Contents

1 Review of Propositional Logic

Task: Recall enough propositional logic to see how it matches up with set theory.

Definition: A <u>proposition</u> is any declarative sentence that is either true or false.

1.1 Connectives

	Connectives		Notation in Maths
and	$\wedge$		
or	$\vee$	"Inclusive or"	
not	$\neg$	Sometimes denoted $\sim$	
implies	$\rightarrow$	if/then; called implication	$\Rightarrow$
if and only if	$\leftrightarrow$	Called equivalence	$\Leftrightarrow$

1.1.1 Truth Table of the Connectives

Let P, Q be propositions:

P	Q	P$\wedge$Q
F	F	F
F	T	F
T	F	F
T	T	T

P	Q	P$\vee$Q
F	F	F
F	T	T
T	F	T
T	T	T

P	$\neg$P
F	T
T	F

P	Q	P$\rightarrow$Q
F	F	T
F	T	T
T	F	F
T	T	T

P	Q	P$\leftrightarrow$Q
F	F	T
F	T	F
T	F	F
T	T	T

Priority of the Connectives

Highest to Lowest: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

1.2 Important Tautologies

$$\begin{array}{rcll}
(P \rightarrow Q) & \leftrightarrow & (\neg P \vee Q) & \\
(P \leftrightarrow Q) & \leftrightarrow & [(P \rightarrow Q) \wedge (Q \rightarrow P)] & \\
\neg(P \wedge Q) & \leftrightarrow & (\neg P \vee \neg Q) & \multirow{2}{*}{$\left.\right\}$ \text{De Morgan Laws}} \\
\neg(P \vee Q) & \leftrightarrow & (\neg P \wedge \neg Q) &
\end{array}$$

As a result, $\neg$ and $\vee$ together can be used to represent all of $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$.

Less obvious: One connective called the sheffer stroke P|Q (which stands for "not both P and Q" or "P nand Q") can be used to represent all of $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ since $\neg$P $\leftrightarrow$ P|P and P $\vee$ Q $\leftrightarrow$ (P|P) | (Q|Q).

Recall if P$\rightarrow$Q is a given implication, Q$\rightarrow$P is called the <u>converse</u> or P$\rightarrow$Q. $\neg$Q $\rightarrow$ $\neg$P.

1.3 Indirect Arguments/Proofs by Contradiction/Reductio as absurdum

Based on the tautology (P→Q) ↔ (¬Q → ¬P)

Example: Famous argument that $\sqrt{2}$ is irrational.

Proof:

Suppose $\sqrt{2}$ is rational, then it can be expressed is fraction form $\frac{a}{b}$. Let us **assume** that our fraction is in the lowest term, **i.e.** their only common divisor is 1.

Then,

$$\sqrt{2} = \frac{a}{b}$$

Squaring both sides, we have

$$2 = \frac{a^2}{b^2}$$

Multiplying both sides by B^2 yields

$$2b^2 = a^2$$

Since a^2b^2, we can conclude that a^2 is even because whatever the value of b^2 has to be multiplied by 2. If a^2 is even, then a is also even. Since a is even, no matter what the value of a is, we can always find an integer that if we divide a by 2, it is equal to that integer. If we let that integer be k, then $\frac{a}{b} = k$ which means that $a = 2k$.

Substituting the value of $2k$ to a, we have $2b^2 = (2k)^2$ which means that $2b^2 = 4k^2$. dividing both sides by 2 we have $b^2 = 2k^2$. That means that the value b^2 is even, since whatever the value of k you have to multiply it by 2. Again, is b^2 is even, then b is even.

This implies that both a and b are even, which means that both the numerator and the denominator of our fraction are divisible by 2. This contradicts our **assumption** that $\frac{a}{b}$ has no common divisor except 1. Since we found a contradiction, our assumption is, therefore, false. Hence the theorem is true.

qed