Assignment 1

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1. (a) we learn that $a = 2$, $b = 4$ and $f(n) = 1$, so we can see

$$n^{\log_b a} = n^{\log_4 2} = \Theta(n^{\frac{1}{2}}) \tag{1}$$

According to this, we can see

$$f(n) = O(n^{\log_b a - \varepsilon}) \tag{2}$$

and the following condition is available

$$\varepsilon = 1 \tag{3}$$

so the solution is

$$T(n) = \Theta(n^{\frac{1}{2}}) \tag{4}$$

(b) we learn that $a = 2$, $b = 4$ and $f(n) = \sqrt{n}$, so we can see

$$n^{\log_b a} = n^{\log_4 2} = \Theta(n^{\frac{1}{2}})$$

According to this, we can see

$$f(n) = \Theta(n^{\log_b a} \lg^k n) \tag{5}$$

and the following condition is available

$$k = 0 \tag{6}$$

so the solution is

$$T(n) = \Theta(n^{\frac{1}{2}} \lg n) \tag{7}$$

(c) we learn that $a = 2$, $b = 4$ and $f(n) = n$, so we can see

$$n^{\log_b a} = n^{\log_4 2} = \Theta(n^{\frac{1}{2}}) \tag{8}$$

According to this, we can see

$$f(n) = \Omega(n^{\log_b a + \varepsilon}) \tag{9}$$

and the following condition is available

$$\varepsilon = 1 \tag{10}$$

so the solution is

$$T(n) = \Theta(n) \tag{11}$$

(d) we learn that $a = 2$, $b = 4$ and $f(n) = n^2$, so we can see

$$n^{\log_b a} = n^{\log_4 2} = \Theta(n^{\frac{1}{2}}) \tag{12}$$

According to this, we can see

$$f(n) = \Omega(n^{\log_b a + \varepsilon}) \tag{13}$$

and the following condition is available

$$\varepsilon = 13 \tag{14}$$

so the solution is

$$T(n) = \Theta(n^2) \tag{15}$$

2. it can be used towards $T(n) = 4T(\frac{n}{2}) + n^2 \lg n$. Since we can see that $a = 4$, $b = 2$ and $f(n) = n^2 \lg n$, it is a case 2 problem and $k = 1$, so the solution is $T(n) = \Theta(n^2 \lg^2 n)$, we can tell that $O(n^3)$ is proper.