Semester project TMA4215

Candidate number
and
Canditate number two

22.09.2011

1 Task

We consider minimization problems of the type

$$\min_{x \in R^n} g(\mathbf{x}), \mathbf{g}(\mathbf{x}) := -\mathbf{b^T x} + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{x^T H x} + \frac{\mathbf{1}}{\mathbf{12}}\mathbf{x^T C(x) x},$$

here $\mathbf{b} \in \mathbb{R}^\mathbf{n}$ and, H is a $n \times n$ symmetric and positive definite matrix and $C(\mathbf{x})$ is a diagonal matrix with diagonal entries $c_i x_i^2, i = 1, ..., n$. Here $c_i > 0$ are the components of a vector $\mathbf{c} \in \mathbb{R}^\mathbf{n}$ and x_i are the components of $\mathbf{x}$.

2 Generation of the data

```
function [b, H, c] = data
%DATA Generate the data for the problem with different dimension N of the ma
% b and c are real vectors and H a symmetric positive definite matrix
% If you want to changer the datas of the problem, you have to modify this
% file. Be careful with the size of the matices, b abd c are column vectors
% and H a square matrix, with the same size than b or c.

b = [ 1; 0];
c = [ 1; 1];
H = [ 1, 1 ; 1, 1];

end
```

3 Gradient and Hessian of g

```
function [ g ] = problem ( X )
%PROBLEM Generate the function g using the data given by DATA.

[ b, H, c ] = data;
N = size(H,1);
C = zeros (N, N);
for i = 1:N
        C(i,i) = c(i) * X(i) * X(i);
end

g = -b'*X + 0.5 * X' * H * X + 1/12 * X' * C * X;

end

function [ nablaG ] = grad( X )
%NABLAG Return the gradient of the function G of the project

[b, H, c] = data;
dim = size(H,1);

for i=1:dim
    C(i,i) = c(i)*X(i)*X(i);
```

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end

nablaG = -b + H*X + 1/3*C*X;
end

function [ HessG ] = hessian( X )
%HESSG Return the heesian of the function G of the project

[~, H, c] = data;
N = size(H,1);
C = zeros(N,N);
for i=1:N
    C(i,i) = c(i)*X(i)*X(i);
end

HessG = H + C;

end
```

4 Existance of minimum

$$\nabla^2 g(x) = H + C$$

Let $u \in R^n$, then

$$u(\nabla^2 g(x))u = u(H + C)u = uHu + uCu$$

Because H is positiv definit, $uHu > 0$.

$$uCu = \sum_{i=1}^{n} u_i c_i x_i^2 u_i = \sum_{i=1}^{n} c_i x_i^2 u_i^2$$

Since $c_i > 0$, $uCu > 0$, so $u(\nabla^2 g(x))u > 0 \Rightarrow$ positive definit.

5 Plot for $n = 2$

6 Steepest decent method

7 Equivalence of steepest decent method and forward Euler method

8 Improved steepest decent method

9 Newton method

10 Combination of steepest decent methode and Newton method