(Depurar) In[ ]:=

```
(*Cálculo de los coeficientes de la ecuación de sexto grado
para "Hibridación de polaritones fonónicos superficiales I"*)
                                                                número i

ClearAll[e1, e2, wl1, wl2, wt1, wt2, eq1, sols, Omega1, Omega2, params, e1, wl1, wl2, wt1,
 wt2, eq1, sols, Omega1, Omega2, params, einf2, einf3, wl3, wt3, c, kx, w, d, arraysol]
eq1 = FullSimplify[Solve[{1 + r1 == r2 + t2,
      Y1 (1 - r1) == Y2 (t2 - r2), t2 * Exp[-d * kx] + r2 * Exp[d * kx] == t3 * Exp[-d * kx],
      Y2 (t2 * Exp[-d * kx] - r2 * Exp[d * kx]) == Y3 * t3 * Exp[-d * kx]}, {r1, r2, t2, t3}]]
```

(Depurar) Out[ ]=

$$\left\{\left\{r1 \to \frac{(Y1+Y2)\,(Y2-Y3)+e^{2\,d\,kx}\,(Y1-Y2)\,(Y2+Y3)}{(Y1-Y2)\,(Y2-Y3)+e^{2\,d\,kx}\,(Y1+Y2)\,(Y2+Y3)},\right.\right.$$

$$r2 \to \frac{2\,Y1\,(Y2-Y3)}{(Y1-Y2)\,(Y2-Y3)+e^{2\,d\,kx}\,(Y1+Y2)\,(Y2+Y3)},$$

$$t2 \to \frac{2\,e^{2\,d\,kx}\,Y1\,(Y2+Y3)}{(Y1-Y2)\,(Y2-Y3)+e^{2\,d\,kx}\,(Y1+Y2)\,(Y2+Y3)},$$

$$\left.\left.t3 \to \frac{4\,e^{2\,d\,kx}\,Y1\,Y2}{(Y1-Y2)\,(Y2-Y3)+e^{2\,d\,kx}\,(Y1+Y2)\,(Y2+Y3)}\right\}\right\}$$

In[ ]:=

(Depurar) In[ ]:=

```
ClearAll[e1, wl1, wl2, wt1, wt2, eq1, sols, Omega1,
 Omega2, params, einf2, einf3, wl3, wt3, c, kx, w, d, arraysol]

(*Funciones dieléctricas*)
e2 = einf2 * (wl2^2 - w^2) / (wt2^2 - w^2);
e3 = einf3 * (wl3^2 - w^2) / (wt3^2 - w^2);

(*Ecuación de dispersión*)
eq = Sinh[kx * d] * (e1 * e3 + e2^2) + Cosh[kx * d] * (e1 * e2 + e2 * e3) == 0
```

borra todo · seno hiperbólico · coseno hiperbólico

$$\left(\text{einf2}\,\text{einf3}\left(-w^2+\text{wl2}^2\right)\left(-w^2+\text{wl3}^2\right)\left(-w^2+\text{wt2}^2\right)+\text{e1}\,\text{einf2}\left(-w^2+\text{wl2}^2\right)\left(-w^2+\text{wt3}^2\right)\left(-w^2+\text{wt2}^2\right)\right)\text{Cosh}[d\,kx]+$$

coseno hiperbólico

$$\left(\text{e1}\,\text{einf3}\left(-w^2+\text{wl3}^2\right)\left(-w^2+\text{wt2}^2\right)^2+\text{einf2}^2\left(-w^2+\text{wl2}^2\right)^2\left(-w^2+\text{wt3}^2\right)\right)\text{Sinh}[d\,kx]==0$$

seno hiperbólico

(Depurar) Out[ ]=

$$\left(\frac{\text{e1}\,\text{einf2}\left(-w^2+\text{wl2}^2\right)}{-w^2+\text{wt2}^2}+\frac{\text{einf2}\,\text{einf3}\left(-w^2+\text{wl2}^2\right)\left(-w^2+\text{wl3}^2\right)}{\left(-w^2+\text{wt2}^2\right)\left(-w^2+\text{wt3}^2\right)}\right)\text{Cosh}[d\,kx]+$$

$$\left(\frac{\text{einf2}^2\left(-w^2+\text{wl2}^2\right)^2}{\left(-w^2+\text{wt2}^2\right)^2}+\frac{\text{e1}\,\text{einf3}\left(-w^2+\text{wl3}^2\right)}{-w^2+\text{wt3}^2}\right)\text{Sinh}[d\,kx]==0$$

(Depurar) Out[ ]=

$$\left(\text{einf2}\,\text{einf3}\left(-w^2+\text{wl2}^2\right)\left(-w^2+\text{wl3}^2\right)\left(-w^2+\text{wt2}^2\right)+\text{e1}\,\text{einf2}\left(-w^2+\text{wl2}^2\right)\left(-w^2+\text{wt3}^2\right)\left(-w^2+\text{wt2}^2\right)\right)\text{Cosh}[d\,kx]+$$

$$\left(\text{e1}\,\text{einf3}\left(-w^2+\text{wl3}^2\right)\left(-w^2+\text{wt2}^2\right)^2+\text{einf2}^2\left(-w^2+\text{wl2}^2\right)^2\left(-w^2+\text{wt3}^2\right)\right)\text{Sinh}[d\,kx]==0$$

(Depurar) In[ ]:=

```
expr1 = Expand[(einf2 einf3 (-w^2 + wl2^2) (-w^2 + wl3^2) (-w^2 + wt2^2) +
                expande factores
      e1 einf2 (-w^2 + wl2^2) (-w^2 + wt3^2) (-w^2 + wt2^2))];
expr2 = Expand[(e1 einf3 (-w^2 + wl3^2) (-w^2 + wt2^2)^2 + einf2^2 (-w^2 + wl2^2)^2 (-w^2 + wt3^2))];
        expande factores

newexpr1 = Collect[expr1, w]
           agrupa coeficientes
newexpr2 = Collect[expr2, w]
           agrupa coeficientes
```

(Depurar) Out[ ]=

$$(-e1\ einf2 - einf2\ einf3)\ w^6 + einf2\ einf3\ wl2^2\ wl3^2\ wt2^2 + e1\ einf2\ wl2^2\ wt2^2\ wt3^2 + w^4\ (e1\ einf2\ wl2^2 + einf2\ einf3\ wl2^2 + einf2\ einf3\ wl3^2 + e1\ einf2\ wt2^2 + einf2\ einf3\ wt2^2 + e1\ einf2\ wt3^2) + w^2\ (-einf2\ einf3\ wl2^2\ wl3^2 - e1\ einf2\ wl2^2\ wt2^2 - einf2\ einf3\ wl2^2\ wt2^2 - einf2\ einf3\ wl3^2\ wt2^2 - e1\ einf2\ wl2^2\ wt3^2 - e1\ einf2\ wt2^2\ wt3^2)$$

(Depurar) Out[ ]=

$$(-einf2^2 - e1\ einf3)\ w^6 + e1\ einf3\ wl3^2\ wt2^4 + einf2^2\ wl2^4\ wt3^2 + w^4\ (2\ einf2^2\ wl2^2 + e1\ einf3\ wl3^2 + 2\ e1\ einf3\ wt2^2 + einf2^2\ wt3^2) + w^2\ (-einf2^2\ wl2^4 - 2\ e1\ einf3\ wl3^2\ wt2^2 - e1\ einf3\ wt2^4 - 2\ einf2^2\ wl2^2\ wt3^2)$$

(Depurar) In[ ]:=

$$\mathbf{(-einf2^2 - e1\ einf3)\ w^6 + e1\ einf3\ wl3^2\ wt2^4 + einf2^2\ wl2^4\ wt3^2 + w^4\ (2\ einf2^2\ wl2^2 + e1\ einf3\ wl3^2 + 2\ e1\ einf3\ wt2^2 + einf2^2\ wt3^2) + w^2\ (-einf2^2\ wl2^4 - 2\ e1\ einf3\ wl3^2\ wt2^2 - e1\ einf3\ wt2^4 - 2\ einf2^2\ wl2^2\ wt3^2)}$$

(Depurar) Out[ ]=

$$(-einf2^2 - e1\ einf3)\ w^6 + e1\ einf3\ wl3^2\ wt2^4 + einf2^2\ wl2^4\ wt3^2 + w^4\ (2\ einf2^2\ wl2^2 + e1\ einf3\ wl3^2 + 2\ e1\ einf3\ wt2^2 + einf2^2\ wt3^2) + w^2\ (-einf2^2\ wl2^4 - 2\ e1\ einf3\ wl3^2\ wt2^2 - e1\ einf3\ wt2^4 - 2\ einf2^2\ wl2^2\ wt3^2)$$

(Depurar) In[•]:=

```
newequation = Collect[ newexpr1 * Cosh[kx * d] + newexpr2 * Sinh[kx * d], w];
              agrupa coeficientes  coseno hiperbólico  seno hiperbólico
ClearAll[e1, wl1, wl2, wt1, wt2, eq1, sols, Omega1, Omega2,
borra todo
 params, einf2, einf3, wl3, wt3, c, kx, w, d, arraysol, RaizOmega, Omega]
(*Coeficientes analíticos*)
Coefficient[newequation, w, 6]
coefficiente
Coefficient[newequation, w, 4]
coefficiente
Coefficient[newequation, w, 2]
coefficiente
Coefficient[newequation, w, 0]
coefficiente
eqcubica1 = u^3 * Coefficient[newequation, w, 6] + u^2 * Coefficient[newequation, w, 4] +
                 coefficiente                          coefficiente
   u * Coefficient[newequation, w, 2] + Coefficient[newequation, w, 0] == 0
       coefficiente                       coefficiente
```

(Depurar) Out[•]=

$$(-e1\ einf2 - einf2\ einf3)\ Cosh[d\ kx] + \left(-einf2^2 - e1\ einf3\right) Sinh[d\ kx]$$

(Depurar) Out[•]=

$$\left(e1\ einf2\ wl2^2 + einf2\ einf3\ wl2^2 + einf2\ einf3\ wl3^2 + e1\ einf2\ wt2^2 + einf2\ einf3\ wt2^2 + e1\ einf2\ wt3^2\right) Cosh[d\ kx] + \left(2\ einf2^2\ wl2^2 + e1\ einf3\ wl3^2 + 2\ e1\ einf3\ wt2^2 + einf2^2\ wt3^2\right) Sinh[d\ kx]$$

(Depurar) Out[•]=

$$\left(-einf2\ einf3\ wl2^2\ wl3^2 - e1\ einf2\ wl2^2\ wt2^2 - einf2\ einf3\ wl2^2\ wt2^2 - einf2\ einf3\ wl3^2\ wt2^2 - e1\ einf2\ wl2^2\ wt3^2 - e1\ einf2\ wt2^2\ wt3^2\right) Cosh[d\ kx] + \left(-einf2^2\ wl2^4 - 2\ e1\ einf3\ wl3^2\ wt2^2 - e1\ einf3\ wt2^4 - 2\ einf2^2\ wl2^2\ wt3^2\right) Sinh[d\ kx]$$

(Depurar) Out[•]=

$$einf2\ einf3\ wl2^2\ wl3^2\ wt2^2\ Cosh[d\ kx] + e1\ einf2\ wl2^2\ wt2^2\ wt3^2\ Cosh[d\ kx] + e1\ einf3\ wl3^2\ wt2^4\ Sinh[d\ kx] + einf2^2\ wl2^4\ wt3^2\ Sinh[d\ kx]$$

(Depurar) Out[•]=

$$\begin{aligned} &einf2\ einf3\ wl2^2\ wl3^2\ wt2^2\ Cosh[d\ kx] + e1\ einf2\ wl2^2\ wt2^2\ wt3^2\ Cosh[d\ kx] + \\ &\ e1\ einf3\ wl3^2\ wt2^4\ Sinh[d\ kx] + einf2^2\ wl2^4\ wt3^2\ Sinh[d\ kx] + \\ &\ u^3\ \left((-e1\ einf2 - einf2\ einf3)\ Cosh[d\ kx] + \left(-einf2^2 - e1\ einf3\right) Sinh[d\ kx]\right) + \\ &\ u^2\ \Big(\left(e1\ einf2\ wl2^2 + einf2\ einf3\ wl2^2 + einf2\ einf3\ wl3^2 + e1\ einf2\ wt2^2 + einf2\ einf3\ wt2^2 + e1\ einf2\ wt3^2\right) Cosh[d\ kx] + \\ &\qquad \left(2\ einf2^2\ wl2^2 + e1\ einf3\ wl3^2 + 2\ e1\ einf3\ wt2^2 + einf2^2\ wt3^2\right) Sinh[d\ kx]\Big) + \\ &\ u\ \Big(\left(-einf2\ einf3\ wl2^2\ wl3^2 - e1\ einf2\ wl2^2\ wt2^2 - einf2\ einf3\ wl2^2\ wt2^2 - einf2\ einf3\ wl3^2\ wt2^2 - e1\ einf2\ wl2^2\ wt3^2 - e1\ einf2\ wt2^2\ wt3^2\right) Cosh[d\ kx] + \\ &\qquad \left(-einf2^2\ wl2^4 - 2\ e1\ einf3\ wl3^2\ wt2^2 - e1\ einf3\ wt2^4 - 2\ einf2^2\ wl2^2\ wt3^2\right) Sinh[d\ kx]\Big) == 0 \end{aligned}$$

(Depurar) In[ ]:=

```
(*Constantes para coeficientes numéricos*)
ClearAll[e1, wl1, wl2, wt1, wt2, eq1, sols, Omega1, Omega2,
 params, einf2, einf3, wl3, wt3, c, kx, w, d, arraysol, RaizOmega, Omega]
einf3 = 2.1;
einf2 = 2.5;
wl3 = 0.15423634285090115; (*en eV*)
wt3 = 0.1304313767517267;
wl2 = 0.11604920973347545;
wt2 = 0.0957158011904306;
e1 = 1;
Coefficient[newequation, w, 6]
Coefficient[newequation, w, 4]
Coefficient[newequation, w, 2]
Coefficient[newequation, w, 0]
(*Ahora vamos a construir una ecuación cúbica con w^2→ u*)
eqcubica = u^3 * Coefficient[newequation, w, 6] + u^2 * Coefficient[newequation, w, 4] +
   u * Coefficient[newequation, w, 2] + Coefficient[newequation, w, 0] == 0
finalsolutions = FullSimplify[NSolve[{eqcubica}, {u}]];
Sort[finalsolutions];
finalsolutions[[1]]
finalsolutions[[2]]
finalsolutions[[3]]
```

(Depurar) Out[ ]=

$$-7.75\,\text{Cosh}[d\,kx] - 8.35\,\text{Sinh}[d\,kx]$$

(Depurar) Out[ ]=

$$0.342797\,\text{Cosh}[d\,kx] + 0.363105\,\text{Sinh}[d\,kx]$$

(Depurar) Out[ ]=

$$-0.0047448\,\text{Cosh}[d\,kx] - 0.00508909\,\text{Sinh}[d\,kx]$$

(Depurar) Out[ ]=

$$0.0000206569\,\text{Cosh}[d\,kx] + 0.0000234777\,\text{Sinh}[d\,kx]$$

(Depurar) Out[ ]=

$$0.0000206569\,\text{Cosh}[d\,kx] + u^3\,(-7.75\,\text{Cosh}[d\,kx] - 8.35\,\text{Sinh}[d\,kx]) + u\,(-0.0047448\,\text{Cosh}[d\,kx] - 0.00508909\,\text{Sinh}[d\,kx]) + u^2\,(0.342797\,\text{Cosh}[d\,kx] + 0.363105\,\text{Sinh}[d\,kx]) + 0.0000234777\,\text{Sinh}[d\,kx] == 0$$

(Depurar) Out[•]=

$$\left\{u \to \frac{1}{155.\,\text{Cosh}[d\,kx] + 167.\,\text{Sinh}[d\,kx]}\left(2.28531\,\text{Cosh}[d\,kx] + 2.4207\,\text{Sinh}[d\,kx] + \left(-2.91711\times 10^{49} - 1.1914\times 10^{50}\,\text{Cosh}[2\,d\,kx] - 1.21277\times 10^{50}\,\text{Sinh}[2\,d\,kx]\right)\Big/\left(2.9568\times 10^{150}\,\text{Cosh}[d\,kx] - 1.20439\times 10^{151}\,\text{Cosh}[3\,d\,kx] + 3.85646\times 10^{150}\,\text{Sinh}[d\,kx] - 1.20676\times 10^{151}\,\text{Sinh}[3\,d\,kx] + 1.\sqrt{\left(-3.58443\times 10^{300} - 3.05068\times 10^{300}\,\text{Cosh}[2\,d\,kx] - 2.08428\times 10^{302}\,\text{Cosh}[4\,d\,kx] - 2.81008\times 10^{301}\,\text{Cosh}[6\,d\,kx] - 4.0795\times 10^{300}\,\text{Sinh}[2\,d\,kx] - 2.08386\times 10^{302}\,\text{Sinh}[4\,d\,kx] - 2.81013\times 10^{301}\,\text{Sinh}[6\,d\,kx]\right)}\right)^{1/3} - 2.15551\times 10^{-51}\left(2.9568\times 10^{150}\,\text{Cosh}[d\,kx] - 1.20439\times 10^{151}\,\text{Cosh}[3\,d\,kx] + 3.85646\times 10^{150}\,\text{Sinh}[d\,kx] - 1.20676\times 10^{151}\,\text{Sinh}[3\,d\,kx] + 1.\sqrt{\left(-3.58443\times 10^{300} - 3.05068\times 10^{300}\,\text{Cosh}[2\,d\,kx] - 2.08428\times 10^{302}\,\text{Cosh}[4\,d\,kx] - 2.81008\times 10^{301}\,\text{Cosh}[6\,d\,kx] - 4.0795\times 10^{300}\,\text{Sinh}[2\,d\,kx] - 2.08386\times 10^{302}\,\text{Sinh}[4\,d\,kx] - 2.81013\times 10^{301}\,\text{Sinh}[6\,d\,kx]\right)}\right)^{1/3}\right)\right\}$$

(Depurar) Out[•]=

$$\left\{u \to \frac{1}{155.\,\text{Cosh}[d\,kx] + 167.\,\text{Sinh}[d\,kx]}\left(2.28531\,\text{Cosh}[d\,kx] + 2.4207\,\text{Sinh}[d\,kx] + \left(\left(1.45856\times 10^{49} + 2.52629\times 10^{49}\,i\right) + \left(5.95698\times 10^{49} + 1.03178\times 10^{50}\,i\right)\text{Cosh}[2\,d\,kx] + \left(6.06384\times 10^{49} + 1.05029\times 10^{50}\,i\right)\text{Sinh}[2\,d\,kx]\right)\Big/\left(2.9568\times 10^{150}\,\text{Cosh}[d\,kx] - 1.20439\times 10^{151}\,\text{Cosh}[3\,d\,kx] + 3.85646\times 10^{150}\,\text{Sinh}[d\,kx] - 1.20676\times 10^{151}\,\text{Sinh}[3\,d\,kx] + 1.\sqrt{\left(-3.58443\times 10^{300} - 3.05068\times 10^{300}\,\text{Cosh}[2\,d\,kx] - 2.08428\times 10^{302}\,\text{Cosh}[4\,d\,kx] - 2.81008\times 10^{301}\,\text{Cosh}[6\,d\,kx] - 4.0795\times 10^{300}\,\text{Sinh}[2\,d\,kx] - 2.08386\times 10^{302}\,\text{Sinh}[4\,d\,kx] - 2.81013\times 10^{301}\,\text{Sinh}[6\,d\,kx]\right)}\right)^{1/3} + \left(1.07775\times 10^{-51} - 1.86672\times 10^{-51}\,i\right)\left(2.9568\times 10^{150}\,\text{Cosh}[d\,kx] - 1.20439\times 10^{151}\,\text{Cosh}[3\,d\,kx] + 3.85646\times 10^{150}\,\text{Sinh}[d\,kx] - 1.20676\times 10^{151}\,\text{Sinh}[3\,d\,kx] + 1.\sqrt{\left(-3.58443\times 10^{300} - 3.05068\times 10^{300}\,\text{Cosh}[2\,d\,kx] - 2.08428\times 10^{302}\,\text{Cosh}[4\,d\,kx] - 2.81008\times 10^{301}\,\text{Cosh}[6\,d\,kx] - 4.0795\times 10^{300}\,\text{Sinh}[2\,d\,kx] - 2.08386\times 10^{302}\,\text{Sinh}[4\,d\,kx] - 2.81013\times 10^{301}\,\text{Sinh}[6\,d\,kx]\right)}\right)^{1/3}\right)\right\}$$

(Depurar) Out[•]=

$$\left\{u \to \frac{1}{155.\,\text{Cosh}[d\,kx] + 167.\,\text{Sinh}[d\,kx]}\left(2.28531\,\text{Cosh}[d\,kx] + 2.4207\,\text{Sinh}[d\,kx] + \left(\left(1.45856\times 10^{49} - 2.52629\times 10^{49}\,i\right) + \left(5.95698\times 10^{49} - 1.03178\times 10^{50}\,i\right)\text{Cosh}[2\,d\,kx] + \left(6.06384\times 10^{49} - 1.05029\times 10^{50}\,i\right)\text{Sinh}[2\,d\,kx]\right)\Big/ \left(2.9568\times 10^{150}\,\text{Cosh}[d\,kx] - 1.20439\times 10^{151}\,\text{Cosh}[3\,d\,kx] + 3.85646\times 10^{150}\,\text{Sinh}[d\,kx] - 1.20676\times 10^{151}\,\text{Sinh}[3\,d\,kx] + 1.\sqrt{\left(-3.58443\times 10^{300} - 3.05068\times 10^{300}\,\text{Cosh}[2\,d\,kx] - 2.08428\times 10^{302}\,\text{Cosh}[4\,d\,kx] - 2.81008\times 10^{301}\,\text{Cosh}[6\,d\,kx] - 4.0795\times 10^{300}\,\text{Sinh}[2\,d\,kx] - 2.08386\times 10^{302}\,\text{Sinh}[4\,d\,kx] - 2.81013\times 10^{301}\,\text{Sinh}[6\,d\,kx]\right)}\right)^{1/3} + \left(1.07775\times 10^{-51} + 1.86672\times 10^{-51}\,i\right)\left(2.9568\times 10^{150}\,\text{Cosh}[d\,kx] - 1.20439\times 10^{151}\,\text{Cosh}[3\,d\,kx] + 3.85646\times 10^{150}\,\text{Sinh}[d\,kx] - 1.20676\times 10^{151}\,\text{Sinh}[3\,d\,kx] + 1.\sqrt{\left(-3.58443\times 10^{300} - 3.05068\times 10^{300}\,\text{Cosh}[2\,d\,kx] - 2.08428\times 10^{302}\,\text{Cosh}[4\,d\,kx] - 2.81008\times 10^{301}\,\text{Cosh}[6\,d\,kx] - 4.0795\times 10^{300}\,\text{Sinh}[2\,d\,kx] - 2.08386\times 10^{302}\,\text{Sinh}[4\,d\,kx] - 2.81013\times 10^{301}\,\text{Sinh}[6\,d\,kx]\right)}\right)^{1/3}\right)\right\}$$

(Depurar) In[1]:=

```
ClearAll[RaizOmega1, Omega1, Omega2, Omega3, RaizOmega2, RaizOmega3, RaizOmega1, kx, d]
```
borra todo

```
d = 14 * 1239.84193;
```

```
RaizOmega1[kx_] := Re[(1 / (155. * Cosh[d * kx] + 167. * Sinh[d * kx])) *
```
parte real · coseno hiperbólico · seno hiperbólico

```
    (2.2853103716432344 * Cosh[d * kx] + 2.42069889281832 * Sinh[d * kx] +
```
coseno hiperbólico · seno hiperbólico

```
   (-2.9171103398636255*^49 - 1.1913958644439183*^50 * Cosh[2 * d * kx] -
```
coseno hiperbólico

```
         1.2127675903895865*^50 * Sinh[2 * d * kx]) /
```
seno hiperbólico

```
    (2.9568047169461383*^150 * Cosh[d * kx] - 1.2043868956753614*^151 * Cosh[3 * d * kx] +
```
coseno hiperbólico · coseno hiperbólico

```
       3.856461191035531*^150 * Sinh[d * kx] -
```
seno hiperbólico

```
     1.2067585480861233*^151 * Sinh[3 * d * kx] + 1. *
```
seno hiperbólico

```
         Sqrt[-3.584428570892242*^300 - 3.050683531422658*^300 * Cosh[2 * d * kx] -
```
raíz cuadrada · coseno hiperbólico

```
    2.0842757544993782*^302 * Cosh[4 * d * kx] - 2.810081855598155*^301 *
```
coseno hiperbólico

```
          Cosh[6 * d * kx] - 4.079502543539876*^300 * Sinh[2 * d * kx] -
```
coseno hiperbólico · seno hiperbólico

```
    2.0838628267045442*^302 * Sinh[4 * d * kx] -
```
seno hiperbólico

```
          2.8101343467294783*^301 * Sinh[6 * d * kx]]) ^ (1 / 3) -
```
seno hiperbólico

```
  2.155507501625528*^-51 *
      (2.9568047169461383*^150 * Cosh[d * kx] - 1.2043868956753614*^151 *
```
coseno hiperbólico

```
Cosh[3 * d * kx] + 3.856461191035531*^150 * Sinh[d * kx] -
1.2067585480861233*^151 * Sinh[3 * d * kx] + 1. *
Sqrt[-3.584428570892242*^300 - 3.050683531422658*^300 * Cosh[2 * d * kx] -
 2.0842757544993782*^302 * Cosh[4 * d * kx] - 2.810081855598155*^301 *
  Cosh[6 * d * kx] - 4.079502543539876*^300 * Sinh[2 * d * kx] -
 2.0838628267045442*^302 * Sinh[4 * d * kx] -
  2.8101343467294783*^301 * Sinh[6 * d * kx]])^(1 / 3))];
RaizOmega2[kx_] := Re[(1 / (155. * Cosh[d * kx] + 167. * Sinh[d * kx])) *
  (2.2853103716432344 * Cosh[d * kx] + 2.42069889281832 * Sinh[d * kx] +
   ((1.4585551699318128*^49 + 2.5262916599641577*^49 * I) +
     (5.956979322219592*^49 + 1.0317790845721548*^50 * I) * Cosh[2 * d * kx] +
     (6.063837951947933*^49 + 1.0502875421638227*^50 * I) * Sinh[2 * d * kx]) /
    (2.9568047169461383*^150 * Cosh[d * kx] - 1.2043868956753614*^151 * Cosh[3 * d * kx] +
     3.856461191035531*^150 * Sinh[d * kx] -
     1.2067585480861233*^151 * Sinh[3 * d * kx] + 1. *
      Sqrt[-3.584428570892242*^300 - 3.050683531422658*^300 * Cosh[2 * d * kx] -
       2.0842757544993782*^302 * Cosh[4 * d * kx] - 2.810081855598155*^301 *
        Cosh[6 * d * kx] - 4.079502543539876*^300 * Sinh[2 * d * kx] -
       2.0838628267045442*^302 * Sinh[4 * d * kx] -
        2.8101343467294783*^301 * Sinh[6 * d * kx]])^(1 / 3) +
     (1.077753750812764*^-51 - 1.8667242544556343*^-51 * I) * (2.9568047169461383*^150 *
        Cosh[d * kx] - 1.2043868956753614*^151 * Cosh[3 * d * kx] +
       3.856461191035531*^150 * Sinh[d * kx] - 1.2067585480861233*^151 * Sinh[3 * d * kx] +
       1. * Sqrt[-3.584428570892242*^300 - 3.050683531422658*^300 * Cosh[2 * d * kx] -
         2.0842757544993782*^302 * Cosh[4 * d * kx] -
         2.810081855598155*^301 * Cosh[6 * d * kx] - 4.079502543539876*^300 * Sinh[2 * d * kx] -
         2.0838628267045442*^302 * Sinh[4 * d * kx] -
```

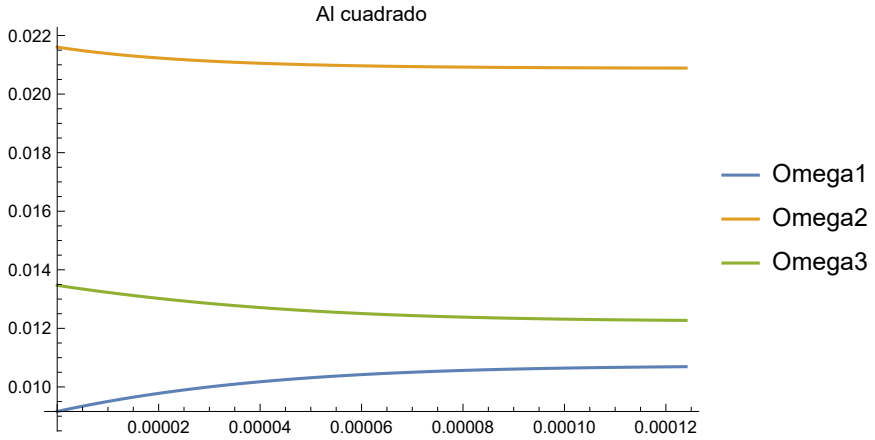
```
      2.8101343467294783*^301 * Sinh[6 * d * kx]])^(1 / 3))];
RaizOmega3[kx_] := Re[(1 / (155. * Cosh[d * kx] + 167. * Sinh[d * kx])) *
     (2.2853103716432344 * Cosh[d * kx] + 2.42069889281832 * Sinh[d * kx] +
   ((1.4585551699318128*^49 - 2.5262916599641577*^49 * I) +
          (5.956979322219592*^49 - 1.0317790845721548*^50 * I) * Cosh[2 * d * kx] +
    (6.063837951947933*^49 - 1.0502875421638227*^50 * I) * Sinh[2 * d * kx]) /
    (2.9568047169461383*^150 * Cosh[d * kx] - 1.2043868956753614*^151 * Cosh[3 * d * kx] +
            3.856461191035531*^150 * Sinh[d * kx] -
    1.2067585480861233*^151 * Sinh[3 * d * kx] + 1. *
              Sqrt[-3.584428570892242*^300 - 3.050683531422658*^300 * Cosh[2 * d * kx] -
      2.0842757544993782*^302 * Cosh[4 * d * kx] - 2.810081855598155*^301 *
                 Cosh[6 * d * kx] - 4.079502543539876*^300 * Sinh[2 * d * kx] -
      2.0838628267045442*^302 * Sinh[4 * d * kx] -
                2.8101343467294783*^301 * Sinh[6 * d * kx]])^(1 / 3) +
   (1.077753750812764*^-51 + 1.8667242544556343*^-51 * I) * (2.9568047169461383*^150 *
              Cosh[d * kx] - 1.2043868956753614*^151 * Cosh[3 * d * kx] +
    3.856461191035531*^150 * Sinh[d * kx] - 1.2067585480861233*^151 * Sinh[3 * d * kx] +
    1. * Sqrt[-3.584428570892242*^300 - 3.050683531422658*^300 * Cosh[2 * d * kx] -
                2.0842757544993782*^302 * Cosh[4 * d * kx] -
      2.810081855598155*^301 * Cosh[6 * d * kx] - 4.079502543539876*^300 * Sinh[2 * d * kx] -
                2.0838628267045442*^302 * Sinh[4 * d * kx] -
      2.8101343467294783*^301 * Sinh[6 * d * kx]])^(1 / 3))];
Omega1[kx_] := Sqrt[RaizOmega1[kx]];
Omega2[kx_] := Sqrt[RaizOmega2[kx]];
Omega3[kx_] := Sqrt[RaizOmega3[kx]];
Plot[{RaizOmega1[kx], RaizOmega2[kx], RaizOmega3[kx]},
  {kx, 0, 0.00012398419843320026},
```

```
 PlotLegends → {"Omega1", "Omega2", "Omega3"}, PlotLabel → "Al cuadrado"]
```
leyendas de representación etiqueta de representación

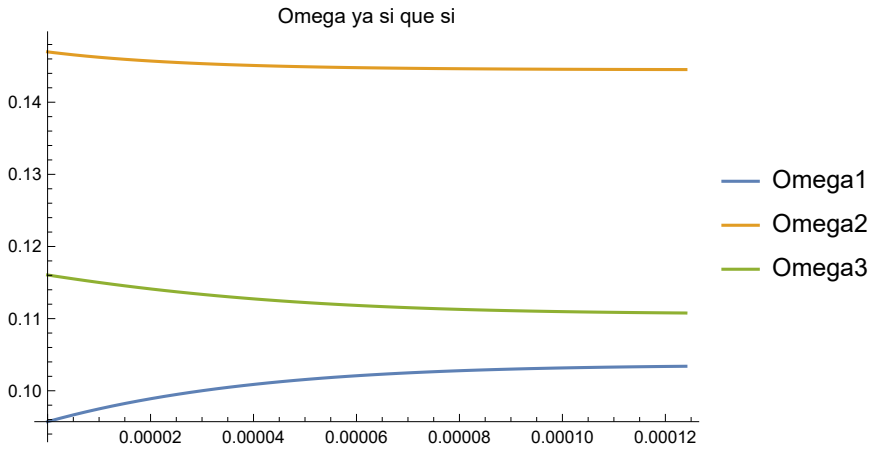
```
Plot[{Omega1[kx], Omega2[kx], Omega3[kx]}, {kx, 0, 0.00012398419843320026},
```
representación gráfica

```
 PlotLegends → {"Omega1", "Omega2", "Omega3"}, PlotLabel → "Omega ya si que si"]
```
leyendas de representación etiqueta de representación

(Depurar) Out[9]=



(Depurar) Out[10]=



```
(*Comprobación de las asíntotas cuando tiende a 0*)
Omega1[kx = 0]
wt2
Omega3[kx = 0]
wl2
Omega2[kx = 0]
Sqrt[(einf3 * wl3^2 + e1 * wt3^2) / (e1 + einf3)]
(**)
Omega3[kx = 10]
Sqrt[einf2 * wl2^2 + e1 * wt2^2] / Sqrt[e1 + einf2]
Omega1[kx = 10]
Sqrt[(einf3 * (wl3^2 + wt2^2) + einf2 * (wl2^2 + wt3^2) -
     Sqrt[-4 * (einf2 + einf3) * (einf3 * wl3^2 * wt2^2 + einf2 * wl2^2 * wt3^2) +
     (einf3 * (wl3^2 + wt2^2) + einf2 * (wl2^2 + wt3^2))^2]) / (2 * (einf2 + einf3))]
Omega2[kx = 10]
Sqrt[(einf3 * (wl3^2 + wt2^2) + einf2 * (wl2^2 + wt3^2) +
      Sqrt[-4 * (einf2 + einf3) * (einf3 * wl3^2 * wt2^2 + einf2 * wl2^2 * wt3^2) +
     (einf3 * (wl3^2 + wt2^2) + einf2 * (wl2^2 + wt3^2))^2]) / (einf2 + einf3)] /
 Sqrt[2]
```

(Depurar) Out[ ]=
0.0957158

(Depurar) Out[ ]=
0.0957158

(Depurar) Out[ ]=
0.116049

(Depurar) Out[ ]=
0.116049

(Depurar) Out[ ]=
0.146979

(Depurar) Out[ ]=
0.146979

(Depurar) Out[ ]=
0.1106217006570942

(Depurar) Out[ ]=
0.110622

(Depurar) Out[ ]=
0.1035836483584822

(Depurar) Out[ ]=
0.103584

(Depurar) Out[ ]=
0.1444924702129221

(Depurar) Out[ ]=

```
0.144492
```

```
(*Para ver que hay errores numéricos en las partes
 imaginarias de las soluciones de un orden muy pequeño*)
ClearAll[RaizOmega1, Omega1, Omega2, Omega3, RaizOmega2, RaizOmega3, RaizOmega1, kx, d]
```
borra todo

```
d = 12 * 1239.84193;
```

```
RaizOmega1[kx_] := Im[(1 / (155. * Cosh[d * kx] + 167. * Sinh[d * kx])) *
```
parte imaginaria coseno hiperbólico seno hiperbólico

```
    (2.2853103716432344 * Cosh[d * kx] + 2.42069889281832 * Sinh[d * kx] +
```
coseno hiperbólico seno hiperbólico

```
   (-2.9171103398636255*^49 - 1.1913958644439183*^50 * Cosh[2 * d * kx] -
```
coseno hiperbólico

```
         1.2127675903895865*^50 * Sinh[2 * d * kx]) /
```
seno hiperbólico

```
   (2.9568047169461383*^150 * Cosh[d * kx] - 1.2043868956753614*^151 * Cosh[3 * d * kx] +
```
coseno hiperbólico coseno hiperbólico

```
         3.856461191035531*^150 * Sinh[d * kx] -
```
seno hiperbólico

```
    1.2067585480861233*^151 * Sinh[3 * d * kx] + 1. *
```
seno hiperbólico

```
           Sqrt[-3.584428570892242*^300 - 3.050683531422658*^300 * Cosh[2 * d * kx] -
```
raíz cuadrada coseno hiperbólico

```
     2.0842757544993782*^302 * Cosh[4 * d * kx] - 2.810081855598155*^301 *
```
coseno hiperbólico

```
           Cosh[6 * d * kx] - 4.079502543539876*^300 * Sinh[2 * d * kx] -
```
coseno hiperbólico seno hiperbólico

```
     2.0838628267045442*^302 * Sinh[4 * d * kx] -
```
seno hiperbólico

```
           2.8101343467294783*^301 * Sinh[6 * d * kx]]) ^ (1 / 3) -
```
seno hiperbólico

```
  2.155507501625528*^-51 *
      (2.9568047169461383*^150 * Cosh[d * kx] - 1.2043868956753614*^151 *
```
coseno hiperbólico

```
           Cosh[3 * d * kx] + 3.856461191035531*^150 * Sinh[d * kx] -
```
coseno hiperbólico seno hiperbólico

```
    1.2067585480861233*^151 * Sinh[3 * d * kx] + 1. *
```
seno hiperbólico

```
           Sqrt[-3.584428570892242*^300 - 3.050683531422658*^300 * Cosh[2 * d * kx] -
```
raíz cuadrada coseno hiperbólico

```
     2.0842757544993782*^302 * Cosh[4 * d * kx] - 2.810081855598155*^301 *
```
coseno hiperbólico

```
           Cosh[6 * d * kx] - 4.079502543539876*^300 * Sinh[2 * d * kx] -
```
coseno hiperbólico seno hiperbólico

```
     2.0838628267045442*^302 * Sinh[4 * d * kx] -
```
seno hiperbólico

```
           2.8101343467294783*^301 * Sinh[6 * d * kx]]) ^ (1 / 3))];
```
seno hiperbólico

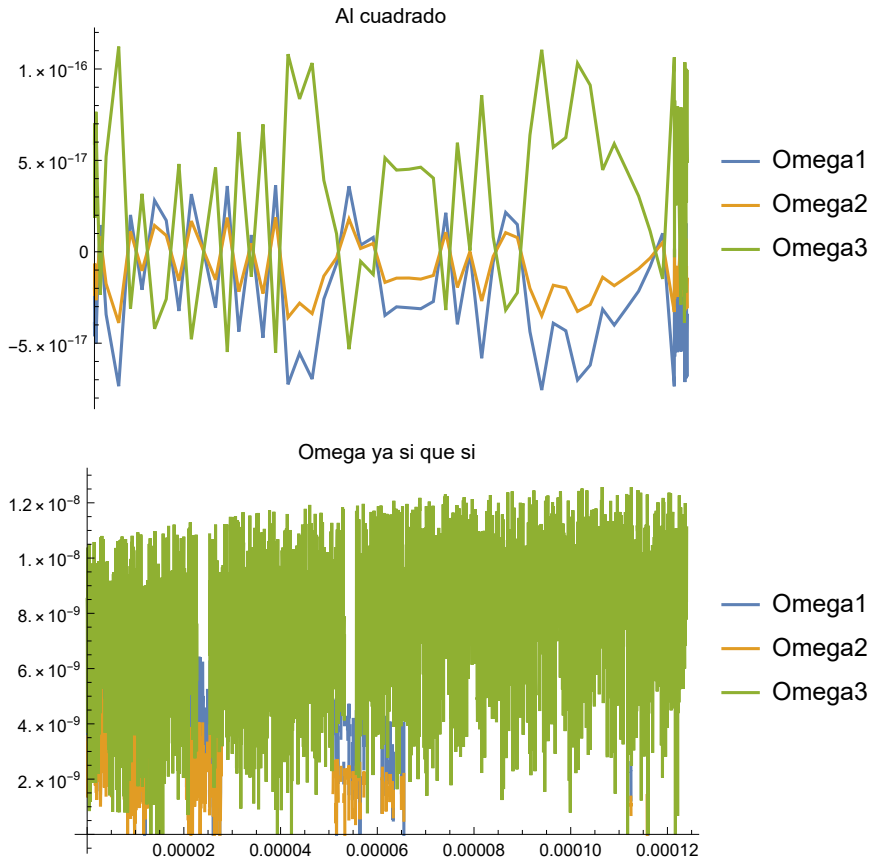
```
RaizOmega2[kx_] := Im[(1 / (155. * Cosh[d * kx] + 167. * Sinh[d * kx])) *
```
parte imaginaria coseno hiperbólico seno hiperbólico

```
(2.2853103716432344 * Cosh[d * kx] + 2.42069889281832 * Sinh[d * kx] +
((1.4585551699318128*^49 + 2.5262916599641577*^49 * I) +
      (5.956979322219592*^49 + 1.0317790845721548*^50 * I) * Cosh[2 * d * kx] +
   (6.063837951947933*^49 + 1.0502875421638227*^50 * I) * Sinh[2 * d * kx]) /
  (2.9568047169461383*^150 * Cosh[d * kx] - 1.2043868956753614*^151 * Cosh[3 * d * kx] +
      3.856461191035531*^150 * Sinh[d * kx] -
   1.2067585480861233*^151 * Sinh[3 * d * kx] + 1. *
      Sqrt[-3.584428570892242*^300 - 3.050683531422658*^300 * Cosh[2 * d * kx] -
   2.0842757544993782*^302 * Cosh[4 * d * kx] - 2.810081855598155*^301 *
      Cosh[6 * d * kx] - 4.079502543539876*^300 * Sinh[2 * d * kx] -
   2.0838628267045442*^302 * Sinh[4 * d * kx] -
      2.8101343467294783*^301 * Sinh[6 * d * kx]]) ^ (1 / 3) +
 (1.077753750812764*^-51 - 1.8667242544556343*^-51 * I) * (2.9568047169461383*^150 *
      Cosh[d * kx] - 1.2043868956753614*^151 * Cosh[3 * d * kx] +
   3.856461191035531*^150 * Sinh[d * kx] - 1.2067585480861233*^151 * Sinh[3 * d * kx] +
   1. * Sqrt[-3.584428570892242*^300 - 3.050683531422658*^300 * Cosh[2 * d * kx] -
      2.0842757544993782*^302 * Cosh[4 * d * kx] -
   2.810081855598155*^301 * Cosh[6 * d * kx] - 4.079502543539876*^300 * Sinh[2 * d * kx] -
      2.0838628267045442*^302 * Sinh[4 * d * kx] -
   2.8101343467294783*^301 * Sinh[6 * d * kx]]) ^ (1 / 3))];

RaizOmega3[kx_] := Im[(1 / (155. * Cosh[d * kx] + 167. * Sinh[d * kx])) *
  (2.2853103716432344 * Cosh[d * kx] + 2.42069889281832 * Sinh[d * kx] +
((1.4585551699318128*^49 - 2.5262916599641577*^49 * I) +
      (5.956979322219592*^49 - 1.0317790845721548*^50 * I) * Cosh[2 * d * kx] +
   (6.063837951947933*^49 - 1.0502875421638227*^50 * I) * Sinh[2 * d * kx]) /
  (2.9568047169461383*^150 * Cosh[d * kx] - 1.2043868956753614*^151 * Cosh[3 * d * kx] +
      3.856461191035531*^150 * Sinh[d * kx] -
```

```
1.2067585480861233*^151 * Sinh[3 * d * kx] + 1. *
                           seno hiperbólico
        Sqrt[-3.584428570892242*^300 - 3.050683531422658*^300 * Cosh[2 * d * kx] -
             raíz cuadrada                                      coseno hiperbólico
  2.0842757544993782*^302 * Cosh[4 * d * kx] - 2.810081855598155*^301 *
                            coseno hiperbólico
          Cosh[6 * d * kx] - 4.079502543539876*^300 * Sinh[2 * d * kx] -
          coseno hiperbólico                          seno hiperbólico
  2.0838628267045442*^302 * Sinh[4 * d * kx] -
                            seno hiperbólico
        2.8101343467294783*^301 * Sinh[6 * d * kx]]) ^ (1 / 3) +
                                  seno hiperbólico
(1.077753750812764*^-51 + 1.8667242544556343*^-51 * I) * (2.9568047169461383*^150 *
                                                          número i
        Cosh[d * kx] - 1.2043868956753614*^151 * Cosh[3 * d * kx] +
        coseno hiperbólico                       coseno hiperbólico
  3.856461191035531*^150 * Sinh[d * kx] - 1.2067585480861233*^151 * Sinh[3 * d * kx] +
                           seno hiperbólico                         seno hiperbólico
  1. * Sqrt[-3.584428570892242*^300 - 3.050683531422658*^300 * Cosh[2 * d * kx] -
       raíz cuadrada                                          coseno hiperbólico
        2.0842757544993782*^302 * Cosh[4 * d * kx] -
                                  coseno hiperbólico
  2.810081855598155*^301 * Cosh[6 * d * kx] - 4.079502543539876*^300 * Sinh[2 * d * kx] -
                           coseno hiperbólico                          seno hiperbólico
        2.0838628267045442*^302 * Sinh[4 * d * kx] -
                                  seno hiperbólico
  2.8101343467294783*^301 * Sinh[6 * d * kx]]) ^ (1 / 3))];
                            seno hiperbólico
Omega1[kx_] := Sqrt[RaizOmega1[kx]];
               raíz cuadrada
Omega2[kx_] := Sqrt[RaizOmega2[kx]];
               raíz cuadrada
Omega3[kx_] := Sqrt[RaizOmega3[kx]];
               raíz cuadrada
Plot[{RaizOmega1[kx], RaizOmega2[kx], RaizOmega3[kx]},
representación gráfica
 {kx, 0, 0.000012398419843320026},
 PlotLegends → {"Omega1", "Omega2", "Omega3"}, PlotLabel → "Al cuadrado"]
 leyendas de representación                    etiqueta de representación
Plot[{Omega1[kx], Omega2[kx], Omega3[kx]}, {kx, 0, 0.000123984198433200 26},
representación gráfica
 PlotLegends → {"Omega1", "Omega2", "Omega3"}, PlotLabel → "Omega ya si que si"]
 leyendas de representación                    etiqueta de representación
```

Out[ ]=




```
(*Cuando tiende a infinito*)
```

(Depurar) In[ ]:=

```
ClearAll[e1, wl1, wl2, wt1, wt2, eq1, sols, Omega1, Omega2,
 params, einf2, einf3, wl3, wt3, c, kx, w, d, arraysol, RaizOmega, Omega]
FullSimplify[Solve[(einf2 einf3 (-w^2 + wl2^2) (-w^2 + wl3^2) (-w^2 + wt2^2) +
    e1 einf2 (-w^2 + wl2^2) (-w^2 + wt3^2) (-w^2 + wt2^2)) +
   (e1 einf3 (-w^2 + wl3^2) (-w^2 + wt2^2)^2 + einf2^2 (-w^2 + wl2^2)^2 (-w^2 + wt3^2)) == 0, w]]
```

(Depurar) Out[ ]=

$$\left\{\left\{w \to -\frac{\sqrt{\text{einf2}\,\text{wl2}^2 + \text{e1}\,\text{wt2}^2}}{\sqrt{\text{e1} + \text{einf2}}}\right\}, \left\{w \to \frac{\sqrt{\text{einf2}\,\text{wl2}^2 + \text{e1}\,\text{wt2}^2}}{\sqrt{\text{e1} + \text{einf2}}}\right\},\right.$$

$$\left\{w \to -\frac{1}{\sqrt{2}}\left(\sqrt{\left(\frac{1}{\text{einf2} + \text{einf3}}\left(\text{einf3}\left(\text{wl3}^2 + \text{wt2}^2\right) + \text{einf2}\left(\text{wl2}^2 + \text{wt3}^2\right) - \right.\right.\right.\right.$$
$$\left.\left.\left.\sqrt{\left(-4\left(\text{einf2} + \text{einf3}\right)\left(\text{einf3}\,\text{wl3}^2\,\text{wt2}^2 + \text{einf2}\,\text{wl2}^2\,\text{wt3}^2\right) + \left(\text{einf3}\left(\text{wl3}^2 + \text{wt2}^2\right) + \text{einf2}\left(\text{wl2}^2 + \text{wt3}^2\right)\right)^2\right)}\right)\right)\right\},$$

$$\left\{w \to \frac{1}{\sqrt{2}}\left(\sqrt{\left(\frac{1}{\text{einf2} + \text{einf3}}\left(\text{einf3}\left(\text{wl3}^2 + \text{wt2}^2\right) + \text{einf2}\left(\text{wl2}^2 + \text{wt3}^2\right) - \right.\right.\right.\right.$$
$$\left.\left.\left.\sqrt{\left(-4\left(\text{einf2} + \text{einf3}\right)\left(\text{einf3}\,\text{wl3}^2\,\text{wt2}^2 + \text{einf2}\,\text{wl2}^2\,\text{wt3}^2\right) + \left(\text{einf3}\left(\text{wl3}^2 + \text{wt2}^2\right) + \text{einf2}\left(\text{wl2}^2 + \text{wt3}^2\right)\right)^2\right)}\right)\right)\right\},$$

$$\left\{w \to -\frac{1}{\sqrt{2}}\left(\sqrt{\left(\frac{1}{\text{einf2} + \text{einf3}}\left(\text{einf3}\left(\text{wl3}^2 + \text{wt2}^2\right) + \text{einf2}\left(\text{wl2}^2 + \text{wt3}^2\right) + \right.\right.\right.\right.$$
$$\left.\left.\left.\sqrt{\left(-4\left(\text{einf2} + \text{einf3}\right)\left(\text{einf3}\,\text{wl3}^2\,\text{wt2}^2 + \text{einf2}\,\text{wl2}^2\,\text{wt3}^2\right) + \left(\text{einf3}\left(\text{wl3}^2 + \text{wt2}^2\right) + \text{einf2}\left(\text{wl2}^2 + \text{wt3}^2\right)\right)^2\right)}\right)\right)\right\},$$

$$\left.\left\{w \to \frac{1}{\sqrt{2}}\left(\sqrt{\left(\frac{1}{\text{einf2} + \text{einf3}}\left(\text{einf3}\left(\text{wl3}^2 + \text{wt2}^2\right) + \text{einf2}\left(\text{wl2}^2 + \text{wt3}^2\right) + \right.\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\sqrt{\left(-4\left(\text{einf2} + \text{einf3}\right)\left(\text{einf3}\,\text{wl3}^2\,\text{wt2}^2 + \text{einf2}\,\text{wl2}^2\,\text{wt3}^2\right) + \left(\text{einf3}\left(\text{wl3}^2 + \text{wt2}^2\right) + \text{einf2}\left(\text{wl2}^2 + \text{wt3}^2\right)\right)^2\right)}\right)\right)\right\}\right\}$$

In[ ]:=

In[ ]:=

In[ ]:=