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Non-linear mode coupling in non-axisymmetric droplet oscillations

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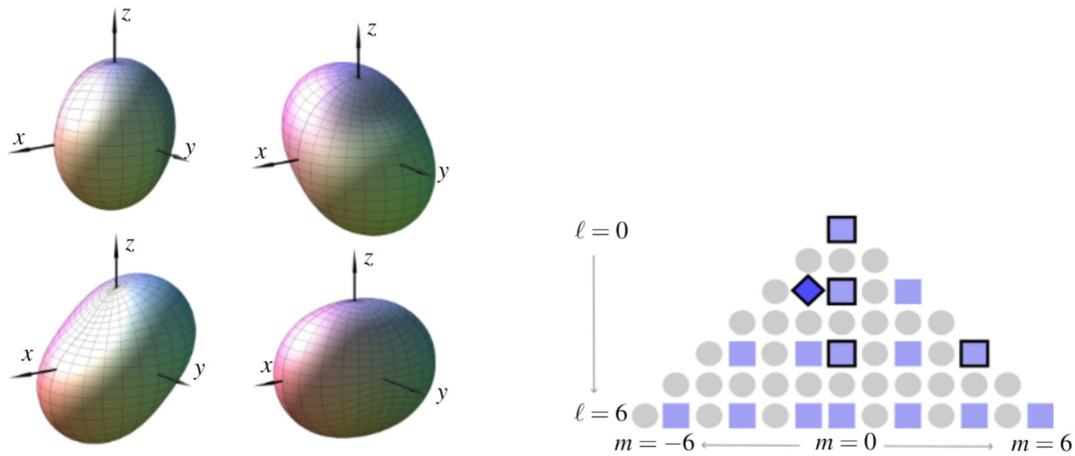
We investigate the non-linear and non-axisymmetric oscillations of inviscid droplets, focusing on the modal coupling between different oscillation modes in the absence of gravity. The analysis builds upon the geometrically non-linear theory[1] (GNLT) for droplet surface oscillation, which allows for both arbitrary initial deformations and couplings between various deformation modes. Using the unified transform method of Fokas[2], in[1] the governing equations are transformed into a system of integro-differential equations defined on a time-independent unit sphere **S** resulting in an initial problem with two unknowns: the velocity potential on the surface $q(\zeta,\phi,t)$ and the radius $R(\zeta,\phi,t)$, with the spatial variables ζ (polar coordinate), ϕ (azimuthal coordinate) and the time t. Initially, the droplet is at rest ($q=0$), and the radius is given by

$$R(\zeta,\phi,t=0) = 1 + kY^m_{\ell=2}(\zeta,\phi) \tag{1}$$

(see Figure 1a), where Y^m_ℓ are spherical harmonics with ℓ as the order and m as the degree. Since large amplitudes are to be analyzed, we have uniformly set $k=0.5$. These equations are numerically solved using a Galerkin method, where the unknowns are expressed as

$$R(\zeta,\phi,t) = \sum_{\ell=0}^{6}\sum_{m=-\ell}^{\ell} a^m_\ell(t)Y^m_\ell(\zeta,\phi) \qquad \text{and} \qquad q(\zeta,\phi,t) = \sum_{\ell=0}^{6}\sum_{m=-\ell}^{\ell} b^m_\ell(t)Y^m_\ell(\zeta,\phi). \tag{2}$$

The oscillation results reveal intriguing patterns in modal coupling, showing that it fundamentally operates through symmetry conservation, consistently preserving both reflection and point symmetry throughout the oscillation with the given initial deformation (see Figure 1a).
Due to this symmetry conservation, certain modes (the time-dependent coefficients $a^m_\ell(t)$ and $b^m_\ell(t)$) are zero throughout the oscillation. This is illustrated in Figure 1b for initial deformation mode $m=-1$ and $\ell=2$ (diamond marker). The excited modes are shown in blue. They are contrasted to the modes framed in black which are predicted by a small radius deformation expansion of the GNLT (up to 2[nd] order in ε).
During the meeting a theoretical analysis of this finding will be presented. Further, the role of symmetry conservation and modal coupling of non-linear and non-axisymmetric droplet oscillations will be detailed.



(a) Initial Deformations with the radius $R = 1 + 0.5\ Y^m_2$ for $m = -2,-1,1$ and 2 (from left to right and top to bottom).

(b) Modal coupling for the initial deformation mode $m=-1$ and $\ell=2$, represented by the diamond marker. The excited modes are shown in blue. The modes framed in black are given by a small radius deformation expansion of the GNLT (2[nd] order in ε).

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[1]Plümacher et al., *Physics of Fluids* **32**, (2020)
[2]Fokas, *Proceedings of the Royal Society A* **453**, 1411 - 1443 (1997)