cubic_1d_demo

March 31, 2024

[ ]:
```julia
using Pkg;
Pkg.activate(".")
using Plots, CSV, DataFrames, Polynomials, LinearAlgebra
```

```
  Activating project at
`~/Documents/Spring2024/NumericalAnalysis/NA-FinalProject/src`
```

0.0.1 Procedure

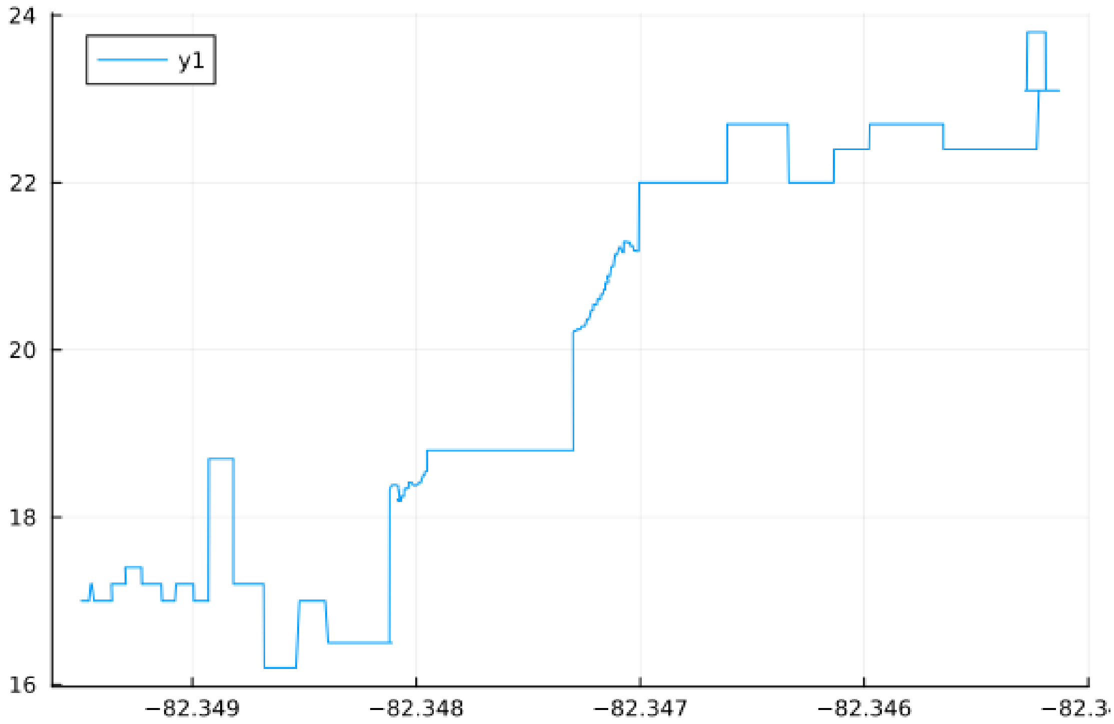
1. Parse data

The spline algorithms only work if the x-values of the data are in ascending order. This also means that if, in the same data recording, I walk forward, and then backwards, the algorithm won't work. To account for this we will sort the data

[ ]:
```julia
# This will create a dataframe from our file
filename = "../data/03-26-Stadium/Location.csv"
data = CSV.File(open(filename)) |> DataFrame

# using the longitude as our x-values and the altitude as our y-values
# dataframes makes it easy to get these values by using the dot syntax

#USE LATITUDE FOR X-VALUES IN OUR TESTING (THIS DATA IS TAKEN GOING E-W, not␣
 ↪N-S)
x = data.longitude;
y = data.altitude;
```

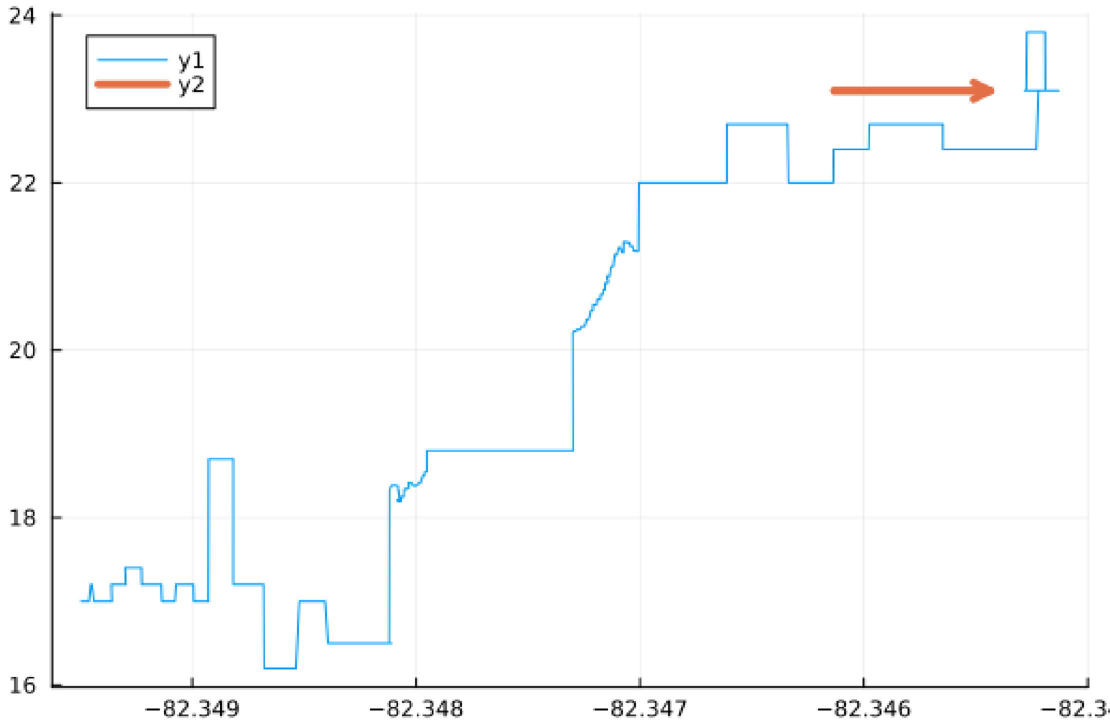
Let me plot the original data to show you what I mean

[ ]:
```julia
plot(x, y)
```



Do you see that at the top right?

```
[ ]: plot!([x[200], x[50]], [y[5], y[5]], arrow=true, l=4)
```



We don't want loops like that, it's hard for the algorithm to build fitting functions when they occur

```
[ ]: #sort the x values to account for any wobble in the data
x_sorted = sort(x, rev=true);
```

2. Sample some points from the sorted data

```
[ ]: """
    get_spaced_points(x_sorted, y, n=10; rev=true)

Takens in sorted x and y values, returns 'n' number of equally spaced out
points from the data. If inputted x-values are in increasing order, set␣
 ↪rev=false
"""
function get_spaced_points(x_sorted, y, n=10; rev=true)
    len = length(x_sorted)
    indices = round.(Int, LinRange(1, len, n))

    if rev
        xs = reverse(x_sorted[indices])
        ys = reverse(y[indices])
    else
        xs = x_sorted[indices]
        ys = y_sorted[indices]
```

```
    end

    return xs, ys
end
```

get_spaced_points

[ ]:
```
xs, ys = get_spaced_points(x_sorted, y);
```

3. Setup cubic interpolation method

This is taken directly from the textbook, it appears to be working currently. He uses a different naming convention. To convert –> His 't' == my 'xs' , His 'y' == my 'ys'

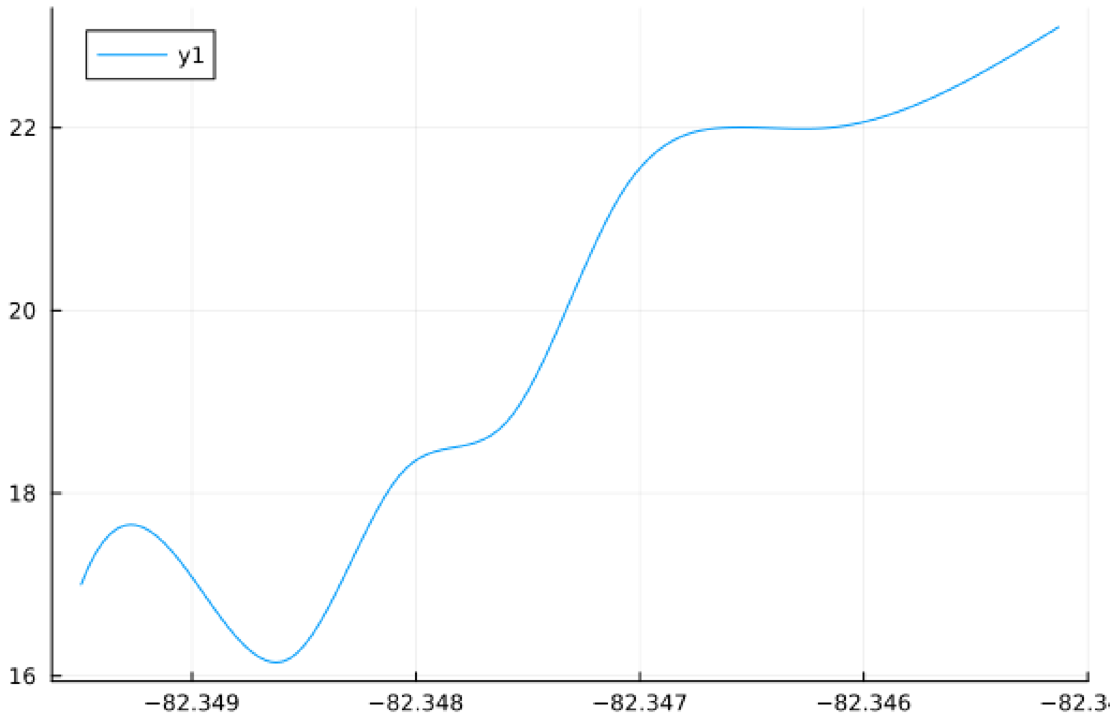
This takes in a sample and returns a function that fits these samples. To plot, we feed into the returned function our data points

[ ]:
```
"""
    spinterp(t,y)

Construct a cubic not-a-knot spline interpolating function for data
values in `y` given at nodes in `t`.
"""
function spinterp(t,y)
    n = length(t)-1
    h = [ t[k+1]-t[k] for k in 1:n ]

    # Preliminary definitions.
    Z = zeros(n,n);
    In = I(n);  E = In[1:n-1,:];
    J = diagm(0=>ones(n),1=>-ones(n-1))
    H = diagm(0=>h)

    # Left endpoint interpolation:
    AL = [ In Z Z Z ]
    vL = y[1:n]

    # Right endpoint interpolation:
    AR = [ In H H^2 H^3 ];
    vR = y[2:n+1]

    # Continuity of first derivative:
    A1 = E*[ Z J 2*H 3*H^2 ]
    v1 = zeros(n-1)

    # Continuity of second derivative:
    A2 = E*[ Z Z J 3*H ]
    v2 = zeros(n-1)
```

```
    # Not-a-knot conditions:
    nakL = [ zeros(1,3*n) [1 -1 zeros(1,n-2)] ]
    nakR = [ zeros(1,3*n) [zeros(1,n-2) 1 -1] ]

    # Assemble and solve the full system.
    A = [ AL; AR; A1; A2; nakL; nakR ]
    v = [ vL; vR; v1; v2; 0; 0 ]
    z = A\v

    # Break the coefficients into separate vectors.
    rows = 1:n
    a = z[rows]
    b = z[n.+rows];  c = z[2*n.+rows];  d = z[3*n.+rows]
    S = [ Polynomial([a[k],b[k],c[k],d[k]]) for k in 1:n ]

    # This function evaluates the spline when called with a value
    # for x.
    return function (x)
        if x < t[1] || x > t[n+1]    # outside the interval
            return NaN
        elseif x==t[1]
            return y[1]
        else
            k = findlast(x .> t)    # last node to the left of x
            return S[k](x-t[k])
        end
    end
end
```

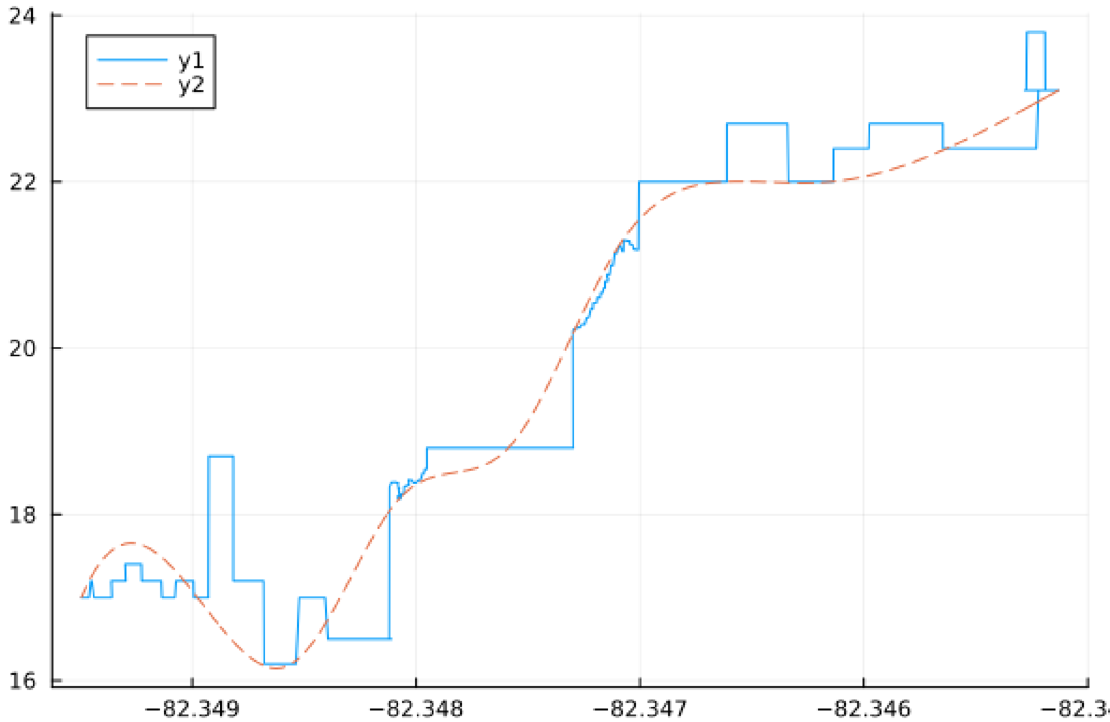
spinterp

[ ]:
```
cspline_f = spinterp(xs, ys)
plot(x_sorted, cspline_f.(x_sorted))
```



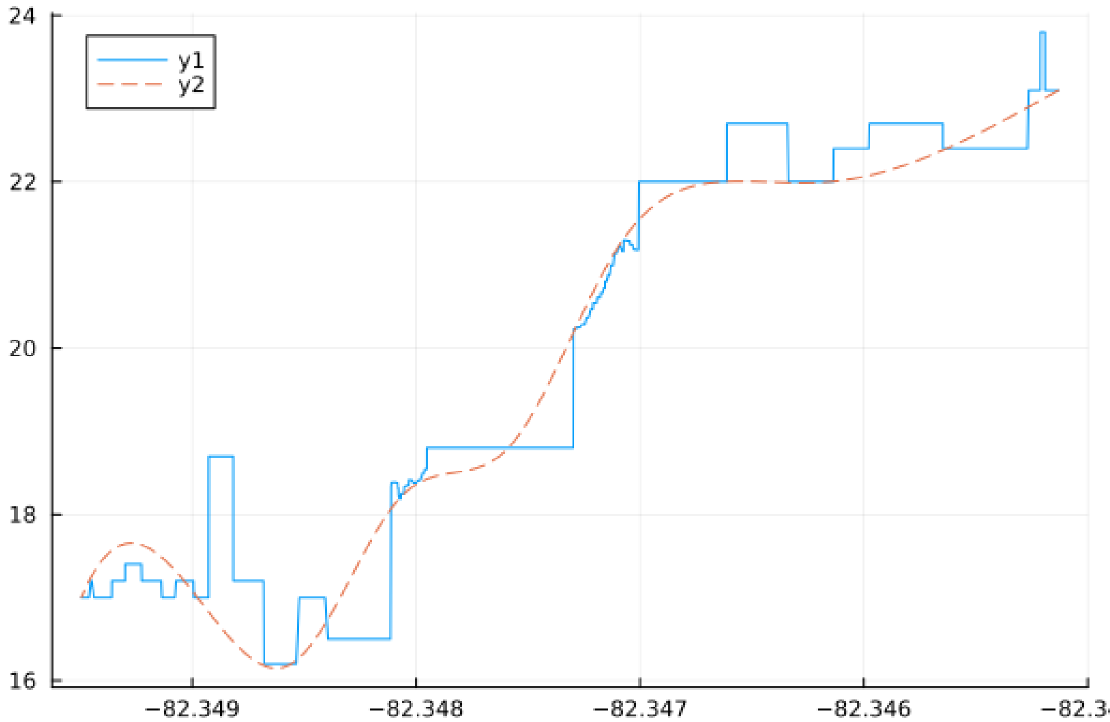
Plotting it over top of our original data

[ ]:
```
plt = plot(x, y)
plot!(plt, x, cspline_f.(x), ls=:dash)
```



Since I built the model using the sorted x-values, it's probably more accurate to show it overlaid on that plot instead

[ ]:
```
plt = plot(x_sorted, y)
plot!(plt, x_sorted, cspline_f.(x_sorted), ls=:dash)
```



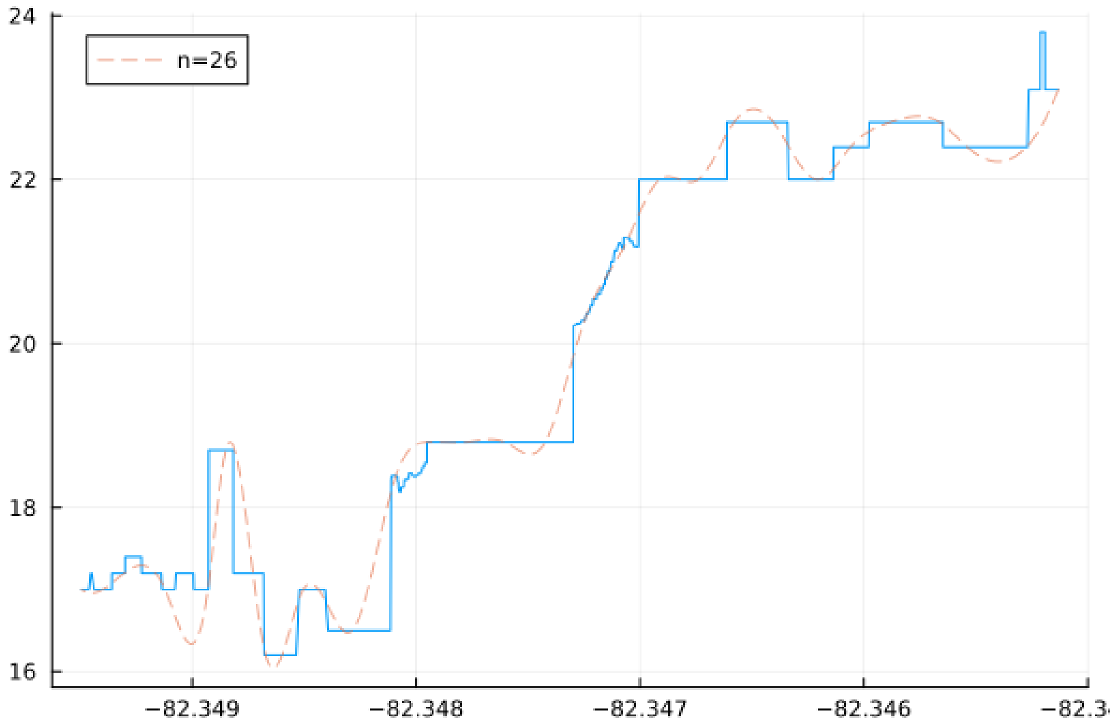
4. Piece together into an easily usable function

I've opted for a function that plots an overlay of the spline onto a preexisting plot. I think this makes it easy to plot a bunch of splines at different values of n. It moreover allows for you to easily change the styling of the original plot without having to edit the function every time or pass in a bunch of parameters.

[ ]:
```
function plot_cubic_spline(plt, x, y, n=10; alpha=0.6)
    x_sorted = sort(x, rev=true);
    xs, ys = get_spaced_points(x_sorted, y, n);
    cspline_f = spinterp(xs, ys)
    plot!(plt, x_sorted, cspline_f.(x_sorted), label="n=$n",ls=:dash,␣
 ↪alpha=alpha)
end
```

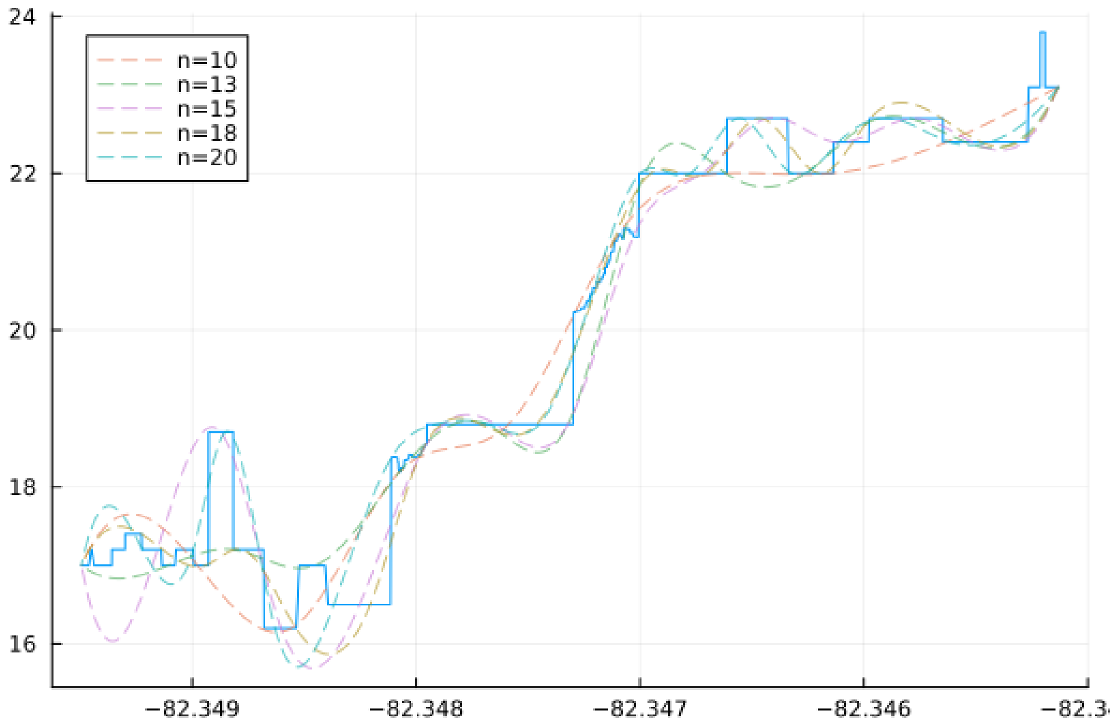
```
plot_cubic_spline (generic function with 2 methods)
```

[ ]:
```
plt = plot(x_sorted, y, label=:none)
plot_cubic_spline(plt, x, y, 26)
```



Demo showing how to overlay multiple

[ ]:
```
ns = [10, 13, 15, 18, 20]

plt = plot(x_sorted, y, label=:none)

for n in ns
     plot_cubic_spline(plt, x, y, n)
end

plt
```


24
22
20
18
16
n=10
n=13
n=15
n=18
n=20
−82.349
−82.348
−82.347
−82.346

[ ]:

[ ]: