Asset Allocation with a High Dimensional Latent Factor Stochastic Volatility Model

Yufeng Han
Tulane University

We investigate the implications of time-varying expected return and volatility on asset allocation in a high dimensional setting. We propose a dynamic factor multivariate stochastic volatility (DFMSV) model that allows the first two moments of returns to vary over time for a large number of assets. We then evaluate the economic significance of the DFMSV model by examining the performance of various dynamic portfolio strategies chosen by mean-variance investors in a universe of 36 stocks. We find that the DFMSV dynamic strategies significantly outperform various benchmark strategies out of sample. This outperformance is robust to different performance measures, investor's objective functions, time periods, and assets.

Mounting evidence suggests that both the expected return and volatility of asset returns vary over time. On the one hand, many studies find that the expected returns have time-varying components that are partially predictable.[1] On the other hand, considerable effort has been devoted to the modeling of time-varying volatility, especially in the context of daily return data.[2] As a result of this quite conclusive evidence, recent attention has moved to examining the implications of these time-varying properties on problems that are of practical importance, such as the optimal asset allocation. However, the majority of the existing literature in this area focuses only on either one of the time-varying aspects. For example, studies on return predictability and asset allocation only consider

I am grateful to Kerry Back, Suman Banerjee, David Bates, Nai-fu Chen, Shane Corwin, Cheol Eun, Heber Farnsworth, Frank Fehle, Jeff Fleming, Thomas George, Shingo Goto, John Hund, Narayanan Jayaraman, Christopher Jones, Praveen Kumar, David Lesmond, Marc Lipson, Greg Niehaus, Thomas Noe, John Scruggs, Paul Schultz, Ken Singleton (the editor), Jason Smith, Christopher Stivers, Sheri Tice, Sergey Tsyplakov, Anand Vijh, Yihong Xia, Guofu Zhou, seminar participants at the 2003 AFA meetings, Georgia Institute of Technology, Tulane University, University of California at Irvine, University of Georgia, University of Houston, University of Iowa, University of Notre Dame, University of South Carolina and Washington University in St. Louis for helpful comments, and especially to Siddhartha Chib for his advice and encouragement, and an anonymous referee for insightful suggestions that have substantially improved the article. This article was previously circulated under the title "The Economic Value of Volatility Modeling: Asset Allocation with a High Dimensional Dynamic Latent Factor Multivariate Stochastic Volatility Model." Address correspondence to Yufeng Han, A. B. Freeman School of Business, Tulane University, 7 McAlister Drive, New Orleans, LA 70118, or e-mail: yhan@tulane.edu.

[1] The empirical literature on return predictability is enormous. Some recent examples are Avramov (2002), Cremers (2002), Goyal and Welch (2003), and Lettau and Ludvigson (2001).

[2] For the (G)ARCH model, see the review by Bollerslev, Engle, and Nelson (1994) and references therein; for the stochastic volatility model, see Kim, Shephard, and Chib (1998), the review by Ghysels, Harvey, and Renault (1995) and references therein.

doi:10.1093/rfs/hhj002 Advance Access publication October 28, 2005

time-varying expected return and assume constant volatility.[3] In contrast, it is customary to assume constant expected returns in the volatility-timing literature.[4] Only a handful of studies examine the asset allocation problem and allow the first two moments to vary over time—for example, Gomes (2002), Han (2004), Johannes, Polson, and Stroud (2002), and Marquering and Verbeek (2001)—but the methodology of these studies is limited to the case of one risky asset only.

The asset allocation problem is inherently a practical problem, and in practice, investors, especially institutional investors, routinely deal with a large number of assets. Despite active research in this area, however, the implications of time-varying expected return and volatility remain unknown with a large number of assets. Consider an investor allocating funds among a wide variety of stocks. Given that the expected return and volatility of stock returns vary over time, how could she take advantage of the time-varying investment opportunity? To answer this question, it is necessary to have a model that incorporates both time-varying expected return and volatility for multiple assets. The model must be practical and straightforward to estimate, even when the number of assets is large. Unfortunately, as discussed above, there is no such model in the existing literature. Our first contribution is to propose a dynamic factor multivariate stochastic volatility (DFMSV) model that allows the first two moments of returns to vary over time for a large number of assets.

A number of distinctive features of the model should be noted. First, in line with the extensive literature on the factor model including the arbitrage pricing theory (APT) model, we utilize unobserved factors to parsimoniously capture the dynamics of the first two moments of returns. However, it should be noted that the factors in our model are assumed to follow a richer time-series process than is normally assumed. In particular, we assume that the factors not only have time-varying volatilities, but also admit autoregressive processes. Both extensions are practically important. Letting the conditional covariance matrix be time-varying has precedence [Chib, Nardari, and Shephard (2002)], but the assumption that the factors follow a vector autoregressive process is new and is motivated by the empirical evidence that many predictive variables, such as the dividend yield and Treasury-bill yield, display strong serial dependence.

Second, our model is considerably more tractable than multivariate generalized autoregressive conditional heteroskedasticity (GARCH) models that tend to be parameter-rich and difficult to estimate for anything more

[3] See, for example, Barberis (2000), Handa and Tiwari (2004), Kandel and Stambaugh (1996), Lynch and Balduzzi (2000), Pesaran and Timmermann (1995), and Tamayo (2002).

[4] Volatility timing is used here generically referring to asset allocation with time-varying volatility. Examples are Aguilar and West (2000), Copeland and Copeland (1999), Fleming, Kirby, and Ostdiek (2001, 2003), and Graham and Harvey (1996).

than a handful of assets. [For example, the proposed GARCH framework of Pojarliev and Polasek (2000) is limited to three MSCI regional indices.] However, with our framework, it is possible to consider the universe of assets defined by a large collection of securities. Such an analysis, we believe, would be of great interest to practitioners who routinely deal with a large number of assets. The estimation of a multivariate GARCH model of this size does not appear to be feasible. Indeed, Bollerslev (2001) points out that the multivariate stochastic volatility (MSV) model outlined in Chib, Nardari, and Shephard (2002), on which our model is based, currently offers the best prospects for modeling multivariate time-varying volatility.

Third, our analysis utilizes Bayesian techniques in estimation. The primary reason for this is that our model contains a large number of unobservables (namely, the factors and volatility terms) that make finding the likelihood function extremely difficult. By utilizing the Bayesian approach, we only need to compute various components of the likelihood function conditioned on the latent variables and thus avoid the direct computation of the likelihood function. Our estimation method has been extensively tested and reliably estimates the parameters even when the model is high dimensional, containing hundreds of parameters.

With the DFMSV model, we study the asset allocation problem of risk-averse investors faced with a collection of 36 assets and examine the economic gains that accrue from timing volatility and expected return. To mimic the real-world portfolio problem, the 36 assets are chosen from stocks in six categories, ranging from large cap to small cap, and including both growth and value stocks. We focus primarily on the mean-variance portfolio problem and ignore the hedging demand induced by the time-varying investment opportunity, which is computationally intractable.[5] However, the assumption of a mean-variance preference is for simplicity only.[6] We assume that mean-variance investors allocate funds among the 36 stocks and a risk-free asset, and rebalance their portfolio weights periodically given the forecasts of the conditional mean and covariance of future returns, which are estimated from the DFMSV model. Two dynamic strategies are constructed: return-volatility timing strategy where investors forecast both the conditional mean and covariance, and volatility-timing strategy where investors only forecast the conditional covariance. We also consider three active investors: "day traders" who rebalance their portfolios daily, "swing traders" who rebalance their portfolios weekly, and "position traders" who rebalance their

[5] Recent studies, such as Aït-Sahalia and Brandt (2001) and Gomes (2002), also suggest that the hedging demand is typically a very small component of the optimal portfolio allocation.

[6] Pulley (1981) shows that its approximation to a general utility function is adequate when the holding periods are short; Kroll, Levy, and Markowitz (1984) further show that the holding periods may be extended to as long as one year. Johannes, Polson, and Stroud (2002) also provide justification for using a conditional mean-variance framework with stochastic volatility.

portfolios once per month. As a benchmark, we also consider passive investors who follow buy-and-hold strategies.

We calculate the *ex post* returns on the dynamic portfolios formed in this manner and evaluate the out-of-sample performance with the Sharpe ratio and several other performance measures such as the risk-adjusted abnormal return ($\mathcal{M}2$) and the performance fee, defined as the management fee that the passive investors would be willing to pay to switch from the buy-and-hold strategy to the dynamic strategy.

We find that all active investors obtain considerable economic gains with both dynamic strategies, and day traders make the most profits with the return-volatility strategy. For example, when a day trader maximizes her expected utility while timing both the expected return and volatility, she would earn as high as 15.8% more than the passive investors would each year on a risk-adjusted basis. Alternatively, the passive investors would be willing to pay an annual fee of as much as 18% to switch from the buy-and-hold strategy to this dynamic strategy. In addition, the outperformance is robust to various benchmark strategies and investor's objective functions. Moreover, analysis of the subperiod performance reveals that the dynamic strategies perform well in bad times when the buy-and-hold strategies perform poorly. Another interesting result is that timing the expected return is effective only in daily rebalance, not in weekly and monthly rebalance, presumably because the dynamics of the latent factors are not persistent.

Transaction costs (TCs) are an important consideration for any dynamic strategy. In the absence of reliable estimates of appropriate TCs, we consider the breakeven TCs, which are the maximum TCs that can be imposed before making the dynamic strategies less desirable than the buy-and-hold strategies. For day traders engaging in the return-volatility timing, the breakeven TCs may be too small. But in other cases, they should be high enough for investors to implement the dynamic strategies.

We also conduct the same analysis with a completely different set of assets and more recent time periods. We analyze the performance of dynamic strategies constructed from 35 Dow Jones sector indices and find similar outperformance results. For all investors, the two dynamic strategies significantly outperform the buy-and-hold strategies year by year. Furthermore, the breakeven TCs are much higher, which makes it easier to implement the dynamic strategies even for the day traders.

The rest of the article is organized as follows. Section 1 describes the model and provides a brief description of the estimation procedure. Section 2 sets up the optimal mean-variance portfolio choice problem, presents various portfolio performance measures, and discusses the issue of TCs. In Section 3, we discuss how one can sequentially update the forecasts of the first two moments of returns. Section 4 describes the data and presents the estimation results of the DFMSV model. In Section 5, we evaluate the portfolio performance of the DFMSV dynamic strategies. Section 6 concludes. The technical details are described in the appendices.

1. The Dynamic Factor Multivariate Stochastic Volatility Model

To allow for time-varying expected return and covariance in a high dimensional setting, it is crucial to have a model that is parsimoniously specified and yet flexible enough to capture many empirical regularities. To this end, we adopt the latent factor perspective that parsimoniously captures the covariance of returns, as other ways of modeling the covariance in a high dimensional setting are prone to estimation inefficiencies. [Chan, Karceski, and Lakonishok (1999) find that covariance estimates using cross-products tend to overfit the data.] We adopt this perspective also because it avoids the measurement-error and potential specification-bias problems that may arise from replacing the unobserved factors with observable proxies.

The MSV model outlined by Chib, Nardari, and Shephard (2002) extends the classic factor model to incorporate time-varying covariance. Building on their work, we consider a further extension in which the factors have Markov dynamics. This extended model, which we call the DFMSV model, incorporates both time-varying expected return and covariance. The DFMSV model can also be fitted in high dimensions, thus providing us with the basis for constructing portfolios from a large universe of stocks. Portfolio construction of this type was previously infeasible.

1.1 Model specification

Let $r_t = \left(r_{1t},...,r_{pt}\right)$ denote a vector of p returns observed at time t, and $f_t = (f_{1t},...,f_{kt})$ denote k latent factors. Similar to the classic factor model, the returns are determined by the latent factors and p idiosyncratic shocks. What is interesting is that both the latent factors and the shocks are assumed to have time-varying volatility. Furthermore, we assume that the factors follow an AR(1) process. Specifically, the model for the returns and factors is given by

$$r_t = \mathbf{B}f_t + V_t^{\frac{1}{2}}\varepsilon_t, \tag{1}$$

$$\begin{aligned} &f_t = c + \mathbf{A}f_{t-1} + S_t^{\frac{1}{2}}\zeta_t, \\ &\varepsilon_t \sim \mathcal{N}\left(0, I_p\right), \ \zeta_t \sim \mathcal{N}(0, I_k), \end{aligned} \tag{2}$$

where $\mathbf{B}$ is a $(p \times k)$ matrix of coefficients, and $\mathbf{A}$ a $(k \times k)$ matrix. To identify the latent factors, both $\mathbf{B} = \left\{b_{ij}\right\}$ and $\mathbf{A} = \left\{a_{ij}\right\}$ are subject to constraints. In particular, $b_{ii} = 1$ and $b_{ij} = 0$, for $j > i$,[7] and $a_{ij} = 0$, for $i \neq j$. We also assume that $a_{ii} \in (-1, 1)$ to ensure that the autoregressive processes are stationary. The time-varying conditional covariances V_t and S_t are assumed to be diagonal

$$V_t = V_t(h_t) = \mathrm{diag}\{e^{h_{1t}},...,e^{h_{pt}}\}, \quad p \times p, \tag{3}$$

$$S_t = S_t(h_t) = \mathrm{diag}\{e^{h_{p+1t}},...,e^{h_{p+kt}}\}, \quad k \times k. \tag{4}$$

The terms h_{jt}, which may be interpreted as the conditional log variances, are unobserved and are assumed to each follow a three-parameter autoregressive process

$$h_{jt} - \mu_j = \phi_j(h_{jt-1} - \mu_j) + \sigma_j \eta_{jt}, \quad \eta_{jt} \overset{i.i.d.}{\sim} \mathcal{N}(0, 1). \tag{5}$$

Thus, the shocks are conditionally independent given the latent log variances $\{h_{jt}\}$. Correlations between the returns given $\{h_{jt}\}$ are determined by the loading matrix and latent factors. If we view the latent factors as state variables, then the covariances of returns (much as in the APT model) are determined by contributions to the state variables, not by the idiosyncratic shocks.

It should be noted that the MSV model has been applied to the portfolio problem in currency markets by Aguilar and West (2000). Although it is referred to as the "dynamic factor model," the model considered there does not have the Markov dynamics nor is it applied to a high dimensional setting. The dynamic factor extension is motivated by the empirical evidence that many economic variables that have predictive power display strong serial dependence. Therefore, the latent factors in the model can be expected to inherit that serial dependence. Indeed, we find strong support for this feature in our empirical analysis. It also includes the model proposed by Johannes, Polson, and Stroud (2002) as a univariate special case; in their univariate model, the latent factor is assumed to follow an AR(1) process without the stochastic volatility.

1.2 Estimation procedure

The DFMSV model is in many ways quite parsimoniously specified. Nonetheless, in a high dimensional setting, the model still contains many parameters. For instance, with 36 return series and three factors, the total number of parameters in the model is 222. In addition, there are typically thousands of latent variables (latent factors and log variances). To deal with models this large, it is necessary to have an estimation

[7] The identification problem is the familiar rotational indeterminacy problem. [See, e.g., Geweke and Zhou (1996), for a detailed discussion.] Intuitively, under these restrictions, the first k assets help to pin down the k latent factors.

procedure that is efficient and scalable in the number of assets and factors. We develop such a method by extending the work of Chib, Nardari, and Shephard (2002) to accommodate dynamic factors.

Our estimation procedure is based on Markov chain Monte Carlo (MCMC) methods [Chib and Greenberg (1996)]. The MCMC method is a simulation-based method designed to sample densities that are otherwise intractable. The method generates sample draws from a target distribution, which is the posterior distribution of parameters, by a recursive Monte Carlo sampling process: the transition kernel of a Markov process is constructed such that its limiting invariant distribution is the target distribution; the Markov chain is then iterated a large number of times in a Monte Carlo simulation. After a burn-in period, the Markov chain converges, and the sampled draws are collected as variates from the target distribution.

By Bayes theorem, the posterior distribution is proportional to the product of the prior and likelihood function. In the presence of latent variables, as in the DFMSV model, however, the likelihood function is hardly available. To overcome the problem, we shift the focus from the posterior distribution of the parameters to that of the parameters and latent variables (*augmented* posterior distribution).

Therefore, to estimate the DFMSV model, the MCMC sampling procedure is applied to the augmented posterior distribution of the parameters, latent factors, and log variances. Clearly, this means that the target distribution is extremely high dimensional since there are many thousands of latent variables in the model. To deal with this high dimensionality, it is crucial to exploit the model structure carefully to ensure that the sampling process is computationally efficient. One key idea in the development of the algorithm is to linearize the model by approximating the distribution of the log of a standard chi-squared random variable with a seven-component mixture of normal distributions.

1. Take the logarithm of the squared returns and factors:

$$z_{jt} = \begin{cases} \log\left(r_{jt} - B_j f_t\right)^2 = h_{jt} + \log \varepsilon_{jt}^2, & j \le p, \\ \log(f_{it} - c_i - a_{ii} f_{it-1})^2 = h_{jt} + \log \zeta_{it}^2, & i = j - p, \quad j \ge p+1, \end{cases} \tag{6}$$

   where B_j denotes the jth row of $\mathbf{B}$, and f_{it}, c_i, and ζ_{it} are the ith element of f, c, and ζ_t, respectively.

2. Approximate the log of the chi-squared random variables $\log \varepsilon_{jt}^2$ and $\log \zeta_{it}^2$ with the mixture of normals:

$$z_{jt} = h_{jt} + m_{s_{jt}} + v_{s_{jt}} e_{jt}, \quad e_{jt} \sim \mathcal{N}(0, 1), \quad j = 1,..., p+k, \tag{7}$$

   where s_{jt} is a discrete random variable with a support of $\{1, 2, ..., 7\}$, and $m_{s_{jt}}$ and $v_{s_{jt}}^2$ are the mean and variance of the

s_{jt}-th normal component.[8] Conditioned on the factors and the parameters of the model, each series of transformed observations z_{jt} becomes conditionally independent.

The above model for the observations, along with the evolution equation for h_{jt} [Equation (5)], leads to a conditionally Gaussian state-space model. Thus, conditioned on the latent factors and other parameters of the model, the latent h_{jt} and the parameters in the AR(1) process of h_{jt} can easily be sampled separately for each series. Given h_{jt}, another key step is to sample the matrices **A** and **B** marginalized over the latent factors, and then sample all the factors in one block. The MCMC sampling procedures are detailed in the appendices.

2. Optimal Portfolio Choice Problem

2.1 Dynamic portfolio choice in a mean-variance framework

Consider risk-averse investors with preferences defined over the conditional mean and covariance of returns. Let $\mu_{t+1|t} \equiv E[r_{t+1}|\mathcal{F}_t]$ and $\Sigma_{t+1|t} \equiv E\left[\left(r_{t+1} - \mu_{t+1|t}\right)\left(r_{t+1} - \mu_{t+1|t}\right)'\middle|\mathcal{F}_t\right]$ denote, respectively, the conditional mean and covariance of future stock returns, given the current information $\mathcal{F}_t$. The key issue is to forecast $\mu_{t+1|t}$ and $\Sigma_{t+1|t}$. We devote the next section (Section 3) to discussing how these two moments are estimated from the DFMSV model and sequentially updated as new information becomes available.

Let $\mu_{p,t+1}$ and $\sigma^2_{p,t+1}$ denote the conditional mean and variance of the portfolio returns $r_{p,t+1}$. We imagine that the investors can operate under three different mean-variance objectives:

1. Maximum expected utility—maximizing an expected mean-variance utility function. Assume the investors invest \$1 at time t. Then the investors solve the following utility maximization problem,

$$\max_{w_t}\left\{E[u(W_{t+1})] = \mu_{p,t+1} - \frac{\gamma}{2}\sigma^2_{p,t+1}\right\}, \tag{8}$$

with

$$\mu_{p,t+1} = \mathbf{w}_t'\mu_{t+1|t} + (1 - \mathbf{w}_t'1)r_f, \quad \text{and} \quad \sigma^2_{p,t+1} = \mathbf{w}_t'\Sigma_{t+1|t}\mathbf{w}, \tag{9}$$

where $\mathbf{w}_t$ is a vector of p portfolio weights,[9] γ the coefficient of the absolute risk aversion, and r_f the return on the riskless asset. The portfolio weights are

[8] For details, see Kim, Shephard, and Chib (1998). The biggest advantage of this approximation is that the seven normal distributions are independent of the parameters in the model.

[9] We have examined the case with no short selling and obtained similar results, although the performance is a bit weaker.

$$\mathbf{w}_t = \frac{1}{\gamma}\Sigma_{t+1|t}^{-1}\left(\mu_{t+1|t} - 1r_f\right). \tag{10}$$

2. Minimum volatility—minimizing the conditional variance for a given level of conditional expected return. The investors solve the following quadratic problem at time t,

$$\min_{\hat{\mathbf{w}}_t}\left\{\sigma_{p,t+1}^2 = \hat{\mathbf{w}}_t'\Sigma_{t|t+1}\hat{\mathbf{w}}_t\right\} \tag{11}$$

$$\text{s.t. } \hat{\mathbf{w}}_t'\mu_{t+1|t} + \left(1 - \hat{\mathbf{w}}_t'1\right)r_f = \mu_p^*, \tag{12}$$

where $\hat{\mathbf{w}}_t$ is the vector of portfolio weights and μ_p^* the target expected return. Define $\kappa_t = \left(\mu_{t+1|t} - 1r_f\right)'\Sigma_{t+1|t}^{-1}\left(\mu_{t+1|t} - 1r_f\right)$. The solution is

$$\hat{\mathbf{w}}_t = \Sigma_{t+1|t}^{-1}\left(\mu_{t+1|t} - 1r_f\right)\frac{\mu_p^* - r_f}{\kappa_t}. \tag{13}$$

3. Maximum expected return—maximizing the conditional expected return for a given level of conditional volatility. The investors solve the following problem at time t,

$$\max_{\tilde{\mathbf{w}}_t}\left\{\mu_{p,t+1} = \tilde{\mathbf{w}}_t'\mu_{t+1|t} + (1 - \tilde{\mathbf{w}}_t'1)r_f\right\} \tag{14}$$

$$\text{s.t. } \tilde{\mathbf{w}}_t'\Sigma_{t+1|t}\tilde{\mathbf{w}}_t = \left(\sigma_p^*\right)^2, \tag{15}$$

where $\tilde{\mathbf{w}}_t$ is the vector of portfolio weights and σ_p^* the target level of volatility. The solution is

$$\tilde{\mathbf{w}}_t = \Sigma_{t+1|t}^{-1}\left(\mu_{t+1|t} - 1r_f\right)\sqrt{\frac{\left(\sigma_p^*\right)^2}{\kappa_t}}. \tag{16}$$

If an investor believes that the conditional expected return and covariance are constant, the optimal portfolio weights will be constant over time, which we refer to as "static strategies." On the other hand, when the conditional expected return and covariance are perceived to be time-varying, the investors will rebalance their portfolio weights accordingly, thus following "dynamic strategies."

2.2 Performance measures of dynamic trading strategies

The most commonly used performance measure is the Sharpe ratio. As is well known, the static strategies will all yield the same Sharpe ratio, but the dynamic strategies may yield different Sharpe ratios. We compute and compare the *ex post* Sharpe ratios using the sample means μ_p and standard deviations (SD) σ_p from the realized portfolio returns, $\mathcal{SR} = (\mu_p - r_f)/\sigma_p$.

However, the Sharpe ratio does not take into account time-varying conditional volatility because the sample SD overestimates the conditional risk an investor faces when she follows dynamic strategies. Consequently, the realized Sharpe ratio underestimates the performance of dynamic strategies. In addition, the Sharpe ratio cannot quantify the economic gains of the dynamic strategies over the buy-and-hold strategies. Therefore, we also use several other portfolio performance measures, which allow us to analyze the robustness of the performance and get a more accurate measurement.

A performance measure that is directly related to the Sharpe ratio, but also quantifies the outperformance is the $\mathcal{M}2$ measure developed by Modigliani and Modigliani (1997).[10] The $\mathcal{M}2$ measure is the abnormal return that the dynamic strategy would have earned if it had the same risk as the benchmark. It is defined as

$$\mathcal{M}2 = \frac{\sigma_b}{\sigma_p}(\mu_p - r_f) - (\mu_b - r_f).$$

If we define "leverage factor" $w = \sigma_b/\sigma_p$, the ratio of the SDs of the benchmark and dynamic portfolios, we can rewrite $\mathcal{M}2$ as

$$\mathcal{M}2 = \left[w\mu_p + (1-w)r_f\right] - \mu_b = \mu_p' - \mu_b,$$

where μ_p' is the average return of a portfolio constructed from the dynamic portfolio and the risk-free asset. In other words, the dynamic portfolio is levered downwards or upwards, so that it has the same volatility as the benchmark portfolio. Then it becomes clear that $\mathcal{M}2$ is the risk-adjusted abnormal return. $\mathcal{M}2$ is also directly related to the Sharpe ratio as $\mathcal{M}2 = \sigma_b(\mathcal{SR}_p - \mathcal{SR}_b)$.

Following Fleming, Kirby, and Ostdiek (2001), we consider a performance-fee measure, which measures a hypothetical management fee that an investor would be willing to pay to switch from a buy-and-hold strategy to an outperforming dynamic strategy, without being worse off. To estimate the performance fee, we find a constant Φ such that the average realized utility generated by the dynamic strategy, subject to expenses Φ,

[10] Graham and Harvey (1997) proposed a similar measure called $\mathcal{GH}2$. The difference between their $\mathcal{GH}2$ and the $\mathcal{M}2$ measures, as pointed out in note 6 of their article, is that the $\mathcal{M}2$ measure ignores the correlation between the risk-free asset and other assets. As the risk-free rates are constant in our setting, the two measures are identical.

is the same as that of the buy-and-hold strategy. That is, the performance fee Φ solves the following problem

$$\begin{aligned}\frac{1}{T}\sum_{t=0}^{T-1}\left[(R_{d,t+1}-\Phi)-\frac{\gamma}{2(1+\gamma)}(R_{d,t+1}-\Phi)^2\right]\\=\frac{1}{T}\sum_{t=0}^{T-1}\left[R_{b,t+1}-\frac{\gamma}{2(1+\gamma)}R_{b,t+1}^2\right],\end{aligned}\tag{17}$$

where $R_{d,t+1}$ and $R_{b,t+1}$ denote the realized gross returns of the dynamic and buy-and-hold strategies, respectively, and γ is the coefficient of the relative-risk aversion, which is set to 6 in our empirical analysis.

Another utility-based measure is the certainty equivalent rate of return (CER). The CER is defined as the risk-free rate that gives the investor the same utility as the average utility generated by the investor's trading strategy. The difference between the CERs of the dynamic and buy-and-hold strategies measures the incremental value of the dynamic strategy over the buy-and-hold strategy. It turns out that this measure yields values similar to the performance fee measure; therefore, for the sake of conciseness, it is not reported.

Another measure is the success rate, defined as the percentage of times in which the dynamic strategy beats the buy-and-hold strategy [see, e.g., Pojarliev and Polasek (2000)]. The success rate indicates how often the dynamic strategy yields higher returns than the benchmark, and whether the outperformance is merely due to large gains on some rare occasions (positive outliers). We denote the success rate $\mathcal{Z}$.

The last measure considered is the performance of a zero-cost portfolio constructed by longing the dynamic strategy and meanwhile shorting the buy-and-hold strategy. Unlike $\mathcal{M}2$, which measures the risk-adjusted abnormal return, the average return of this portfolio represents the unadjusted abnormal return over the benchmark. In addition, a positive skewness suggests that the zero-cost portfolio has more large positive returns than large negative returns, which means that the dynamic strategy beats the buy-and-hold strategy with large margins more often than it loses with large margins.

2.3 Transaction costs

TCs are an important consideration whenever we follow a dynamic strategy. Unfortunately, the appropriate estimates of TCs are not readily available in most cases. Academic research concerning TCs has used a wide range of estimates. For example, Balduzzi and Lynch (1999) argue that 0.5% is a reasonable estimate of the one-way trading costs for an investor who trades individual stocks directly [see also Bhardwaj and Brooks (1992)], and 0.01% [1 basis point (*bp*)] is the ballpark for an institutional investor who can use futures contracts to hold equity [see, e.g., Fleming, Kirby, and Ostdiek (2003) and Fleming, Ostdiek, and Whaley (1996)]. Marquering and Verbeek (2001) consider three levels of TCs, 0.1, 0.5, and 1%, representing low, medium, and high costs, respectively. This is of no surprise, as the true TCs are notoriously difficult to estimate. TCs contain three components: commission fees, ask-bid spreads, and price impacts. Unless the transaction is very large, the price impact would be negligible. On the other hand, the ask-bid spreads are a major component but difficult to estimate reliably. Furthermore, many trades are executed at prices better than the quoted prices (effective spreads). Knez and Ready (1996) estimate that the effective spreads for large stocks are about 0.20–0.30%. Therefore, for one-way transactions, the costs are 0.10–0.15%. However, the spreads are estimated with market orders only; hence, the actual spreads may be smaller if limit orders are used. Finally, commission fees have been decreasing over the years, especially so with the growth of Internet trading. For example, Scottrade offers a flat commission fee as low as $7 per trade regardless of how many shares traded. If you buy 1000 shares at $35 per share, the commission is merely 0.02%. On the other hand, the full-service brokerage firms charge hefty fees for trading through them. Therefore, different investors may have very different TCs.

Because of the issues discussed above, we do not take a stand as to what the appropriate TCs should be. Instead, we calculate the "breakeven" TCs that make the investors indifferent between the dynamic and buy-and-hold strategies in terms of utility. We assume that the TCs equal to a fixed percentage (τ) of the value traded for all stocks. Consequently, the costs are $\tau\left|w - \frac{w(1+r_t)}{w\left(r_t - r_f\right)+r_f+1}\right|$ for the static strategies and $\tau\left|w_t - \frac{w_{t-1}(1+r_t)}{w_{t-1}\left(r_t - r_f\right)+r_f+1}\right|$ for the dynamic strategies. We denote the breakeven TCs τ^{be}. If an investor has TCs lower than τ^{be}, she will be better off with the dynamic strategies; otherwise, she should follow the buy-and-hold strategies.

3. Prediction of the Conditional Mean and Covariance

This section discusses how to forecast future expected returns and covariances conditioned only on previous returns and parameters, and how to sequentially update the forecasts as new information becomes available.[11]

3.1 Daily rebalance

The covariances of returns are determined by the latent factors and latent log variances. Therefore, to forecast the conditional covariances of future

[11] The sequential updating process is carried out with the parameters being fixed. Ideally we would like to update the parameters at the same time because the parameters may slowly change over time, but it is computationally intractable to do so. An alternative approach is to re-estimate the model periodically [see, e.g., Johannes, Polson, and Stroud (2002)], although it is computationally costly. Our results may be conservative in this aspect.

returns, we first estimate the predictive distributions of the latent factors and log variances. For instance, to make a one-day-ahead prediction of the daily conditional covariances, $\Sigma_{t+1|t}$, note that

$$\begin{aligned}\Sigma_{t+1|t} &= \mathrm{var}(r_{t+1}|\mathcal{F}_t, \psi)\\ &= E[\mathrm{var}(r_{t+1}|f_{t+1}, h_{t+1})|\mathcal{F}_t, \psi] + \mathrm{var}[E(r_{t+1}|f_{t+1}, h_{t+1})|\mathcal{F}_t, \psi]\\ &= \int V(h_{t+1})d\pi(h_{t+1}|\mathcal{F}_t, \psi) + B\mathrm{var}(f_{t+1}|\mathcal{F}_t, \psi)B',\end{aligned} \tag{18}$$

where $\pi(h_{t+1}|\mathcal{F}_t, \psi)$ is the predictive distribution of h_{t+1}, $\mathrm{var}(f_{t+1}|\mathcal{F}_t, \psi)$ is the variance of the predictive distribution of f_{t+1}, and ψ are the parameters, fixed at their posterior means. The second equality follows from the relation between the unconditional and conditional (co)variances. Intuitively, the covariances of returns equal the expected value of the conditional covariances plus the covariances of the conditional means conditioned on the latent variables. As the predictive distributions of f_{t+1} and h_{t+1} are not available in close form, the conditional covariances are estimated by Monte Carlo simulation as

$$\Sigma_{t+1|t} = \frac{1}{M}\sum_{i=1}^{M} V\left(\tilde{h}_{t+1}^{(i)}\right) + B\Omega_{t+1|t}\left(\tilde{f}_{t+1}^{(i)}\right)B', \tag{19}$$

where $\Omega_{t+1|t}\left(\tilde{f}_{t+1|t}^{(i)}\right)$ is the sample covariance of $\tilde{f}_{t+1}^{(i)}$, and $\tilde{f}_{t+1}^{(i)}$ and $\tilde{h}_{t+1}^{(i)}$ are sample draws from the predictive density $\pi(f_{t+1}, h_{t+1}|\mathcal{F}_t, \psi)$, generated through their respective AR(1) transition processes

$$\pi(f_{t+1}, h_{t+1}|\mathcal{F}_t, \psi) = \int \pi(f_{t+1}, h_{t+1}|f_t, h_t, \mathcal{F}_t, \psi)\pi(f_t, h_t|\mathcal{F}_t, \psi)df_t dh_t. \tag{20}$$

Namely, given sample draws $f_t^{(i)}$ and $h_t^{(i)}$ from the filtering distribution $\pi(f_t, h_t|\mathcal{F}_t, \psi)$, the predictive draws are obtained by directly sampling from their transition distributions. The sample draws $f_t^{(i)}$ and $h_t^{(i)}$, called particles in the filtering literature, can be generated via Bayes theorem,

$$\pi(f_t, h_t|\mathcal{F}_t, \psi) \propto f(r_t|f_t, h_t)\pi(f_t, h_t|\mathcal{F}_{t-1}, \psi). \tag{21}$$

Therefore, the predictive and filtering distributions are updated sequentially. The prior beliefs about the latent factors and log variances at time t are the predictive distributions at time $t-1$. The priors are then updated to form the filtering distributions by incorporating the new information about returns at time t. The filtering distributions in turn are used to form the predictive distributions at time t, which are again the priors at time $t+1$, and the updating process repeats again for $t+1$.

Because the DFMSV model is a highly nonlinear system, the commonly used linear Kalman filter cannot be applied; thus, we employ a nonlinear filter referred to as the auxiliary particle filter proposed by Pitt and Shephard (1999). Similar to the Kalman filter, the particle filter uses an iterative procedure to produce particles. Given the particles of f_{t-1} and h_{t-1} conditioned on the information at time $t-1$, $\mathcal{F}_{t-1}$ the particle filter produces particles of f_t and h_t conditioned on the information at time t, $\mathcal{F}_t$, by recursively applying Equations (20) and (21). The detailed procedure of the auxiliary particle filter is described in Appendix C.

Having obtained the predictive sample draws $\tilde{f}^{(i)}_{t+1}$ and $\tilde{h}^{(i)}_{t+1}$, the one-step-ahead conditional means $\mu_{t+1|t}$ are simply estimated as

$$\mu_{t+1|t} = E[r_{t+1}|\mathcal{F}_t, \psi] = \frac{B}{M}\sum_{i=1}^{M}\tilde{f}^{(i)}_{t+1}\,. \tag{22}$$

To implement the dynamic strategies, an investor would forecast the conditional mean and covariance of returns period by period, for example, $(\mu_{t+1|t}, \Sigma_{t+1|t}), (\mu_{t+2|t+1}, \Sigma_{t+2|t+1}),\ldots$. This is achieved by updating first the distribution of the latent factors and log volatilities through the particle filter, and then the forecasts of the conditional mean and covariance via Equations (19) and (22) period by period.

3.2 Weekly and monthly rebalance

Similar to the one-day-ahead forecasts of the daily conditional covariance, the j-day-ahead forecasts of the daily conditional covariance, $\Sigma_{t+j|t}$, are generated from the following expression

$$\begin{aligned}\Sigma_{t+j|t} &= E\left[\mathrm{var}\left(r_{t+j}|\, f_{t+j}, h_{t+j}\right)|\mathcal{F}_t, \psi\right] + \mathrm{var}\left(Bf_{t+j}|\mathcal{F}_t, \psi\right)\\ &= \int V\left(h_{t+j}\right)d\pi\left(h_{t+j}|\mathcal{F}_t, \psi\right) + B\mathrm{var}\left(f_{t+j}|\mathcal{F}_t, \psi\right)B'\\ &= \frac{1}{M}\sum_{i=1}^{M} V\left(\tilde{h}^{(i)}_{t+j}\right) + B\Omega_{t+j|t}\left(\tilde{f}^{(i)}_{t+j}\right)B',\end{aligned} \tag{23}$$

where $\Omega_{t+j|t}\left(\tilde{f}^{(i)}_{t+j}\right)$ is the sample covariance of $\tilde{f}^{(i)}_{t+j}$, and $\tilde{f}^{(i)}_{t+j}$ and $\tilde{h}^{(i)}_{t+j}$ are sample draws from the predictive density $\pi\left(f_{t+j}, h_{t+j}|\mathcal{F}_t, \psi\right)$ and are generated from the particles $f^{(i)}_t$ and $h^{(i)}_t$ by propagating the AR(1) processes to date $t+j$. Because the daily returns $r_{t+1}, \ldots, r_{t+s}$ are conditionally independent conditioned on the sample draws $\tilde{f}_{t+1}, \ldots, \tilde{f}_{t+s}$ and $\tilde{h}_{t+1}, \ldots, \tilde{h}_{t+s}$, and the parameters ψ, the forecasts of the conditional covariance of the s-day's return, $r^s_{t+s} = r_{t+1} + \ldots + r_{t+s}$, are

$$
\begin{aligned}
\Sigma^{s}_{t+s|t} &= \mathrm{var}\left(r^{s}_{t+s}|\mathcal{F}_t, \psi\right) \\
&= E\left[\mathrm{var}\left(r^{s}_{t+s}|\, f_{t+1}, ..., f_{t+s}, h_{t+1}, ..., h_{t+s}\right)|\mathcal{F}_t, \psi\right] \\
&\quad + \mathrm{var}\left(E\left[r^{s}_{t+s}|\, f_{t+1}, ..., f_{t+s}, h_{t+1}, ..., h_{t+s}\right]|\mathcal{F}_t, \psi\right) \\
&= \sum_{j=1}^{s}\left\{E\left[\mathrm{var}\left(r_{t+j}|\, f_{t+j}, h_{t+j}\right)|\mathcal{F}_t, \psi\right] + \mathrm{var}\left(Bf_{t+j}|\mathcal{F}_t, \psi\right)\right\} \\
&= \sum_{j=1}^{s} \Sigma_{t+j|t}.
\end{aligned} \tag{24}
$$

Namely, $\Sigma^{s}_{t+s|t}$ is the sum of the daily conditional covariance forecasts within the s days.

Similarly, the expected returns over the s-day period are

$$
\begin{aligned}
\mu^{s}_{t+s|t} &= E\left[r^{s}_{t+s}|\mathcal{F}_t, \psi\right] = \sum_{j=1}^{s} E\left[r_{t+j}|\mathcal{F}_t, \psi\right] \\
&= \sum_{j=1}^{s} \mu_{t+j|t} = \frac{B}{M}\sum_{j=1}^{s}\sum_{i=1}^{M}\tilde{f}^{(i)}_{t+j}.
\end{aligned} \tag{25}
$$

For weekly and monthly rebalance, s is about 5 and 20, respectively, but is adjusted for holidays and number of weeks in a month. Just like the day traders, swing and position investors sequentially update their forecasts of the conditional expected return and covariance via the particle filter and Equations (24) and (25).

4. Estimation of the DFMSV Model

4.1 Data description

To form a well-diversified portfolio, we choose 36 stocks in our empirical analysis. However, it is important to note that our estimation procedure is highly scalable; the choice of 36 stocks is arbitrary, and we have done some preliminary studies with as many as 50 assets. The stocks we pick fall into six categories: large & growth; large & value; medium & growth; medium & value; small & growth; and small & value. In each category, we pick 6 stocks with the market values somewhat evenly distributed within the category.

Daily returns on the 36 stocks from January 2, 1990, to December 29, 2000, are collected from the Center for Research in Security Prices. The whole sample period is divided into two periods: an estimation period (January 2, 1990–December 29, 1995, 1517 observations) and a testing period (January 2, 1996–December 29, 2000, 1263 observations).

The Tickers and some summary statistics of the 36 stocks are listed in Table 1. In general, the growth stocks have higher average returns, volatilities, and market betas than the value stocks, and the small stocks are more volatile than the large and medium cap stocks. In addition, the average returns of the small stocks are more dispersed, containing both the lowest and highest annualized daily returns (–16.65 versus 41.2%). As expected, the skewness and kurtosis deviate from those of a normal distribution, and in many cases, the differences are large. There are, however, no apparent patterns of skewness and kurtosis either over growth versus value or across different market caps. The last columns report the first-order autocorrelations, which are small and negative for the majority of the stocks, especially for the small growth stocks.

4.2 Model estimation

We estimate several DFMSV models, with the number of factors from one to three. Statistically, the best DFMSV model is the DFMSV2 model (2-factor). However, in the portfolio analysis that follows, we examine all the three DFMSV models because Han (2004) finds that the best statistical models do not necessarily outperform simpler models in terms of portfolio performance. In a similar spirit, Aït-Sahalia and Brandt (2001) argue that one should choose predictive variables that are important for optimal portfolio weights, rather than for moments of the conditional distribution of returns, when considering portfolio choice problems.

As expected, both the latent factors and the return shocks display very persistent volatilities. However, the Markov process of the latent factors is not quite persistent. For example, the posterior means of the persistent coefficients in the DFMSV2 model are 0.1 (a_{11}) and 0.2 (a_{22}), both of which are nonetheless statistically significant. This is the key difference between our latent factor model and models using observed economic variables. The latent factors can explain a much larger portion of the variations in the expected returns than the observed predictive variables, because the latent factors are not forced to be as highly serially correlated as those of predictive variables. For example, with the predictive variables, the R^2s are usually less than 4% [Ferson, Sarkissian, and Simin (1999)], whereas the average R^2s in the DFMSV2 model are 17%. Another observation worth noting is that the first factor seems more important, and all stocks have significant loadings on it. By contrast, the loadings on the second factor are smaller in general, and even insignificant for some stocks. The first factor is also related to the market portfolio—it has a correlation of 0.91 with the Standard & Poor's 500 index, whereas the second factor only has a correlation of 0.27.

Table 1
Stock composition in the portfolio

Growth stocks							Value stocks						
Stock	Mean	SD	Skewness	Kurtosis	β	ρ_1	Stock	Mean	SD	Skewness	Kurtosis	β	ρ_1
Large													
GE	22.12	24.18	0.02	4.94	1.12	0.009	IBM	13.87	32.40	–0.10	9.93	1.08	–0.032
INTC	30.35	42.57	–0.51	7.90	1.60	0.013	PG	15.49	28.50	–3.46	74.91	0.76	–0.004
AMGN	37.50	42.68	–0.17	6.53	1.25	0.013	PEP	16.16	29.43	0.32	6.98	0.78	–0.063
AMAT	34.07	58.90	0.19	4.68	2.03	–0.007	CAT	12.73	32.13	–0.05	6.32	0.87	0.027
GMH	16.33	35.57	0.28	6.68	0.86	0.025	BUD	16.57	24.19	–0.12	5.06	0.55	–0.022
ALTR	37.30	66.90	–0.28	6.47	2.13	–0.009	EMR	15.22	23.77	0.23	4.45	0.76	0.014
Medium													
BBY	36.98	64.19	–1.22	18.33	1.41	0.055	CPB	10.34	28.49	0.11	7.88	0167	–0.073
LUV	27.91	39.45	0.05	5.61	1.06	–0.028	DE	10.09	32.80	0.08	5.73	0.80	–0.013
AVY	13.54	28.46	–0.36	8.46	0.68	0.038	JCI	13.30	27.42	0.19	6.22	0.76	–0.004
SYMC	13.24	66.24	–0.60	12.36	1.55	0.033	MYG	7.53	35.83	–0.67	17.88	0.68	–0.011
COMS	17.98	65.24	–1.42	21.60	1.61	0.045	DQE	11.36	17.24	1.00	17.76	0.25	0.063
TKLC	30.28	83.76	0.09	8.89	1.48	–0.120	MBI	15.38	28.79	0.06	8.94	0.83	0.104
Small													
ROST	14.68	51.91	0.24	7.58	0.95	–0.002	SMTC	41.20	86.73	0.22	5.54	1.57	–0.109
AHA	31.66	77.38	0.80	9.33	1.66	–0.033	JEC	17.35	30.58	0.30	7.25	0.58	0.118
MDP	13.35	28.46	0.06	6.96	0.89	–0.017	LANC	15.72	33.55	–0.06	9.35	0.47	–0.077
PBKS	16.32	40.73	0.03	8.12	0.77	–0.153	TBL	28.52	50.46	0.20	8.15	1.03	0.070
PKD	–6.17	54.57	0.33	4.81	0.93	–0.065	KWD	6.44	34.71	–0.66	17.21	0.51	0.121
ELMG	5.63	55.59	–0.37	10.65	0.83	–0.045	PRD	–16.65	39.93	–0.45	16.11	0.92	–0.015

Note: This table lists the Ticker, sample mean, sample SD, skewness, kurtosis, market beta, and the first-order autocorrelation (ρ_1) of the 36 stocks. The stocks are grouped into six categories: large cap & growth; medium cap & growth; small cap & growth; large cap & value; medium cap & value; and small cap & value. The sample means and SDs are annualized and reported in percentage. The sampling period is from January 2, 1990, to December 29, 2000, with a total of 2780 daily returns for each stock.

5. Performance Analysis of the Dynamic Trading Strategies

Given the parameter estimates, we assume that all active investors follow the sequential learning process and update their portfolio weights periodically. Of course, the key question is whether the DFMSV dynamic strategies could generate any economic gains in a high dimensional setting. To this end, we compare the out-of-sample performance of the dynamic strategies to that of the buy-and-hold strategies, which are constructed by first forming a value-weighted portfolio of the 36 stocks, and then combining it with the risk-free asset under the three mean-variance scenarios. As another benchmark, we assume the investors can follow an *ex ante* optimal static strategy based on the sample mean and covariance of the estimation period.

5.1 Economic gains of the daily trading strategies

Table 2 compares the out-of-sample performance for the day traders. Both types of dynamic strategies, return-volatility timing and volatility timing, are compared with the buy-and-hold strategies under the three mean-variance scenarios: maximum expected utility (panel A), minimum volatility (panel B), and maximum expected return (panel C). It clearly shows that the DFMSV dynamic strategies substantially outperform the benchmark under all scenarios. First, the average realized returns are significantly higher in the DFMSV dynamic portfolios, which is also suggested by the positive mean returns of the zero-cost portfolios. The dynamic strategies also yield Sharpe ratios several times higher than those of the buy-and-hold strategies. Moreover, the performance fees and risk-adjusted abnormal returns are substantial. For instance, the passive investor would be willing to pay an annual fee of as much as 1805 *bps* to switch to the DFMSV1 dynamic strategy when she maximizes the expected utility. Interestingly, the risk-adjusted abnormal returns as measured by $\mathcal{M}2$ are in the same magnitude as the performance fees, which suggests that the economic gains are robust. For example, the same dynamic strategy yields 1583 *bps* of risk-adjusted abnormal returns. In addition, the success rates for the dynamic strategies are close to one-half, and many exceed 0.5, suggesting that the outperformance of the dynamic strategies is not due to extreme values. On the other hand, the positive skewnesses of the zero-cost portfolios suggest that, when the dynamic strategies beat the buy-and-hold strategies, they often outperform with large margins.

Of the two dynamic strategies, the return-volatility timing yields much stronger performance under all but one scenario. These results suggest that expected return timing contributes significantly to the outperformance of the return-volatility timing strategies and likely plays a dominant role. The only exception is the minimum volatility scenario where

Table 2
Portfolio performance with daily rebalance

Model	μ (%)	σ (%)	$\mathcal{SR}$	Φ (bp)	$\mathcal{M}2$ (bp)	$\mathcal{Z}$	Δr μ_d(%)	Δr sk_d	τ^{be} (bp)	$\Delta\Phi$ (bp)	$\Delta\mathcal{M}2$ (bp)
Panel A—Maximum expected utility strategy											
BH	8.57	12.44	0.233								
Static	13.86	15.08	0.543	310.9	385.7	0.519	5.29	0.223	38.6		
Return-volatility timing											
DFMSV1	29.50	15.82	1.506	1805.3	1583.1	0.519	20.93	1.349	7.1	1494.3	1197.5
DFMSV2	34.51	26.49	1.088	946.4	1063.7	0.523	25.93	0.448	2.3	635.5	678.0
DFMSV3	56.43	37.36	1.359	1042.3	1400.2	0.508	47.85	0.986	1.5	731.3	1014.5
Volatility timing											
DFMSV1	9.44	7.36	0.512	388.7	346.8	0.494	0.86	0.551	15.0	77.7	–38.9
DFMSV2	12.34	7.38	0.904	677.9	834.0	0.500	3.77	0.437	21.9	366.9	448.3
DFMSV3	11.00	10.66	0.500	365.7	331.2	0.489	2.42	0.314	7.1	54.8	–54.5
Panel B—Minimum volatility strategies											
BH	6.92	10.48	0.119								
Static	7.01	2.47	0.543	320.6	444.6	0.484	0.09	0.341	247.2		
Return-volatility timing											
DFMSV1	8.37	5.68	0.476	378.1	373.7	0.492	1.45	0.67	3.0	57.5	–70.9
DFMSV2	8.20	3.11	0.813	428.8	727.0	0.492	1.28	0.478	5.7	108.2	282.5
DFMSV3	7.55	2.27	0.825	376.9	739.8	0.489	0.63	0.441	6.7	56.3	295.2
Volatility timing											
DFMSV1	9.42	5.24	0.715	497.1	624.8	0.495	2.5	0.527	27.6	176.4	180.2
DFMSV2	10.37	5.24	0.897	592.3	815.6	0.507	3.45	0.442	28.0	271.7	371.0
DFMSV3	8.07	3.98	0.604	397.7	508.1	0.492	1.16	0.357	21.0	77.0	63.6
Panel C—Maximum expected return strategies											
BH	16.06	19.95	0.521								
Static	13.68	14.74	0.543	305	44.6	0.485	–2.38	0.282	38.8		
Return-volatility timing											
DFMSV1	25.11	15.02	1.295	1423.5	1543.4	0.495	9.05	1.554	5.2	1118.5	1498.8
DFMSV2	27.13	15.61	1.375	1570.9	1703.6	0.504	11.06	0.675	5.1	1265.9	1659.0
DFMSV3	27.73	16.09	1.371	1585.2	1696.1	0.488	11.67	0.803	4.3	1280.2	1651.5
Volatility timing											
DFMSV1	14.94	14.66	0.633	438.8	222.8	0.492	–1.12	0.545	8.7	133.8	178.2
DFMSV2	19.12	14.64	0.919	858.2	793.8	0.500	3.06	0.489	14.3	553.2	749.2
DFMSV3	14.26	15.59	0.551	285.5	59.5	0.479	–1.81	0.324	3.8	–19.5	14.9
All-risky	18.96	22.43	0.592								

Note: This table evaluates the out-of-sample performance of the DFMSV models with daily rebalance. BH, value-weighted buy-and-hold strategy; Static, static strategy based on the sample mean and covariances; DFMSVk, dynamic strategies based on the DFMSV models with k factors; All-risky, buy-and-hold strategy without the risk-free asset. The mean-variance scenarios are: (i) maximum expected utility with a risk aversion of 6; (ii) minimum volatility with a target expected return of about 10%; and (iii) maximum expected return with a target volatility of 12%. μ_p, average return; σ_p, volatility; $\mathcal{SR}$, Sharpe ratio; Φ, performance fee; $\mathcal{M}2$, M2 measure; $\mathcal{Z}$, success rate; Δr, returns on the zero-cost portfolio; sk_d, skewness; τ^{be}, breakeven TC; $\Delta\Phi$ & $\Delta\mathcal{M}2$, differences in Φ and $\mathcal{M}2$ between the dynamic and static strategies, respectively.

the volatility-timing strategies seem to perform slightly better. This result is of no surprise, as the objective of this scenario is to minimize volatility given a fixed expected return.

When compared with the static strategies, the dynamic strategies still outperform. The last two columns in the table report the differences in the performance fee and $\mathcal{M}2$ measures, respectively, between the dynamic and static strategies. The differences are mostly positive and considerable, with only a few negative numbers mainly from the $\mathcal{M}2$ measure, but under no circumstance are the differences in the two measures both negative. The last row of the table reports the performance of a buy-and-hold strategy with no risk-free asset. Even though it performs better than other buy-and-hold strategies with the risk-free asset, it is easily dominated by the dynamic strategies.

The results discussed above are obtained without TCs. In the presence of TCs, the performance of both dynamic strategies deteriorates. Table 2 also lists the breakeven TCs of the dynamic strategies. In general, the return-volatility timing strategies sustain lower breakeven costs than the volatility-timing strategies, suggesting higher turnover rates for the former strategies. Not surprisingly, the highest breakeven costs are obtained with the volatility-timing strategies under the minimum volatility scenario, as the portfolio returns are least volatile. The highest τ^{be} among the return-volatility timing strategies is about 7 *bps*, which is arguably low. On the other hand, the highest τ^{be} among the volatility-timing strategies is 28 *bps*, which should make the dynamic strategies easily implementable.

5.2 Economic gains of the weekly and monthly trading strategies

Results above suggest that, for a day trader, return-volatility timing may not be desirable in the presence of TCs. Swing and position traders trade less frequently, so one may think TCs would be less of a problem. However, forecasting over longer periods is inevitably less accurate. Therefore, it remains to be seen whether there are any economic gains with less frequent rebalance.

Table 3 reports the out-of-sample performance of weekly rebalance strategies. As suspected, the performance of the return-volatility timing strategies is mixed, with better performance under the minimum volatility scenario, slightly better performance under the maximum expected utility scenario, and underperformance under the maximum expected return scenario. On the other hand, the volatility-timing strategies still significantly outperform the benchmark strategies under all scenarios despite the longer forecasting periods. These seemingly puzzling results can be understood once we note that the AR(1) process of the latent factors is not persistent, whereas the dynamics of the volatilities are very persistent. Low persistence means that any deviation from the long-term mean is short lived; therefore, timing expected return with weekly rebalance may not capture the short-lived variations in the expected return. In fact, it hurts the performance, presumably because it chases the random noises in the expected return.

With less frequent trading, the weekly volatility-timing strategies yield much higher breakeven TCs than the daily trading strategies, and thus

Table 3
Portfolio performance with weekly rebalance

Model	μ (%)	σ (%)	$\mathcal{SR}$	Φ (bp)	$\mathcal{M}2$ (bp)	$\mathcal{Z}$	Δr		τ^{be} (bp)	$\Delta\Phi$ (bp)	$\Delta\mathcal{M}2$ (bp)
							μ_d(%)	sk_d			
Panel A—Maximum expected utility strategy											
BH	8.51	11.72	0.242								
Static	11.83	16.57	0.372	–83.5	151.6	0.519	3.32	0.120	–18.9		
Return-volatility timing											
DFMSV1	9.34	8.57	0.428	275.7	217.9	0.496	0.83	0.418	15.3	359.1	66.3
DFMSV2	8.03	9.86	0.240	73.1	–3.3	0.508	–0.48	–0.009	3.1	156.6	–154.9
DFMSV3	9.50	14.58	0.263	–128.0	24.2	0.488	0.99	–1.643	–2.6	–44.5	–127.4
Volatility timing											
DFMSV1	9.97	7.48	0.576	391.8	390.4	0.488	0.58	0.165	44.1	475.2	238.8
DFMSV2	11.10	7.28	0.746	513.1	590.1	0.473	2.59	1.022	52.3	596.5	438.5
DFMSV3	10.95	9.00	0.587	414.3	404.0	0.519	2.45	0.408	30.9	497.8	252.4
Panel B—Minimum volatility strategies											
BH	6.88	9.91	0.122								
Static	6.62	2.56	0.372	249.5	246.6	0.473	–0.26	0.441	377.8		
Return-volatility timing											
DFMSV1	7.48	5.65	0.321	259.7	197.1	0.481	0.60	0.488	12.0	10.2	–49.6
DFMSV2	8.19	5.39	0.467	338.6	341.6	0.485	1.31	0.233	16.1	89.1	95.0
DFMSV3	8.47	3.51	0.797	416.9	668.5	0.500	1.59	0.461	30.1	167.4	421.9
Volatility timing											
DFMSV1	8.81	4.81	0.652	418.2	524.4	0.512	0.68	0.800	80.8	168.7	277.7
DFMSV2	9.10	4.73	0.725	449.8	596.6	0.496	2.22	0.960	78.3	200.3	350.0
DFMSV3	8.17	3.96	0.632	377.0	504.6	0.488	1.29	0.522	69.7	127.5	257.9
Panel C—Maximum expected return strategies											
BH	15.85	18.60	0.548								
Static	11.52	15.73	0.372	–134.2	–327.1	0.481	–4.34	0.140	–32.3		
Return-volatility timing											
DFMSV1	11.44	14.04	0.411	10.4	–254.4	0.462	–4.42	0.339	0.3	144.6	72.7
DFMSV2	11.40	14.76	0.389	–56.0	–295.9	0.458	–4.45	–0.152	–1.2	78.2	31.2
DFMSV3	13.86	15.95	0.514	79.1	–63.4	0.469	–2.00	–0.266	1.4	213.3	263.7
Volatility timing											
DFMSV1	14.39	14.00	0.623	309.1	140.1	0.477	–4.46	0.511	20.3	443.3	467.1
DFMSV2	15.94	13.82	0.744	479.6	364.4	0.462	0.09	0.932	28.0	613.8	691.5
DFMSV3	14.39	14.14	0.617	296.8	128.3	0.500	–1.47	0.325	15.0	431.0	455.4
All-risky	18.68	20.87	0.623								

Note: This table evaluates the out-of-sample performance of the DFMSV models with weekly rebalance. BH, value-weighted buy-and-hold strategy; Static, static strategy based on the sample mean and covariances; DFMSVk, dynamic strategies based on the DFMSV models with k factors; All-risky, buy-and-hold strategy without the risk-free asset. The mean-variance scenarios are: (i) maximum expected utility with a risk aversion of 6; (ii) minimum volatility with a target expected return of about 10%; and (iii) maximum expected return with a target volatility of 12%. μ_p, average return; σ_p, volatility; $\mathcal{SR}$, Sharpe ratio; Φ, performance fee; $\mathcal{M}2$, M2 measure; $\mathcal{Z}$, success rate; Δr, returns on the zero-cost portfolio; sk_d, skewness; τ^{be}, breakeven TC; $\Delta\Phi$ & $\Delta\mathcal{M}2$, differences in Φ and $\mathcal{M}2$ between the dynamic and static strategies, respectively.

should be easily implemented. For example, the highest τ^{be}s are as high as 81 *bps*, again obtained under the minimum volatility scenario. Other results are similar to those of the daily rebalance. For example, most dynamic strategies outperform the static strategies (positive $\Delta\Phi$ or $\Delta\mathcal{M}2$), and many also outperform the all-risky portfolio.

Table 4 reports the out-of-sample performance of monthly rebalance strategies. Surprisingly, even with monthly rebalance, the performance of the volatility-timing strategies remains strong and matches that of the more frequently rebalanced strategies. In addition, the return-volatility timing strategies perform much better, close to the performance of the volatility-timing strategies. The improved performance of the former strategies is probably because of the fact that the random noises chased by the weekly rebalance strategies are evened out in the monthly horizon. The dynamic strategies also significantly outperform the static strategies and many outperform the all-risky portfolio. Finally, the breakeven TCs are definitely beyond the range of the normal TCs in most cases, and the highest τ^{be} is about 368 *bps*.

5.3 Diagnosis of return-timing performance

From the above analysis, it is clear that the return-volatility timing strategies yield the best performance in daily rebalance, but underperform the volatility-timing strategies in weekly and monthly rebalance. This suggests that the expected return timing plays a vital role in daily rebalance, not in weekly and monthly rebalance. In this subsection, we examine the return-timing performance using two return-timing-related performance measures.

A return-timing investor would adjust her portfolio weights according to her forecast of the future expected returns; she would invest more in risky assets when the future expected returns are high, and less otherwise. As a result, her portfolio weights would be positively correlated with the stock returns if the return timing is ever successful.[12] The second to last column in Table 5 reports the correlations between the total portfolio weights on the risky assets and an equal-weighted portfolio of the risky assets. Interestingly, the correlations are positive in the daily rebalance, but negative in the weekly and monthly rebalance.

Grinblatt and Titman (1993) have shown that the total abnormal performance, including timing and selectivity performance, can be measured by the sum of the covariances between the portfolio weights and the returns on the risky assets in the portfolio

$$\mathcal{PM} = \sum_{j=1}^{p} \mathrm{cov}(w_j, r_j) = \sum_{j=1}^{p} \{E[w_j r_j] - E[w_j]E[r_j]\}. \tag{26}$$

[12] There is no definite sign of correlation if the investor follows volatility timing strategies. We find both positive and negative correlations, with a majority of negative correlations.

Table 4
Portfolio performance with monthly rebalance

Model	μ (%)	σ (%)	$\mathcal{SR}$	Φ (bp)	$\mathcal{M}2$ (bp)	$\mathcal{Z}$	Δr $\mu_d(\%)$	Δr sk_d	τ^{be} (bp)	$\Delta\Phi$ (bp)	$\Delta\mathcal{M}2$ (bp)
Panel A—Maximum expected utility strategy											
BH	8.42	10.82	0.254								
Static	26.09	50.48	0.404	–8441.2	162.6	0.567	17.67	0.025			
Return-volatility timing											
DFMSV1	8.84	7.39	0.430	233.8	189.8	0.467	0.42	0.269	102.3	8675.0	8630.9
DFMSV2	10.87	7.44	0.700	434.5	481.8	0.483	2.45	0.249	160.2	8875.6	8923.0
DFMSV3	9.95	9.48	0.451	236.1	212.9	0.533	1.53	–0.070	53.0	8677.2	8654.1
Volatility timing											
DFMSV1	9.16	7.43	0.469	263.1	232.5	0.500	0.74	0.116	124.7	8704.2	8673.6
DFMSV2	11.47	7.14	0.812	507.6	604.0	0.483	3.05	0.241	215.0	8948.8	9045.1
DFMSV3	10.39	8.85	0.533	315.7	301.8	0.517	1.97	–0.033	104.1	8756.9	8743.0
Panel B—Minimum volatility strategies											
BH	6.81	9.09	0.125								
Static	6.92	3.09	0.404	233.1	253.8	0.433	0.11	0.689	546.9		
Return-volatility timing											
DFMSV1	8.65	4.55	0.655	372.4	481.3	0.533	1.84	0.352	197.8	139.3	227.5
DFMSV2	9.92	4.71	0.904	495.3	707.7	0.533	3.11	0.250	236.1	262.2	453.9
DFMSV3	7.58	4.01	0.475	278.9	317.9	0.500	0.77	0.450	146.6	45.8	64.1
Volatility timing											
DFMSV1	8.75	4.33	0.712	388.6	533.6	0.550	1.94	0.341	314.5	155.5	279.8
DFMSV2	10.05	4.34	1.009	517.7	803.5	0.517	3.24	0.354	367.5	284.6	549.7
DFMSV3	7.84	3.80	0.571	310.5	405.5	0.500	1.03	0.357	238.7	77.4	151.6
Panel C—Maximum expected return strategies											
BH	15.71	17.57	0.571								
Static	17.87	30.17	0.404	–1821.5	–293.3	0.500	2.16	0.336	–348.6		
Return-volatility timing											
DFMSV1	13.27	13.43	0.566	163.5	–8.7	0.450	–2.44	0.410	40.1	1985.0	284.5
DFMSV2	17.04	13.70	0.830	517.4	455.1	0.383	1.33	0.489	105.9	2338.8	748.4
DFMSV3	12.38	14.63	0.459	–31.6	–197.9	0.433	–3.33	–0.070	–4.8	1789.8	95.3
Volatility timing											
DFMSV1	13.66	13.47	0.593	198.4	38.3	0.450	–2.05	0.308	57.2	2019.8	331.6
DFMSV2	17.77	13.24	0.914	628.2	601.4	0.417	2.06	0.548	155.4	2449.7	894.7
DFMSV3	13.34	13.86	0.553	132.3	–32.3	0.467	–2.37	–0.048	29.2	1953.8	261.0
All-risky	18.53	19.86	0.647								

Note: This table evaluates the out-of-sample performance of the DFMSV models with monthly rebalance. BH, value-weighted buy-and-hold strategy; Static, static strategy based on the sample mean and covariances; DFMSVk, dynamic strategies based on the DFMSV models with k factors; All-risky, buy-and-hold strategy without the risk-free asset. The mean-variance scenarios are: (i) maximum expected utility with a risk aversion of 6; (ii) minimum volatility with a target expected return of about 10%; and (iii) maximum expected return with a target volatility of 12%. μ_p, average return; σ_p, volatility; $\mathcal{SR}$, Sharpe ratio; Φ, performance fee; $\mathcal{M}2$, M2 measure; $\mathcal{Z}$, success rate; Δr, returns on the zero-cost portfolio; sk_d, skewness; τ^{be}, breakeven TC; $\Delta\Phi$ & $\Delta\mathcal{M}2$, differences in Φ and $\mathcal{M}2$ between the dynamic and static strategies, respectively.

Intuitively, this sum is the expected return of the investor's portfolio, less what the expected return would be if her portfolio weights and asset returns were uncorrelated. Because investors do not engage in any stock-picking activities in our setting, the abnormal returns are solely

Table 5
Performance of expected return timing

Model	α (bp)	t-statistic	β_1	t-statistic	β_2	t-statistic	$\mathcal{TM}$ (bp)	Correlation*	$\mathcal{PM}$ (bp)
Panel A—Daily rebalance									
Maximum expected utility									
DFMSV1	–18.2	–0.02	0.24	6.88	15.07	7.19	2332.8	0.095	1905.7
DFMSV2	1433.5	1.1	0.11	1.9	9.16	2.53	1417.7	0.088	2375.3
DFMSV3	3353	1.82	0.1	1.22	10.94	2.14	1693.8	0.049	4529.1
Minimum volatility									
DFMSV1	–410.5	–1.52	0.13	8.49	6.06	5.78	665.4	0.083	157.3
DFMSV2	23.3	0.15	0.02	2.16	2.07	3.5	227.7	0.076	193.4
DFMSV3	8.2	0.07	0.02	3.72	1.61	3.73	176.7	0.037	146
Maximum expected return									
DFMSV1	–1348.6	–1.93	0.18	9.18	7.79	10.14	3101.8	0.105	1494.2
DFMSV2	550.2	0.72	0.06	2.81	3.85	4.56	1531.2	0.098	1783.4
DFMSV3	625.7	0.79	0.08	3.59	3.76	4.33	1495.8	0.054	1879.3
Panel B—Weekly rebalance									
Maximum expected utility									
DFMSV1	250.9	0.76	0.48	13.35	–0.21	–0.18	–28.4	–0.005	–165.1
DFMSV2	612.3	1.58	0.49	11.71	–3.83	–2.92	–526.1	0.009	–272.1
DFMSV3	681.8	1.01	0.48	6.62	–3.25	–1.42	–445.9	–0.024	–149.6
Minimum volatility									
DFMSV1	81.8	0.34	0.32	10.42	0.5	0.45	48.8	–0.057	–98.1
DFMSV2	242.2	0.95	0.19	5.6	–0.28	–0.24	–27.9	–0.032	5.2
DFMSV3	307.8	1.87	0.13	5.98	–0.61	–0.8	–59.5	–0.058	127.1
Maximum expected return									
DFMSV1	14.2	0.03	0.52	14.83	0.08	0.1	28.3	–0.037	–294.3
DFMSV2	495.2	0.78	0.44	10.66	–1.1	–1.18	–379.1	–0.017	–231.1
DFMSV3	1008.9	1.34	0.36	7.37	–1.63	–1.5	–565.4	–0.041	154.2
Panel C—Monthly rebalance									
Maximum expected utility									
DFMSV1	276	1.06	0.51	8.55	–0.86	–0.7	–100.6	–0.256	–233.1
DFMSV2	553.3	2.02	0.49	7.82	–1.45	–1.12	–169.7	–0.071	–15
DFMSV3	265.7	0.64	0.53	5.57	0.14	0.07	16	–0.075	–122.2
Minimum volatility									
DFMSV1	287.5	1.71	0.36	7.67	–0.38	–0.34	–31.1	–0.148	–18.2
DFMSV2	472.9	2.57	0.34	6.63	–1.07	–0.88	–88.1	–0.023	103.3
DFMSV3	129.5	0.72	0.25	4.91	0.4	0.34	33.1	–0.109	–29.9
Maximum expected return									
DFMSV1	281.8	0.6	0.6	9.69	–0.4	–0.45	–124.2	–0.223	–233.1
DFMSV2	846	1.64	0.58	8.6	–0.94	–0.96	–290	–0.048	150.1
DFMSV3	30.6	0.05	0.51	5.82	0.41	0.33	127.9	–0.094	–164.5

Note: This table reports performance analysis of the expected return timing. The coefficients α, β_1, and β_2 are estimated in the quadratic regression: $r_{d,t} - r_f = \alpha + \beta_1(r_{b,t} - r_f) + \beta_2(r_{b,t} - r_f)^2 + u_t, u_t \sim \mathcal{N}(0, \sigma^2)$. $\mathcal{TM}$ and $\mathcal{PM}$ are timing performance measures defined in the text. *correlations of the total weights on the 36 stocks in the dynamic portfolios and returns on the equal-weighted portfolio.

contributed by the timing activities. However, this sum is a biased measure of the expected return timing because of the presence of volatility timing, under which the sign of the correlation is ambiguous (see note 12). The last column in Table 5 reports this measure. As expected, the daily rebalance strategies generate large positive numbers, whereas the weekly and monthly rebalance strategies generate mostly negative numbers.

Another timing performance measure is based on the quadratic regression proposed by Treynor and Mazuy (1966) (hereafter $\mathcal{TM}$)

$$r_{d,t} - r_f = \alpha + \beta_1\left(r_{b,t} - r_f\right) + \beta_2\left(r_{b,t} - r_f\right)^2 + u_t, \quad u_t \sim \mathcal{N}\left(0, \sigma^2\right). \quad (27)$$

A significantly positive estimate of β_2 indicates successful return timing. Furthermore, under the conditions provided by Admati et al. (1986), $\beta_2 \mathrm{var}\left(r_{b,t}\right)$ is the abnormal return of return timing.[13] Table 5 also reports the results of the quadratic regressions. As expected, the daily rebalance strategies generate significantly positive β_2 estimates, and subsequently positive timing performance as measured by $\mathcal{TM}$, whereas the weekly and monthly rebalance strategies generate insignificant and mostly negative β_2 estimates and $\mathcal{TM}$ measures.

5.4 Characteristics of the dynamic strategies

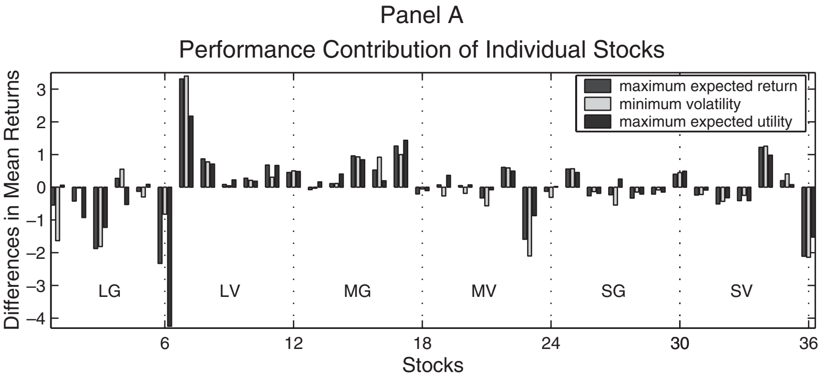
Small stocks often yield positive abnormal returns. It is of interest to check if the outperformance of the dynamic strategies is driven by the small stocks. Panel A of Figure 1 plots, for each stock, the average difference in returns between the DFMSV2 daily return-volatility timing and the buy-and-hold strategies. We find that most gains come from the large value and medium growth stocks, whereas the large growth stocks suffer large losses. Panel B plots the correlations between the portfolio weights and stock returns. Consistent with the expected return timing, the correlations are mostly positive.

When examining the investment style of the dynamic strategies, we find several interesting observations. First, the dynamic strategies long the large stocks, short the small stocks, and, to a lesser degree, also short the medium stocks. Second, the dynamic strategies take much larger positions in the growth stocks than in the value stocks as a whole. Finally, within the six groups, the large value stocks have the largest positions, followed by the large growth, the medium growth, the small growth, the medium value, and the small value stocks, with the last two groups in short positions.

5.5 Subperiod performance

Analysis of subperiod performance tells us how the dynamic strategies perform over different periods, which is especially interesting as the markets went from boom to recession during the testing period. Table 6 reports the performance results each year from 1996 to 2000. For the sake of conciseness, we only report the results of the DFMSV2 model under the minimum volatility scenario. Several interesting observations emerge

[13] The $\mathcal{TM}$ measure may be biased because the benchmark portfolio, the buy-and-hold strategy, is not known to be mean-variance efficient.



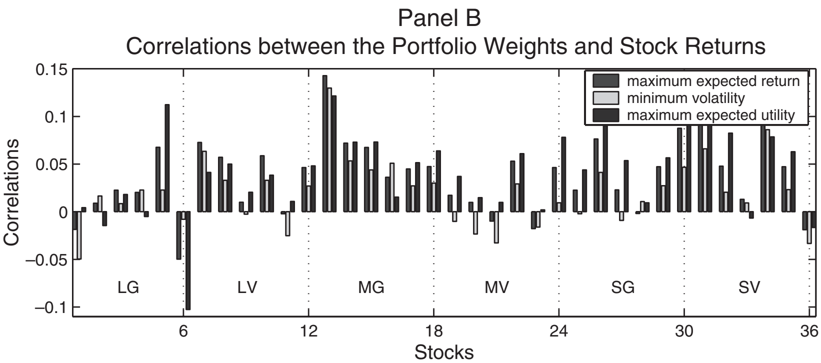


Figure 1
Characteristics of dynamic strategies
This figure shows the characteristics of the DFMSV2 daily return-volatility timing strategies under the three mean-variance objectives. Panel A, differences in the mean returns of each stock between the dynamic strategies and the buy-and-hold strategies. Panel B, correlations between individual portfolio weights and stock returns.

from our analysis. First, the performance of the buy-and-hold strategies reflects the changes in the market conditions. For example, in daily rebalance, the Sharpe ratios dropped from 0.946 to 0.605 from 1996 to 1997, kept almost constant for the next three years, and then sharply dropped to 0.119 in 2000, when the recession started. Second, the two dynamic strategies seem to perform very well in the recession period (2000)—in most cases, there were only slight performance drops from 1999 to 2000, in sharp contrast to the buy-and-hold strategies. The results suggest that the two DFMSV dynamic strategies precisely anticipated negative changes in the market conditions and successfully counteracted those negative changes.

Table 6
Yearly performance of the DFMSV2 dynamic strategy under the minimum volatility scenario

	Buy-and-hold			Return-volatility timing						Volatility timing					
Year	μ (%)	σ (%)	$\mathcal{SR}$	μ (%)	σ (%)	$\mathcal{SR}$	Φ (bp)	$\mathcal{M}2$ (bp)	τ^{be} (bp)	μ (%)	σ (%)	$\mathcal{SR}$	Φ (bp)	$\mathcal{M}2$ (bp)	τ^{be} (bp)
Daily rebalance															
1996	10.31	4.91	0.946	14.1	2.89	2.916	425.7	2663.2	5.13	12.05	3.91	1.630	200.0	335.8	11.54
1997	10.12	7.35	0.605	12.87	2.92	2.462	411.9	3343.2	5.13	9.42	4.70	0.798	26.6	142.2	1.42
1998	11.00	8.29	0.644	10.82	2.94	1.752	161.9	2263.2	2.02	10.28	5.01	0.920	58.6	229.2	2.91
1999	11.20	8.98	0.615	8.62	3.11	0.950	–43.9	928.0	–0.55	11.35	4.98	1.142	183.6	472.8	8.84
2000	6.92	10.48	0.119	8.20	3.11	0.813	428.8	1703.6	5.72	10.37	5.24	0.897	592.3	815.6	27.98
Weekly rebalance															
1996	11.45	5.06	1.105	10.40	3.99	1.141	–75.4	18.3	–4.37	11.29	4.14	1.314	9.8	105.9	1.80
1997	9.50	6.36	0.574	6.87	4.35	0.234	–198.8	–216.6	–9.09	8.21	4.25	0.556	–61.6	–11.3	–11.85
1998	11.10	7.03	0.746	10.14	4.52	0.950	–7.7	143.0	–0.37	9.53	4.37	0.842	–65.3	66.9	–12.79
1999	11.15	8.01	0.662	10.41	4.70	0.969	52.3	245.6	2.59	9.81	4.34	0.910	2.0	198.7	0.37
2000	6.88	9.91	0.104	8.19	5.39	0.433	338.6	326.5	16.17	9.10	4.73	0.686	449.8	577	78.92
Monthly rebalance															
1996	10.43	4.99	0.953	11.12	3.84	1.419	99.2	232.7	72.17	10.85	3.8	1.363	72.4	204.6	65.48
1997	10.19	7.25	0.624	10.47	4.31	1.114	130.7	355.5	101.45	10.02	4.13	1.055	90.4	312.5	83.87
1998	11.06	7.94	0.679	9.77	4.83	0.849	–6.4	135.0	–3.33	9.89	4.45	0.949	16.3	214.4	12.23
1999	11.17	7.79	0.706	10.47	4.63	1.037	51.6	257.9	26.05	10.58	4.24	1.16	73.9	353.5	56.21
2000	6.81	9.09	0.125	9.92	4.71	0.904	495.3	707.7	246.5	10.05	4.34	1.009	517.7	803.5	384.4

Note: The yearly performance is calculated over the period from the beginning of the testing period (January 2, 1996) to the end of the target year. The table only shows the performance of the DFMSV2 strategies under the minimum volatility scenario. We report the average realized return $(\mu_p\%)$, the realized volatility (σ), the realized Sharpe ratio $(\mathcal{SR})$, the performance fee (Φ), the $\mathcal{M}2$ measure, and the breakeven TCs (τ^{be}). The testing period is January 2, 1996, through December 29, 2000.

5.6 More robustness analysis

The first robustness test is to compare the performance of the dynamic strategies to other benchmarks. For a third benchmark, we assume that an investor estimates a univariate stochastic volatility model for each return series and uses the sample correlations of the estimation period as the constant correlations among returns. Another two benchmarks are naive dynamic strategies based on the expanding and rolling windows of returns, respectively. We find that the two DFMSV dynamic strategies outperform these additional benchmark strategies as well. Interestingly, the expanding and rolling window strategies significantly underperform the buy-and-hold strategies, suggesting that dynamic strategies that use information in naive ways may perform very poorly out of sample. In a second test, we assess the sensitivity of the outperformance to variations in the investor's objective functions (e.g., degree of risk aversion, target volatility, and expected return). The performance fees and risk-adjusted abnormal returns increase as the target expected return or volatility increases, decrease as the risk aversion increases, but remain positive throughout.

5.7 Performance analysis with Dow Jones Sector Indices

As another robustness test, we analyze a different set of returns at different time periods. To this end, we choose Dow Jones sector indices because many sector indices have corresponding exchange-Traded Funds (ETFs) being traded in the markets, and ETFs have much lower TCs. Another important difference is that these indices are portfolios. Returns on Dow Jones sector indices obtained from DataStream are available only from January 3, 1992. Therefore, we take the first six-year period from January 3, 1992, to December 31, 1997, as the estimation period, and the next six-year period from January 2, 1998, to December 29, 2003, as the testing period. Again, the testing period contains both boom periods (1998–1999) and recession periods (2000–2003).

Table 7 reports the yearly performance of the dynamic strategies formed from 35 Dow Jones sector indices. As with stocks, the performance of the buy-and-hold strategies reflects changes in the market conditions—the Sharpe ratios kept declining from 1998 to 2002 but rebounded slightly in 2003, and there were sharp drops in 2000 when the market turned bearish. By contrast, the two dynamic strategies did not show any significant decline in the Sharpe ratios in 2000, though there were large drops in 1999. However, without any doubt, they significantly outperform the buy-and-hold strategies year by year regardless of the rebalance frequencies. Furthermore, the breakeven TCs are one order of magnitude higher for both dynamic strategies if compared to the case of stocks. Given the lower TCs of trading ETFs, day traders may have little problem profiting from the return-volatility timing strategies.

Table 7
Yearly performance of the DFMSV2 dynamic strategy under the minimum volatility scenario (Dow Jones sectors)

	Buy-and-hold			Return-volatility timing						Volatility timing					
Year	μ (%)	σ (%)	$\mathcal{SR}$	μ (%)	σ (%)	$\mathcal{SR}$	Φ (bp)	$\mathcal{M}2$ (bp)	τ^{be} (bp)	μ (%)	σ (%)	$\mathcal{SR}$	Φ (bp)	$\mathcal{M}2$ (bp)	τ^{be} (bp)
Daily rebalance															
1998	6.88	6.96	0.173	6.62	0.88	1.080	117.0	1317.9	5.2	6.75	0.90	1.201	129.7	1402.1	21.0
1999	5.91	6.69	0.036	6.33	0.91	0.720	173.7	1048.9	6.2	6.14	0.96	0.485	154.3	891.6	23.9
2000	3.41	7.17	–0.316	6.41	0.94	0.794	452.2	1136.0	17.6	5.98	1.01	0.308	408.3	787.5	62.1
2001	1.31	7.20	–0.605	6.39	0.94	0.774	661.5	1124.9	27.0	5.85	0.95	0.191	607.1	704.6	93.1
2002	–0.77	7.38	–0.873	6.17	0.93	0.537	855.0	963.6	35.0	5.66	0.95	–0.012	803.8	557.9	122.3
2003	0.70	7.07	–0.703	6.15	0.89	0.536	692.3	946.1	28.3	5.76	0.90	0.096	653.2	634.7	101.9
Weekly rebalance															
1998	6.35	6.65	0.075	6.45	0.86	0.707	139.1	1055.4	46.8	6.39	0.81	0.662	132.6	1025.1	80.2
1999	5.91	6.71	0.009	5.99	0.91	0.153	140.2	687.9	53.2	5.88	0.92	0.035	129.5	608.8	75.3
2000	3.42	7.31	–0.333	5.93	0.90	0.091	408.9	651.5	157.3	6.01	0.89	0.179	416.7	716.0	239.9
2001	1.43	7.54	–0.587	5.75	0.86	–0.120	599.4	494.9	233.3	5.81	0.84	–0.052	605.3	545.5	340.3
2002	–0.83	7.33	–0.912	5.53	0.83	–0.389	794.1	299.9	311.4	5.61	0.83	–0.294	802.3	369.8	450.1
2003	0.62	6.98	–0.750	5.62	0.79	–0.288	644.0	384.0	257.0	5.68	0.79	–0.218	649.8	433.2	368.0
Monthly rebalance															
1998	6.95	7.94	0.161	6.27	0.96	0.628	108.8	1065.3	314.0	6.32	0.94	0.687	113.0	1112.5	362.2
1999	5.90	6.47	0.036	5.95	0.84	0.329	126.2	779.6	345.6	5.99	0.83	0.387	130.7	817.5	384.0
2000	3.35	6.27	–0.371	6.00	0.81	0.408	379.6	822.8	1028.9	6.06	0.80	0.494	386.0	876.3	1087.9
2001	1.24	6.32	–0.701	5.73	0.74	0.076	564.4	615.3	1380.7	5.76	0.74	0.118	567.4	641.3	1531.8
2002	–0.84	6.34	–1.027	5.63	0.74	–0.050	763.9	535.4	1930.4	5.66	0.74	–0.012	766.7	559.7	2107.6
2003	0.63	6.06	–0.832	5.72	0.69	0.076	616.6	613.2	1611.8	5.75	0.69	0.120	619.6	639.9	1749.0

Note: The yearly performance is calculated over the period from the beginning of the testing period (January 2, 1998) to the end of the target year. The table only shows the performance of the DFMSV2 strategies under the minimum volatility scenario. We report the average realized return $(\mu_p\%)$, the realized volatility (σ), the realized Sharpe ratio $(\mathcal{SR})$, the performance fee (Φ), the $\mathcal{M}2$ measure, and the breakeven TCs (τ^{be}). The testing period is January 2, 1998, through December 29, 2003.

6. Conclusion

This article investigates the implications of time-varying expected return and volatility on asset allocation in a high dimensional setting. It proposes a DFMSV model as a new framework for active portfolio management, including volatility and expected return timing, and undertakes extensive analysis to evaluate the economic significance of the DFMSV model in a setting appealing to practitioners. Our model seems to be the first one that is capable of handling a large number of assets and, at the same time, which allows the first two moments to vary over time for each asset. In contrast, previous studies are often confined to a handful of assets, such as a few market indices.

There are a number of possible extensions to the DFMSV model. One extension is to add more structures into the dynamics of the expected return, such as imposing pricing restrictions.[14] Another approach is to establish the risk-return relation between the first two moments. This task has proven difficult and controversial. It is our ongoing effort to extend the model to accommodate the linkage. Another extension is to replace the Gaussian shocks in both the return and factor dynamics with the student-t shocks and also to add discrete jumps. One may also relax the orthogonality assumption of factors and assume stochastic covariance for the factors. In a similar spirit, one may relax the assumption of uncorrelated idiosyncratic shocks and impose another factor structure on the covariance matrix of returns to overcome the problem of high dimensionality. Intuitively, it suggests that the idiosyncratic shocks may have industry-specific components that can be captured by the second factor structure.

In the portfolio analysis, we restrict attention to stock returns, but the methodology may be extended to include bond returns and exchange rates as well. It would be of interest to examine how the DFMSV model performs with mixed assets that include stocks, bonds, and foreign currencies.

Finally, one prominent feature of the DFMSV model is that it allows the expected return, volatility, and correlation to vary over time and thus can be applied to many interesting problems, such as asset pricing tests, performance evaluation, risk management, and so on. For example, it can be used to examine whether the co-movements of international markets are due to the contagion problem with increased correlations among different markets, or time-varying country variances [Forbes and Rigobon (2002)].

[14] Previous work imposes asset-pricing restrictions in the prior of the intercept. See, for example, Pástor (2000) and Pástor and Stambaugh (2000). In the DFMSV model, it is possible to add intercepts and impose pricing-motivated priors on them; however, there are several caveats. First, there seems to be an identification issue, and further restrictions are required. Specifically, not all the intercepts can be identified. There are $p + k$ unknown mean parameters, but only p mean equations. (The k factors are latent.) Second, adding the intercepts increases the number of parameters drastically, and those are difficult parameters to estimate for not having well-behaved density functions.

Appendix A: Prior Specification

We assume a normal prior $\mathcal{N}(b_{ij}|b_0, B_0)$ for each of the free elements of **B**, and a centered normal prior $I_{[-1,1]}\mathcal{N}(a_{jj}|a_0, A_0)$ for each of the elements of **A**. Similarly, we assume the factor expected return c follows a normal prior $\mathcal{N}(c|c_0, C_0)$. As for the parameters in the latent log volatility h_{jt}, we assume a normal prior $\mathcal{N}(\mu_0, S_0)$ for the mean μ_j and an inverse gamma prior $\mathcal{IG}(\delta_0, \nu_0)$ for the volatility σ_j. For the persistent coefficient ϕ_j, we formulate the prior in terms of $\phi_j = 2\phi_j^* - 1$, where ϕ_j^* is distributed as beta with parameters $(\phi^{(1)}, \phi^{(2)})$. This implies that the prior on $\phi_j \in (-1,1)$ is

$$\pi(\phi_j) = \kappa[0.5(1+\phi)j]^{\phi^{(1)}-1}\left[0.5(1-\phi_j)\right]^{\phi^{(2)}-1}, \quad \phi^{(1)}, \phi^{(2)} > 0.5, \tag{A1}$$

where $\kappa = 0.5\frac{\Gamma(\phi^{(1)}+\phi^{(2)})}{\Gamma(\phi^{(1)}\Gamma(\phi^{(2)})}$. We set the hyperparameters to reasonable values, but the algorithm is fairly robust to the prior specifications and initial values of the parameters.

Appendix B: Estimation Algorithms

For ease of notation, we use y_t in place of r_t to denote stock returns, and let $Y^t = (y_{t+1}, ..., y_n)$ and $Y_t = (y_1, ..., y_t)$. Let b denote the free elements of the loading matrix **B**, θ_j, the vector consisting of the parameters in h_{jt}, $(\mu_j, \phi_j, \sigma_j)$ and $\xi = (\{a_{jj}\}, b)$. Denote $h_{j.} = \{h_{jt}\}_{t=1}^n$, and $\{h_{j.}\} = \{h_{j.}\}_{j=1}^{p+k}$. Assume similar notations for s_{jt} and z_{jt}. Then the MCMC algorithm may be summarized as follows:

1. Initialize $\{h_{j.}\}$ and c;
2. Sample $[\xi, \{f_t\}|y, c, \{h_{j.}\}]$
3. Compute $z_{j.}$ and repeat for each $j \le p+k$;
   (a) Sample $s_{j.}$ from $[s_{j.}|z_{j.}, h_{j.}]$,
   (b) Sample $[\theta_j, h_{j.}]$.
4. Sample c from $[c|\{f_t\}, \{h_{j.}\}]$; and
5. Go to Step 2 and repeat.

In Step 2, we sample ξ and $\{f_t\}$ jointly from the distribution $\pi(\xi, \{f_t\}|y, c, \{h_{j.}\})$ by the method of composition. This is done by first drawing ξ from $\pi(\xi|y, c, \{h_{j.}\})$, marginalized over $\{f_t\}$, and then drawing $\{f_t\}$ from $\pi(\{f_t\}|y, \xi, c, \{h_{j.}\})$. The posterior density of ξ is

$$\pi(\xi|y,c,\{h_{j.}\}) \propto \pi(\xi)\prod_{t=1}^{n} p(y_t|Y_{t-1},\xi,c,\{h_{j.}\}), \tag{A2}$$

where $p(y_t|Y_{t-1}, \xi, c, \{h_{j.}\})$ can be obtained from Kalman filter recursions. To sample for ξ, we employ the Metropolis-Hastings algorithm with a multivariate student-t density $f_T(\xi|m, \Sigma, \nu)$ as the proposal. Following Chib and Greenberg (1998), we take the mean m to be the mode of the likelihood function $l = \log\{\prod_{t=1}^n p(y_t|Y_{t-1}, \xi, c, \{h_{j.}\})\}$ and the dispersion matrix Σ the inverse of the negative Hessian matrix of l. To sample $\{f_t\}$ jointly, we rewrite the joint density of the $\{f_t\}$ in reverse time order as

$$p(f_n|y, \psi) \times p(f_{n-1}|y, f_n, \psi) \times \cdots \times p(f_0|y, f_1, ..., f_n, \Psi), \tag{A3}$$

where ψ represents other conditioning variables including c, ξ, and $\{h_{j.}\}$. We now derive the density of a typical term $p(f_t|y, f_{t+1}, ..., f_n, \psi)$. Denote $f^{t+1} = (f_{t+1}, ..., f_n)$.

$$p\left(f_t|y, f^{t+1}, \psi\right) \propto p(f_t|Y_t, \psi)p(f_{t+1}|f_t, \psi)p\left(Y^{t+1}, f^{t+2}|f_t, f_{t+1}, Y_t, \psi\right) \\ \propto p(f_t|Y_t, \psi)p(f_{t+1}|f_t, Y_t, \psi), \tag{A4}$$

where both densities can be obtained from Kalman filter recursions. Thus, to sample the joint posterior distribution of $\{f_t\}$ given ψ, we first draw $\tilde{f}_n$ from $[f_n|y, \psi]$, then draw $\tilde{f}_{n-1}$ from $\left[f_{n-1}|y, \tilde{f}_n, \psi\right]$ and so on, until $\tilde{f}_1$ is drawn from $\left[f_1|y, \tilde{f}_2, ..., \tilde{f}_n, \psi\right]$.

To sample f_0 from $p(f_0|y, f_1, ..., f_n, \psi)$, we assume that the prior of f_0 is the steady-state density of f_t, $f_0 \sim \mathcal{N}(\mu_0, \Sigma_0)$, where $\mu_0 = (I - A)^{-1}c$, and $\sum_0$ is a diagonal matrix with jth element as $\left(1 - a_j^2\right)^{-1} e^{\mu_j + \frac{\sigma_j^2}{2(1-\phi_j^2)}}$. Thus, f_0 is sampled from the Bayesian normal updating scheme,

$$p(f_0|y, f_1, ..., f_n, \psi) \propto \pi(f_0|\mu_0, \Sigma_0)\phi_k(f_1|f_0, \psi) = \phi_k\left(f_0|\hat{f}_0, F_0\right), \tag{A5}$$

where $F_0 = \left(\Sigma_0^{-1} + AS_1^{-1}A\right)^{-1}$, $\hat{f}_0 = F_0\left[\Sigma_0^{-1}\mu_0 + AS_1^{-1}\left(\tilde{f}_1 - c\right)\right]$, and $\phi_k(\cdot)$ denotes the k-dimensional normal density function.

In Step 3, having sampled $\{f_t\}$ and ξ, we compute z_{jt} and sample $\left(s_{j.}, \theta_j, h_{j.}\right)$ for each j using the univariate SV sampler of Chib, Nardari, and Shephard (2002). Similarly, we denote $Z_{jt} = \left(z_{j1}, ..., z_{jt}\right)$ and $Z_j^{t+1} = \left(z_{jt+1}, ..., z_{jn}\right)$. We first sample $s_{j.}$ from $\left[s_{j.}|z_{j.}, h_{j.}\right]$

$$\pi\left(s_{j.}|z_{j.}, h_{j.}\right) = \prod_{t=1}^{n} \pi\left(s_{jt}|z_{jt}, h_{jt}\right), \tag{A6}$$

where $\pi\left(s_{jt}|z_{jt}, h_{jt}\right) \propto \pi\left(s_{jt}\right)\phi\left(z_{jt}|h_{jt} + m_{s_{jt}}, v_{s_{jt}}^2\right)$.

Next, we sample $\left[\theta_j, h_{j.}\right]$ by drawing θ_j from $\left[\theta_j|z_{j.}, s_{j.}\right]$ followed by a draw of $h_{j.}$ from $\left[h_{j.}|z_{j.}, s_{j.}, \theta_j\right]$. The posterior density of θ_j is

$$\pi\left(\theta_j|z_{j.}, s_{j.}\right) \propto \pi\left(\theta_j\right)p\left(z_{j.}|s_{j.}, \theta_j\right), \tag{A7}$$

where $p\left(z_{j.}|s_{j.}, \theta_j\right) = p\left(z_{j1}|s_{j.}, \theta_j\right)\prod_{t=2}^{n} p\left(z_{jt}|Z_{jt-1}, s_{j.}, \theta_j\right)$, and $p\left(z_{jt}|Z_{jt-1}, s_{j.}, \theta_j\right)$ can be computed from the output of Kalman filter recursions. To sample θ_j, we use the Metropolis-Hastings algorithm with a multivariate student-t proposal similar to the way we sample ξ. To sample $h_{j.}$ from $\left[h_{j.}|z_{j.}, s_{j.}, \theta_j\right]$ for each j, we employ the simulation smoother of de Jong and Shephard (1995), which uses the backward Kalman recursion to generate sample draws from the smoothing distributions.

In Step 4, after sampling $\{f_t\}, \{h_{j.}\}$, we can easily sample c by the Bayesian normal updating scheme.

$$\pi\left(c|\xi, \{\theta_j\}, \{f_t\}, \{h_{j.}\}\right) \propto \pi(c)p_k\left(f_0|\xi, \{\theta_j\}\right)\prod_{t=1}^{n} \phi_k\left(f_t|\xi, f_{t-1}, \{h_{jt}\}, c\right). \tag{A8}$$

Appendix C: Auxiliary Particle Filter

Similar to the linear Kalman filter, the auxiliary particle filter sequentially updates the latent variables, including the factors and log variances. Let ψ^* denote the posterior means of the parameters.

1. Given values $\left\{h_{t-1}^{(1)}, f_{t-1}^{(1)}, ..., h_{t-1}^{(M)}, f_{t-1}^{(M)}\right\}$ from $(h_{t-1}, f_{t-1}|\mathcal{F}_{t-1}, \psi^*)$, calculate

$$\hat{h}_{j,t}^{*(g)} = \mu_j^* + \phi_j^*\left(h_{j,t-1}^{(g)} - \mu_j^*\right), \quad \hat{f}_t^{*(g)} = Af_{t-1}^{(g)},$$
$$\omega_g = \phi_p\left(y_t|B\hat{f}_t^{*(g)}, V_t\left(\hat{h}_t^{*(g)}\right), \psi^*\right), \quad j \le p+k, g = 1,...,M, \tag{A9}$$

and sample R times the integers $1, 2, ..., M$ with probability proportional to $\{\omega\}$. Let the sampled indices be $k_1, ..., k_R$ and associate these with $\hat{h}_t^{*(k_1)}, ..., \hat{h}_t^{*(k_R)}$.

2. For each value of k_j from Step 1, simulate the values $\left\{h_t^{*(1)}, f_t^{(i)}, ..., h_t^{*(R)}, f_t^{(R)}\right\}$ from the volatility process and state evolution process as

$$h_{j,t}^{*(g)} = \mu_j^* + \phi_j^*\left(h_{j,t-1}^{(g)} - \mu_j^*\right) + \sigma_j^*\eta_{j,t}^{*(g)}, \quad f_t^{*(g)} = Af_{t-1}^{(g)} + \zeta_t^{(g)}, \quad g = 1, \dots, R, \tag{A10}$$

where $\eta_{j,t}^{(g)} \sim N(0,1)$ and $\zeta_t^{(g)} \sim N\left[0, S\left(h_t^{*(g)}\right)\right]$.

3. Resample the values $\left\{h_t^{*(1)}, f_t^{*(1)}, \dots, h_t^{*(R)}, f_t^{*(R)}\right\}M$ times with replacement, using probabilities proportional to

$$\frac{\phi_p\left[y_t|Bf_t^{*(g)}, V\left(h_t^{*(g)}\right)\right]}{\phi_p\left[y_t|Bf_t^{*(k_g)}, V\left(h_t^{*(k_g)}\right)\right]} \quad g = 1, \dots, R, \tag{A11}$$

to produce the desired filtered sample $\left\{h_t^{(1)}, f_t^{(1)}, \dots, h_t^{(M)}, f_t^{(M)}\right\}$ from $(h_t, f_t|\mathcal{F}_t, \psi^*)$.

References

Admati, A. R., S. Bhattacharya, P. Pfleiderer, and S. A. Ross, 1986, "On Timing and Selectivity," *Journal of Finance*, 41(3), 715–730.

Aguilar, O., and M. West, 2000, "Bayesian Dynamic Factor Models and Portfolio Allocation," *Journal of Business and Economic Statistics*, 18(3), 338–357.

Aït-Sahalia, Y., and M. W. Brandt, 2001, "Variable Selection for Portfolio Choice," *Journal of Finance*, 56, 1297–1351.

Avramov, D., 2002, "Stock Return Predictability and Model Uncertainty," *Journal of Financial Economics*, 64(3), 431–467.

Balduzzi, P., and A. W. Lynch, 1999, "Transaction Costs and Predictability: Some Utility Cost Calculations," *Journal of Financial Economics*, 52, 47–78.

Barberis, N., 2000, "Investing for the Long Run when Returns are Predictable," *Journal of Finance*, 55, 225–264.

Bhardwaj, R. K., and L. D. Brooks, 1992, "The January Anomaly: Effects of Low Share Price, Transaction Costs, and Bid-Ask Bias," *Journal of Finance*, 47(2), 553–575.

Bollerslev, T., 2001, "Financial Econometrics: Past Developments and Future Challenges," *Journal of Econometrics*, 100(1), 41–51.

Bollerslev, T., R. F. Engle, and D. B. Nelson, 1994, "ARCH Models," in R. F. Engle and D. McFadden (ed.), *Handbook of Econometrics*, Elsevier, North-Holland, Amsterdam, Vol. IV, pp. 2959–3038.

Chan, L. K. C., J. Karceski, and J. Lakonishok, 1999, "On Portfolio Optimization: Forecasting Covariances and Choosing the Risk Model," *The Review of Financial Studies*, 12(5), 937–974.

Chib, S., and E. Greenberg, 1996, "Markov Chain Monte Carlo Simulation Methods in Econometrics," *Econometric Theory*, 12, 409–431.

Chib, S., and E. Greenberg, 1998, "Analysis of Multivariate Probit Models," *Biometrika*, 85(2), 377–361.

Chib, S., F. Nardari, and N. Shephard, 2005, "Analysis of High Dimensional Multivariate Stochastic Volatility Models," *Journal of Econometrics*, forthcoming.

Copeland, M. M., and T. E. Copeland, 1999, "Market Timing: Style and Size Rotation using VIX," *Financial Analysts Journal*, 55, 73–81.

Cremers, M., 2002, "Stock Return Predictability: a Bayesian Model Selection Perspective," *The Review of Financial Studies*, 15(4), 1223–1250.

de Jong, P., and N. Shephard, 1995, "The Simulation Smoother for Time Series Models," *Biometrika*, 82(2), 339–350.

Ferson, W. E., S. Sarkissian, and T. Simin, 2002, "Spurious Regressions in Financial Economics," NBER Working Paper No. W914.

Fleming, J., C. Kirby, and B. Ostdiek, 2001, "The Economic Value of Volatility Timing," *Journal of Finance*, 56(1), 329–351.

Fleming, J., C. Kirby, and B. Ostdiek, 2003, "The Economic Value of Volatility Timing Using 'Realized' Volatility," *Journal of Financial Economics*, 67(3), 473–509.

Fleming, J., B. Ostdiek, and R. E. Whaley, 1996, "Trading Costs and The Relative Rates of Price Discovery in The Stock, Futures, and Option Markets," *Journal of Futures Markets*, 16, 353–387.

Forbes, K. J., and R. Rigobon, 2002, "No Contagion, Only Interdependence: Measuring Stock Market Co-movements," *Journal of Finance*, 57(5), 2223–2261.

Geweke, J., and G. Zhou, 1996, "Measuring the Pricing Error of the Arbitrage Pricing Theory," *The Review of Financial Studies*, 9(2), 557–587.

Ghysels, E., A. C. Harvey, and E. Renault, 1995, "Stochastic Volatility," in G. S. Maddala and C. R. Rao (ed.), *Handbook of Statistics*, Elsevier, North-Holland, Amsterdam, Vol. 14, pp. 119–191.

Gomes, F., 2002, "Exploiting Short-Run Predictability," working paper, London Business School.

Goyal, A., and I. Welch, 2003, "Predicting the Equity Premium With Dividend Ratios," *Management Science*, 49(5), 639–654.

Graham, J. R., and C. R. Harvey, 1996, "Market Timing Ability and Volatility Implied by Investment Newsletters' Asset Allocation Recommendations," *Journal of Financial Economics*, 42, 397–421.

Graham, J. R., and C. R. Harvey, 1997, "Grading the Performance of Market-Timing Newsletters," *Financial Analysts Journal*, 53(6), 54–68.

Grinblatt, M., and S. Titman, 1993, "Performance Measurement without Benchmarks: An Examination of Mutual Fund Returns," *Journal of Business*, 66(1), 47–68.

Han, Y., 2005, "Return Predictability and Model Mis-Specification: When Can an Investor Profit from Return Predictability," working paper, Tulane University.

Handa, P., and A. Tiwari, 2005, "Does Stock Return Predictability Imply Improved Asset Allocation and Performance? Evidence from the U.S. Stock Market (1954–2002)" *Journal of Business*, forthcoming.

Johannes, M., N. Polson, and J. Stroud, 2002, "Sequential Optimal Portfolio Performance: Market and Volatility Timing," working paper, University of Chicago.

Kandel, S., and R. F. Stambaugh, 1996, "On the Predictability of Stock Returns: An Asset-Allocation Perspective," *Journal of Finance*, 51(2), 385–424.

Kim, S., N. Shephard, and S. Chib, 1998, "Stochastic Volatility: Optimal Likelihood Inference and Comparison with ARCH Models," *Review of Economic Studies*, 65, 361–393.

Knez, P. J., and M. J. Ready, 1996, "Estimating the Profits from Trading Strategies," *The Review of Financial Studies*, 9(4), 1121–1163.

Kroll, Y., H. Levy, and H. Markowitz, 1984, "Mean-Variance Versus Direct Utility Maximization," *Journal of Finance*, 39, 47–61.

Lettau, M., and S. Ludvigson, 2001, "Consumption, Aggregate Wealth and Expected Stock Returns," *Journal of Finance*, 56(3), 815–849.

Lynch, A. W., and P. Balduzzi, 2000, "Predictability and Transaction Costs: The Impact on Rebalancing Rules and Behavior," *Journal of Finance*, 66(5), 2285–2309.

Marquering, W., and M. Verbeek, 2001, "The Economic Value of Predicting Stock Index Returns and Volatility," working paper, Tilburg University.

Modigliani, F., and L. Modigliani, 1997, "Risk-Adjusted Performance," *Journal of Portfolio Management*, 23(2), 45–54.

Pástor, L., 2000, "Portfolio Selection and Asset Pricing Models," *Journal of Finance*, 55, 179–223.

Pástor, L., and R. Stambaugh, 2000, "Comparing Asset Pricing Models: An Investment Perspective," *Journal of Financial Economics*, 56, 335–381.

Pesaran, M. H., and A. Timmermann, 1995, "Predictability of Stock Returns: Robustness and Economic Significance," *Journal of Finance*, 50(4), 1201–1228.

Pitt, M. K., and N. Shephard, 1999, "Filtering via Simulation: Auxiliary Particle Filter," *Journal of the American Statistical Association*, 94, 590–599.

Pojarliev, M., and W. Polasek, 2000, "Portfolio Construction Using Multivariate Time Series Forecasts," working paper, Institute of Statistics and Econometrics, University of Basel.

Pulley, L. B., 1981, "A General Mean-Variance Approximation to Expected Utility for Short Holding Periods," *Journal of Financial and Quantitative Analysis*, 16, 361–373.

Tamayo, A., 2002, "Stock Return Predictability, Conditional Asset Pricing Models and Portfolio Selection," working paper, London Business School.

Treynor, J. L., and K. Mazuy, 1966, "Can Mutual Funds Outguess the Market?" *Harvard Business Review*, 44, 131–136.