1 Hibridación de polaritón plasmónico de grafeno y polaritón fonónico superficial

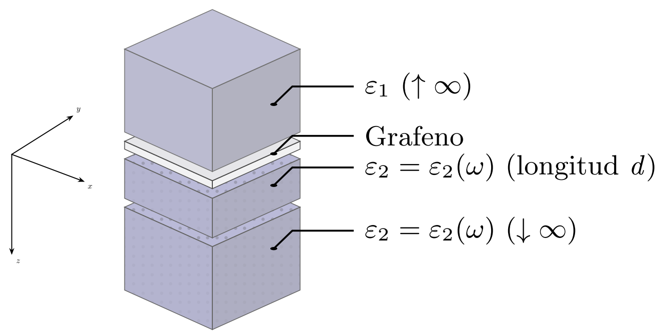

Figure 1: Esquema del sistema Hibridación de polaritones fonónicos superficiales.

Descripción. Hibridación de polaritón plasmónico de grafeno y polaritón fonónico superficial: heteroestructura aire/grafeno/lámina polar de espesor d/sustrato de un segundo material polar. (Se puede intercambiar la posición de los dos materiales polares.)

Ecuación característica (forma original).

$$\begin{aligned}
&-\varepsilon_{\infty,2}(\omega-\omega_{L2})(\omega+\omega_{L2})(\omega-\omega_{T2})(\omega+\omega_{T2})\Big[\varepsilon_{\infty,3}\,\omega^2(\omega-\omega_{L3})(\omega+\omega_{L3})+\varepsilon_1\,\omega^2(\omega-\omega_{T3})(\omega+\omega_{T3})+2c\,k_x \\
&\quad-\Big[\varepsilon_{\infty,3}(\varepsilon_1\omega^2-2c\,k_x\,\omega_D)(\omega-\omega_{L3})(\omega+\omega_{L3})(\omega-\omega_{T2})^2(\omega+\omega_{T2})^2+\varepsilon_{\infty,2}^2(\omega^3-\omega\,\omega_{L2}^2)^2(\omega^2-\omega_{T3}^2)\Big]\tanh(d
\end{aligned} \tag{1}$$

Forma polinómica en ω^2. Sea $u=\omega^2$. Tras expandir y reagrupar, (1) se escribe como

$$a_8\,u^4-a_6\,u^3+a_4\,u^2-a_2\,u+a_0=0. \tag{2}$$

Definiciones "de interfaz" y acoplos. Introducimos $\kappa := \coth(dk_x)$ y las frecuencias efectivas

$$\omega^2_{(2|1)}(\kappa) = \frac{\varepsilon_{\infty,2}\,\kappa\,\omega^2_{L2} + \varepsilon_1\,\omega^2_{T2}}{\varepsilon_{\infty,2}\,\kappa + \varepsilon_1}, \tag{3}$$

$$\omega^2_{(2|3)}(\kappa) = \frac{\varepsilon_{\infty,2}\,\kappa\,\omega^2_{L2} + \varepsilon_{\infty,3}\,\omega^2_{T2}}{\varepsilon_{\infty,2}\,\kappa + \varepsilon_{\infty,3}}, \tag{4}$$

$$\omega^2_{(3|2)}(\kappa) = \frac{\varepsilon_{\infty,3}\,\kappa\,\omega^2_{L3} + \varepsilon_{\infty,2}\,\omega^2_{T3}}{\varepsilon_{\infty,2}\,\kappa + \varepsilon_{\infty,3}}. \tag{5}$$

Producto "fonones superficiales" (par 2|3):

$$\Delta^{(2|3)}_{\text{sph}} = \frac{\varepsilon_{\infty,2}\kappa\,\omega^2_{L2}\,\omega^2_{T3} + \varepsilon_{\infty,3}\,\omega^2_{L3}\,\omega^2_{T2}}{\varepsilon_{\infty,2}\kappa + \varepsilon_{\infty,3}}. \tag{6}$$

Acoplo geométrico (tiende a 0 cuando $k_x \to \infty$):

$$\Delta(\kappa) = \frac{\varepsilon^2_{\infty,2}\,(1-\kappa)}{(\varepsilon_{\infty,2}\kappa + \varepsilon_1)(\varepsilon_{\infty,2}\kappa + \varepsilon_{\infty,3})}. \tag{7}$$

Parámetro plasmónico efectivo (grafeno):

$$\omega^2_p(k_x) = \frac{2c\,\omega_D}{\varepsilon_{\infty,2}\kappa + \varepsilon_1}\,k_x. \tag{8}$$

Coeficientes del polinomio (2)**.** Usando las definiciones anteriores,

$$a_8 = 1 + \Delta(\kappa), \tag{9}$$

$$a_6 = \omega^2_{(2|3)}(\kappa) + \omega^2_{(3|2)}(\kappa) + \omega^2_{(2|1)}(\kappa) + \Delta(\kappa)\big(2\omega^2_{L2} + \omega^2_{T3}\big) + \omega^2_p, \tag{10}$$

$$a_4 = \omega^2_{(2|1)}(\kappa)\big(\omega^2_{(2|3)}(\kappa) + \omega^2_{(3|2)}(\kappa)\big) + \Delta(\kappa)\big(\omega^4_{L2} + 2\omega^2_{T3}\omega^2_{L2}\big) + \Delta^{(2|3)}_{\text{sph}} + \omega^2_p\big(\omega^2_{(2|3)}(\kappa) + \omega^2_{(3|2)}(\kappa) + \omega^2_{T2}\big), \tag{11}$$

$$a_2 = \big(\omega^2_{(2|1)}(\kappa) + \omega^2_p\big)\Delta^{(2|3)}_{\text{sph}} + \Delta(\kappa)\,\omega^4_{L2}\omega^2_{T3} + \omega^2_p\,\omega^2_{T2}\big(\omega^2_{(2|3)}(\kappa) + \omega^2_{(3|2)}(\kappa)\big), \tag{12}$$

$$a_0 = \omega^2_p\,\omega^2_{T2}\,\Delta^{(2|3)}_{\text{sph}}. \tag{13}$$

Chequeo de consistencia (sin grafeno). Si $\omega_p = 0$ (i.e., sin grafeno), (2) reduce *instantáneamente* al caso puramente fonónico (POLAR4), con los mismos a_i pero sin los términos proporcionales a ω^2_p.

Límites asintóticos.

$k_x \to 0$ $\big(\tanh(dk_x) \to 0\big)$. La ecuación original se reduce a

$$\varepsilon_{\infty,2}(\omega-\omega_{L2})(\omega+\omega_{L2})(\omega-\omega_{T2})(\omega+\omega_{T2})\Big[\varepsilon_{\infty,3}\omega^2(\omega-\omega_{L3})(\omega+\omega_{L3})+\varepsilon_1\omega^2(\omega-\omega_{T3})(\omega+\omega_{T3})+2c\,k_x\omega_D(-\omega^2+\omega^2_{T3}$$

cuyas raíces (asíntotas) son

$$\boxed{\omega = 0, \qquad \omega = \omega_{L2}, \qquad \omega = \omega_{T2}, \qquad \omega = \sqrt{\frac{\varepsilon_{\infty,3}\omega_{L3}^2 + \varepsilon_1\omega_{T3}^2}{\varepsilon_1 + \varepsilon_{\infty,3}}} \equiv \omega_{(3|1)}}.$$

$k_x \to \infty$**.** Aquí $\kappa = \coth(dk_x) \to 1$, $\Delta(\kappa) \to 0$ y $\omega_p^2 \to \infty$. Las cuatro soluciones de (2) se organizan en dos pares:

(i) Par fonónico (interfase $2|3$*)* evaluado en $\kappa = 1$:

$$\boxed{u = \omega^2 \ \to \ \frac{\omega_{(2|3)}^2 + \omega_{(3|2)}^2 \ \mp \ \sqrt{\left(\omega_{(2|3)}^2 + \omega_{(3|2)}^2\right)^2 - 4\,\Delta_{\text{sph}}^{(2|3)}}}{2}}. \tag{14}$$

(ii) Par grafeno–film (interfase $2|1$*)* con gran ω_p:

$$\boxed{u = \omega^2 \ \to \ \frac{\omega_{(2|1)}^2 + \omega_p^2 \ \mp \ \sqrt{\left(\omega_{(2|1)}^2 + \omega_p^2\right)^2 - 4\,\omega_p^2\,\omega_{T2}^2}}{2}}. \tag{15}$$

Límite analítico ($\omega_p^2 \gg \omega_{(2|1)}^2, \omega_{T2}^2$).

$$\boxed{\omega_- \ = \ \omega_{T2}\left[1 - \frac{\omega_{(2|1)}^2 - \omega_{T2}^2}{2\,\omega_p^2} + O(\omega_p^{-4})\right] \to \omega_{T2}}, \tag{16}$$

$$\boxed{\omega_+ \ = \ \omega_p \ + \ \frac{\omega_{(2|1)}^2 - \omega_{T2}^2}{2\,\omega_p} + O(\omega_p^{-3}) \to \omega_p}. \tag{17}$$

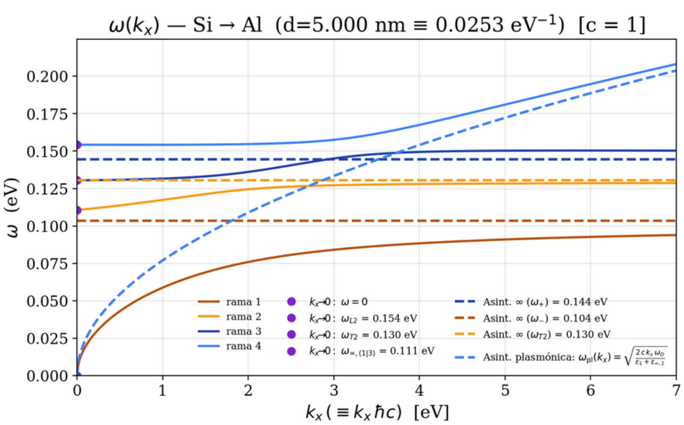

Figure 2: Representación

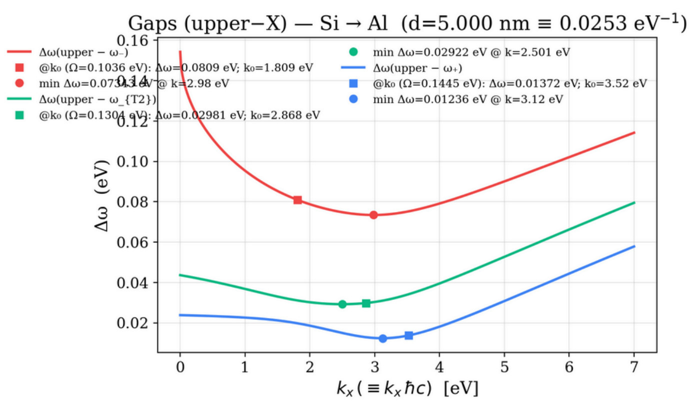

Figure 3: Diferencias de $\omega_{up} - \omega_{low}$