SOLUTION ASSIGNMENT 3 CMPT300 SUMMER 2015

1. Consider Smaug's world as described in assignment 2.

Solution code posted separately

2. Consider a series of processes that are sharing a variety of resources. To avoid or recover from deadlocks it is necessary to be able to determine if a system is likely to deadlock. This decision may be made based on the 'state' of the system and the Banker's algorithm.
   a) **[10 points]** Consider the system with the state given below. In point form, explain what the terms in these equations represent in terms of the system resources and the processes presently running in the system.

      If process i is will need 4 resources of type j then Rij = 4. The maximum number of resources needed by process I during its execution (not at all times)

      NOTE: THE MATRIX Rij IN YOUR TEXT IS R-C THE NUMBER OF RESOURCES OF EACH TYPE EACH PROCESS MAY REQUEST IN ADDITION TO THE RESOURCES OF EACH TYPE IT ALREADY HOLDS.

      If process i is using 2 resources of type j then Cij = 2. Process i is presently using these 2 resources.

      The total number of resources of a particular type i that exist in the system

      The total number of unallocated resources of a particular type i in the system
   b) **[20 points]** Use the banker's algorithm to determine if the state given in C is a safe state for this system. For this state of the system can you state that any of the processes will deadlock. If you can state that processes will deadlock which process will deadlock?

$$R = \begin{bmatrix} 5 & 4 & 1 & 4 \\ 6 & 3 & 3 & 1 \\ 2 & 3 & 0 & 2 \\ 1 & 4 & 1 & 3 \\ 3 & 0 & 2 & 4 \\ 2 & 7 & 1 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 2 & 2 & 0 & 1 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \quad A = \begin{bmatrix} 0 & 1 & 2 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 6 & 8 & 4 & 6 \end{bmatrix}$$

$$R - C = \begin{bmatrix} 4 & 3 & 0 & 2 \\ 5 & 2 & 3 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 0 & 1 & 3 \\ 1 & 6 & 1 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 0 & 1 & 2 & 1 \end{bmatrix}$$

Numbers shown in read indicate resources that may be oversubscribed

[4 3 0 2] > A cannot run

[5 2 3 1] > A cannot run

[0 1 0 1] <= A runs to completion

When this third process runs to completion A becomes

$A = [2 \quad 3 \quad 2 \quad 2]$ after adding the third row of C

[4 3 0 2] > A cannot run

[5 2 3 1] > A cannot run

[1 2 1 2] <= A runs to completion

When this fourth process runs to completion A becomes

$A = [2 \quad 5 \quad 2 \quad 3]$ after adding the fourth row of C

[4 3 0 2] > A cannot run

[5 2 3 1] > A cannot run

[2 0 1 3] > A can now run to completion

When this fifth process runs to completion A becomes

$A = [3 \quad 5 \quad 3 \quad 4]$ after adding the fifth row of C

[4 3 0 2] > A cannot run

[5 2 3 1] > A cannot run

[1 6 1 1] > A cannot run

The remaining processes (1^{st} 2^{nd} 6^{th}) may deadlock

The state is unsafe

3. Consider a hard disk. The disk has N=128 tracks and 24 sectors per track. The seek time per track is 1 ms. The rotation speed of the disk is 7200rpm.
   a) [10 points] Assume that once the head is in position above the appropriate track you need to wait an average of half a rotation of the disk before the head can read your data. What are the maximum and minimum rotational latency of this disk? What is the average rotational latency of this disk?
      - ***Rotational latency is the time it takes the disk to rotate to bring the desired track under the read or write head.***
      - ***If the disk rotates at 7200rpm, 1 rotation takes 1/7200 minutes or***
      - ***[1/(7200/60)]s =[1/120]s = 8.33 milliseconds. Thus the maximum rotational latency, one rotation of the disk to get back to a point on the track that was just missed when the head finished its seek, is 8.33 milliseconds.***
      - ***The minimum rotational latency is 0s, the read or write head finishes its seek exactly synchronized to read the first bit of data that passes under the read head.***
      - ***The average rotational latency, assuming that it is equally likely to have any rotational latency 0<=rotational latency<=8.33 milliseconds, is 4.17 milliseconds***
   b) The following sequence of disk access requests have been received by the computer systems. The numbers are the track numbers. Numbering starts at 0 (the centremost track on the disk).
      24, 68, 3, 17, 57, 49, 91, 121
      Determine the actual average seek length for this particular series of accesses based on each of the following algorithms. Assume the read head starts at track 48
      i. **[10 points]** Shortest seek first (move the fewest number of tracks to the next entry)

ii. **[10 points]** Circular Scan (towards the outside of the disk first, direction of increasing track number)

iii. **[10 points]** LOOK algorithm (towards the outside of the disk first, direction of increasing track number

Shortest seek first : sequence of accesses **49, 57, 68, 91, 121, 24, 17, 3**

- ***48 to 49 seek length 1***
- ***49 to 57 seek length 8***
- ***57 to 68 seek length 11***
- ***68 to 91 seek length 23***
- ***91 to 121 seek length 30***
- ***121 to 24 seek length 97***
- ***24 to 17 seek length 7***
- ***17 to 3 seek length 14***

Average seek length (1+8+11+23+30+97+7+14)/8 = 191/8 =23.875

Circular Scan: sequence of accesses **49, 57, 68, 91, 121, 3, 17, 24**

- ***48 to 49 seek length 1***
- ***49 to 57 seek length 8***
- ***57 to 68 seek length 11***
- ***68 to 91 seek length 23***
- ***91 to 121 seek length 30***
- ***121 to 127 seek length 6***
- ***127 to 0 seek length 127 (not included in average)***
- ***0 to 3 seek length 3***
- ***3 to 17 seek length 14***
- ***17 to 24 seek length 7***

  Average seek length (1+8+11+23+30+6+3+14+7)/8
  =103/8= 12.875

LOOK: sequence of accesses **49, 57, 68, 91, 121, 24, 17, 3**

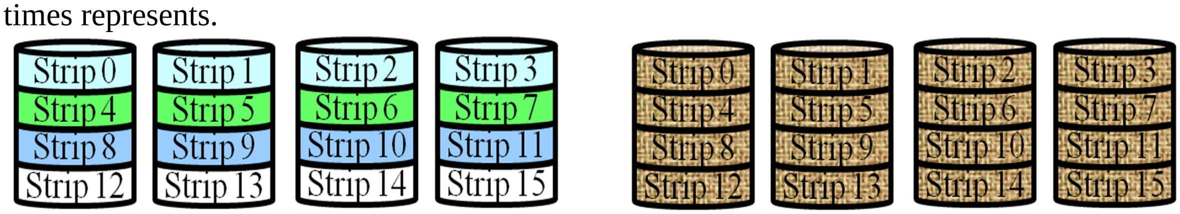
- ***48 to 49 seek length 1***
- ***49 to 57 seek length 8***
- ***57 to 68 seek length 11***
- ***68 to 91 seek length 23***
- ***91 to 121 seek length 30***
- ***121 to 24 seek length 97***
- ***24 to 17 seek length 7***
- ***17 to 3 seek length 14***

Average seek length (1+8+11+23+30+97+7+14)/8 =191/8= 23.875

c) [20 points] Assume that we have 8 of these disks, and we are using RAID 1 to combine these 8 disks. For this particular RAID system 1strip holds 1 track of data. The data for a particular file is loaded into tracks in the following order. Track 0 disk 0, Track 0 disk 1, Track 0 disk 2, Track 0 disk 3, Track 1 disk 0, Track 1 disk 1, … , Track 1 disk 3, …, Track 128 disk 0, …, Track 128 disk 3 and the same data is simultaneously loaded onto Track 0 disk 4, Track 0 disk 5, Track 0 disk 6, Track 0 disk 7, Track 1 disk 4, Track 1 disk 5, … , Track 1 disk 7, …, Track 128 disk 4, …, and Track 128 disk 7. (This is illustrated in the diagram below). The second 4 four disks are used to create a second copy of the data. Assume that the raid controller is capable of simultaneously accessing all disks. The raid controller contains eight buffers that each hold one track of data. Each of these buffers is reserved for the use of one of the eight disks. Assume that

i. Transferring one track of data from the disk, to one of the RAID controller's internal buffers takes 0.4 seconds
ii. A track of data must be completely loaded into the buffer of the RAID controller before it can be transferred to memory using the DMA
iii. All RAID controller buffers must be emptied (transferred to memory using the DMA) before any RAID controller buffer can be refilled.
iv. The DMA transfer rate (RAID controller buffer to memory is 4 GBytes per second (1 GByte is 2^{30} bytes)
v. The DMA setup time is negligible.
vi. One DMA transfer is used to transfer the contents of one buffer to memory
vii. the data being read is contiguous beginning at the start of Track 0 on platter 0 and filling a total of 14 tracks.
viii. Each track holds 60 Mbytes of data.

How long does it take to transfer the file from the disk to memory? How long would it take to transfer the file if the data were stored on track 0 to track 14 on disk 0 and disk 4? How would your answer differ if you were reading data from memory and writing it to the RAID 1 disks? When you answer each of these questions show step by step how you determined the length of time each transfer would take. Indicate in words what each quantity included in your calculated times represents.



The length of time taken to transfer one track is

Amount of data on one track/DMA transfer rate + DMA setup time

($60*2^{20}$)bytes /($4 * 2^{30}$)bytes per second + 0 (negligible) = 15/1024=0.01465

Tracks 0 on disks 0-3 can be read to the buffer in the RAID controller simultaneously which will take 0.4s read time.

Because we have two copies of the data, one on disks 0-3 and the other on disks 4-7 it is possible to simultaneously read tracks 4-7 from tracks 1 of disks 4-7. This will occur during the same 0.4s read time

*The 8 tracks of data must then be transferred to memory, it takes 0.01465s to transfer each track, or a total of **0.117s** transfer time*
After transferring this data from the RAID controller to the memory track 2 on disks 0-3 and track 3 on disks 4 and 5 can be read simultaneously, which will take 0.4 s read time.
Again, the data must then be transferred to memory. This time there are 6 tracks of data to transfer. It takes 0.01465s to transfer each track, or a total of 0.08789
Total read time (transfer from disk to RAID controller) is =0.8 s,
*Total transfer time is **0.117+0.08789=0.205s.***
*Thus it takes **1.05s** to transfer the data from disk to memory using the RAID 1.*
If instead we read tracks 0 to 14 on disk 0 (duplicated on disk 4) only 2 reads can be done simultaneously. We can read track 0 from disk 0 while reading track 1 from disk 4,
*then we can read track 2 from disk 0 and track 3 from disk 4, and so on. It take a total of 7 pairs of reads. Thus the file would take 7*0.4 =2.8s to transfer from disk to disk controller buffer. Adding the total transfer time of **0.205s** give a total transfer time of **3.05s***
If we are writing to disk rather than reading from disk, then each time we transfer data from the RAID controller buffer to the disks we need to write the buffer of data to two disks, for example one copy to disk 1 and one copy to the same track on disk 5. This means we can write a maximum of four tracks of data at one time.
*Writing 14 tracks of data from the data buffers to the RAID will therefore take 4*0.4s=1.6s because it takes 4 writes (4+4+4+2 tracks) to write the 14 tracks to disk*
*Adding the time to transfer 14 tracks from memory to the disk controller (same as transferring from the disk controller to memory) gives a total write time of **1.805s***

4. [20 points] Consider a disk with N tracks numbered from 0 to N-1. Assume that requested sectors are distributed randomly and evenly over the disk.
   a) Calculate the probability of a seek of length j when the head is currently positions over track t. HINT: determine the total number of combinations, recognizing that all track positions for the destination of the seek are equally likely.

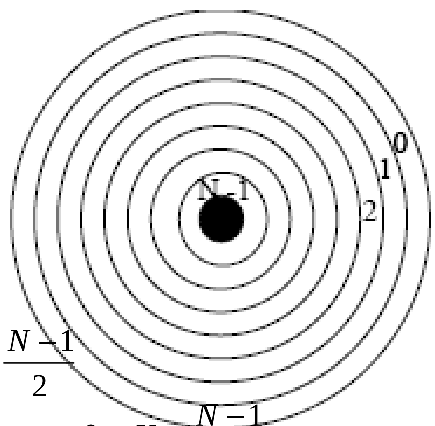
<u>NUMBER OF WAYS TO SEEK J TRACKS STARTING AT TRACK T</u>

K T	0	1	2	3 …	**N-3**	N-2	N-1	# *of waysJ* (to seek *starting at track T)*
0	1	1	1	1	1	1	1	N
1	1	2	1	1	1	1	0	N
2	1	2	2	1	1	0	0	N
3	1	2	2	2	0	0	0	N
…								
N-4	1	2	2	2	0	0	0	N
N-3	1	2	2	1	1	0	0	N
N-2	1	2	1	1	1	1	0	N
N-1	1	1	1	1	1	1	1	N
∑ − = 1 0 # *N T of seekwaysT* (to seek J tracks)	N	2(N-1)	2(N-2)	2(N-3)	2(N-[N-3]) Or 2(3)	2*2	2*1	

Total number of possible seeks

$$\sum_{T=0}^{N-1}\sum_{K=0}^{N-1}\#\ of\ \ ways=\sum_{T=0}^{N-1}N=N^2$$

N ways to seek starting at track T (for each track T)
N tracks T on which to begin seeking.



Probability of a seek of length K starting at track T

$$E(x)=\frac{\#\ of\ \ seek\ \ ways}{\sum_{K=0}^{N-1}\#\ of\ \ ways}=\begin{cases}0 & N-K\le T<K\ and\ N>K>\frac{N-1}{2}\\ \frac{2}{N} & N-K>T\ge K\ and\qquad 0<K\le\frac{N-1}{2}\\ \frac{1}{N} & otherwise\end{cases}$$

b) Calculate the probability of a seek of length K

$$E(x) = \frac{\sum_{K=0}^{N-1} \# \, of \;\; seek \;\; ways}{\sum_{T=0}^{N-1} \sum_{K=0}^{N-1} \# \, of \;\; ways} = \frac{2(N-K)}{N^2}$$

c) Calculate the average number of tracks traversed by a seek HINT: Use the formula for expected value

$$E(x) = \sum_{i=0}^{N-1} i \sum \Pr[x = i]$$

$$E(x) = \sum_{J=0}^{N-1} J \, \frac{2(N-J)}{N^2} = \sum_{J=0}^{N-1} \left\{ \frac{2J}{N} - \frac{2J^2}{N^2} \right\}$$

$$\sum_{T=0}^{N-1} \frac{2T}{N} = \frac{2}{N} * N * (N-1)/2 = N-1$$

$$\sum_{T=0}^{N-1} \frac{2T^2}{N^2} = \frac{(N-1)*(2N-1)}{3N}$$

$$E(x) = \sum_{J=0}^{N-1} \left\{ \frac{2J}{N} - \frac{2J^2}{N^2} \right\} = (N-1) - \frac{(N-1)*(2N-1)}{3N}$$

$$= \frac{3N^2 - 3N - 2N^2 + 3N - 1}{3N}$$

$$= \frac{N^2 - 1}{3N}$$