COMP3520 Assignment 1

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1. Using the C language a structure to define a pixel and an array of $M * N$ pixels would be:

```
struct Pixel {
    unsigned char red;
    unsigned char green;
    unsigned char blue;
};

struct Pixel DISPLAY[M * N];
```

The point **P=(X, Y)** would be located at offset $y * N + x$ from &DISPLAY.

For example:

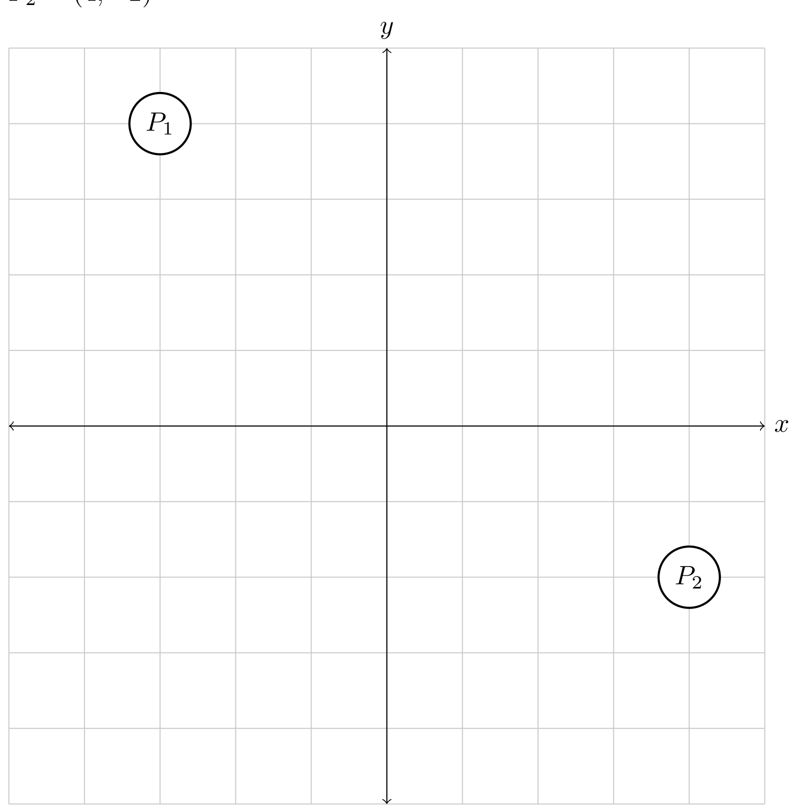
```
struct Pixel* p = DISPLAY + y * N + x;
p->red = 255;
```

2. Screen dimensions: 29.44cm x 16.56cm
Resolution: 1920 x 1080
$29.44/1920 = 0.01534$
$16.56/1080 = 0.01534$
The distance between pixels is 0.1534mm.

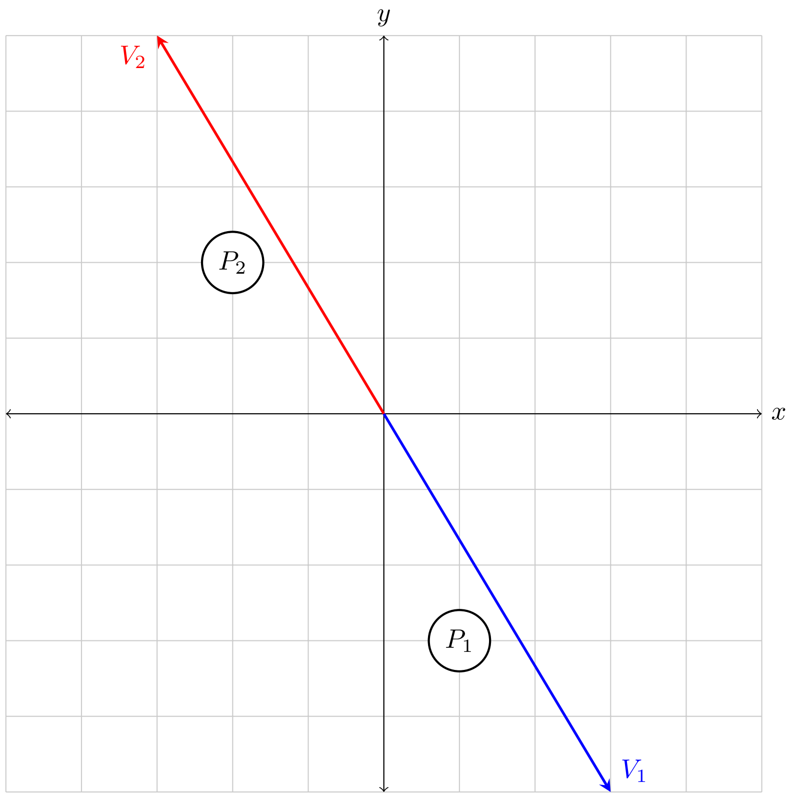
The smallest objects the human eye can see is about 0.1mm.

https://learn.genetics.utah.edu/content/cells/scale/

3. a. $P_1 = (-3, 4)$
$P_2 = (4, -2)$



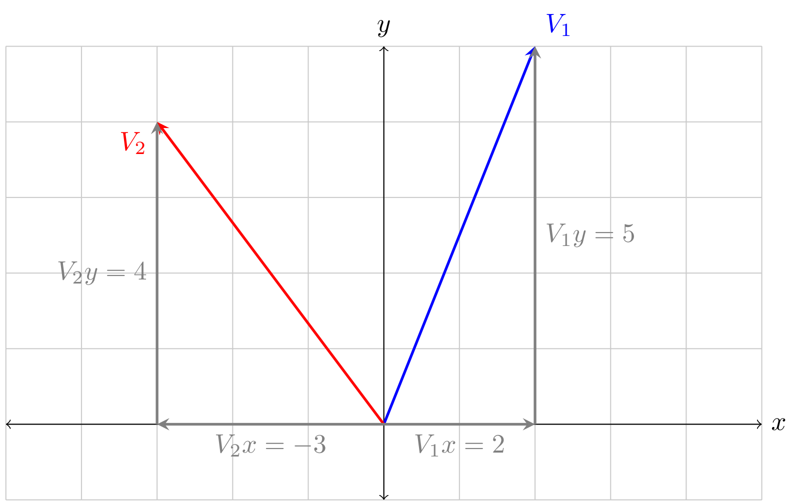
b. $P_1 = (1, -3)$
$P_2 = (-2, 2)$
$V_1 = P_1 - P2 = \begin{bmatrix} 1-(-2) \\ -3-2 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$
$V_2 = P_2 - P1 = \begin{bmatrix} -2-1 \\ 2-(-3) \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$



The two vectors V_1 and V_2 are mirrors of each other. Adding them together will give a vector of $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Adding V_2 to P_1 will give P_2 and adding V_1 to P_2 will give P_1.

c. The dot product of the vector $V_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ and the vector $V_2 = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ can be solved as follows:

$V_1 \cdot V_2 = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 4 \end{bmatrix} = (2 \times -3) + (5 \times 4) = 14$
There is no difference between $V_1 \cdot V_2$ and $V_2 \cdot V_1$ because the dot product is commutative.



d. $V_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

$$V_2 = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$$

i. $V_1 - V_2 = \begin{bmatrix} 2-3 \\ 4-6 \\ 6-7 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$

ii. $|V_1| = \sqrt{2^2+4^2+6^2} = \sqrt{56}$
$|V_2| = \sqrt{3^2+5^2+7^2} = \sqrt{83}$

iii. $V_1 \cdot V_2 = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} = 2(3) + 4(5) + 6(7) = 68$

iv. $V_1 \times V_2 = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \times \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 4(7)-6(5) \\ 6(3)-2(7) \\ 2(5)-4(3) \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 2 \end{bmatrix}$

$V_2 \times V_1 = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} \times \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 5(6)-7(4) \\ 7(2)-3(6) \\ 3(4)-5(2) \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 2 \end{bmatrix}$

4. $M_1 = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \end{bmatrix}$

$M_2 = \begin{bmatrix} 1 & 2 & 1/2 \\ -1/2 & -1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

i. The inverse matrix M_1^{-1} does not exist because it is not a square matrix.
The inverse matrix M_2^{-1} can be found as follows:
Create a matrix of minors for M_2:

$$\begin{bmatrix} (-1)(2)-(1)(1) & (-1/2)(2)-(1)(0) & (-1/2)(1)-(-1)(0) \\ (2)(2)-(1/2)(1) & (1)(2)-(1/2)(0) & (1)(1)-(2)(0) \\ (2)(1)-(1/2)(-1) & (1)(1)-(1/2)(-1/2) & (1)(-1)-(2)(-1/2) \end{bmatrix} = \begin{bmatrix} -3 & -1 & -1/2 \\ 7/2 & 2 & 1 \\ 5/2 & 5/4 & 0 \end{bmatrix}$$

Create a matrix of co-factors:

$$\begin{bmatrix} -3 & 1 & -1/2 \\ -7/2 & 2 & -1 \\ 5/2 & -5/4 & 0 \end{bmatrix}$$

Transpose that matrix:

$$\begin{bmatrix} -3 & -7/2 & 5/2 \\ 1 & 2 & -5/4 \\ -1/2 & -1 & 0 \end{bmatrix}$$

Multiply the matrix by the determinant of the original matrix:
$\det M_2 = 1((-1)(2)-(1)(1)) - 2((-1/2)(2)-(1)(0)) + (1/2)((-1/2)(1)-(-1)(0)) = -4/5$

$$M_2^{-1} = -4/5 \begin{bmatrix} -3 & -7/2 & 5/2 \\ 1 & 2 & -5/4 \\ -1/2 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 12/5 & 14/5 & -2 \\ -4/5 & -8/5 & 1 \\ -2/5 & 4/5 & 0 \end{bmatrix}$$

ii. $M_1^T = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ -2 & -4 \end{bmatrix}$

$M_2^T = \begin{bmatrix} 1 & -1/2 & 0 \\ 2 & -1 & 1 \\ 1/2 & 1 & 2 \end{bmatrix}$

iii. The determinant $\det M_1$ does not exist because it is not a square matrix.
$\det M_2 = 1((-1)(2)-(1)(1)) - 2((-1/2)(2)-(1)(0)) + (1/2)((-1/2)(1)-(-1)(0)) = -4/5$

iv. $M_1 \cdot M_2 = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1/2 \\ -1/2 & -1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

$M_{11} = (1)(1) + (3)(-1/2) + (-2)(0) = -1/2$
$M_{12} = (1)(2) + (3)(-1) + (-2)(1) = -3$

$M_{13} = (1)(1/2) + 3(1) + (-2)(2) = -1/2$

$M_{21} = (2)(1) + (-1)(-1/2) + (4)(0) = 5/2$
$M_{22} = (2)(2) + (-1)(-1) + (4)(1) = 9$
$M_{23} = (2)(1/2) + (-1)(1) + (4)(2) = 8$

$$M_1 \cdot M_2 = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \end{bmatrix} = \begin{bmatrix} -1/2 & -3 & -1/2 \\ 5/2 & 9 & 8 \end{bmatrix}$$

v. No the inner product or M_1 and M_2 is not commutative. The inner dimensions of the two matrices would not be the same.