

EXAMPLE 14 How many bit strings of length n contain exactly r 1s?

Solution: The positions of r 1s in a bit string of length n form an r -combination of the set $\{1, 2, 3, \dots, n\}$. Hence, there are $C(n, r)$ bit strings of length n that contain exactly r 1s. ◀

EXAMPLE 15 Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department?

Solution: By the product rule, the answer is the product of the number of 3-combinations of a set with nine elements and the number of 4-combinations of a set with 11 elements. By Theorem 2, the number of ways to select the committee is

$$C(9, 3) \cdot C(11, 4) = \frac{9!}{3!6!} \cdot \frac{11!}{4!7!} = 84 \cdot 330 = 27,720.$$

Exercises

1. List all the permutations of $\{a, b, c\}$.
2. How many different permutations are there of the set $\{a, b, c, d, e, f, g\}$?
3. How many permutations of $\{a, b, c, d, e, f, g\}$ end with a ?
4. Let $S = \{1, 2, 3, 4, 5\}$.
 - a) List all the 3-permutations of S .
 - b) List all the 3-combinations of S .
5. Find the value of each of these quantities.

a) $P(6, 3)$	b) $P(6, 5)$
c) $P(8, 1)$	d) $P(8, 5)$
e) $P(8, 8)$	f) $P(10, 9)$
6. Find the value of each of these quantities.

a) $C(5, 1)$	b) $C(5, 3)$
c) $C(8, 4)$	d) $C(8, 8)$
e) $C(8, 0)$	f) $C(12, 6)$
7. Find the number of 5-permutations of a set with nine elements.
8. In how many different orders can five runners finish a race if no ties are allowed?
9. How many possibilities are there for the win, place, and show (first, second, and third) positions in a horse race with 12 horses if all orders of finish are possible?
10. There are six different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?
11. How many bit strings of length 10 contain
 - a) exactly four 1s?
 - b) at most four 1s?
 - c) at least four 1s?
 - d) an equal number of 0s and 1s?
12. How many bit strings of length 12 contain
 - a) exactly three 1s?
 - b) at most three 1s?
 - c) at least three 1s?
 - d) an equal number of 0s and 1s?
13. A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?
14. In how many ways can a set of two positive integers less than 100 be chosen?
15. In how many ways can a set of five letters be selected from the English alphabet?
16. How many subsets with an odd number of elements does a set with 10 elements have?
17. How many subsets with more than two elements does a set with 100 elements have?
18. A coin is flipped eight times where each flip comes up either heads or tails. How many possible outcomes
 - a) are there in total?
 - b) contain exactly three heads?
 - c) contain at least three heads?
 - d) contain the same number of heads and tails?
19. A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes
 - a) are there in total?
 - b) contain exactly two heads?
 - c) contain at most three tails?
 - d) contain the same number of heads and tails?
20. How many bit strings of length 10 have
 - a) exactly three 0s?
 - b) more 0s than 1s?
 - c) at least seven 1s?
 - d) at least three 1s?

- 21** How many permutations of the letters $ABCDEFGG$ contain
- the string BCD ?
 - the string $CFGA$?
 - the strings BA and GF ?
 - the strings ABC and DE ?
 - the strings ABC and CDE ?
 - the strings CBA and BED ?
- 22.** How many permutations of the letters $ABCDEFGH$ contain
- the string ED ?
 - the string CDE ?
 - the strings BA and FGH ?
 - the strings AB , DE , and GH ?
 - the strings CAB and BED ?
 - the strings BCA and ABF ?
- 23.** How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? [*Hint*: First position the men and then consider possible positions for the women.]
- 24.** How many ways are there for 10 women and six men to stand in a line so that no two men stand next to each other? [*Hint*: First position the women and then consider possible positions for the men.]
- 25.** One hundred tickets, numbered $1, 2, 3, \dots, 100$, are sold to 100 different people for a drawing. Four different prizes are awarded, including a grand prize (a trip to Tahiti). How many ways are there to award the prizes if
- there are no restrictions?
 - the person holding ticket 47 wins the grand prize?
 - the person holding ticket 47 wins one of the prizes?
 - the person holding ticket 47 does not win a prize?
 - the people holding tickets 19 and 47 both win prizes?
 - the people holding tickets 19, 47, and 73 all win prizes?
 - the people holding tickets 19, 47, 73, and 97 all win prizes?
 - none of the people holding tickets 19, 47, 73, and 97 wins a prize?
 - the grand prize winner is a person holding ticket 19, 47, 73, or 97?
 - the people holding tickets 19 and 47 win prizes, but the people holding tickets 73 and 97 do not win prizes?
- 26.** Thirteen people on a softball team show up for a game.
- How many ways are there to choose 10 players to take the field?
 - How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?
 - Of the 13 people who show up, three are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?
- 27** A club has 25 members.
- How many ways are there to choose four members of the club to serve on an executive committee?
 - How many ways are there to choose a president, vice president, secretary, and treasurer of the club, where no person can hold more than one office?
- 28.** A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?
- *29.** How many 4-permutations of the positive integers not exceeding 100 contain three consecutive integers $k, k + 1, k + 2$, in the correct order
- where these consecutive integers can perhaps be separated by other integers in the permutation?
 - where they are in consecutive positions in the permutation?
- 30.** Seven women and nine men are on the faculty in the mathematics department at a school.
- How many ways are there to select a committee of five members of the department if at least one woman must be on the committee?
 - How many ways are there to select a committee of five members of the department if at least one woman and at least one man must be on the committee?
- 31.** The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain
- exactly one vowel?
 - exactly two vowels?
 - at least one vowel?
 - at least two vowels?
- 32.** How many strings of six lowercase letters from the English alphabet contain
- the letter a ?
 - the letters a and b ?
 - the letters a and b in consecutive positions with a preceding b , with all the letters distinct?
 - the letters a and b , where a is somewhere to the left of b in the string, with all the letters distinct?
- 33.** Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women?
- 34** Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have more women than men?
- 35.** How many bit strings contain exactly eight 0s and 10 1s if every 0 must be immediately followed by a 1?
- 36.** How many bit strings contain exactly five 0s and 14 1s if every 0 must be immediately followed by two 1s?
- 37.** How many bit strings of length 10 contain at least three 1s and at least three 0s?
- 38.** How many ways are there to select 12 countries in the United Nations to serve on a council if 3 are selected from a block of 45, 4 are selected from a block of 57, and the others are selected from the remaining 69 countries?