# Assignment 8

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1. Given $(a, n)$ we can firstly calculate $(a, \frac{n}{2})$ and then we can calculate $(a, n)$ with $a^{\frac{n}{2}} \times a^{\frac{n}{2}}$. We assume the time cost is T(n). Then we need $T(\frac{n}{2})$ to finish the sub problem and another O(1) to merge the sub problem.

$$T(n) = T(\frac{n}{2}) + O(1)$$

   Through the master solution we can learn that $T(n) = O(\lg n)$.

2. According to t2's decription, we can learn that what we indeed obtain are two queues in order. Every time we choose the median in each queue to compare and if $b[mid] > a[mid]$ then we can delete the elements larger than $b[mid]$ as well as the elements smaller than $a[mid]$. So we can see that we can narrow the range down to half after an O(1) operation. We assume the time complexity is T(n).

$$T(n) = T(\frac{n}{2}) + O(1)$$

   Through the master solution we can learn that $T(n) = O(\lg n)$.

3. Lemma:
   You must walk $2^n$km and then turn back to walk another $2^{n+1}$km. We suppose that you finally find the treasure during your nth turn, then the distance you have already walked is smaller than $1 + 2 + .... + 2^{n-1}$ and is greater than $1 + 2 + .... + 2^{n-2} + 2^{n-2}$.

$$3 \times 2^{n-2} - 1 < dist \leq 2^n - 1$$
$$2^{n-3} < x \leq 3 \times 2^{n-3}$$
$$\frac{dist}{x} < 2$$

   So the competitive ratio is 2.
   Proof:
   And this time, there are m directions to go, so let's divide the m roads into $\lceil \frac{m}{2} \rceil$ groups. Each group contains 2 roads and each group could be regarded as an endless road.

$$\frac{dist}{x} < 2 \times \lceil \frac{m}{2} \rceil \leq m + 1$$

   So the competitive ratio is O(m).