MA1477 Veckotest v. 44 Lösningsförslag

1. a) $9(x-1)^2 - (3x-1)^2 - 3(1-2x)$

$= 9(x^2 - 2x + 1) - (9x^2 - 6x + 1) - 3 + 6x$

$= 9x^2 - 18x + 9 - 9x^2 + 6x - 1 - 3 + 6x$

$= -6x + 5$

b) $\frac{2x}{3} + \frac{2(1-x)}{6} + \frac{x-1}{2} = \frac{2x}{3} \cdot \frac{2}{2} + \frac{2(1-x)}{6} + \frac{(x-1)}{2} \cdot \frac{3}{3}$

$= \frac{4x + 2(1-x) + 3(x-1)}{6} = \frac{4x + 2 - 2x + 3x - 3}{6}$

$= \frac{5x - 1}{6}$

c) $\frac{27^2 \cdot 9^{-1} \cdot 6^2}{3^8 \cdot 3^{-2}} = \frac{(3^3)^2 \cdot 9^{-1} \cdot (2 \cdot 3)^2}{3^{8-2}} = \frac{3^6 \cdot 9^{-1} \cdot 2^2 \cdot 3^2}{3^6}$

$= \frac{2^2 \cdot 3^2}{9} = \frac{2^2 \cdot 3^2}{3^2} = 2^2 = 4$

d) $\frac{\sqrt{50} - \sqrt{98}}{\sqrt{8}} = \frac{\sqrt{2 \cdot 25} - \sqrt{2 \cdot 49}}{\sqrt{2 \cdot 4}} = \frac{\sqrt{2} \cdot \sqrt{25} - \sqrt{2} \cdot \sqrt{49}}{\sqrt{2} \cdot \sqrt{4}} =$

$= \frac{\sqrt{2}(\sqrt{25} - \sqrt{49})}{\sqrt{2} \cdot \sqrt{4}} = \frac{\sqrt{25} - \sqrt{49}}{\sqrt{4}} = \frac{5 - 7}{2} = \frac{-2}{2} = -1$

2. a) $4x^2 - 9 = 0$

$\Leftrightarrow (2x)^2 - 3^2 = 0$

$\Leftrightarrow (2x+3)(2x-3) = 0$

$\boxed{x = -\frac{3}{2} \text{ eller } x = \frac{3}{2}}$

b) $x^2 + 3x - 2 = 0$

$x = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 + 2}$ (pq)

$x = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{8}{4}}$

$x = -\frac{3}{2} \pm \sqrt{\frac{17}{4}}$

$\boxed{x = -\frac{3}{2} + \frac{\sqrt{17}}{2}, \quad x = -\frac{3}{2} - \frac{\sqrt{17}}{2}}$

c) $(x^2-4)(x-1)(x^2-5x+6) = 0$

$\Leftrightarrow (x+2)(x-2)(x-1)(x^2-5x+6) = 0$ (nollproduktsmetoden)

↑ noll när $x = -2$ ↑ noll när $x = 2$ noll när $x = 1$

$x^2 - 5x + 6 = 0$

$x = -\frac{5}{2} \pm \sqrt{\left(\frac{5}{2}\right)^2 - 6}$ (pq)

$x = -\frac{5}{2} \pm \sqrt{\frac{25}{4} - \frac{24}{4}}$

$x = -\frac{5}{2} \pm \sqrt{\frac{1}{4}}$

$x = -\frac{5}{2} \pm \frac{1}{2}$

$\boxed{x = -2, \quad x = 2, \quad x = 1, \quad x = -3,}$

$x = -2$ är en dubbelrot.

3. $f(x) = ax^2 + bx + c$

$f(0) = 3 \Rightarrow a\cdot 0^2 + b\cdot 0 + c = 3$ ger $c = 3$

$f(1) = 2 \Rightarrow a\cdot 1^2 + b\cdot 1 + 3 = 2$ ger $a + b = -1$ (*)

$f(-2) = 3 \Rightarrow a\cdot(-2)^2 + b\cdot(-2) + 3 = 3$ ger $4a - 2b = 0$ (**)

Från (*) får vi att $a = -b - 1$. Insättning av detta i (**) ger

$$4(-b-1) - 2b = 0$$

$$\Leftrightarrow -4b - 4 - 2b = 0$$

$$\Leftrightarrow -6b = 4$$

$$\Leftrightarrow b = -\frac{4}{6} = -\frac{2}{3}$$

Vidare vet vi att $a + b = -1$. Det ger

$$a - \frac{2}{3} = -1$$

$$\Leftrightarrow a = -1 + \frac{2}{3} = -\frac{1}{3}$$

Svar: $f(x) = -\frac{1}{3}x^2 - \frac{2}{3}x + 3$

4. a) $x^2(x+3) - 1(x+3) = 0$

$\Leftrightarrow (x+3)(x^2-1) = 0$

$\Leftrightarrow (x+3)(x+1)(x-1) = 0$

$\boxed{x = -3, \quad x = -1, \quad x = 1}$

b) $(x^2+2x+1)(x^2-1) + (x^2+2x+1)(2x^2-10) = 0$

$\Leftrightarrow (x^2+2x+1)(x^2-1+2x^2-10) = 0$

$\Leftrightarrow (x+1)^2(3x^2-11) = 0$

↑ noll när $x = -1$

↖

$3x^2 - 11 = 0$

$3x^2 = 11$

$x^2 = \frac{11}{3}$

$x = \pm\sqrt{\frac{11}{3}}$

$\boxed{x_1 = -1, \quad x_2 = \sqrt{\frac{11}{3}}, \quad x_3 = -\sqrt{\frac{11}{3}}}$

5.

a) $p(x) = 4x^2 - 32x + 60$

$= 4(x^2 - 8x + 15)$

pq-formeln för att finna nollställen till $x^2 - 8x + 15 = 0$

$$x = \frac{8}{2} \pm \sqrt{\left(\frac{8}{2}\right)^2 - 15}$$

$$x = 4 \pm \sqrt{4^2 - 15}$$

$$x = 4 \pm \sqrt{16 - 15}$$

$$x = 4 \pm 1$$

$$x_1 = 3, \quad x_2 = 5$$

$$\boxed{p(x) = 4(x - 3)(x - 5)}$$

b) $p(x) = 20x^2 - 245$

$= 5(4x^2 - 49)$

$= 5((2x)^2 - 7^2)$

$= 5(2x + 7)(2x - 7)$

$$\boxed{p(x) = 5(2x + 7)(2x - 7)}$$