UNIVERSITY OF NAIROBI

UNIVERSITY EXAMINATIONS-2023/2024

DEPARTMENT OF MATHEMATICS

THIRD YEAR EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN GEOSPATIAL ENGINEERING

FGE 373: NUMERICAL METHODS

Date: **Time: 2 hours**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions.
2. The maximum possible marks that can be earned in this paper is 70 marks.
3. Ensure that you have **THREE** printed pages.

QUESTION 1. **(30 MARKS)**

(a) Covert the octal number $(56)_8$ to a hexadecimal number. (4 marks)

(b) State the stopping criteria of the bisection method. (3 marks)

(c) Estimate $\sqrt{2}$ using Newton Raphson method assuming $x_0 = 1$ (4 marks)

(d) Evaluate the sum $S = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to 4 significant digits and find its absolute and relative errors. (4 marks)

(e) Using the following data find $f'(6.0)$ and $f''(6.3)$ (6 marks)

x	6.0	6.1	6.2	6.3	6.4
f(x)	0.1750	-0.1998	-0.2223	-0.2422	-0.2596

(f) Find the least squares approximating polynomial of degree 2 for $f(x) = \cos \pi x$ in $[0, 1]$. (5 marks)

(g) Determine the number of iterations necessary to solve $e^x - 3x = 0$ with accuracy 10^{-2} using $a_0 = 1.5$ and $b_0 = 1.6$ (4 marks)

QUESTION 2. (20 MARKS)

(a) State the intermediate value theorem. (2 marks)

(b) Apply the Newton Raphson method with $x_0 = 0.8$ to the equation

$$f(x) = x^3 - x^2 - x + 1 = 0$$

and verify that the convergence is only of first order. Then, apply the Newton Raphson method

$$x_{k+1} = x_k - m\frac{f(x_k)}{f'(x_k)}$$

with $m = 2$ and verify that the convergence is of second order. (10 marks)

(c) Perform four iterations (rounded to four decimal places) using Gauss-Seidel method for solving the system of equations (8 marks)

x_0 :

$$\begin{bmatrix} -8 & 1 & 1 \\ 1 & -5 & 1 \\ 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 16 \\ 7 \end{bmatrix}$$

with $x^{(0)} = 0$. The exact solution is $x = (-1 - 4 - 3)^T$

QUESTION 3. (20 MARKS)

(a) Define numerical differentiation. (2 marks)

(b) Find $y'(1)$ for the following data points of a polynomial $y = f(x)$. (8 marks)

x	0	2	4	6	8
y	4	8	15	7	6

(c) Find the smallest eigenvalue in magnitude and the corresponding eigenvector of the matrix

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

using four iteration of the inverse power method. (10 marks)

QUESTION 4. (20 MARKS)

(a) Calculate all solutions of the system

$$x^2 + y^2 = 1.12$$

$$xy = 0.23$$

correct to three decimal places. (10 marks)

(b) (i)

Find the Lagrange interpolating polynomial of degree 2 approximating the function $y = \ln x$ defined by the following table of values. Hence determine the value of ln2.7. (6 marks)

ii. Estimate the error in the value of y obtained in $b(i)$ above. (4 marks)

x	2.0	2.5	3.0
y = ln x	0.69315	0.91629	1.09861

QUESTION 5. (20 MARKS)

(a) The table below gives the readings from a laboratory experiment. Fit

Time t	2	3	5	6	9
Reading y	7	17	49	71	161

(i) a linear function

(ii) a quadratic polynomial

to the above data by method of least squares and determine which of these two is a better approximation.

(12 marks)

(b) Solve the following system of equations using Gauss elimination method with partial pivoting.

(8 marks)

$$x + y + z = 6$$

$$3x + 3y + 4z = 20$$

$$2x + y + 3z = 13$$

UNIVERSITY OF NAIROBI

UNIVERSITY EXAMINATIONS-2023/2024

THIRD YEAR EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN ELECTRICAL AND INFORMATION ENGINEERING AND BACHELOR OF SCIENCE IN GEOSPATIAL AND SPACE ENGINEERING

FEE 371/FGE 371:ENGINEERING MATHEMATICS IIIA

DATE: 5TH FEBRUARY 2023 **TIME: 2 HOURS**

INSTRUCTIONS TO CANDIDATES

1. Attempt question **ONE** and any other **TWO** questions.
2. Symbols have their usual meaning.
3. Mobile phones are **NOT** allowed in the examination hall.
4. The maximum possible marks that can be earned in this paper is 70.

QUESTION ONE **(30 MARKS)**

(a) Find the domain of

(i) $f(x,y,z) = \frac{3x-4y+2z}{\sqrt{9-x^2-y^2-z^2}}$ **(3mks)**

(ii) $g(x,y,t) = \frac{\sqrt{2t-4}}{x^2-y^2}$ **(3mks)**

(b) Find the limit of:

(i) $\lim_{(x,y)\to(2,-1)}(x^2-2xy+3y^2-4x+3y-6)$ **(3mks)**

~~f + (4)~~ 4+3 +8 +-3 -6

(ii) $\lim_{(x,y)\to(2,-1)}\frac{2x+3y}{4x-3y}$ **(3mks)**

(c) Use the limit definition of partial derivative to find the derivative of

$$f(x,y) = x^2-3xy+2y^2-4x+5y-12.$$

(4mks)

(d) Find the equation of the tangent plane to the surface defined by the function

$$f(x,y) = \sin(2x)\cos(3y) \quad \text{at point} \quad \left(\frac{\pi}{3},\frac{\pi}{4}\right).$$

Product rule
u'v + v'u

(5mks)

(e) Show that the function $f(x,y) = 2x^2-4y$ is differentiable at $(2,-3)$. **(7mks)**

(f) Define the term critical point. **(2mks)**

Gradient of function = 0

QUESTION TWO (20 MARKS)

(a) Find $A - B$ and $2A - 3B$ if

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & -2 & -3 \\ 1 & 0 & -1 \end{bmatrix}.$$

(4mks)

(b) Find AB if

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & -3 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 3 \\ 1 & -1 \\ -2 & 4 \end{bmatrix}.$$

(3mks)

(c) Find the transpose of A and B. Hence confirm that $(AB)^T = B^T A^T$ given that

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & -1 & 0 \\ 1 & 2 & -2 \end{bmatrix}.$$

(4mks)

(d) If

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \\ -1 & 2 & 1 \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \qquad \mathbf{d} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix},$$

find the set of equations for the $x, y,$ and z represented by $A\mathbf{x} = \mathbf{d}$. **(5mks)**

(e) Find the inverse of A A^{-1} given that

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & 5 \\ -1 & -1 & 2 \end{bmatrix}.$$

(4mks)

QUESTION THREE (20 MARKS)

(a) Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

(6mks)

(b) Write down the set of equations given by $A\mathbf{x} = \mathbf{d}$ where,

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -2 & 2 \\ 1 & 0 & 1 \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \qquad \mathbf{d} = \begin{bmatrix} 6 \\ 3 \\ -9 \end{bmatrix}.$$

Hence find A^{-1} and calculate $A^{-1}\mathbf{d}$ and calculate the solution of the equation. **(7mks)**

(c) A general $n \times n$ matrix is given by $A = [a_{ij}]$. Show that $A + A^T$ is a symmetric matrix and $A - A^T$ is a skew-symmetric matrix. Express the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 \\ -2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}.$$

as the sum of a symmetric matrix.

(7mks)

QUESTION FOUR **(20 MARKS)**

(a) Evaluate $\int_C (xdy - y^2dx)$, where C is positively oriented square bounded by lines $x = \pm 1$ and $y = \pm 1$. **(5mks)**

(b) Let $\vec{F}$ be a vector field over simply connected region D whose component functions have continuous second order partial derivatives then $\vec{F}$ is conservative if and only if curl $\vec{F} = \vec{0}$. Prove.

(5mks)

(c) State and prove the Gauss divergence theorem. **(5mks)**

(d) Evaluate $\int_C \vec{F} d\vec{r}$, where $\vec{F} = xy\vec{i} + yz\vec{j} + zx\vec{k}$ and C is the twisted cube given by $x = t, y = t^2, z = t^3$ and $0 \leq t \leq 1$. **(5mks)**

QUESTION FIVE **(20 MARKS)**

(a) (i) Define the continuity of a function of two variables. **(2mks)**

(ii) Show that the function $f(x,y) = 4x^3y^2$ and $g(x,y) = \cos(4x^3y^2)$ are continuous everywhere. **(5mks)**

(b) Solve

(i) $f(x,y) = x^2 - 3xy + 2y^2 - 4x + 5y - 12.$ **(3mks)**

(ii) $g(x,y) = \sin(x^2y - 2x + 4).$ **(3mks)**

(c) Find the equation of the tangent plane to the surface defined by the function

$$f(x,y) = 2x^2 - 3xy + 8y^2 + 2x - 4y + 4 \qquad \text{at point} \qquad (2,-1).$$

(4mks)

(d) Prove

$$\lim_{(x,y)\to(4,3)} \sqrt{25 - x^2 - y^2} = 0.$$

(3mks)

UNIVERSITY OF NAIROBI

DEPARTMENT OF GEOSPATIAL & SPACE TECHNOLOGY

FGE 349: GEOSPATIAL SURFACE MODELLING

CONTINUOUS ASSESSMENT TEST

Attempt **ALL** Questions

Date: 11th December 2022 Time: 2 Hours

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intermediate models

Question 1

a) Describe the scope of Digital Terrain Modelling (DTM). DTM G **(10 marks)**

- Manipulation,
- Interpolation –
- Visualization –

b) Using suitable examples, describe different spatial circumstances that would necessitate that interpolation be done. **(15 marks)**

- Need of elevation Z at single unsampled pts, rectangular grid, irregular
- Resampling – Densification or coarsening of rectangular grids
- the data we have doesn't cover the area of interest completely
- Contour interpolation – Need location of pts along contour lines to compute the

c) Local deterministic methods of interpolation use only information from the nearest data points. In this regard, distinguish between bilinear and cubic convolution interpolation. **(10 marks)**

refined; smoothest

smooth transition of d. pts.
– Uses average of the data at any corner of rectangle (4 neighboring pts)
– The closer the pt the weight the corner increases the closer to the

Convolution. Converges to the ... function

Question 2

a) One way of evaluating the accuracy of interpolation is by computing the *Cross-Validation*. Explain how this is accomplished 1st law of Geography **(15 marks)**

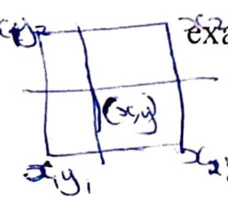
b) Using a suitable example, explain what trend surface interpolation refers to.

Local & Deterministic

IDW exact

(10 marks)

c) The Inverse Distance Weighted (IDW) approach assumes that each measured point has a local influence that diminishes with distance. Using suitable examples discuss this statement. **(10 marks)**



- use all measured pts to create a surface.
- If the avg (squared) difference b/w the actual value & the pred. isn't that large,
- & use poly to estimate values at non-grid positions
- Pred: MSE of prediction (Root mean ...)

TIN

Contour

x	y	(z) elevation
1	6	
2		
3		
4		
5		
6		

Row

Average squared

Question 3

geostatistical technique used for spatial interpolation, either in contour... of pred

a) Distinguish between Kriging and Co-Kriging. *(10 marks)*

b) Using signal strength as a proxy for measuring accessibility/customer preference, describe how one would in an innovative and timely manner, compare and contrast the quality of Safaricom and Airtel mobile service providers within the Main Campus, University of Nairobi. *(20 marks)*

used to calculate weights

Commonly empl

UNIVERSITY OF NAIROBI

DEPARTMENT OF GEOSPATIAL & SPACE TECHNOLOGY
FIRST SEMESTER 2023/2024

THIRD YEAR: Continuous Assessment Test

FGE 311: INTRODUCTION TO GEODESY

DATE: 6th Dec. 2023 DURATION: 1HR 20MIN

Answer **ANY TWO** the questions.

Useful constants: Parameters of: (1) WGS84 system: a=6 378 137m, 1/f = 298.25722356
(2) Clarke 1880: a=6 378 249.14m, 1/f = 293.46

The expression: $$P_n(t) = \frac{1}{2^n n!}\frac{d^n}{dt^n}(t^2-1)^n$$

Question One

projection

a) How has the definition of geodesy changed since the time of *Friedrich Robert Helmert* in 1880? *(3 marks)*

b) What are some of the goals of geodesy? *(3 marks)*

-Establishment
-measurement & represen

c) Briefly, explain the problem of geodesy *(3 marks)*

-gravity field.
-geom

d) Explain how Eratosthenes determined the radius of the earth pointing out the error possibilities with his method. *(4 marks)*

e) Briefly, highlight the contribution of the following persons in development of Geodesy.
(i) C. F. Gauss
(ii) Isaac Newton Newtons Law *(3 marks)*

f) Explain how the controversy between Cassini and Newton with regard to the shape of the earth was resolved. *(4 marks)*

Question Two

astronomic latitude Φ
longitude Λ
H

a) Distinguish between the local astronomic coordinate system and the local geodetic (ellipsoidal) coordinate system. (6 marks)

b) Compute the equivalent Cartesian coordinates x_i given the following curvilinear coordinates in the WGS-84 system. φ= 01°24 13.0"N, λ= 36° 52' 19.0" E, h= 1742.96 m *(10 marks)*

$y = (v+h)\cos\phi \sin\lambda$
$z = [v(1-e^2)+h]\sin\phi$

-78 3954.1875

c) Determine the components of the deflection of the vertical η and ξ and the undulation N given the following coordinates of the same point:

lat φ= 01°24 15.234″S, λ= 36° 52' 09.205″ E, h= 1752.967 m and Φ= 01°24 15.001″S, Λ= 36° 52' 09.215″ E, H= 1752.943 m

(4 marks)

Question Three

a) Show that the reciprocal of the distance *l* satisfies the Laplace equation. (4 marks)

b) Derive the radius of curvature in the meridian normal section. (5 marks)

c) Explain the two main components of gravity potential. What are there mathematical expressions? (7 marks)

d) Using the. RODRIGUES' formula, find the Legendre polynomial, $P_2(t)$ given that $P_0(t)$ = 1. (4 marks)

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314 20
51

FIRST SEMESTER EXAMINATIONS 2023/2024

THIRD YEAR EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN GEOSPATIAL ENGINEERING

FGE 311: INTRODUCTION TO GEODESY

DATE: **06TH FEBRUARY 2024** **TIME: 10.30 – 12.30**

INSTRUCTIONS:
Answer **QUESTION ONE** and any other **TWO** questions. Notations used carry their usual meaning unless otherwise defined.

Useful constants and expressions: WGS84 system: a=6378137m, 1/f = 298.25722356;

$$P_n(t) = \frac{1}{2^n n!}\frac{d^n}{dt^n}(t^2-1)^n \quad ;$$

$\frac{1}{f} = 298$
$f = .$

QUESTION ONE **TOTAL MARKS = 30**

a) V (x,y,z) is a differentiable scalar function. Determine its gradient. 2 marks

b) **F**(x,y,z) = a_1**i**+a_2**j**+a_3**k** is a differentiable vector function (vector field) for all points P(x,y,z). Determine the curl **F**. 3 marks

c) Show that the inverse of the distance *l* satisfies the Laplace equation. 5 marks

d) The earth's gravity potential is composed of two main potentials. Describe these potentials, giving their expressions. Further, show that one of these components does not satisfy the Laplace equation. 5 marks

- gravity potential field & Centrifugal potential

3

e) In the early years of development of geodesy, there was a controversy between the views of Newton and Cassini. What was this controversy and how was it solved? 5 marks

8
5/13

f) Before the advent of satellites, Arc measurements were the main methods of determination of the figure of the earth. Explain how these arc measurements were done. 5 marks

oblate
Peru (Syene. prola

Eslab Lapsland oblate

6,366.30 using maps

g) What are harmonic functions? Using the RODRIGUES' formula, find the Legendre polynomial, $P_2(t)$ given that $P0(t) = 1$. 5 marks

QUESTION TWO TOTAL MARKS = 20

a) Explain the goals of geodesy. 5 marks

b) Name any three institutions that are maintained by the IAG. 3 marks

c) Distinguish between a reference ellipsoid and a geoid. 7 marks

d) Explain the Free air gravity reduction, giving an expression for its computation. 5 marks

QUESTION THREE TOTAL MARKS = 20

a) Derive an expression for the radius of curvature in the meridian normal section. 3 marks

b) List three different types of heights used in geodesy. 3 marks

c) At a given point, the ellipsoidal normal deviates from the plumb line by 5.23" in the north-south direction and -0.72" in the east-west direction. If the astronomic latitude and longitude of this point are 4° 13′ 15.2" and 33° 58′ 29.0" respectively, determine the geodetic coordinates of this point. 9 marks

d) Briefly, discuss the astro- geodetic method of geoid determination. 5 marks

QUESTION FOUR TOTAL MARKS = 20

a) Distinguish between the natural coordinate systems and the conventional geodetic coordinate systems. 10 marks

b) Distinguish between the Cardanian and Eulerian rotation matrices. 5 marks

c) Explain the Bouguer gravity reduction method. 5 marks

QUESTION FIVE TOTAL MARKS = 20

a) Compute the equivalent Cartesian coordinates $\mathbf{x}_i$ given the following curvilinear coordinates in the WGS-84 system. φ= -01°17 20.0"S, λ= 35° 12' 19.3" E, h= 1542.24 m 16 marks

b) Which of the following expressions describes the earth's gravitational potential outside the attracting masses? 4 marks

$$V_i(r,\theta,\lambda) = \sum_{n=0}^{\infty} r^n \sum_{m=0}^{\infty} (a_{nm} P_{nm}(\cos\theta)\cos m\lambda + b_{nm} P_{nm}(\cos\theta)\sin m\lambda)$$

and

$$V_e(r,\theta,\lambda) = \sum_{n=0}^{\infty} r^{-(n+1)} \sum_{m=0}^{\infty} (a_{nm} P_{nm}(\cos\theta)\cos m\lambda + b_{nm} P_{nm}(\cos\theta)\sin m\lambda)$$

$x = (v+h)\cos\phi$
$y = (v+h)\cos\phi$
$z = v(1-e^2)$

$v = \frac{a}{(1-e^2\sin^2\phi)^{\frac{1}{2}}}$

$e^2 = f(2-f)$

UNIVERITY OF NAIROBI

DEPARTMENT OF GEOSPATIAL & SPACE TECHNOLOGY

BSc Geospatial Engineering (Year 3)

FGE 341 – Photogrammetry I A   C.A.T.   14/12/2023

Answer all the three questions   Time Allowed: 1 Hour

Q.1. (a) State FOUR differences between Aerial photogrammetry and Close-Range photogrammetry. (8 marks)

Camera, X-rays, Taken from various stations

(b) With the aid of a dual flow-chart, illustrate the analogue and digital routes of the photogrammetric mapping process, starting from "taking the photograph" to "producing the map". (12 marks)

Q.2. (a) In photogrammetry, a vertical photograph is considered to be a "record of directions". With reference to a simple diagram, explain how that is so, and how the said directions may be reconstructed with the aid of the photograph. (10 marks)

(b) List FIVE pieces of information that are usually displayed in the margin of an aerial photograph and state their respective uses in photogrammetry. (10 marks)

- Flying Height - at the
- Clock - time in which photograph
- Serial number - easy
- Photo number -
- Fudicial - Help

Q.3. (a) On a vertical photograph, the image of an airport runway is 160mm long. On a map of scale 1:25,000, the runway measures 102mm. What is the scale of the photograph? (4 marks)

(b) On a vertical photograph taken from a height of 1000m above datum, the images of the top and base of a vertical telecommunications mast are located at distances of 122mm and 115mm respectively away from the principal point. Assuming that the base is at an altitude of 500m, calculate the height of the mast. (6 marks)

(c) A photograph taken with a camera of focal length 150mm, from a height 3000m above datum, has a tilt of 3°. An point-image q lies on the principal line, upward of the axis of tilt, at distance of 60mm from the principal point. Assuming that the object-point is at datum level, compute:

(i) The scale at q

(ii) The tilt displacement of q. (10 marks)

$(iq)^2 \sin t \cos \lambda$

cycle

$S = \frac{f}{H}(\cos t - \tan \alpha \sin t)$ $\frac{160}{102}$

$iq = ir + pq$; $ir = f \tan \frac{t}{2} + pq \cos 30^\circ$

1cm = 25,000

102 / (102 × 25000) cm on ground

H 2 cm → 31875

31875

UNIVERSITY OF NAIROBI
DEPARTMENT OF GEOSPATIAL AND SPACE TECHNOLOGY
FGE 345: REMOTE SENSING SYSTEMS
CONTINUOUS ASSESSMENT TEST

Date: 23/01/24 Time: 2 – 3 pm

Attempt all questions

1a) The use of space has been expanding over the years. Using an area of application explain the role of space. **(10 marks)**

- satellites

b) Describe how electromagnetic energy can be modeled. **(10 marks)**

- Wave model - uses electron radiation
- Photon model - when you want to quantify

c) Distinguish between framing and scanning systems. **(10 marks)**

$c = \lambda f$

$Q = h\nu$

2a) On the basis of spatial resolution, give 3 different examples of operational multispectral sensors; highlight their characteristics and a possible area of application. **(10 marks)**

b) Using an example, explain how spectral reflectance curves can be used. **(5 marks)**

3a) Within the context of the remote sensing process, describe what the stage of transmission, reception and processing involves. **(10 marks)**

5th
- Interpretation & Analysis
- Application

b) Using a clear illustration distinguish among the various generic types of satellites. **(5 marks)**

- Geostationary Earth
- Low Earth orbit
- Median Earth orbit
- High Ellipsoid orbit

3a)
- Energy source / illumination
- Radiation & atmosphere
- Interaction with the Target

Stefan Boltzmann's law

σT^4

Passive & Act

Sensor

oriented