Assignment 9

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1. If the deleted edge is in not in the minimum spanning tree, then there is no need for us to build another MST.
   If the deleted edge is one edge in the given MST, then find another minimum unused edge to become one edge of the MST(the added edge can not generate a circle with the other edges in the MST).
   The time complexity is O(E).

2. Assume the MST is M. After adding one edge to the M, the new graph is called $M\prime$. Now consider find the MST in graph $M^{'}$. Find the largest edge in the edges of $M\prime$ which once we delete, the $M^{''}$ graph will be acyclic. For $M^{'}$, it has v edges. So the time complexity is O(v).

3. How to build a shortest-path forest:
   We can simply realize it through performing 1 time Dijkstra algorithm to each point. And we can design a 3 dimension array to store the path. For example, if we start from the point i and end with the point j, the path is preserved in the array p[i][j].
   How to generate the path:
   Every time we reach a new point, we put the points in its successor into its path and then additionally add itself to the end of the path.

4. Firstly, generate the Dijkstra forest in both S and T, which cost $O(Elgv)$ while using the binomial heap to maintain the minimum set.
   Secondly, we can see now we have at most $\frac{v^2}{2}$ conditions, because we need to find the minimum one in $s_{ui} + w_{ij} + t_{jv}$, and (i,j) have at most $\frac{v^2}{2}$ choices. Thus, in order to find the minimum one, the time complexity is $O(n^2)$.
   So the time complexity is $O(Elgv)$.