Helical Turbulence - Bridging the Gap between 2D and 3D Turbulence

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Based on vanishing or active vortex stretching, our theoretical understanding of 2D and 3D turbulence reveals fundamental differences. To gain further insights, we study helically-symmetric flows, which allow us to precisely study the transition between 2D and 3D turbulence. The helical coordinate system is defined by its direction r, $\xi = az + b\phi$, and $\eta = -bz + ar^2\phi$, where a and b are constants, $a^2 + b^2 > 0$, and (r, ϕ, z) are cylindrical coordinates. The velocity vector, using helical basis vectors, is defined as $u = u^r e_r + u^\eta e_{\perp \eta} + u^\xi e_\xi$ (see figure 1).

Helical symmetry is characterized by flow-variables that are independent of η, thus $\partial/\partial\eta \equiv 0$ holds for all dependent variables. Helical flows exhibit significant differences based on the presence of a velocity component u^η. When $u^\eta = 0$, vortex stretching is switched off. For $u^\eta \neq 0$ vortex stretching is active even though all flow variables only live on the 2D manifold r, ξ (see figure 1). In both scenarios, helical flows are characterized by infinite classes of conservation laws (CL), thereby giving rise to related integral invariants. Specifically, for $u^\eta \neq 0$, the central new invariants emerge from an infinite set of generalized helicity CL, and for $u^\eta = 0$, there exist infinitely many generalized enstrophy CL [1]. 2D and 3D turbulence reveals that local and global invariants play a central role in turbulence and this we also observe for helical turbulence.

For the case with vortex stretching, as in the general 3D case, we expect the anomalous scaling of dissipation for the limit $\nu \to 0$ and the resulting Kolmogorov scaling. The case without vortex stretching is closer to the 2D case and different new energy spectra arise for limiting cases $r \to 0$ and $r \to \infty$.

Additionally, the transition from 2D to 3D turbulence is investigated by analyzing asymptotically small vortex stretching, i.e. $u^\eta \ll 1$. This allows to also theoretically trace the transition between the two cases above using the method of approximate CL. Results and comparison of invariant theory and high-fidelity simulation of helical turbulence will be presented at the meeting.
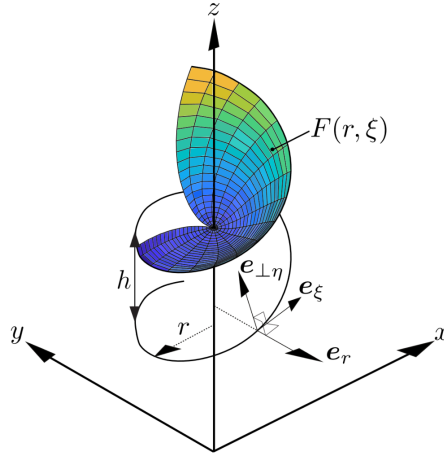
Figure 1: Helical flows live on a 2D manifold (r, ξ) embedded in a 3D space. For illustration, the outer helix shows a line of varying ξ and constant r. h represents the pitch of the helix. On the surface $F(r, \xi)$ all flow variables are defined, i.e. pressure, vorticity and velocity and the last two denote vectors which have three independent components.

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[1]Kelbin et al., Journal of Fluid Mechanics, 721 (2013).