

people where every two people are friends or enemies, there may not be three mutual friends or three mutual enemies (see Exercise 26).

It is possible to prove some useful properties about Ramsey numbers, but for the most part it is difficult to find their exact values. Note that by symmetry it can be shown that  $R(m, n) = R(n, m)$  (see Exercise 30). We also have  $R(2, n) = n$  for every positive integer  $n \geq 2$  (see Exercise 29). The exact values of only nine Ramsey numbers  $R(m, n)$  with  $3 \leq m \leq n$  are known, including  $R(4, 4) = 18$ . Only bounds are known for many other Ramsey numbers, including  $R(5, 5)$ , which is known to satisfy  $43 \leq R(5, 5) \leq 49$ . The reader interested in learning more about Ramsey numbers should consult [MiRo91] or [GrRoSp90].

## Exercises

1. Show that in any set of six classes, each meeting regularly once a week on a particular day of the week, there must be two that meet on the same day, assuming that no classes are held on weekends.
2. Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.
3. A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.
  - a) How many socks must he take out to be sure that he has at least two socks of the same color?
  - b) How many socks must he take out to be sure that he has at least two black socks?
4. A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.
  - a) How many balls must she select to be sure of having at least three balls of the same color?
  - b) How many balls must she select to be sure of having at least three blue balls?
5. Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4.
6. Let  $d$  be a positive integer. Show that among any group of  $d + 1$  (not necessarily consecutive) integers there are two with exactly the same remainder when they are divided by  $d$ .
7. Let  $n$  be a positive integer. Show that in any set of  $n$  consecutive integers there is exactly one divisible by  $n$ .
8. Show that if  $f$  is a function from  $S$  to  $T$ , where  $S$  and  $T$  are finite sets with  $|S| > |T|$ , then there are elements  $s_1$  and  $s_2$  in  $S$  such that  $f(s_1) = f(s_2)$ , or in other words,  $f$  is not one-to-one.
9. What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?
- \*10. Let  $(x_i, y_i), i = 1, 2, 3, 4, 5$ , be a set of five distinct points with integer coordinates in the  $xy$  plane. Show that the midpoint of the line joining at least one pair of these points has integer coordinates.
- \*11. Let  $(x_i, y_i, z_i), i = 1, 2, 3, 4, 5, 6, 7, 8, 9$ , be a set of nine distinct points with integer coordinates in  $xyz$  space. Show that the midpoint of at least one pair of these points has integer coordinates.
12. How many ordered pairs of integers  $(a, b)$  are needed to guarantee that there are two ordered pairs  $(a_1, b_1)$  and  $(a_2, b_2)$  such that  $a_1 \bmod 5 = a_2 \bmod 5$  and  $b_1 \bmod 5 = b_2 \bmod 5$ ?
13. a) Show that if five integers are selected from the first eight positive integers, there must be a pair of these integers with a sum equal to 9.  
b) Is the conclusion in part (a) true if four integers are selected rather than five?
14. a) Show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs of these integers with the sum 11.  
b) Is the conclusion in part (a) true if six integers are selected rather than seven?
15. How many numbers must be selected from the set  $\{1, 2, 3, 4, 5, 6\}$  to guarantee that at least one pair of these numbers add up to 7?
16. How many numbers must be selected from the set  $\{1, 3, 5, 7, 9, 11, 13, 15\}$  to guarantee that at least one pair of these numbers add up to 16?
17. A company stores products in a warehouse. Storage bins in this warehouse are specified by their aisle, location in the aisle, and shelf. There are 50 aisles, 85 horizontal locations in each aisle, and 5 shelves throughout the warehouse. What is the least number of products the company can have so that at least two products must be stored in the same bin?
18. Suppose that there are nine students in a discrete mathematics class at a small college.
  - a) Show that the class must have at least five male students or at least five female students.
  - b) Show that the class must have at least three male students or at least seven female students.
19. Suppose that every student in a discrete mathematics class of 25 students is a freshman, a sophomore, or a junior.
  - a) Show that there are at least nine freshmen, at least nine sophomores, or at least nine juniors in the class.