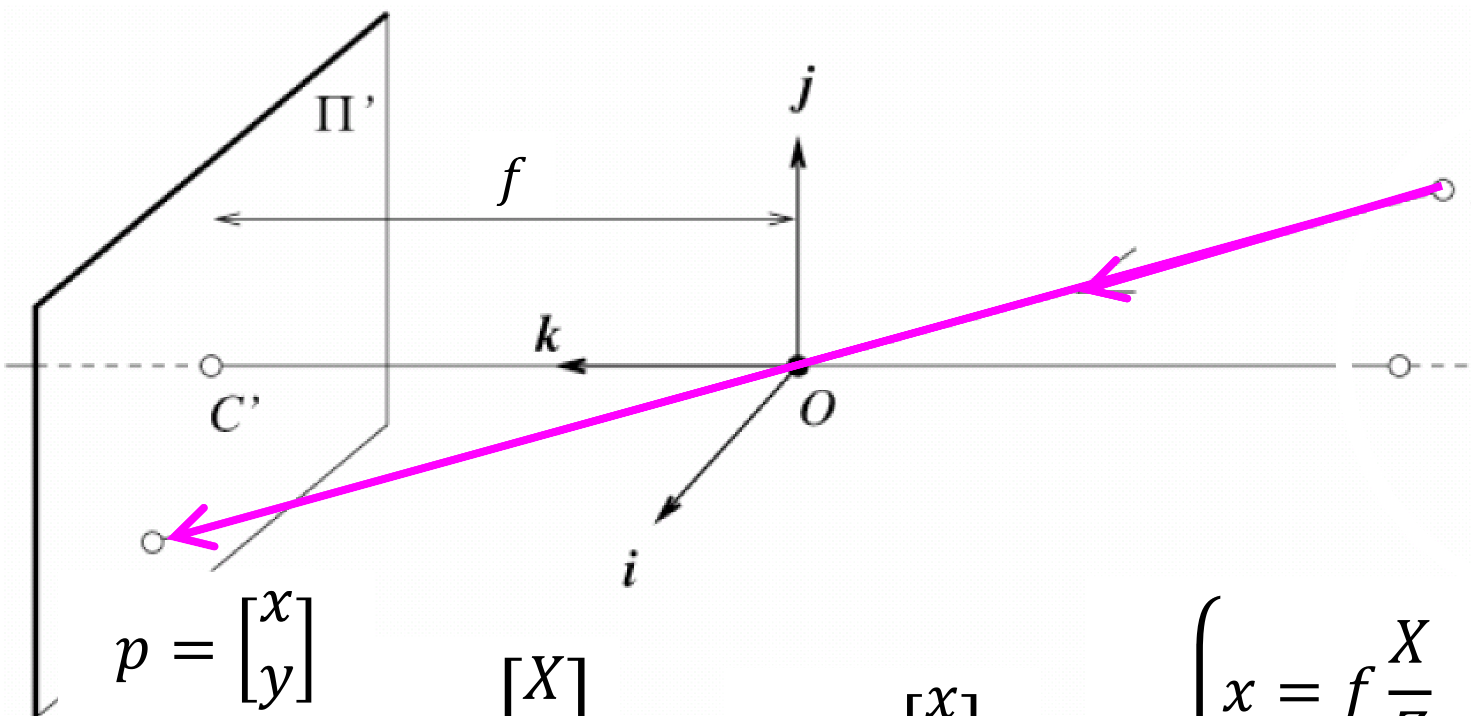
- 3d world mapped to 2d projection in image plane



$$\frac{\boxed{x}}{\boxed{f}} = \frac{\boxed{X}}{\boxed{Z}} \rightarrow \boxed{x} = \boxed{f}\frac{\boxed{X}}{\boxed{Z}}$$

Projection equations of ideal Pinhole

- 3d world mapped to 2d projection in image plane



$$P = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

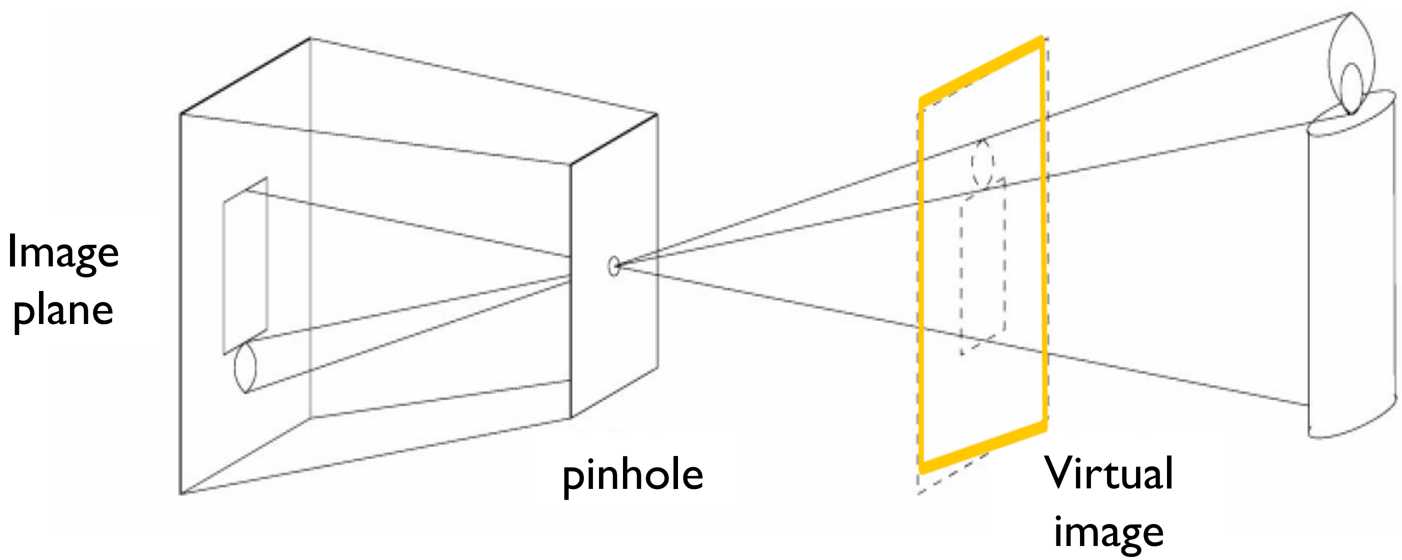
$$p = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow p = \begin{bmatrix} x \\ y \end{bmatrix}$$

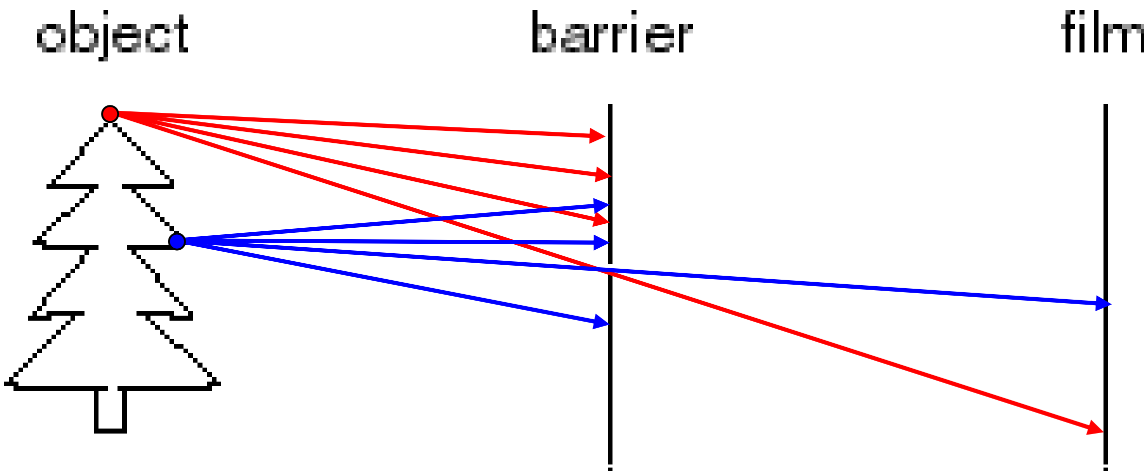
$$\begin{cases} x = f\dfrac{X}{Z} \\ y = f\dfrac{Y}{Z} \end{cases}$$

Pinhole camera

- It is common to draw the image plane **in front** of the focal point
- Moving the image plane merely scales the image

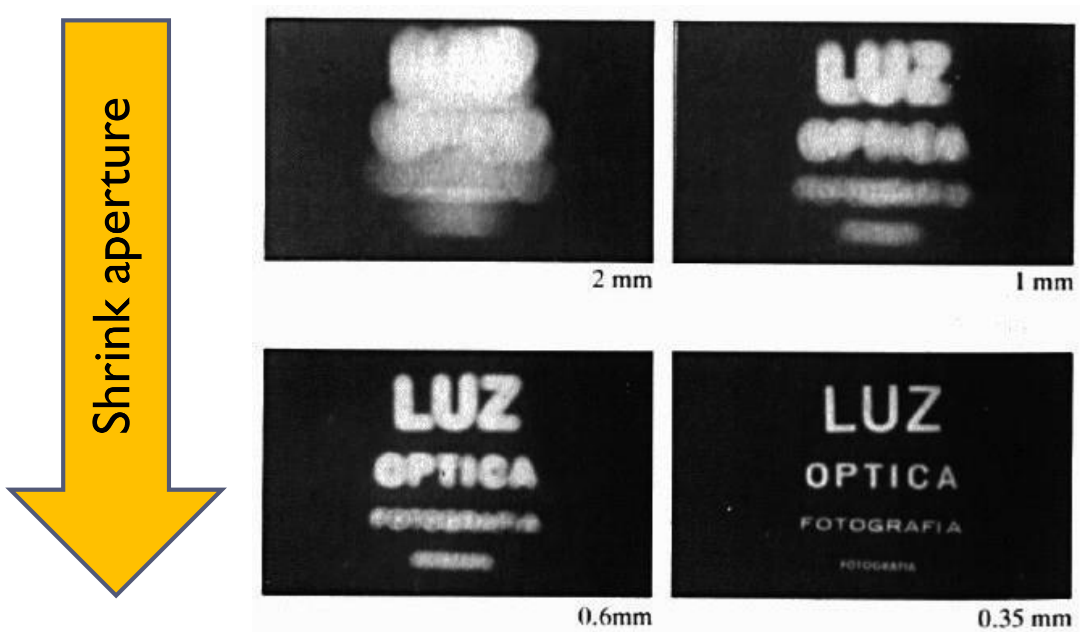


When the camera is not ideal



How does the size of the aperture affect the image?

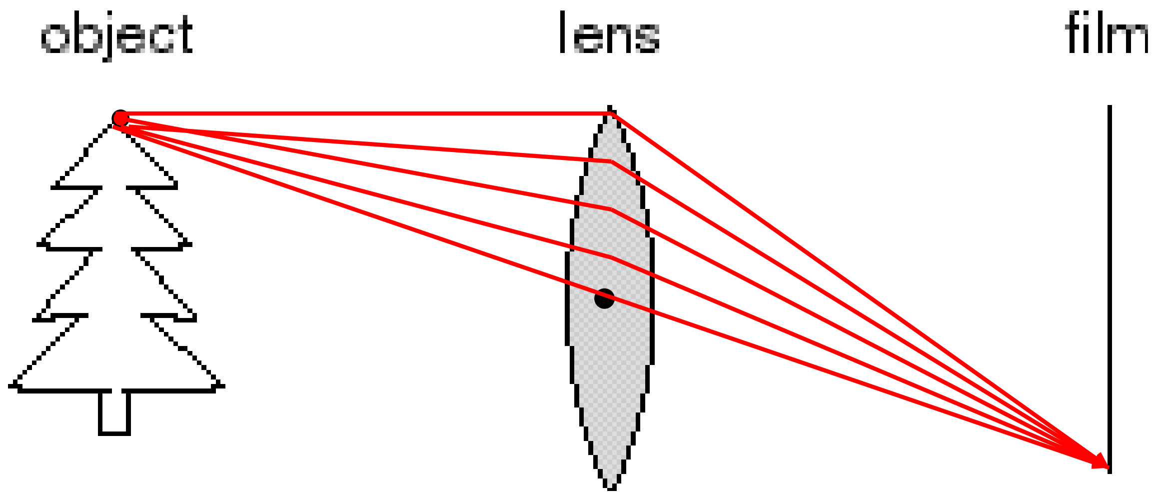
Pinhole size / aperture



Problems with small aperture:

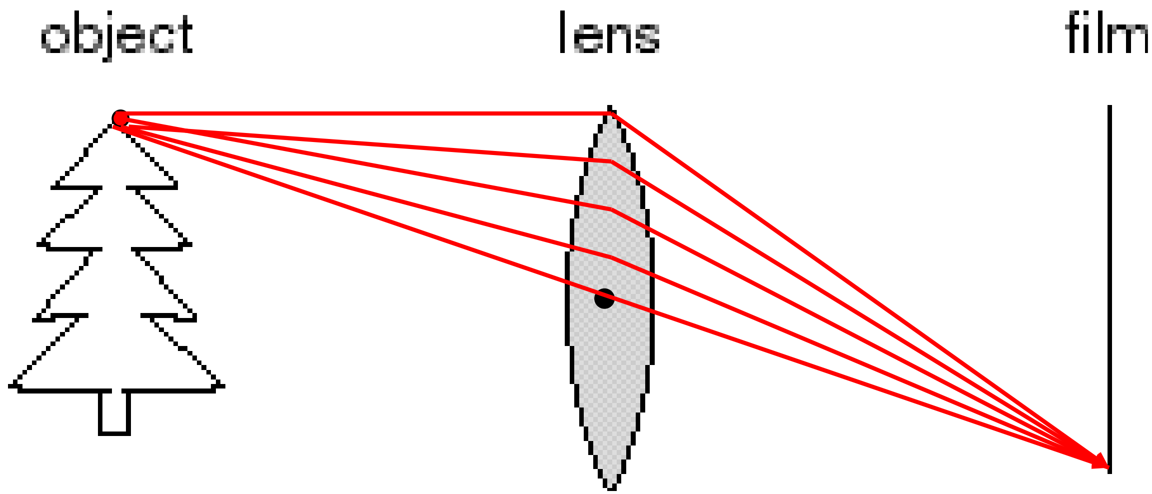
- Less light goes through
- Diffraction effect

Adding a lens



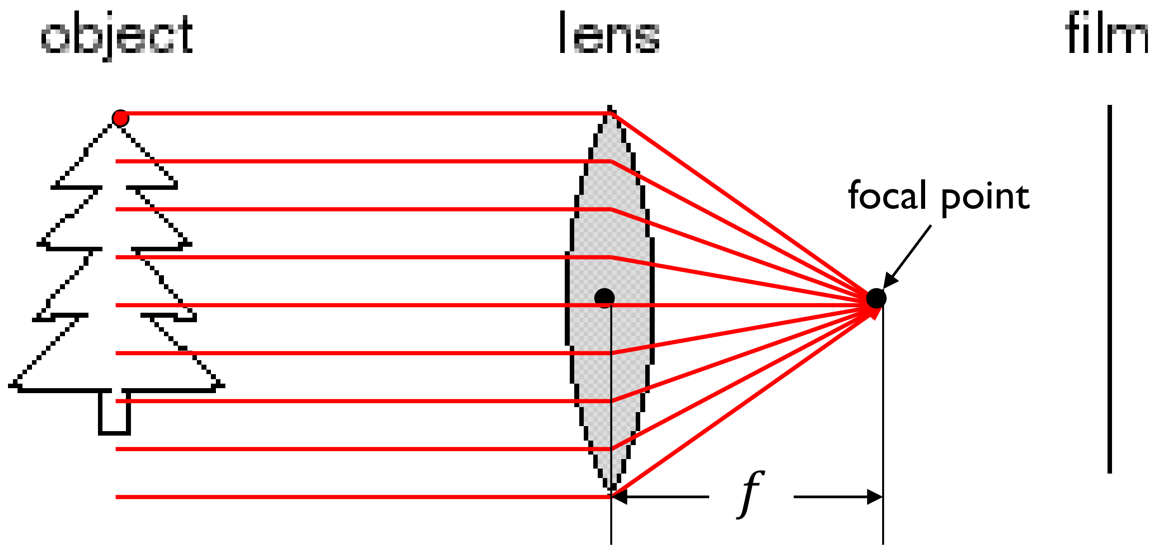
- A lens focuses light onto the film

Adding a lens



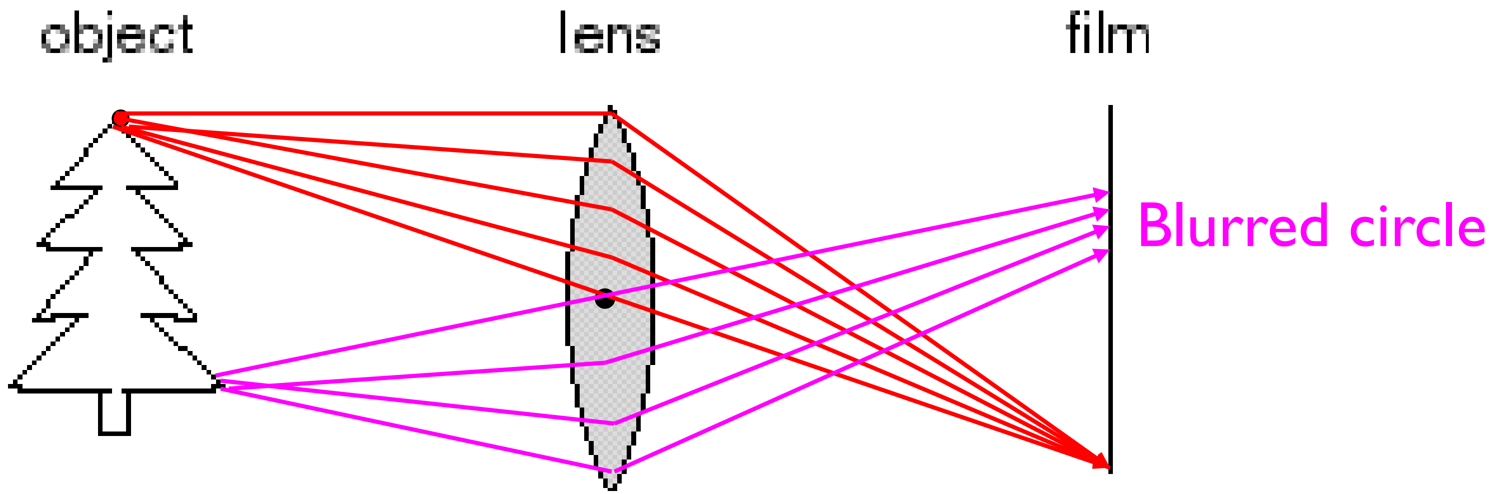
- A lens focuses light onto the film
  - More lights goes through the center than through the boundaries

Adding a lens - focus



- A lens focuses light onto the film
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point on a plane located at the *focal length f*

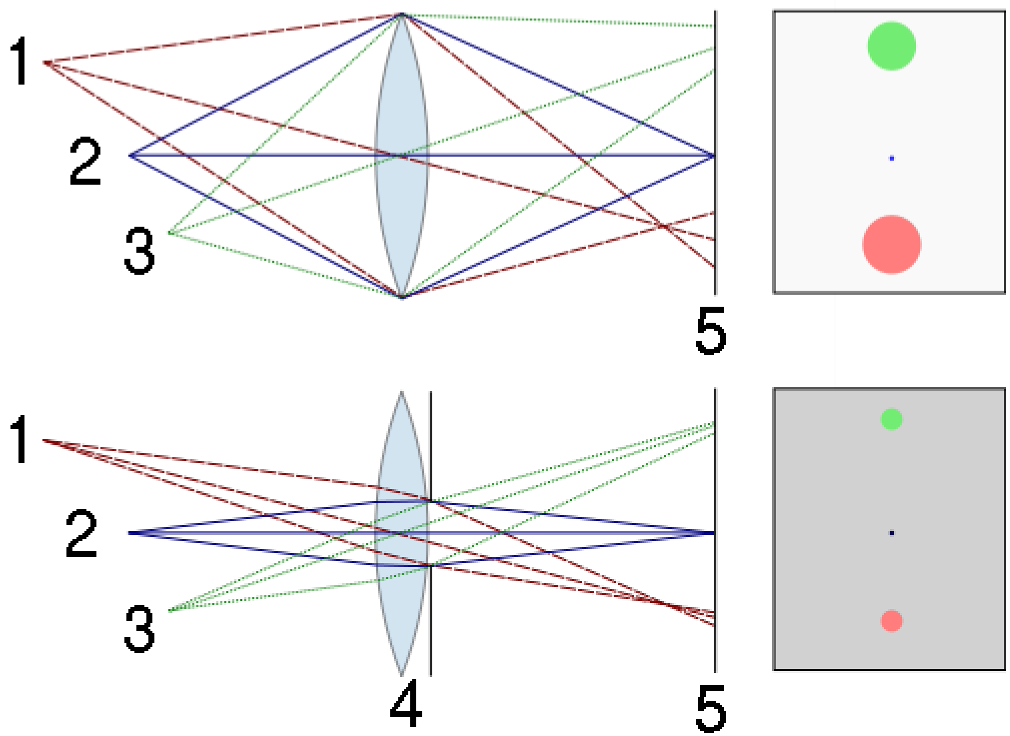
Adding a lens - focus



- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”

A lens with aperture

1. Blurred
2. In focus
3. Blurred

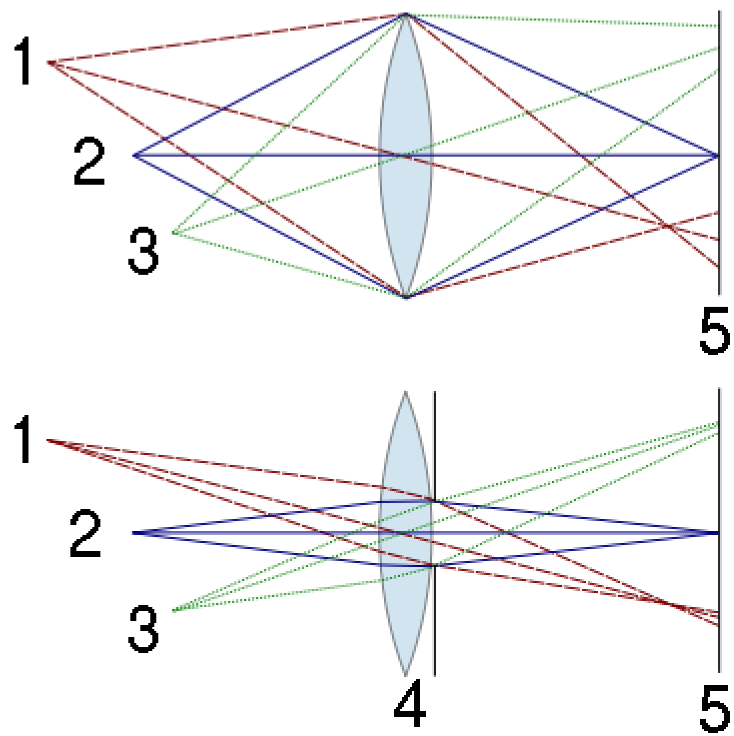


No aperture

Small aperture

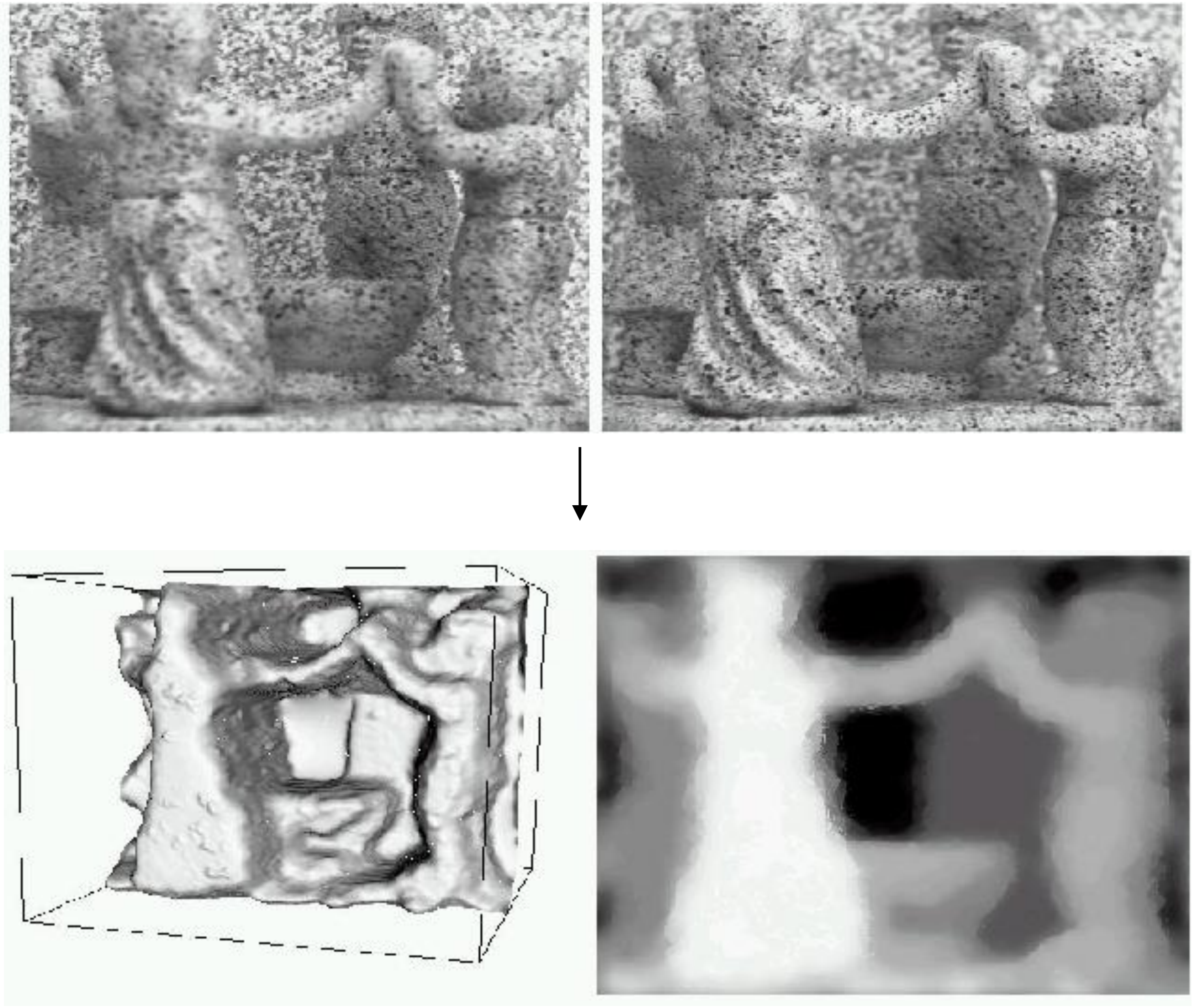
Images from Wikipedia
http://en.wikipedia.org/wiki/Depth_of_field

A lens with aperture



A smaller aperture increases the range in which the object is approximately in focus

Depth from focus



Images from same point of view, different aperture

3d shape / depth estimates

[figs from H. Jin and P. Favaro, 2002]

Depth from defocus

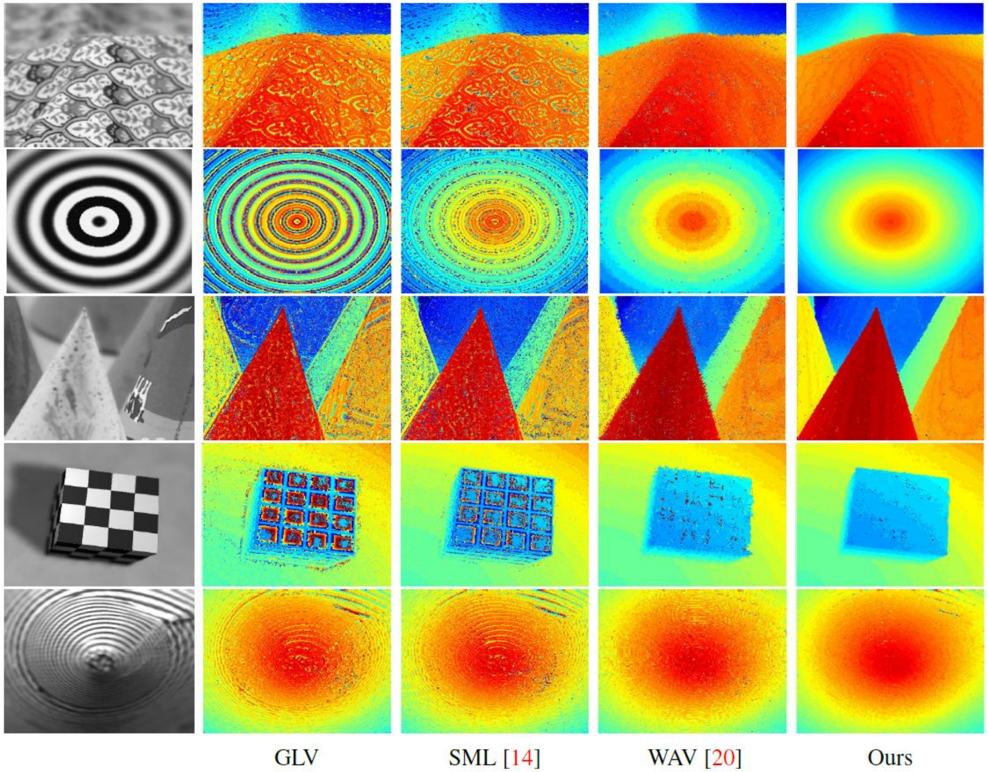


Figure 4: Comparison of focus measures. Color coded depth maps (from near red to far blue) extracted by applying WTA over different focus measures. From top to bottom: ‘Cloth’,‘Synth-Cone’,‘Middlebury-Cones’,‘Cube’ and ‘Real-Cone’. Note the significant improvement obtained using the proposed focus measure.

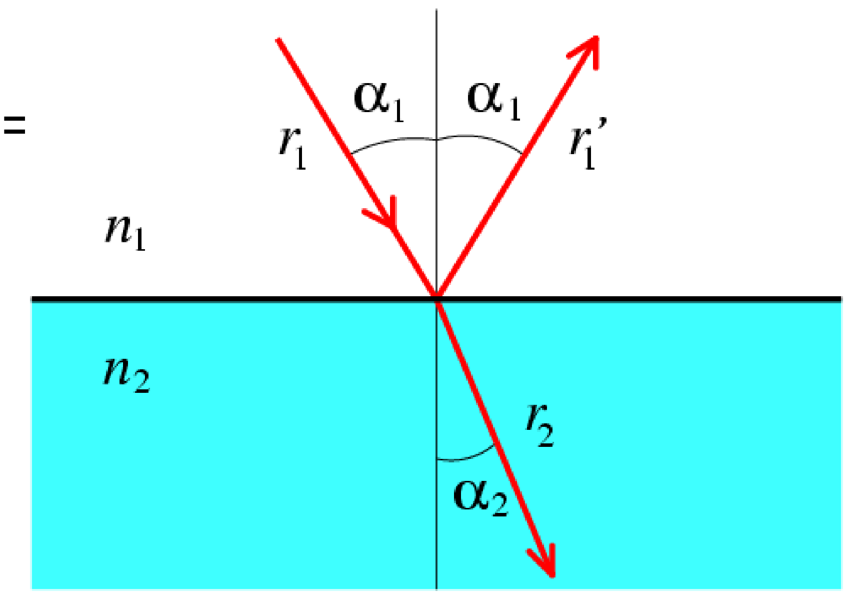
Y. Frommer, R. Ben-Ari and N. Kiryati
Shape from Focus with Adaptive Focus Measure and High Order Derivatives, BMVC 2015

Light passing through a lens

- Optic laws
  - Light travels in straight lines in homogeneous medium
  - Reflection:
    incoming ray, surface normal, and reflected ray are co-planar
  - Refraction: when a ray passes from one medium to another

- Snell's law

$$n_1 \sin \alpha_1 =$$

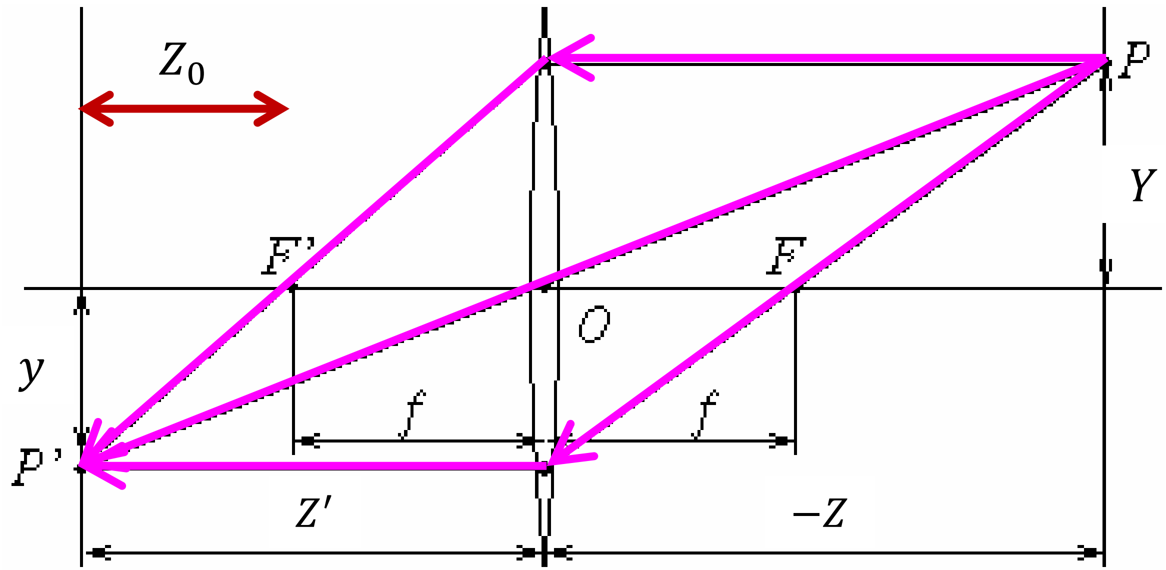


α_1 = incident angle

α_1 = refraction angle

$n_{1,2}$ = index of refraction

Thin lens



$$\begin{cases} Z' = f + Z_0 \\ f = \dfrac{Radius}{2(n-1)} \end{cases}$$

Snell's law

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

Small angles

$$\begin{cases} n_1 \alpha_1 \approx n_2 \alpha_2 \\ n_1 = 1 \quad (air) \\ n_2 = n \;\; (lens) \end{cases}$$

$$\begin{cases} x = Z' \dfrac{X}{Z} \\ y = Z' \dfrac{Y}{Z} \end{cases}$$

Today

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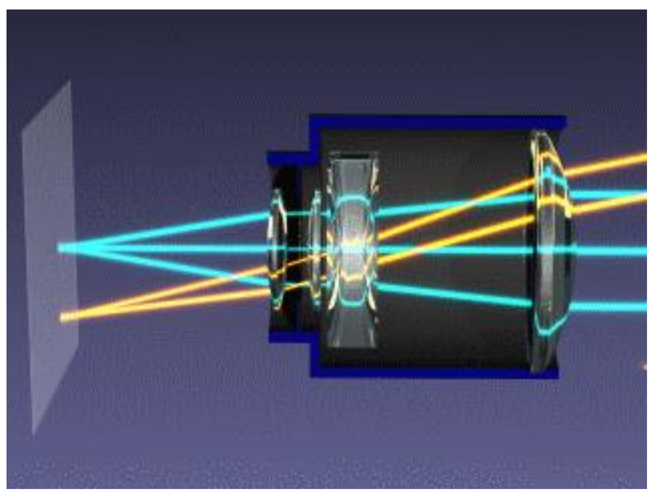
- Cameras & lenses
- The geometry of pinhole cameras
- Other camera models

Cameras and lenses

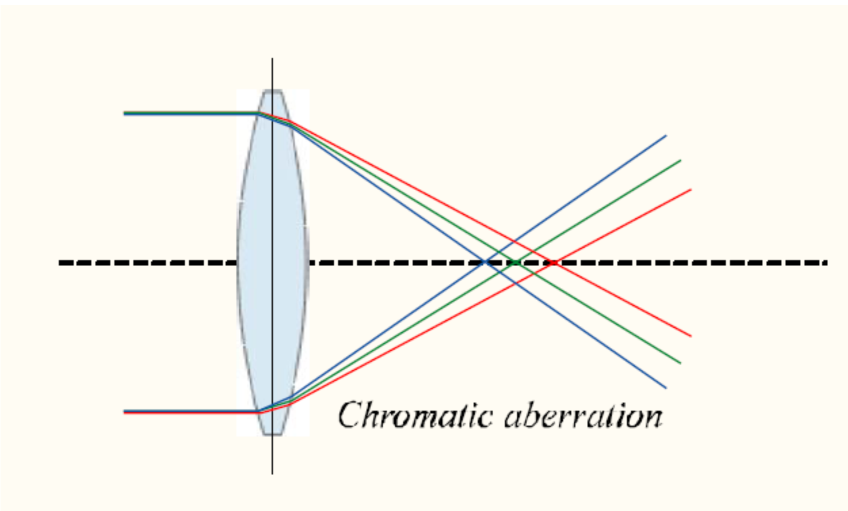


Source wikipedia

Issues with lenses: Chromatic aberration

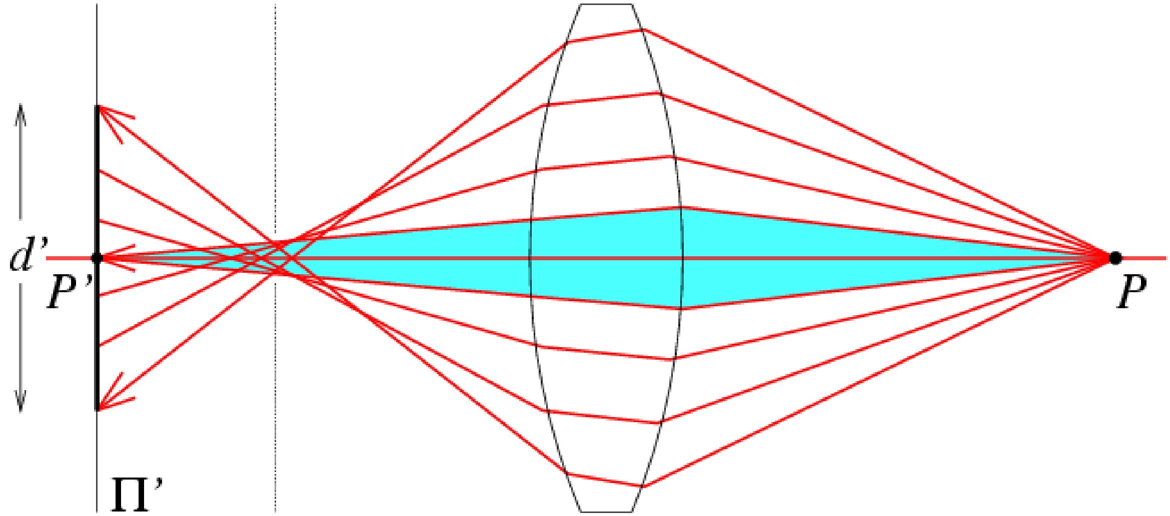
- A lens has different refractive indices for different wavelength: causes color fringing

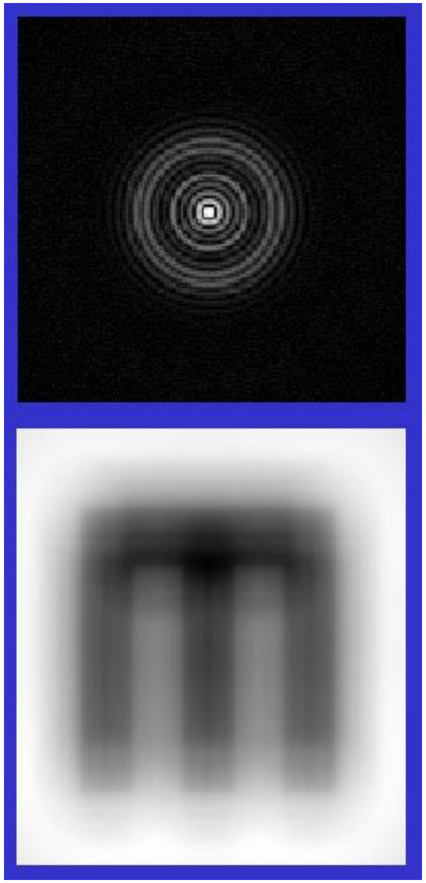
$$f = \frac{Radius}{2(n-1)}$$



Issues with lenses: Chromatic aberration

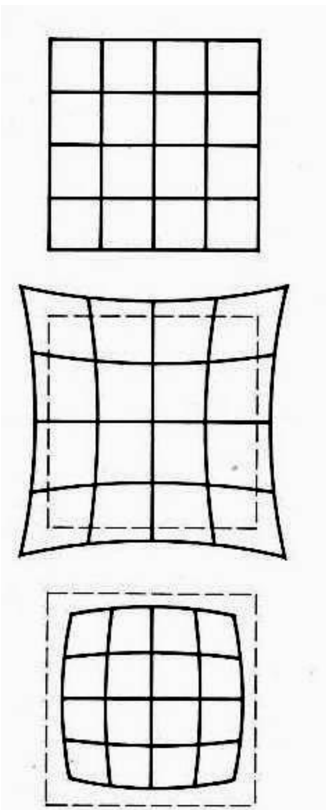
- Rays farther from the optical axis focus closer





Issues with lenses: Chromatic aberration

- Deviations are most noticeable for rays that pass through the edge of the lens



No distortion

Pin cushion

Fisheye

Issues with lenses: vignetting

- A lens focuses light onto the film
  - More lights goes through the center than through the boundaries

Today

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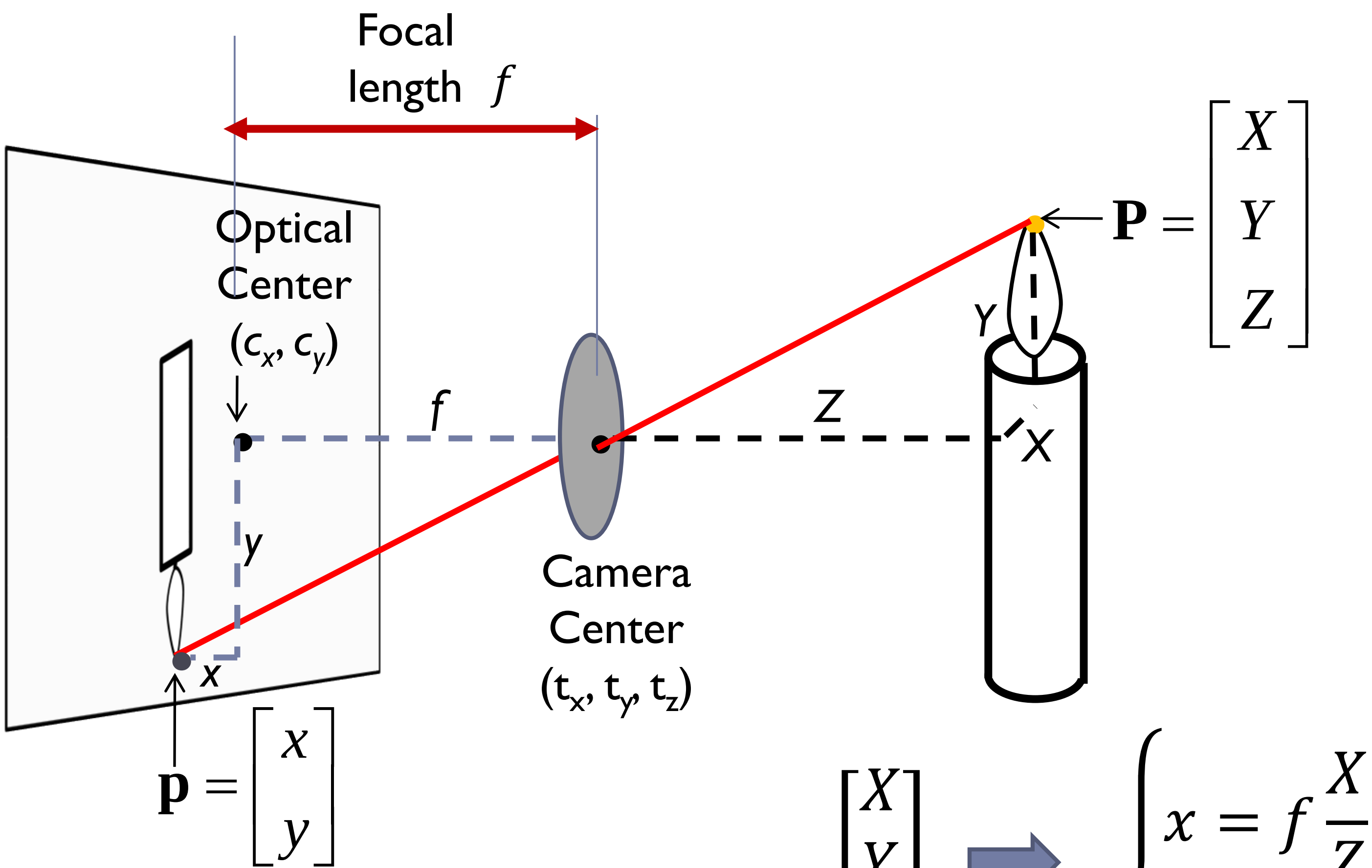
Projection: world coordinates→image coordinates



$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \Rightarrow \begin{cases} x = f\dfrac{X}{Z} \\ y = f\dfrac{Y}{Z} \end{cases}$$

Pinhole camera

- Is this a linear transformation?

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \Rightarrow \begin{cases} x = f\dfrac{X}{Z} \\ y = f\dfrac{Y}{Z} \end{cases}$$

- No – division by Z is not linear!

- How can we make it linear?

Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Using homogeneous coordinates

- Projection is a matrix multiplication using homogeneous coordinates:

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} fX \\ fY \\ Z \end{bmatrix}$$

$$MP = p$$

Projection matrix

$$p' = \begin{bmatrix} \frac{fX}{Z} \\ \frac{fY}{Z} \end{bmatrix}$$

From world to image coordinates

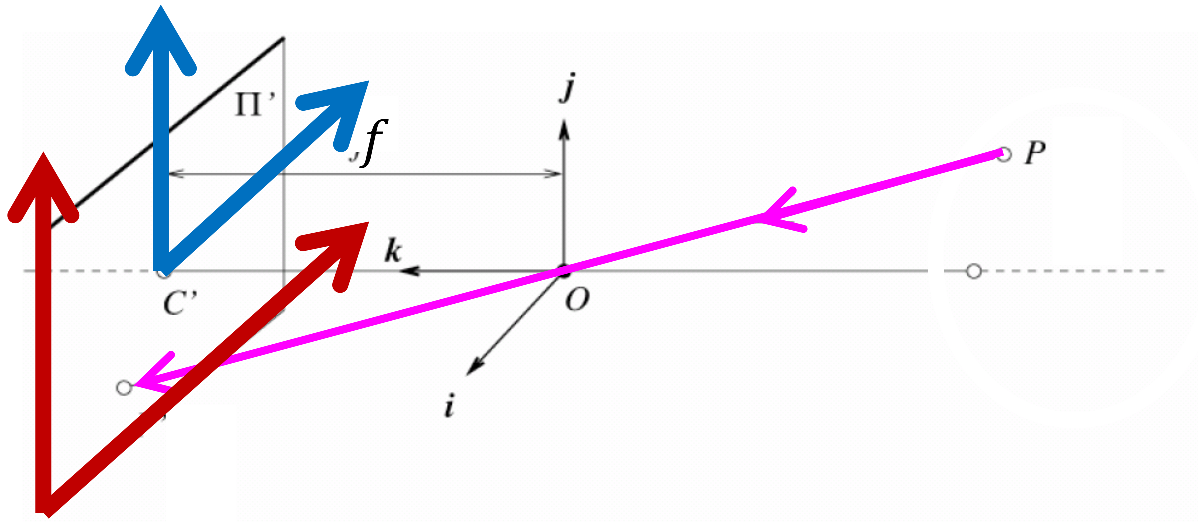
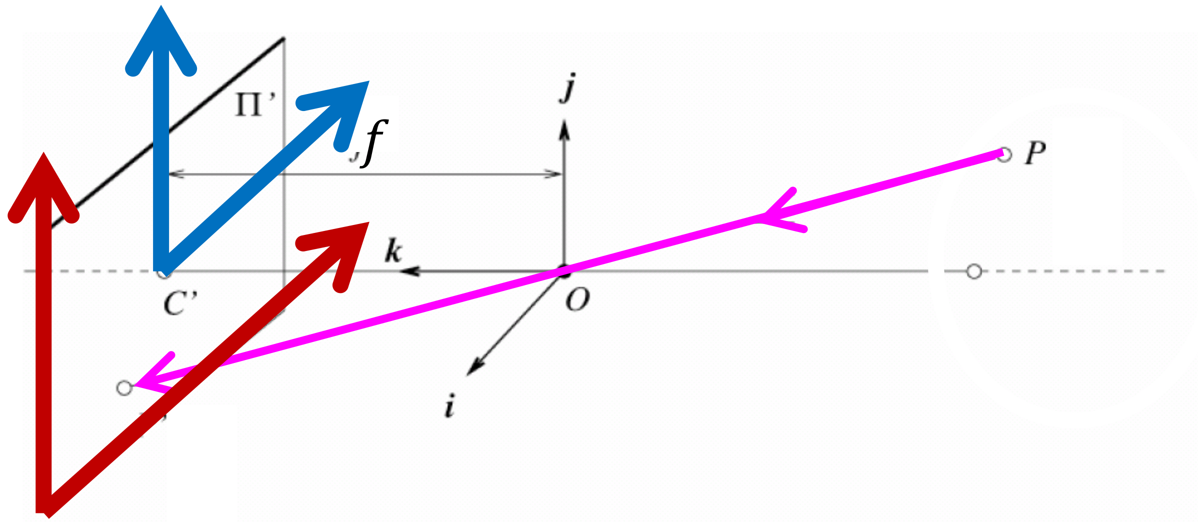


Image coordinate system is not always aligned with optical axis

$$p' = \begin{bmatrix} \frac{fX}{Z} + c_x \\ \frac{fY}{Z} + c_y \end{bmatrix}$$

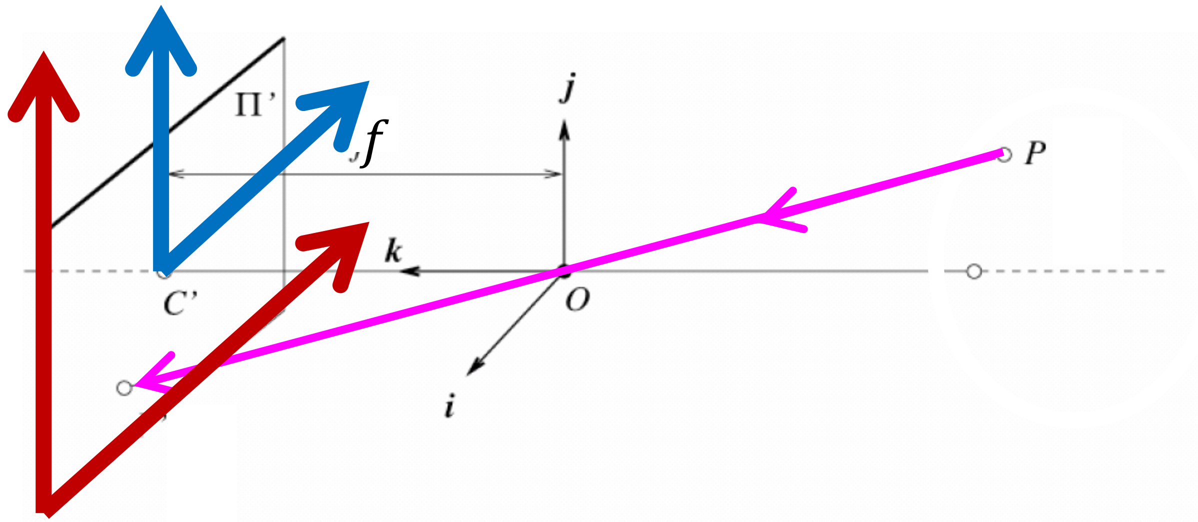
From world to image coordinates



Pixels scale could differ from metric measurements

$$p' = \begin{bmatrix} k\dfrac{fX}{Z} + c_x \\ k\dfrac{fY}{Z} + c_y \end{bmatrix}$$

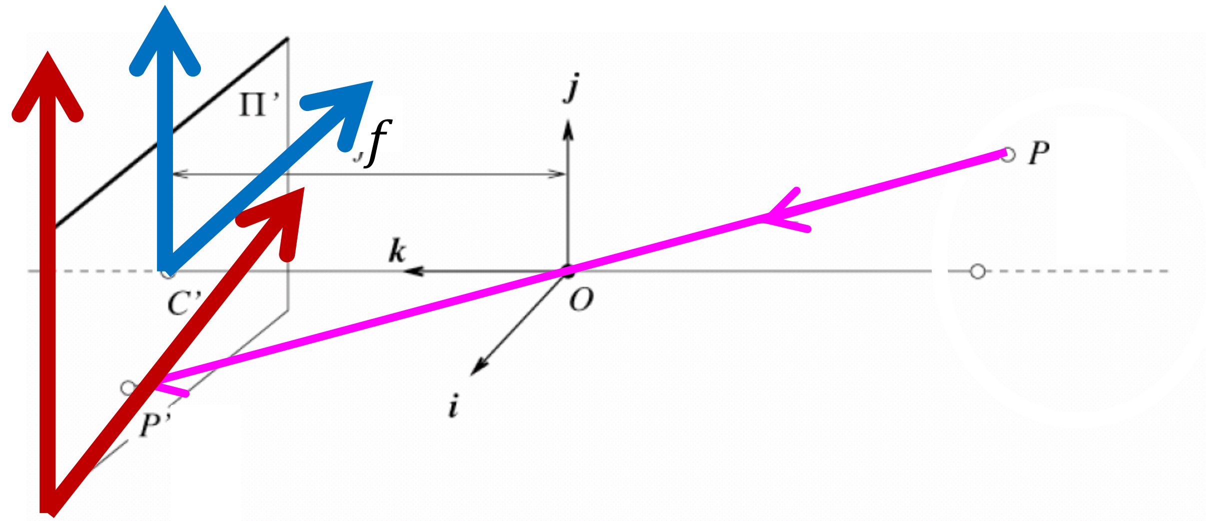
From world to image coordinates



Pixels could be non-square

$$p' = \begin{bmatrix} kf_\alpha \frac{X}{Z} + c_x \\ kf_\beta \frac{Y}{Z} + c_y \end{bmatrix} = \begin{bmatrix} \alpha \frac{X}{Z} + c_x \\ \beta \frac{Y}{Z} + c_y \end{bmatrix}$$

From world to image coordinates



Camera axes could be not-orthogonal

$$p' = \begin{bmatrix} \alpha \frac{X}{Z} + \frac{sY}{Z} + c_x \\ \beta \frac{Y}{Z} + c_y \end{bmatrix}$$

From world to image coordinates

We can write this in matrix form

$$p' = \begin{bmatrix} \alpha \frac{X}{Z} + \frac{sY}{Z} + c_x \\ \beta \frac{Y}{Z} + c_y \\ Z \end{bmatrix} = \begin{bmatrix} \alpha & s & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

The calibration matrix

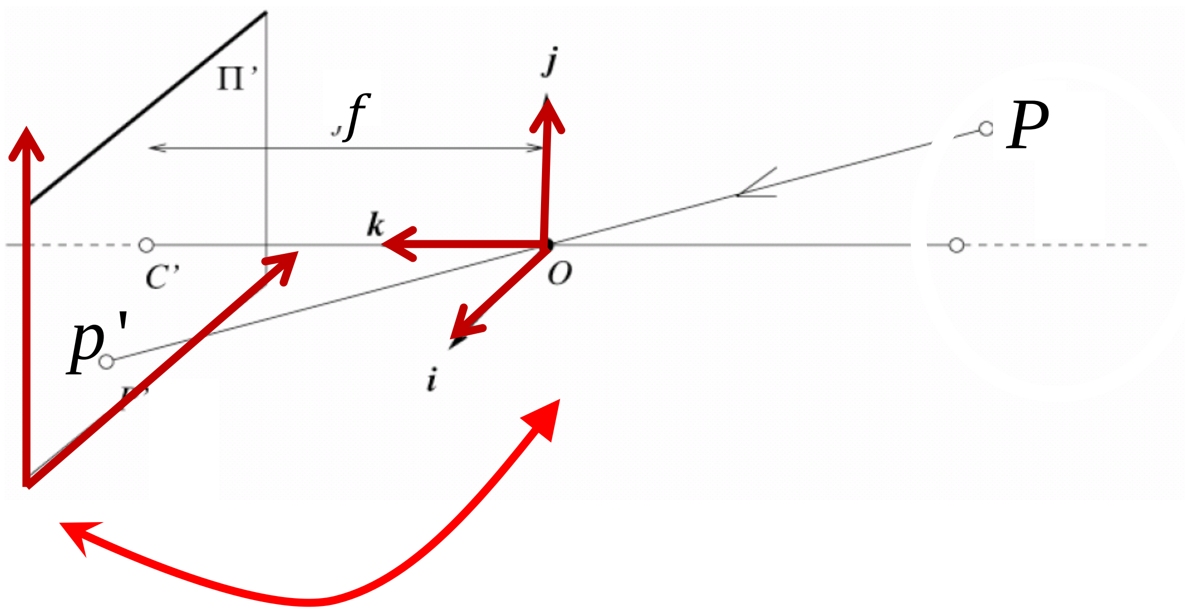
$$p' = \underbrace{\begin{bmatrix} \alpha & s & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{K} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$p' = MP = K\begin{bmatrix} I & 0 \end{bmatrix} P$$

This matrix includes 5 camera parameters and is called:

- Calibration matrix
- Camera matrix

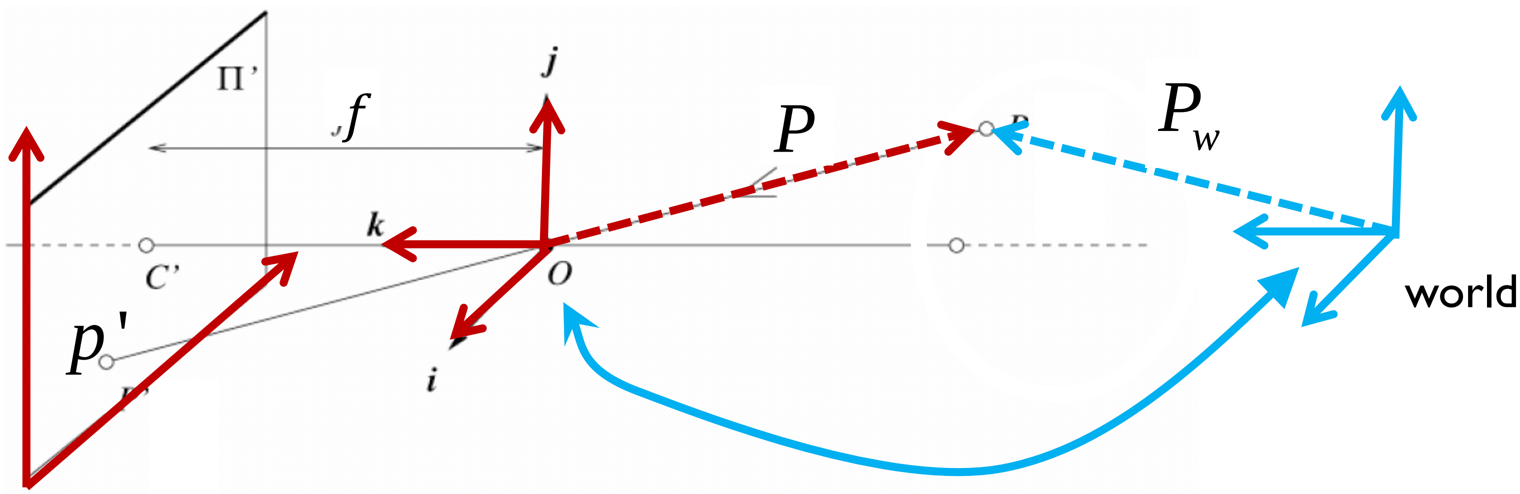
From world to image coordinates



$$p' = MP = K\begin{bmatrix} I & 0 \end{bmatrix} P$$

- So far the world coordinate system was aligned with the lens
- Can we represent the scene in "world" coordinate system?

World coordinates



$$P = [R \quad T] P_w$$

In 4D homogeneous coordinates

$$p' = MP = \boxed{K}\left[\boxed{R \quad T}\right] P_w$$

Internal parameters

External parameters

Camera translation

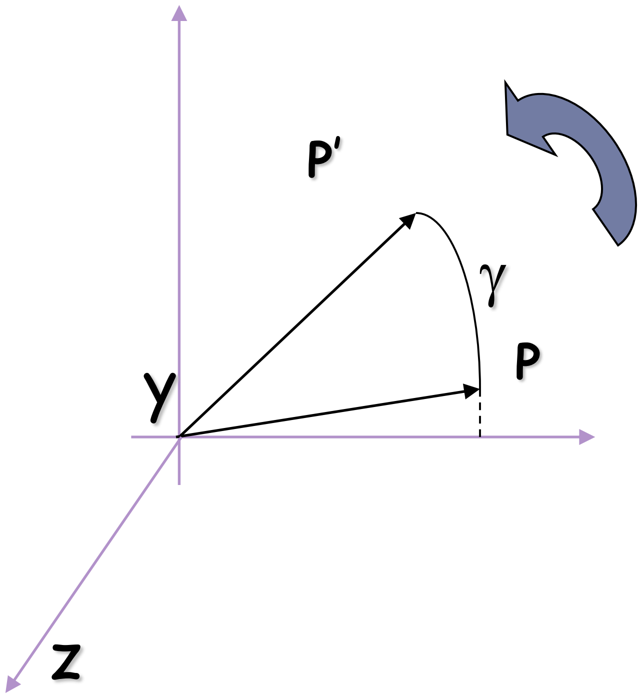
$$p' = MP = K\begin{bmatrix} R & T \end{bmatrix} P_w$$

$$\Downarrow$$

$$w\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & c_x \\ 0 & \beta & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \end{bmatrix} \begin{bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{bmatrix}$$

3D Rotation of Points

Rotation around the coordinate axes, counter-clockwise:



$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

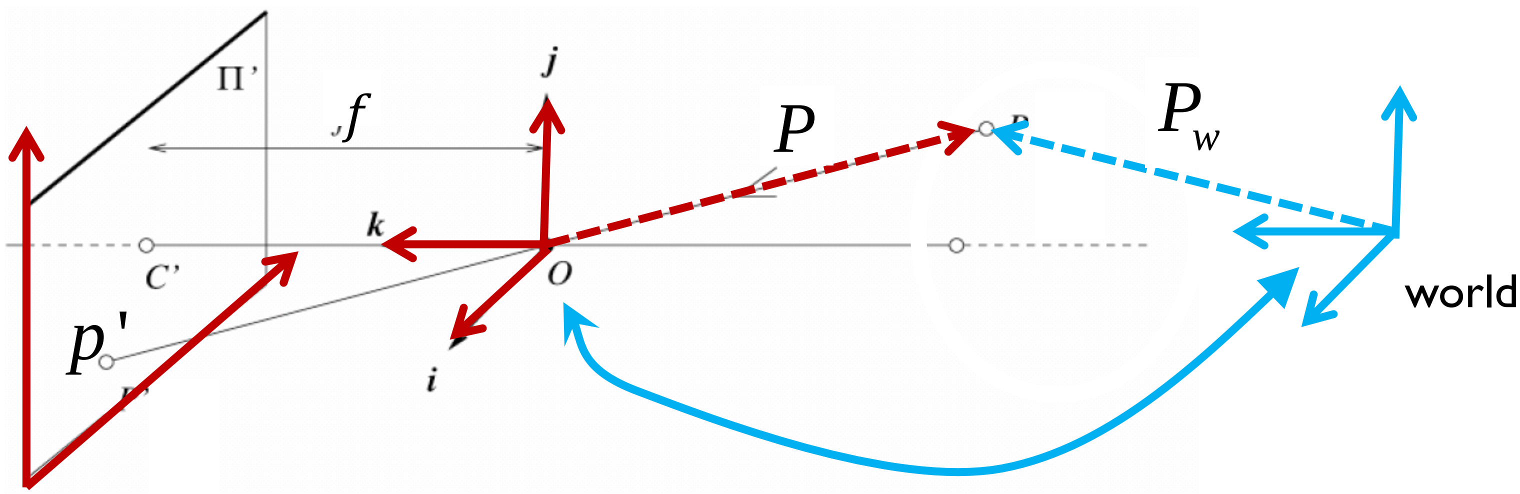
Camera translation and rotation

$$p' = MP = K\begin{bmatrix} R & T \end{bmatrix} P_w$$

$$\Downarrow$$

$$w\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & c_x \\ 0 & \beta & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{bmatrix}$$

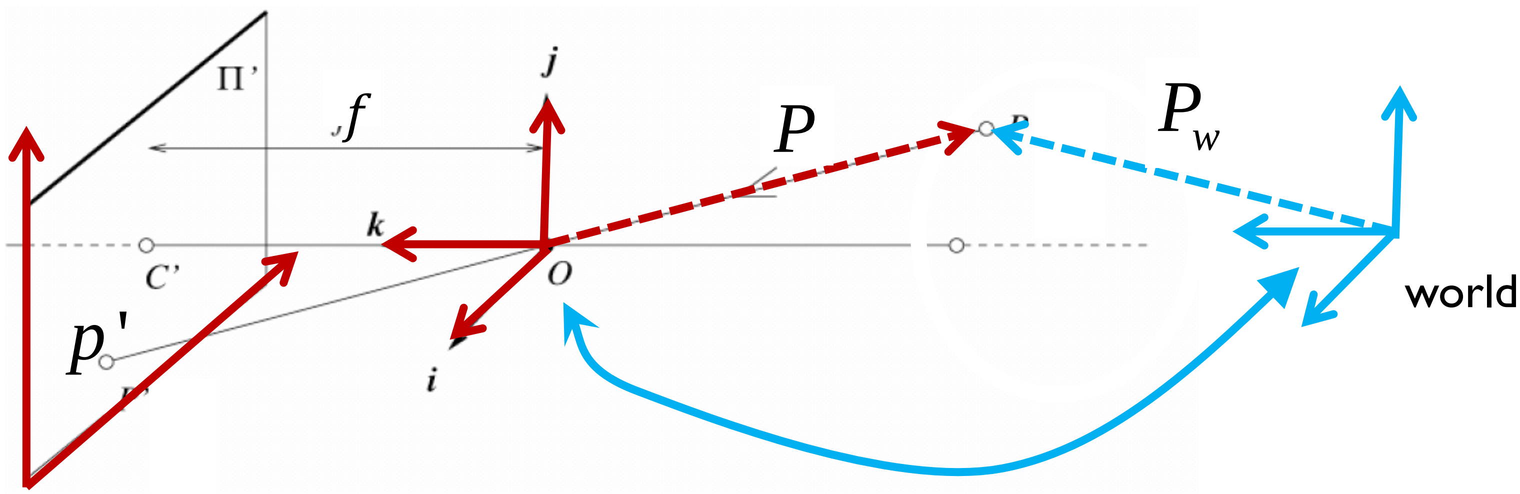
Projective camera equations



$$p'_{3\times1} = MP = K_{3\times3}\left[R_{3\times3} \quad T_{3\times1}\right]_{3\times4} P_{w4\times1}$$

11 degrees of freedom

Projective camera equations



$$p'_{3\times1} = MP = K_{3\times3}\left[R_{3\times3} \quad T_{3\times1}\right]_{3\times4} P_{w4\times1}$$

M is defined up to scale!
Multiplying M by a scalar won't change the image

$$p' \rightarrow \begin{bmatrix} \dfrac{M_1 P}{M_3 P} \\ \dfrac{M_2 P}{M_3 P} \end{bmatrix}$$

Theorem (Faugeras, 1993)

$$M = K\begin{bmatrix} R & T \end{bmatrix} = \begin{bmatrix} KR & KT \end{bmatrix} = \begin{bmatrix} A & b \end{bmatrix}$$

Let $\mathcal{M} = (\mathcal{A} \quad \boldsymbol{b})$ be a 3×4 matrix and let $\boldsymbol{a}_i^T$ $(i = 1, 2, 3)$ denote the rows of the matrix $\mathcal{A}$ formed by the three leftmost columns of $\mathcal{M}$.

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix is that $\mathrm{Det}(\mathcal{A}) \neq 0$.
- A necessary and sufficient condition for $\mathcal{M}$ to be a zero-skew perspective projection matrix is that $\mathrm{Det}(\mathcal{A}) \neq 0$ and

$$(\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0.$$

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix with zero skew and unit aspect-ratio is that $\mathrm{Det}(\mathcal{A}) \neq 0$ and

$$\begin{cases} (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0, \\ (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_1 \times \boldsymbol{a}_3) = (\boldsymbol{a}_2 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3). \end{cases}$$

Properties of projection

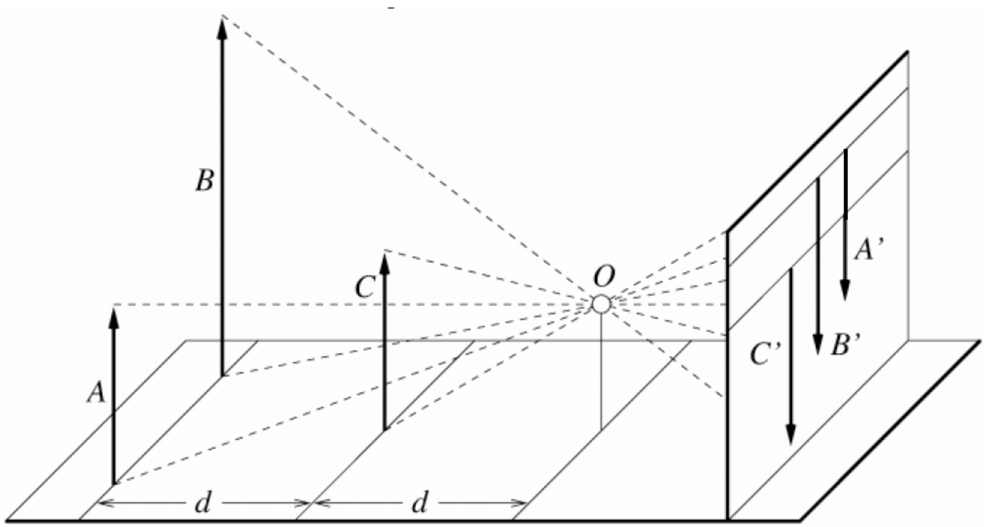
- Points project to points
- Straight lines project to straight lines

Properties of projection

- Angles are not preserved
- Parallel lines meet

Perspective effects

- Far away objects appear smaller

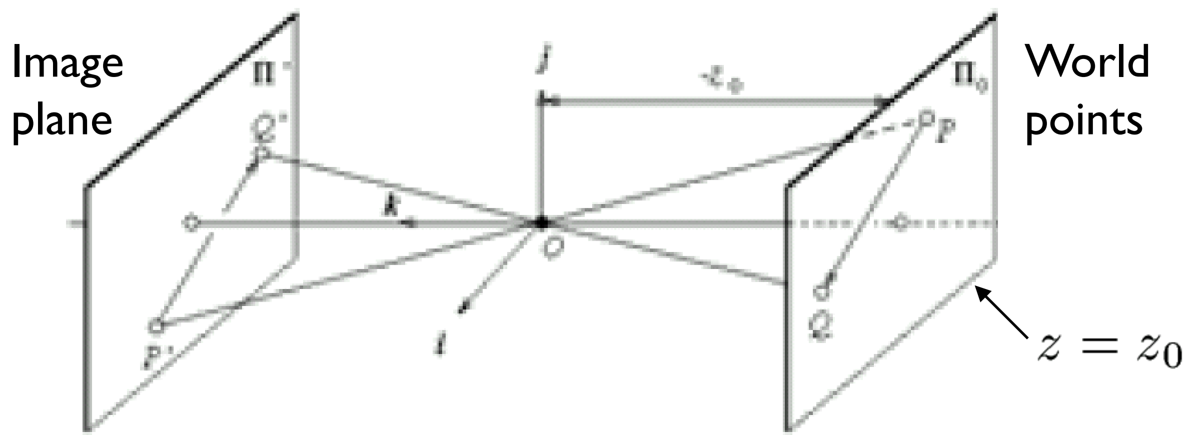


Today

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
- Other camera models

Weak perspective projection

- Assumption:
  All points have the same depth

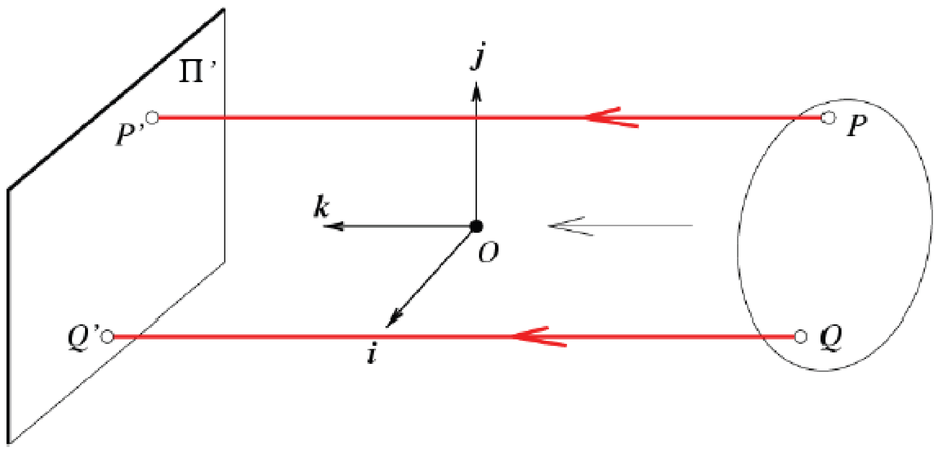


$$\begin{cases} x' = -\dfrac{f}{Z_0} X \\ y' = -\dfrac{f}{Z_0} Y \end{cases}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z_0 \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z_0 / f \end{bmatrix}$$

Orthographic (affine) projection

- Assumption
  Distance from center of projection to image plane is infinite



$$\begin{cases} x' = X \\ y' = Y \end{cases}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Weak perspective example

The kangxi emperor's southern inspection tour (1691-1698) Wang Hui

Affine or perspective?

- Affine
  - Simpler math
  - Accurate enough when object is small and distant
  - Useful for recognition
- Pinhole
  - Used for 3D reconstruction

End – Pinhole camera

Now you know how it works