PHYS105 Matlab Workshop 3: Moments of Inertial and Complex Trajectories

Introduction

This problem concerns a baton made from two point masses (M1 = 1.0 kg and M2 = 2.0 kg) connected by a light, rigid rod of length L = 1.0 m.

In this problem I calculated the centre of mass of the baton, and plotted the baton’s translational trajectory by considering the trajectory of the centre of mass. I also calculated and plotted the rotational trajectories of each of the two point masses.

This enabled me to calculate the combined trajectories of each of the point masses, and plot them alongside the trajectory of the (centre of mass of) the baton.

Part 1

'The baton is rotataed about a point on the rod which lies at a distance R1 from M1. Calculate and plot the moment of inertia as R1 changes from 0 to L. Cal;culate and plot the moment of inertia as R1 changes from 0 to L.

The centre of mass of the baton is given by $R_{\mathrm{cm}} = \dfrac{\sum m_x x_i}{\sum m_i}$

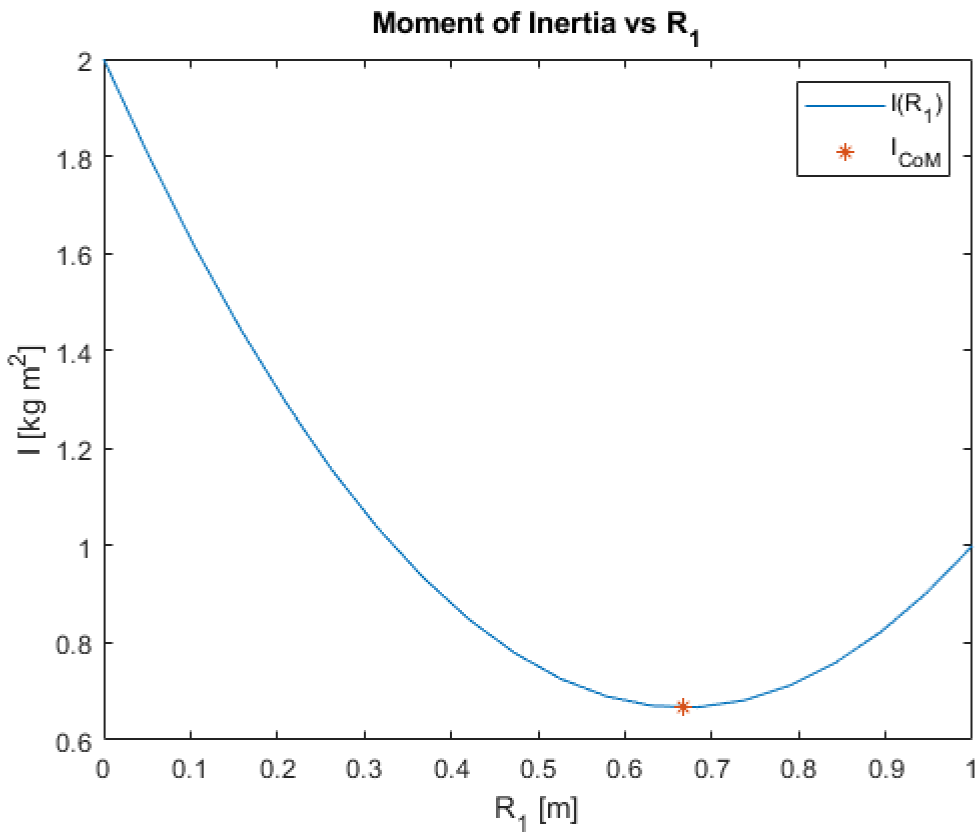
Calculate the centre of mass of the baton (relative to the position of the mass M1).

Finally, calculate the moment of inertia, if the baton rotates about the centre of mass, and plot the point (R cm, I cm) in the same figure as the previous plot.'

```
L = 1.0; %defining the length [m]
M_1 = 1.0; %mass 1 [kg]
M_2 = 2.0; %mass 2 [kg]
R_1 = linspace(0, L, 20); %creating a vector for R1

I = M_1.*(R_1).^2 + M_2.*(L-R_1).^2; %creating a vector for the moment of inertia as a
% function of R1
plot(R_1,I) %plotting I vs R1
hold on
xlabel('R_1 [m]')
ylabel('I [kg m^2]')
title('Moment of Inertia vs R_1')

R_cm = (M_2*L) / (M_1 + M_2); %calculating the centre of mass of the baton relative to
% the position of M1
I_cm = M_1*(R_cm)^2 + M_2*(L-R_cm)^2; %moment of inertia of centre of mass
plot(R_cm, I_cm, '*') %plotting the moment of inertia of centre of mass on the same
% graph
legend('I(R_1)', 'I_C_o_M')
hold off
```




```
disp('The moment of inertia of the centre of mass is at the minima.')
```

```
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```

Part 2

'The baton is thrown with initial velocity $v_0 = 20\,\text{ms}^{-1}$ at an angle of 75 degrees from the horizontal. Calculate and plot the trajectory of the baton.'

By assuming that the total mass of the baton is concentrated at the centre of mass, the trajectory of the centre of mass of the baton can be found. I defined a coordinate system where y is the height above the ground and x is the horizontal distance and plotted the trajectory of the baton using,

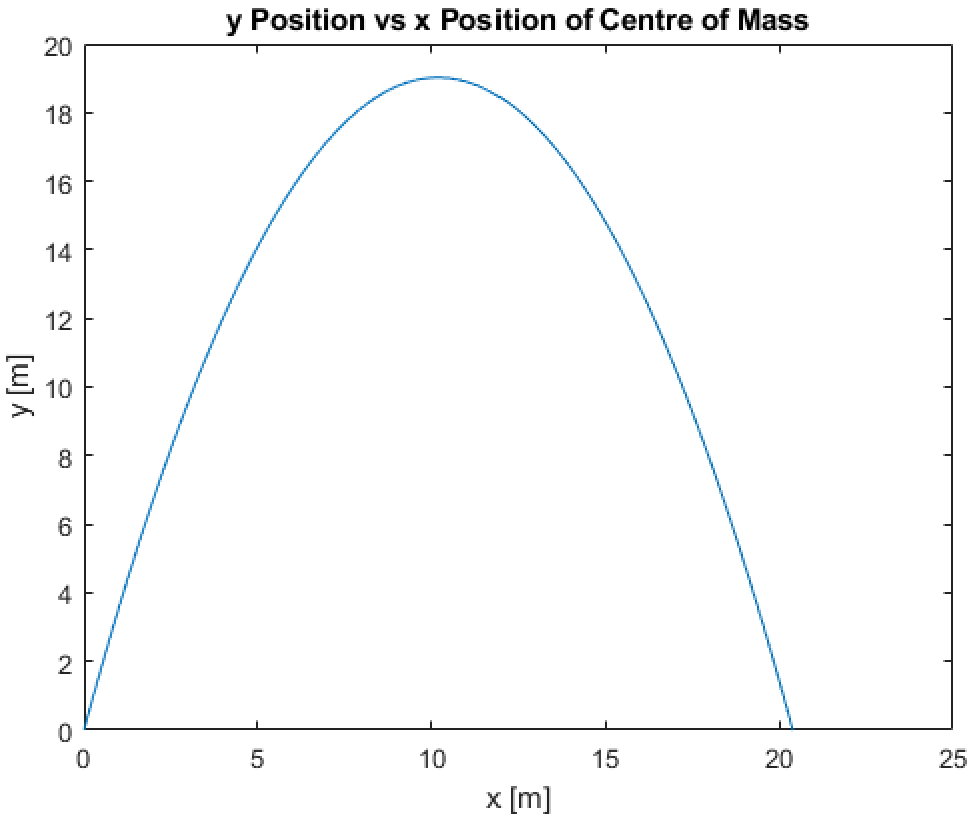
$$y(t) = v_{0y}t + \frac{1}{2}\text{g}t^2$$

and $x(t) = v_{0x}t$

where $g = -9.81\ ms^{-2}$.

```
v_0 = 20; % defining initial velocity variable
deg = 75; %angle of launch in degrees
rad = deg*pi()/180; %calculating angle in radians
v_0y = v_0*sin(rad); %calculating initial velocity in y direction
v_0x = v_0*cos(rad); %calculating initial velocity in x direction
g = -9.81; %defining g, gravity, [m/s^2]
```

```
t_max = (-2)*v_0y / g; %finding the time when the baton is on the ground again
t = linspace(0, t_max, 200); %creating a vector for time [s]
x_t = v_0x*t; %ceating a vector for the translational x position as a function of time
y_t = v_0y*t + 0.5*g*t.^2; %creating a vector for the translational y position as a
% function of t
plot(x_t, y_t) %plotting the translational y position vs the x position
xlabel('x [m]')
ylabel('y [m]')
title('y Position vs x Position of Centre of Mass')
```



Part 3

'The baton is initially horizontal so $\theta_1(0) = 0$ and $\theta_2(0) = \pi$. The rotational velocity is $\omega = 10\,\mathrm{rad}\,s^{-1}$. Calculate the position (the x, y coordinates) of each mass and plot them as a function of time. The angle between each mass and the x-axis will be given by $\theta_i(t) = \omega t + \theta_0$ where θ_i is the initial angle (at t = 0).'

To simplify the problem, I set a new frame of reference where the centre of mass is fixed at x = 0, y = 0.
I then found the x and y positions of the point masses M1 and M2 by converting the polar coordinates to Cartesian coordinates using,

$x = R \cdot \cos(\theta_i)$

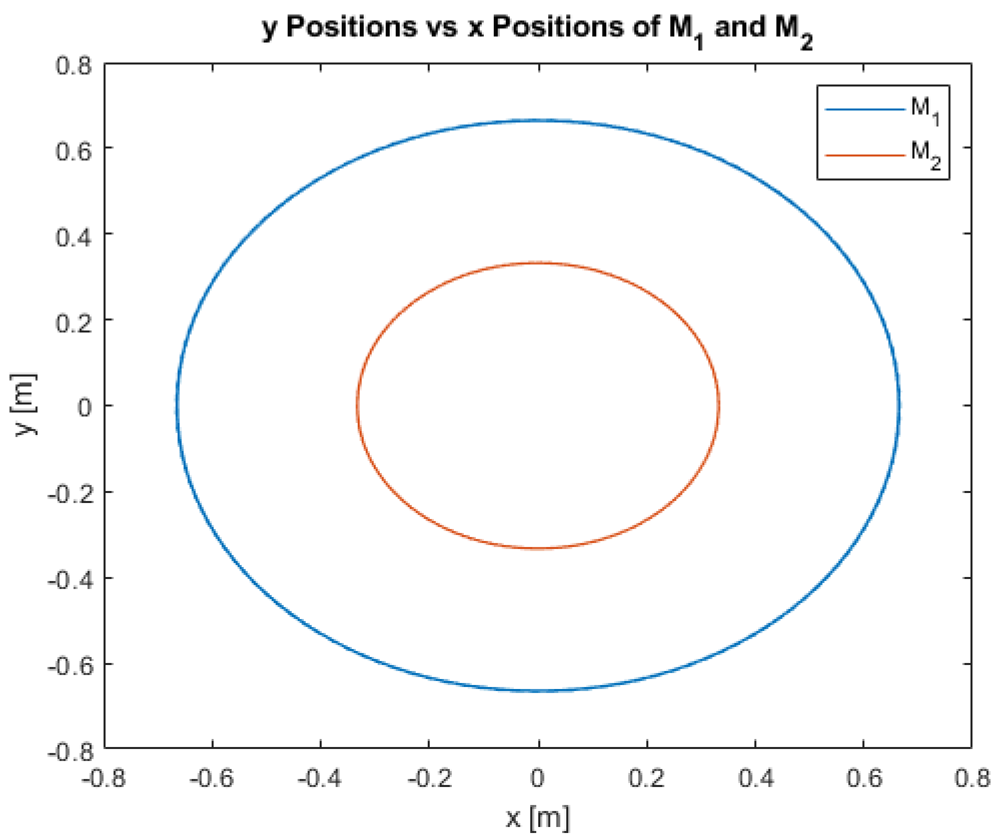
and $y = R \cdot \sin(\theta_i)$.

```
theta_01 = 0; %angle at t=0 for M1
theta_02 = pi(); %angle at t=0 for M2
```

```
w = 10; %defining angular velocity [rad/s]

theta_M1 = w*t + theta_01; %vector for angle of M1
theta_M2 = w*t + theta_02; %vector for angle of M2
x_rM1 = R_cm*cos(theta_M1); %x rotational position of M1
y_rM1 = R_cm*sin(theta_M1); %y rotational position of M1
x_rM2 = (L-R_cm)*cos(theta_M2); %x position of M2
y_rM2 = (L-R_cm)*sin(theta_M2); %y position of M2

plot(x_rM1, y_rM1) %plotting y vs x positions of M1
hold on
plot (x_rM2, y_rM2)%plotting y vs x positions of M2
xlabel('x [m]')
ylabel('y [m]')
title('y Positions vs x Positions of M_1 and M_2')
legend('M_1', 'M_2')
hold off
```

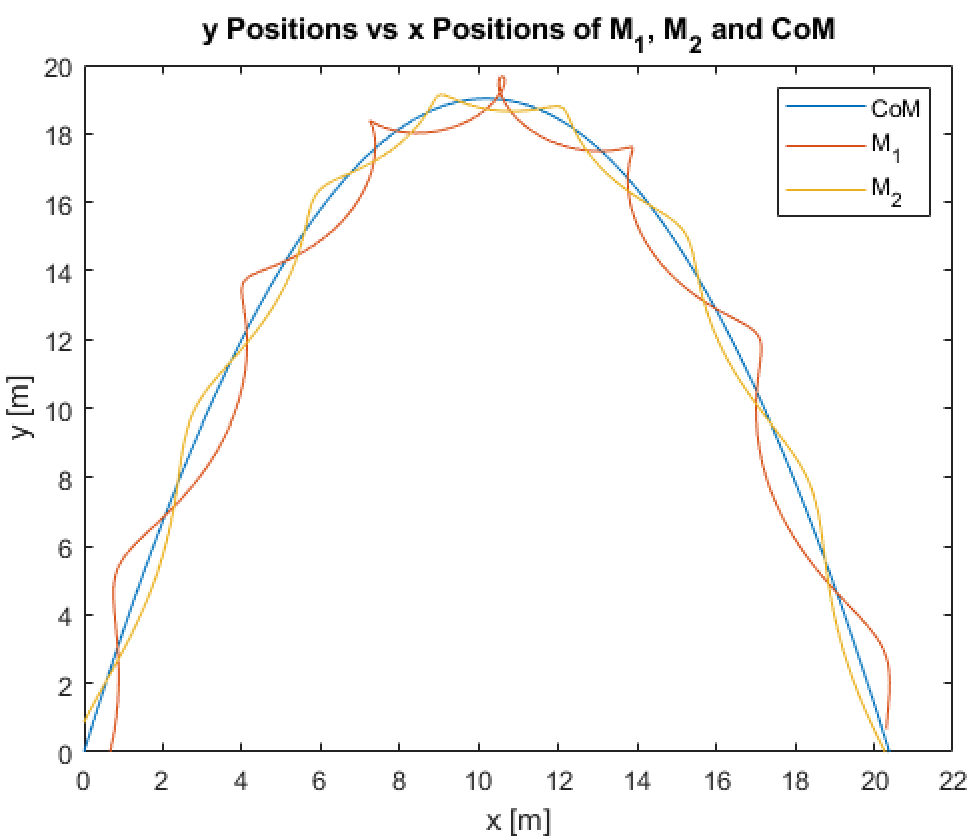


Part 3

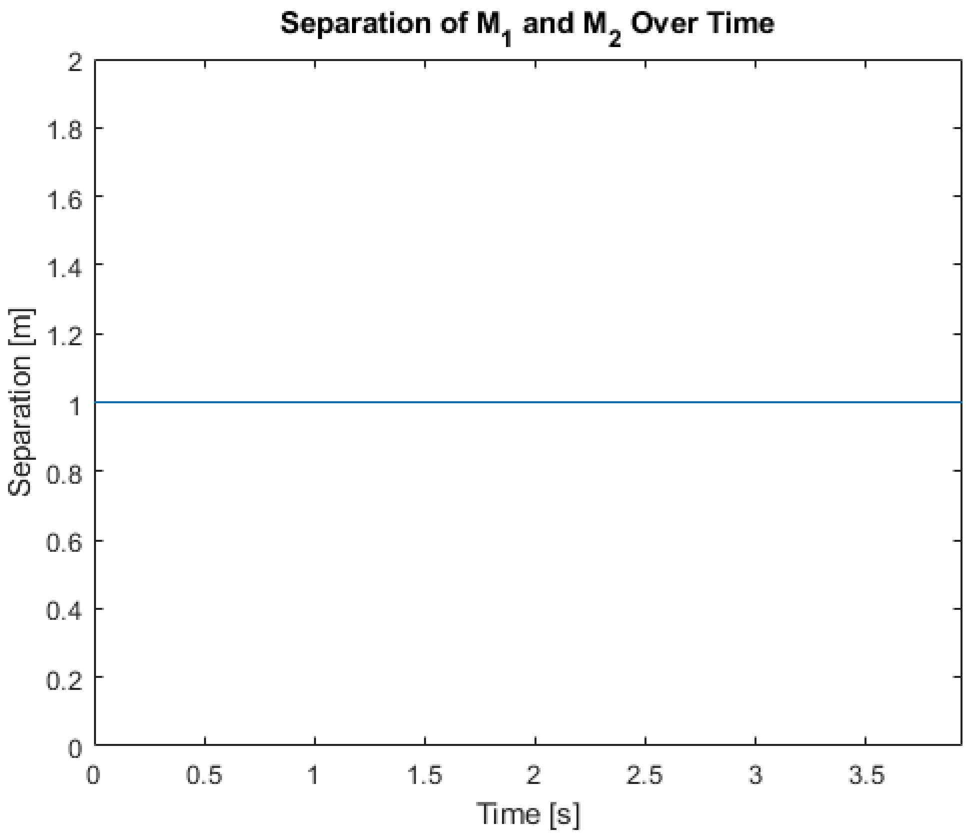
'In a new figure, plot the CoM trajectory and the trajectories of each mass. As a consistency check on your calculation, check that at each step in time, the trajectories of M1 and M2 are separated by distance L.'

I summed the translational and rotational trajectories in x and y for each of M1 and M2 and plotted these trajectories over the plot for the baton's centre of mass. I then plotted the separation of M1 and M2 as a function of time to show that it was comstant and equal to L = 1 m.

```
plot(x_t, y_t) %plotting trajectory of centre of mass
hold on %plotting more on the same graph
x_tM1 = x_t + x_rM1; %summing the translational x coordinates and rotational x
% coordinates
y_tM1 = y_t + y_rM1;
plot(x_tM1, y_tM1) %plotting trajectory of M1
x_tM2 = x_t + x_rM2;
y_tM2 = y_t + y_rM2;
plot(x_tM2, y_tM2) %plotting trajectory of M2
xlabel('x [m]')
ylabel('y [m]')
title('y Positions vs x Positions of M_1, M_2 and CoM')
legend('CoM', 'M_1', 'M_2')
ylim([0 20])
xlim([0 22])
hold off
```




```
diff = sqrt((x_tM1-x_tM2).^2 + (y_tM1-y_tM2).^2); %calculating the distance between
% the two masses
plot(t, diff) %plotting the separation of the two masses as a function of time
ylim([0 2])
xlim([0 t_max])
xlabel('Time [s]')
ylabel('Separation [m]')
title('Separation of M_1 and M_2 Over Time')
```




```
disp('At each step in time, masses M1 and M2 are separated by L = 1 m.')
```

```
At each step in time, masses M1 and M2 are separated by L = 1 m.
```