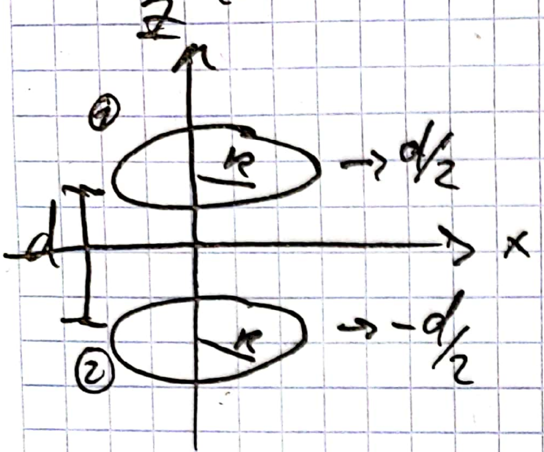
wont:

$$B_0 = \frac{\mu_0 n I R^2}{(R^2+b^2)^{3/2}} = \frac{\mu_0 n I R^2}{\left(R^2+\left(\frac{R}{2}\right)^2\right)^{3/2}}$$



Biot-Savart:

$$B(r) = \frac{\mu_0 I}{4\pi}\int_S \frac{r-R}{|r-R|^3}\times ds$$

Two loops:

$$B_1(r) = \frac{\mu_0 I}{4\pi}\int_{S_1} \frac{r-R}{|r-R|^3}\times ds$$

$$B_2(r) = \frac{\mu_0 I}{4\pi}\int_{S_2} \frac{r-R}{|r-R|^3}\times ds$$

S_1 - line in center of loop ①

S_2 - line in center of loop ②

Superposition:

$S = S_1 + S_2$, r goes along $z \Rightarrow r = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}$

$ds = R\, d\varphi\, \hat{\varphi}$ → cylindrical coordinates

$$R = \begin{pmatrix} R\cos(\varphi) \\ R\sin(\varphi) \\ \frac{d}{2} \end{pmatrix} \Rightarrow r - R = \begin{pmatrix} -R\cos(\varphi) \\ -R\sin(\varphi) \\ z - \frac{d}{2} \end{pmatrix}$$

$$\Rightarrow |r-R|^3 = \left(R^2 \cos(\varphi)^2 + R^2 \sin(\varphi)^2 + \left(z-\frac{d}{2}\right)^2\right)^{\frac{3}{2}}$$

$$= \left(R^2\left(\cos(\varphi)^2 + \sin(\varphi)^2\right) + \left(z-\frac{d}{2}\right)^2\right)^{\frac{3}{2}} =$$

$$= \left(R^2 + \left(z-\frac{d}{2}\right)^2\right)^{\frac{3}{2}}$$

then, $(r-R) \times ds = \begin{pmatrix} -R\cos(\varphi) \\ -R\sin(\varphi) \\ z-\frac{d}{2} \end{pmatrix} \times \begin{pmatrix} -\sin(\varphi) \\ \cos(\varphi) \\ 0 \end{pmatrix} R\, d\varphi =$

a bit of math later $= -R \begin{pmatrix} \left(z-\frac{d}{2}\right)\cos(\varphi) \\ \left(z-\frac{d}{2}\right)\sin(\varphi) \\ R \end{pmatrix} d\varphi$

Integrate over φ from 0 to 2π:

$$\int_0^{2\pi} -R \begin{pmatrix} \left(z-\frac{d}{2}\right)\cos(\varphi) \\ \left(z-\frac{d}{2}\right)\sin(\varphi) \\ R \end{pmatrix} d\varphi = -R \begin{bmatrix} \left(z-\frac{d}{2}\right)\sin(\varphi) \\ -\left(z-\frac{d}{2}\right)\cos(\varphi) \\ R\cdot\varphi \end{bmatrix}\Bigg|_0^{2\pi} =$$

$$= -R\left(\begin{pmatrix} \left(z-\frac{d}{2}\right)\sin(2\pi) \\ -\left(z-\frac{d}{2}\right)\cos(2\pi) \\ R\cdot 2\pi \end{pmatrix} - \begin{pmatrix} \left(z-\frac{d}{2}\right)\sin(0) \\ -\left(z-\frac{d}{2}\right)\cos(0) \\ R\cdot 0 \end{pmatrix}\right) = -R\begin{pmatrix} 0 \\ 0 \\ R\cdot 2\pi \end{pmatrix}$$

$$= -R^2\, 2\pi\, \vec{z}$$

→ plug in:

$$B_1(z) = -\frac{\mu_0 I R^2}{2}\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{-\frac{3}{2}}\hat{z}$$

for n-windings in the loop:

$$B_1(z) = -\frac{\mu_0 R^2 n I}{2\cdot\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{\frac{3}{2}}}\hat{z}$$

for $B_2(z) = -\frac{\mu_0 R^2 n I}{2}\left(R^2+\left(z+\frac{d}{2}\right)^2\right)^{-\frac{3}{2}}\hat{z}$

$$B(z)=B_1+B_2 = -\frac{\mu_0 R^2 n I}{2}\left[\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{-\frac{3}{2}} + \left(R^2+\left(z+\frac{d}{2}\right)^2\right)^{-\frac{3}{2}}\right]$$

simplify denominator: $z=0$ → middle of the loops

$$\Rightarrow \frac{1}{2}\left[\left(R^2\left(-\frac{d}{2}\right)^2\right)^{-\frac{3}{2}} + \left(R^2+\left(\frac{d}{2}\right)^2\right)^{-\frac{3}{2}}\right]$$

$$\frac{d}{2} = \frac{R}{2} = b \Rightarrow \frac{1}{2}\left[\left(R^2-b^2\right)^{-\frac{3}{2}} + \left(R^2+b^2\right)^{-\frac{3}{2}}\right] =$$

$$= \frac{1}{2}\left[2\left(R^2+b^2\right)^{-\frac{3}{2}}\right] = \left(R^2+b^2\right)^{-\frac{3}{2}}$$

Therefore: $B = \frac{\mu_0 n I R^2}{(R^2+b^2)^{\frac{3}{2}}}$