• GP eq. in 1D

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + \frac{1}{2}m\omega^2x^2\psi + g_{1D}|\psi|^2\psi$$

. Dimensionless $\longrightarrow$ $\tilde{t} = \omega t$; $\tilde{x} = \frac{x}{\ell}$; $\psi = \frac{1}{\sqrt{\ell}}\tilde{\psi}$; $\ell = \sqrt{\frac{\hbar}{m\omega}}$

$$i\frac{\partial}{\partial \tilde{t}}\tilde{\psi} = -\frac{1}{2}\frac{\partial^2}{\partial \tilde{x}^2}\tilde{\psi} + \frac{1}{2}\tilde{x}^2\tilde{\psi} + G|\tilde{\psi}|^2\tilde{\psi} \qquad G \equiv \frac{g_{1D}}{\ell\hbar\omega}$$

• GP eq. in 2D

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}\psi + \frac{\partial^2}{\partial y^2}\psi\right) + \frac{1}{2}m(\omega_x^2x^2 + \omega_y^2y^2)\psi + g_{2D}|\psi|^2\psi$$

Dimensionless $\longrightarrow$ $\tilde{t} = \omega_x t$; $\tilde{x} = \frac{x}{\ell_x}$; $\tilde{y} = \frac{y}{\ell_x}$; $\psi = \frac{1}{\ell_x}\tilde{\psi}$; $\ell_x = \sqrt{\frac{\hbar}{m\omega_x}}$; $\alpha = \frac{\omega_y}{\omega_x}$

$$i\frac{\partial}{\partial \tilde{t}}\tilde{\psi} = -\frac{1}{2}\left(\frac{\partial^2}{\partial \tilde{x}^2} + \frac{\partial^2}{\partial \tilde{y}^2}\right)\tilde{\psi} + \frac{1}{2}(\tilde{x}^2 + \alpha^2\tilde{y}^2)\tilde{\psi} + G|\tilde{\psi}|^2\tilde{\psi} \qquad G \equiv \frac{g_{2D}}{\ell_x^2\hbar\omega_x}$$

• GP eq in 3D

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2}{\partial z^2}\psi\right) + \frac{m}{2}(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2)\psi + g_{3D}|\psi|^2\psi$$

Dimensionless: $\tilde{t} = \omega_x t$; $\tilde{x} = \frac{x}{\ell_x}$, $\tilde{y} = \frac{y}{\ell_x}$; $\tilde{z} = \frac{z}{\ell_x}$; $\psi = \frac{\psi}{\ell_x^{3/2}}$; $\alpha_y = \frac{\omega_y}{\omega_x}$; $\alpha_z = \frac{\omega_z}{\omega_x}$

$$i\frac{\partial}{\partial \tilde{t}}\tilde{\psi} = -\frac{1}{2}\left(\frac{\partial^2}{\partial \tilde{x}^2} + \frac{\partial^2}{\partial \tilde{y}^2} + \frac{\partial^2}{\partial \tilde{z}^2}\right)\tilde{\psi} + \frac{1}{2}(\tilde{x}^2 + \alpha_y^2y^2 + \alpha_z^2z^2)\tilde{\psi} + G|\tilde{\psi}|^2\tilde{\psi} \qquad G = \frac{g_{3D}}{\ell_x^3\hbar\omega_x}$$

• The dimensionless eq. is hence of the form (removing "~"):

$$i\frac{\partial}{\partial t}\psi = -\frac{1}{2}\nabla^2\psi + V(\vec{x})\psi(\vec{x}) + G|\psi(\vec{x})|^2\psi(\vec{x})$$

The form is general in any dimension, but $\nabla_{1D}^2 = \frac{\partial^2}{\partial x^2}$; $\nabla_{2D}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$; $\nabla_{3D}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

. Then $\rightarrow$ $H_{KIN} = \frac{|\vec{k}|^2}{2}$

$$H_{POT} = V(\vec{x}) + G|\psi(\vec{x})|^2$$

Real time

$$\psi(\vec{x}, t + \Delta t) = e^{-iH\Delta t}\psi(\vec{x}, t)$$

Imag time

$$\psi(\vec{x}, t + \Delta t) = e^{-H\Delta t}\psi(\vec{x}, t)$$

$e^{uH\Delta t}\psi(\vec{x}, t)$ in general

$$u = \begin{cases} -1 \longrightarrow \text{im. time} \\ -i \longrightarrow \text{re. time} \end{cases}$$

• Algorithm:

$$FFT^{-1}\left[e^{u\frac{k^2}{2}\Delta t}\, FFT\left[e^{uH_{POT}\Delta t}\psi(\vec{x}, t)\right]\right] = \psi(\vec{x}, t + \Delta t)$$

with FFT the Fourier Transform in D dimensions.

- Hence : 1) u is like in 1D
  2) G is a number
  3) Δt is a number
  4) $|\vec{k}|^2$ is also a number $\quad k^2 = k_x^2 + k_y^2$ in 2D
  $k^2 = k_x^2 + k_y^2 + k_z^2$ in 3D