QML Estimation

$y_t = \mu + \epsilon_t$

$\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2 exp(h_t))$

$h_{t+1} = \mu + \phi(h_t - \mu) + \xi_t$

where $\xi_t \sim \mathcal{N}(0, \sigma_\xi^2)$

In [365…

```
import numpy as np
import pandas as pd
import random as rd
import math
import scipy as sp
import matplotlib.pyplot as plt
```

i) Simulate the Model

In [366…

```
from SV_Functions import sv_simul, sv_simul_v2
from scipy.optimize import minimize
```

QML Estimation (Francq, Zakoian)

canonical SV Model

$\epsilon_t = \sqrt{h_t}\eta_t$

$log(h_t) = \omega + \beta log(h_{t-1}) + \sigma v_t$

$\eta_t \sim \mathcal{N}(0,1)$

$v_t \sim \mathcal{N}(0,1)$

State-space model

$log(\epsilon_t^2) = log(h_t) + \mu_Z + u_t$

$log(h_t) = \beta log(h_{t-1}) + \omega + \sigma v_t$

In [367…

```
def sv_simul_can(θ, log_h0):
    ω = θ[0]
    α = θ[1]
    σ = θ[2]
    η = np.random.normal(0, 1, T)
    v  = np.random.normal(0, 1, T)
    ϵ_t = np.empty(0)
    log_h = np.empty(0)

    log_h = np.append(log_h, ω + β * log_h0 + σ * v[0])
    ϵ_t = np.append(ϵ_t, np.sqrt(np.exp(log_h[0])) * η[0])

    for t in range(1, T):
        log_h = np.append(log_h, ω + β * log_h[t-1] + σ * v[t])
```

```
        ϵ_t = np.append(ϵ_t, np.sqrt(np.exp(log_h[t])) * η[t])

    return ϵ_t, log_h
```

In [368…

```
# Inputs
ω = 1e-3
β = 0.9
σ = 0.8
θ = ω, β, σ
log_h0 = ω
T = 500
```

In [369…

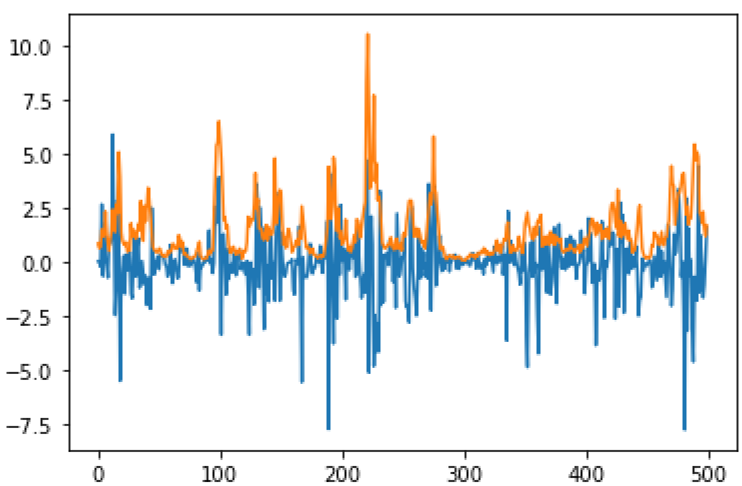
```
data = sv_simul_can(θ, log_h0)
ϵ_t = data[0]
log_h = data[1]
```

In [370…

```
plt.plot(range(T), ϵ_t)
plt.plot(range(T), np.sqrt(np.exp(log_h)))
```

Out[370…

```
[<matplotlib.lines.Line2D at 0x19d68c80a00>]
```



In [371…

```
def obj_sv(θ, ϵ_t):
    α_t = np.empty(0)
    P_t = np.empty(0)
    F_t = np.empty(0)
    K_t = np.empty(0)
    ω = θ[0]
    β = θ[1]
    σ = θ[2]
    μ_Z = -1.270
    σ_Z = np.pi**2 / 2
    a0 = 0
    β0 = 0.8
```

```
    P0 = σ**2
    F0 = P0 + σ_Z**2
    T = len(ϵ_t)
    y_t = np.log(np.square(ϵ_t))

    α_t = np.append(α_t, β0 * a0 + ω)
    P_t = np.append(P_t, σ**2)
    K_t = np.append(K_t, β * P0 * 1/F0)

    for t in range(1,T):
        F_t = np.append(F_t, P_t[t-1] + σ_Z**2)
        K_t = np.append(K_t, β * P_t[t-1] * 1/F_t[t-1])
        α_t = np.append(α_t, β * α_t[t-1] + K_t[t] * (y_t[t-1] - α_t[t-1] -
μ_Z) + ω)
        P_t = np.append(P_t, β**2 * P_t[t-1] - K_t[t]**2 * F_t[t-1] + σ**2)
    F_t = np.append(F_t, P_t[T-1] + σ_Z**2)

    qml = -T/2 * np.log(2*np.pi) - 1/2 * np.sum(np.log(F_t) +
np.square((np.log(np.square(ϵ_t)) - α_t - μ_Z)) / F_t)
    return -qml
```

In [372…

```
def estim_sv(θ0, ϵ_t):
    valinit = θ0
    res = minimize(obj_sv, valinit, args=(ϵ_t), bounds=((1e-5, math.inf), (0,
0.99), (0, math.inf)))
    θ_hat = res.x
    likeli = res.fun
    return θ_hat, likeli
```

In [373…

```
ω0 = 1e-3
β0 = 0.85
σ0 = 0.9
θ0 = ω0, β0, σ0
```

In [374…

```
estimation = estim_sv(θ0, ϵ_t)
θ_hat = estimation[0]
θ_hat
```

Out[374…

```
array([1.e-05, 0.e+00, 0.e+00])
```

SV Model Estimation - Monte Carlo

In [375…

```
M = 2000
```

In [376…

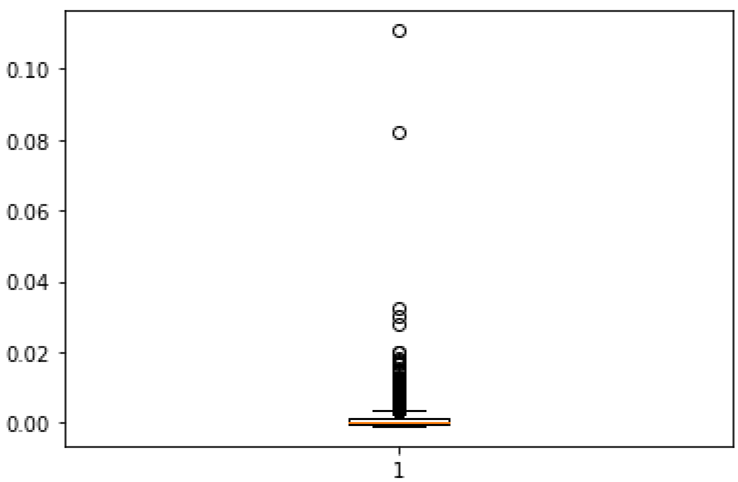
```
ω_spread = np.empty(0)
β_spread = np.empty(0)
σ_spread = np.empty(0)
for j in range(M):
    results = garch_simul(θ, T)
    ϵ_t = results[0]
    estimation = estim_garch(θ0, ϵ_t)
    θ_hat = estimation[0]
    ω_hat = θ_hat[0]
    β_hat = θ_hat[1]
    σ_hat = θ_hat[2]
    ω_spread = np.append(ω_spread, ω_hat - ω)
    β_spread = np.append(β_spread, β_hat - β)
    σ_spread = np.append(σ_spread, σ_hat - σ)
```

In [377…

```
plt.boxplot(ω_spread)
np.seterr(divide = 'ignore')
```

Out[377…

```
{'divide': 'ignore', 'over': 'warn', 'under': 'ignore', 'invalid': 'warn'}
```

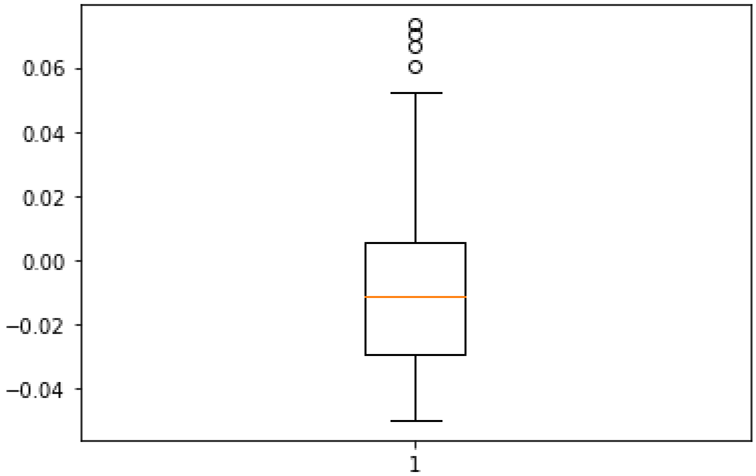


In [378…

```
plt.boxplot(α_spread)
np.seterr(divide = 'ignore')
```

Out[378…

```
{'divide': 'ignore', 'over': 'warn', 'under': 'ignore', 'invalid': 'warn'}
```

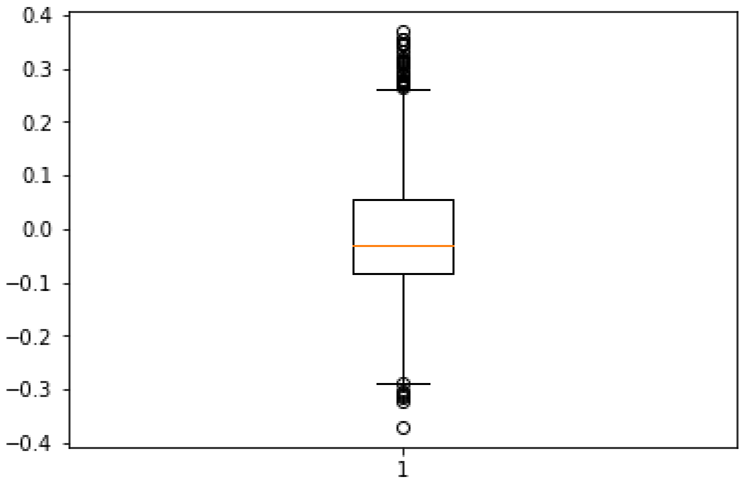


In [379…

```
plt.boxplot(β_spread)
np.seterr(divide = 'ignore')
```

Out[379…

```
{'divide': 'ignore', 'over': 'warn', 'under': 'ignore', 'invalid': 'warn'}
```



Estimate GARCH Models

In [74]:

```
def garch_simul(θ, T):
    η = np.random.normal(0, 1, T)
    ϵ_t = np.empty(0)
    σ2 = np.empty(0)
    ω = θ[0]
    α = θ[1]
    β = θ[2]

    ϵ_t = np.append(ϵ_t, np.sqrt(ω) * η[0])
    σ2 = np.append(σ2, ω)

    for t in range(1, T):
        σ2 = np.append(σ2, ω + α * ϵ_t[t-1]**2 + β * σ2[t-1])
        ϵ_t = np.append(ϵ_t, np.sqrt(σ2[t]) * η[t])
```

```
    return ϵ_t, σ2
```

In [75]:
```
def obj_garch(θ, ϵ_t):
    ω = θ[0]
    α = θ[1]
    β = θ[2]
    T = len(ϵ_t)
    σ2 = np.empty(0)

    σ2= np.append(σ2, ω)
    for t in range(1,T):
        σ2 = np.append(σ2, ω + α * ϵ_t[t-1]**2 + β * σ2[t-1])
    qml = np.mean(np.log(σ2[1:T]) + np.square(ϵ_t[1:T]) / σ2[1:T])
    return qml
```

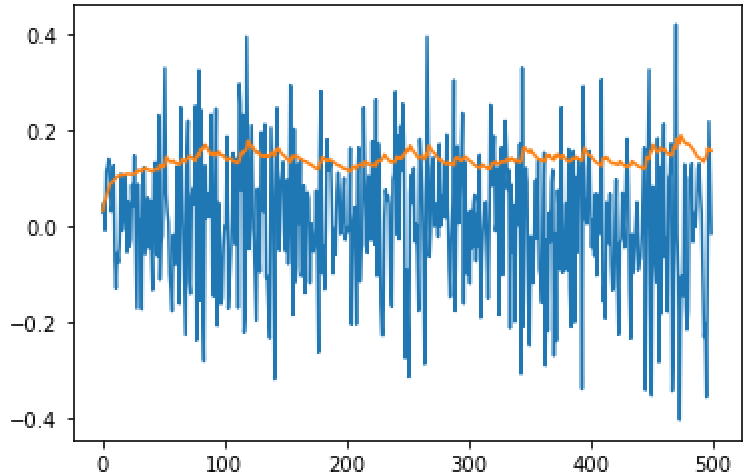
In [76]:
```
def estim_garch(θ, ϵ_t):
    valinit = θ
    res = minimize(obj_garch, valinit, args=(ϵ_t), bounds=((1e-4, math.inf),
(0, math.inf), (0, 0.999)))
    θ_hat = res.x
    likeli = res.fun
    return θ_hat, likeli
```

In [77]:
```
ω = 1e-3
α = 0.05
β = 0.9
θ = ω, α, β
```

In [78]:
```
results = garch_simul(θ, T)
ϵ_t = results[0]
σ2 = results[1]
```

In [79]:
```
plt.plot(range(T), ϵ_t)
plt.plot(range(T), np.sqrt(σ2))
```

Out[79]: [<matplotlib.lines.Line2D at 0x19d62fb5280>]



In [80]:
```
ω0 = 1e-3
α0 = 0.1
β0 = 0.8
θ0 = ω0, α0, β0
```

In [81]:
```
estimation = estim_garch(θ0, ϵ_t)
θ_hat = estimation[0]
θ_hat - θ
```

Out[81]:
```
array([ 0.00146761,  0.03494167, -0.10814077])
```

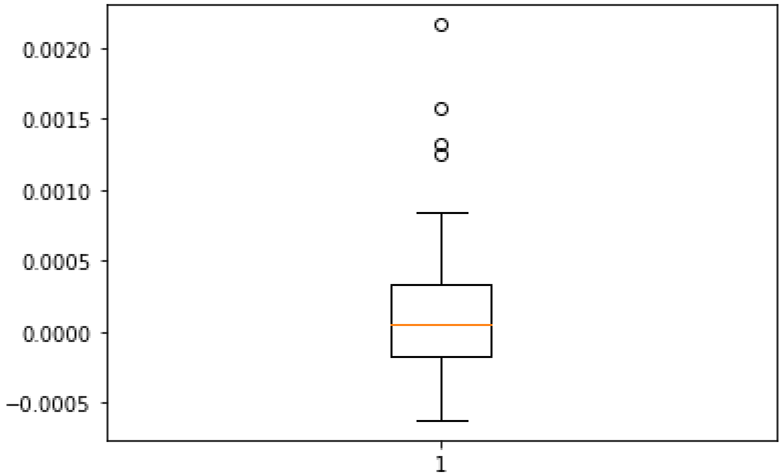
GARCH Monte Carlo Experiment

In [82]:
```
M = 100
```

In [83]:
```
ω_spread = np.empty(0)
α_spread = np.empty(0)
β_spread = np.empty(0)
for j in range(M):
    results = garch_simul(θ, T)
    ϵ_t = results[0]
    estimation = estim_garch(θ0, ϵ_t)
    θ_hat = estimation[0]
    ω_hat = θ_hat[0]
    α_hat = θ_hat[1]
    β_hat = θ_hat[2]
    ω_spread = np.append(ω_spread, ω_hat - ω)
    α_spread = np.append(α_spread, α_hat - α)
    β_spread = np.append(β_spread, β_hat - β)
```

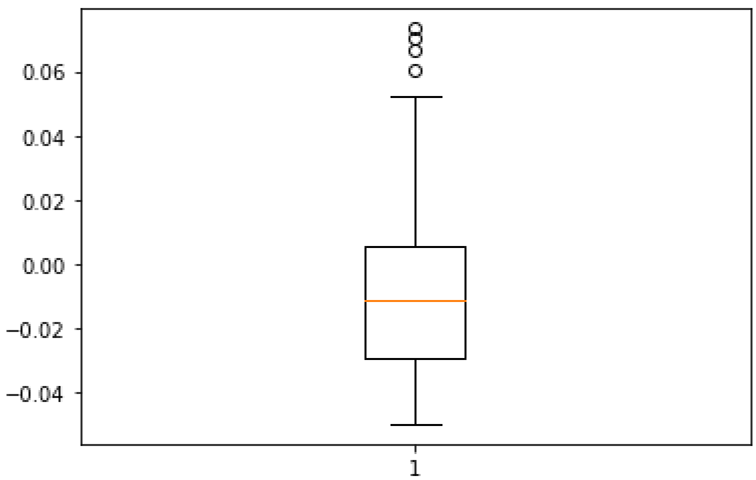
In [84]:
```
plt.boxplot(ω_spread)
np.seterr(divide = 'ignore')
```

Out[84]: {'divide': 'ignore', 'over': 'warn', 'under': 'ignore', 'invalid': 'warn'}



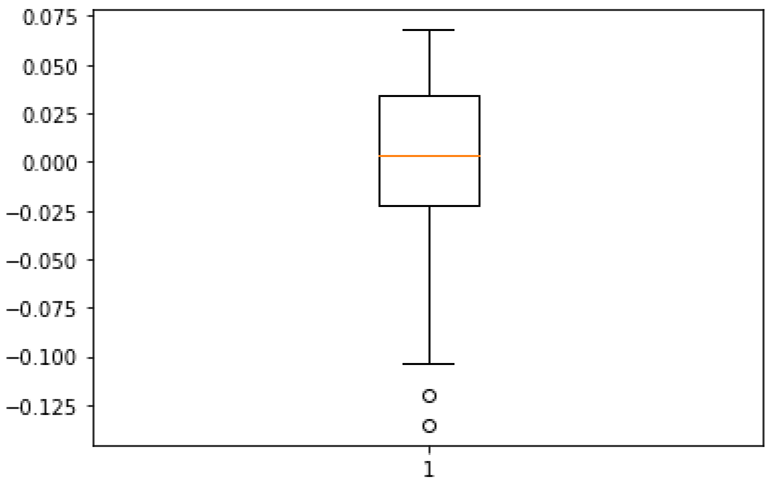
In [85]:
```
plt.boxplot(α_spread)
np.seterr(divide = 'ignore')
```

Out[85]: {'divide': 'ignore', 'over': 'warn', 'under': 'ignore', 'invalid': 'warn'}



In [86]:
```
plt.boxplot(β_spread)
np.seterr(divide = 'ignore')
```

Out[86]: {'divide': 'ignore', 'over': 'warn', 'under': 'ignore', 'invalid': 'warn'}



In [ ]:
```

```